Stochastic Programming Recourse Models for Reconfigurable Multi-Period Storage Allocation in Remanufacturing Facilities

James C. Benneyan\(^1\)*, Thomas P. Cullinane\(^2\), Aysegul Topcu\(^3\)

\(^1, 2, 3\) Department of Mechanical and Industrial Engineering, Northeastern University

360 Huntington Avenue, Boston, Massachusetts 02115, USA

Received March 2010; Accepted May 2010

Abstract—Inventory space requirements in remanufacturing facilities can vary significantly over time and by type of space needed due to variability in recaptured component quality, availability of refurbished components, remanufactured product demand, and returned product rates. Multi-period stochastic programming recourse models are developed to identify optimal adaptive schedules for internal, external, and reconfigured storage space requirements in each time period. Results are compared with expected value models and computational issues are discussed. In most cases, solutions with expected value formulations are found to be as much as 34% higher than stochastic programming recourse solutions. Model size, however, increases significantly with more periods due to the rapid increase in the number of possible scenarios, variables, and constraints.

Keywords—Space planning, facility layout problems, logistics, stochastic optimization, reconfiguration

1. INTRODUCTION

Remanufacturing a product generally is quite different than manufacturing it for the first time, adding several new dimensions to the work that must be performed on a product. Since these processes often depend on the age, wear, and condition of returned products, the number of usable parts retrieved from each item varies significantly, in turn causing fluctuations in inventory and configuration requirements. Consequently, as remanufacturing becomes increasingly important, reconfigurable storage spaces are advantageous for adapting to variable component recapture yields, product condition, availability of refurbished components, and remanufactured product demand (Topcu and Cullinane 2005; Topcu et al. 2007). Stochastic programming recourse (SPR) models that optimize adaptive storage space requirements over multiple time periods are developed in this paper to help decision-makers manage such processes. Two central features of these models are: (1) instead of replacing uncertainties with their expectations, the probability space of possible events is explicitly modeled, and (2) the concept of recourse in later periods is taken into account, (i.e. the ability to take corrective action depending on the outcomes in preceding periods of the above random variables).

The resulting optimal set of decisions produced by these models that minimize total expected costs includes (a) the amount of each type of storage space to allocate in the current time period and (b) a schedule of subsequent recourse actions for each possible future period. A two-period SPR model, for example, identifies an initial storage configuration for period 1 and subsequent recourse actions for period 2 that together minimize the a priori total expected costs (Topcu et al. 2008; Topcu 2009). Introduced independently by Dantzig (1955) and Beale (1955), stochastic programming models have been applied to a variety of problems in which inherent randomness affects decision making, ranging from agricul-
ture to energy, military, and manufacturing applications. While related models have been applied to traditional capacity and production planning problems, to our knowledge no work has been published on reconfigurable remanufacturing applications.

In other industries, SPR models have been used in a wide range of areas including automotive (Jordan and Graves, 1995), electric utilities (Takriti et al, 1996), semiconductors (Barahona et al, 2005; Christie and Wu, 2002; Hood et al, 2003; Karabuk and Wu, 2003), computers (Swaminathan and Tayur, 1998), oil processing industries (Zimberg and Testuri, 2006), and supply chain settings (Gupta and Maranas, 2003; Messina and Bosetti, 2006; Santoso et al, 2005). Solution techniques used to solve such models include network methods (Agizy, 1969), stochastic decomposition (Buchanan et al, 2001; Caballero et al, 2001; Dyer and Stougie, 2006; Haneveld and Van der Vlerk, 1999; Higle and Sen, 1991; Schultz, 2003), approximations based on Monte Carlo sampling (Shapiro and Homem-de-Mello, 1998), Lagrangian relaxation (Takriti et al, 1996), Benders decomposition (MirHassani et al, 2000; Sen and Sherali, 2006), and others.

2. SPR MODEL DEVELOPMENT

2.1 Model Assumptions and Notation

Remanufactured items consist of one or more components reclaimed from returned products. In each time period, a number of returned products are received by a remanufacturing facility, each consisting of one or more recapturable components. After disassembly, each of these components undergoes testing, cleaning, and refurbishing, with a random number deemed as acceptable for reuse. Some of the underlying assumptions of the SPR models are

1. Disassembly, testing, remanufacturing, and reconfiguration durations are assumed small enough that they occur in the same period in which components are salvaged. This assumption is based on several classes of products ranging from electronics (cellular phones) to mechanical components (washing machines).
2. One unit of storage space is assumed to be that occupied by one component.
3. A penalty cost is associated with unmet demand.
4. Excess refurbished components are stored internally or externally or disposed of if capacities are exceeded.
5. Non-salvageable components are disposed of externally and do not need to be stored.
6. A cost is associated with disposal of refurbished components if capacity is exceeded.

The decision variables in this model are

- \( S_1 \) = Internal storage space allocated for the extra refurbished components in period 1,
- \( I_p \) = Number of returned products in period \( p \)
- \( R_{k,p} \) = Amount of space to reconfigure for excess refurbished components in period \( p \) under scenario \( k \),
- \( O_{k,p} \) = External storage space for excess refurbished components in period \( p \) under scenario \( k \),
- \( U_{k,p} \) = Unused internal storage space in period \( p \) under scenario \( k \), and
- \( J_{k,p} \) = Number of disposed components in period \( p \) under scenario \( k \).

The parameters computed by the model while executing are

- \( E_{k,p} \) = Extra number of refurbished components in period \( p \) = \( \max(0, I_p \ast Y_{k,p} - D_p) \) where \( E_{k,1} = 0 \), and
- \( X_{k,p} \) = Unmet demand in period \( p \) under scenario \( k \),

where the subscripts \( p \) and \( k \) indicate the period and outcome, respectively, and with \( R_{k,p}, O_{k,p}, U_{k,p}, J_{k,p} \) and \( X_{k,p} \) being recourse variables. For simplification the subscript \( k \) is dropped when the variables are scenario-independent, (e.g., \( I_p \) and \( S_1 \)). Input cost, demand, and capacity variables include

- \( Y_{k,p} \) = Remanufacturing yield percentage \( k \) in period \( p \),
- \( D_p \) = Product demand in period \( p \),
- Probability of outcome \( k \),
- \( \alpha \) = Cost of internally storing each remaining refurbished component,
\[ x_r = \text{Cost of reconfiguring one unit of storage space}, \]
\[ x_o = \text{Cost of externally storing each remaining refurbished component}, \]
\[ x_u = \text{Opportunity cost of each unused internal unit of space}, \]
\[ \delta = \text{Cost of disposing each remaining refurbished component}, \]
\[ \alpha_n = \text{Cost of unmet demand}, \]
\[ C^I_p = \text{Capacity for internal storage space in period } p, \text{ and} \]
\[ C^a_p = \text{Capacity for external storage space in period } p. \]

Figure 1: Example of discretization of probability space for number of salvaged reusable components for two-period model

Denoting \( I_p \) as the number of returned products in period \( p \), Figure 1 illustrates all possible component yield quantities, \( X \), after disassembly and testing (a binomial random variable, with \( X = \{0, 1, \ldots, I_p\} \)), which for modeling convenience is discretized into a fewer number of yield percentages to make the model more tractable. In the discretized model, each tree path \( k \), or “scenario”, corresponds to a possible percent, \( Y_{k,p} \), of salvaged reusable components occurring with probability \( \pi_k \). For example, in Figure 1 the remanufacturer recaptures \( Y_{1,2} = 60\% \) of a certain component with probability \( 1 = 0.1 \), \( Y_{2,2} = 90\% \) with probability \( 2 = 0.6 \), and \( Y_{3,2} = 75\% \) with probability \( 3 = 0.3 \). Considering each of these possible outcomes in period 2 and their probabilities, an initial decision is made in period 1 about how much internal storage space, \( S_1 \), to allocate. If demand is met in the second period, a quantity of \( E_{k,2} \) excess refurbished components may need to be either stored or discarded (based on the yield and demand).

The objective is to determine the optimal internal, reconfigured, and external storage space for refurbished components that together minimize all associated storage, reconfiguration, penalties, and opportunity costs. The internal storage space is that allocated to store the components and products in the facility. If available, a certain percentage of the non-allocated internal space (total available internal storage space minus the allocated internal storage space) can be used as reconfigured storage space. If the total available internal storage space is completely allocated, then space can be rented or bought externally to store the components. If the internal space needed in period \( p \) exceeds allocated availability, options are to (i) reconfigure other space at a cost of \( c_r \) per unit space, (ii) pay for outside storage space at a cost of \( c_e \) per unit space, (iii) scrap the extra refurbished components at a cost of \( c_s \) per component, (iv) sell the extra refurbished component at a price (essentially disposing the extra refurbished components at a negative cost), or (v) some combination of the above.

Conversely, if available internal space exceeds storage needs, an opportunity cost \( c_o \) is incurred for each unused unit of space. Upper bounds exist on the total available internal and external storage space in each time period, where the total allocated internal and reconfigured space together cannot exceed the internal limit. A decision is made in the first period as to the amount of internal space to allocate for the next period, based on probabilities of future component yields, incoming supply, and demand for remanufactured products (with the facility assumed to start empty in period 0). A schedule of optimal recourse actions for each possible outcome is determined by the SPR model that, together with the above period 1 allocation decision, minimizes the total multi-period expected inventory storage cost.
2.2 Two-Period SPR Model

In a two-period model, after an initial space allocation decision is made in the first period and the number of refurbished components is observed, then reconfiguration, external storage, and disposal recourse decisions are made in period 2. For illustration purposes, assuming the three possible yield scenarios illustrated in Figure 1, one component type is salvageable from returned items, and demand is deterministic and known, then a two-period SPR model can be written as

Minimize \( c_s S_1 + \sum_{k=1}^{3} \pi_k \left( c_r R_{k,2} + c_o O_{k,2} + c_a U_{k,2} + c_j J_{k,2} + \epsilon X_{k,2} \right) \) \hspace{1cm} (1)

subject to

\[
\begin{align*}
I_2 + Y_{k,2} - D_2 &= E_{k,2} - X_{k,2} \quad (2) \\
S_1 + R_{k,2} + O_{k,2} + J_{k,2} - E_{k,2} &= U_{k,2} \quad (3) \\
S_1 + R_{k,2} &\leq C^s_k \quad (4) \\
O_{k,2} &\leq C^o_k \quad (5) \\
S_1, I_2, X_{k,2}, E_{k,2}, R_{k,2}, O_{k,2}, U_{k,2}, J_{k,2} &\geq 0 \quad (6)
\end{align*}
\]

for scenarios \( k = 1, 2, 3 \). The objective function minimizes the total two-period cost, i.e. the internal storage cost plus the expected value of the future reconfigured storage, external storage, opportunity, disposal, and penalty costs. Equation 2 computes either the number of extra components if demand is met or the unmet demand if a penalty is incurred for unmet demand in the second period. Equation 3 forces any unused internal storage space to incur opportunity costs, equations 4 and 5 specify limits for internal and external storage space, and equation 6 ensures non-negativity of all variables. Solving an equivalent expected value (EV) model adds an additional constraint to the model which is the expected yield for extra refurbished components

\[
S_1 = \sum_{k=1}^{3} \pi_k E_{k,2} \cdot \hspace{1cm} (7)
\]

In the multiple period case (for \( p \geq 3 \), given \( m \) outcomes per period, \( n \geq 3 \) periods, and \( m^{n-1} \) scenarios, the above model extends to

Minimize \( c_s S_1 + \sum_{k=1}^{m^{n-1}} \pi_k \sum_{p=2}^{m} \left( c_r R_{k,p} + c_o O_{k,p} + c_a U_{k,p} + c_j J_{k,p} + \epsilon X_{k,p} \right) \) \hspace{1cm} (8)

subject to

\[
\begin{align*}
I_{p} + Y_{k,p} + E_{k,(p-1)} - D_p &= E_{k,p} - X_{k,p} \quad (9) \\
S_1 + R_{k,p} + O_{k,p} + J_{k,p} - E_{k,p} &= U_{k,p} \quad (10) \\
S_1 + R_{k,p} &\leq C^s_p \quad (11) \\
O_{k,p} &\leq C^o_p \quad (12) \\
R_{k,p} = R_{k+1,p} = \cdots = R_{m^{p-1},p} = R_{m^{p-2},p+1,p} = \cdots = R_{m^{p-1},m^{p-1}+1,p} = \cdots \quad (13) \\
O_{k,p} = O_{k+1,p} = \cdots = O_{m^{p-1},p} = O_{m^{p-2},p+1,p} = \cdots = O_{m^{p-1},m^{p-1}+1,p} = \cdots \quad (14) \\
U_{k,p} = U_{k+1,p} = \cdots = U_{m^{p-1},p} = U_{m^{p-2},p+1,p} = \cdots = U_{m^{p-1},m^{p-1}+1,p} = \cdots \quad (15)
\end{align*}
\]
As an example, Figure 2 illustrates an \( n = 3 \) period model with demand and internal, reconfigured, external, and opportunity storage costs assumed constant in all periods. Similarly discretizing the probability space to \( m = 3 \) yields in each period results in the shown \( m^{n-1} = 3^2 = 9 \) possible scenarios, with the “non-anticipativity” constraints now being

\[
R_{1, 2} = R_{2, 2} = R_{3, 2}; \quad R_{4, 2} = R_{5, 2} = R_{6, 2}; \quad R_{7, 2} = R_{8, 2} = R_{9, 2} \tag{20a-20c}
\]
\[
O_{1, 2} = O_{2, 2} = O_{3, 2}; \quad O_{4, 2} = O_{5, 2} = O_{6, 2}; \quad O_{7, 2} = O_{8, 2} = O_{9, 2} \tag{21a-21c}
\]
\[
U_{1, 2} = U_{2, 2} = U_{3, 2}; \quad U_{4, 2} = U_{5, 2} = U_{6, 2}; \quad U_{7, 2} = U_{8, 2} = U_{9, 2} \tag{22a-22c}
\]
\[
J_{1, 2} = J_{2, 2} = J_{3, 2}; \quad J_{4, 2} = J_{5, 2} = J_{6, 2}; \quad J_{7, 2} = J_{8, 2} = J_{9, 2} \tag{23a-23c}
\]
\[
X_{1, 2} = X_{2, 2} = X_{3, 2}; \quad X_{4, 2} = X_{5, 2} = X_{6, 2}; \quad X_{7, 2} = X_{8, 2} = X_{9, 2} \tag{24a-24c}
\]

As an example consider the recourse variables \( R_{1, 2}, R_{2, 2}, \) and \( R_{3, 2} \), which are the amounts of reconfigured storage spaces in scenarios \( k = 1, 2, \) and 3 at the third period. These scenarios are not distinguishable from one another since all three have a common history up through the third period. Thus it seems logical that \( R_{1, 2}, R_{2, 2}, \) and \( R_{3, 2} \) are equal since the same amount of reconfigured storage space should result for the same scenarios.
Again, an initial decision is made for how much internal storage space to allocate based on all available information and scenario probabilities. After the second period, the component yield for that period becomes known and a second decision is made for the recourse variables – the amounts of reconfigured, external storages, and disposal to accommodate this yield, the unused storage space and unmet demand for the current period, and the allocation of internal storage to prepare for the third period. Once the component yield in the third period becomes known, a decision is made again for the recourse variables and the number of returned products to pull into the facility.

3. MODEL EXTENSIONS

The same general approach can be taken to extend the above model to more realistic scenarios with multiple salvaged components, and multiple types of returned products with partially common components (although these extensions significantly increase the number of possible scenarios and model size as discussed below). The decision variables then become

\[ S_{i,1} = \text{Internal storage space allocated for the extra refurbished components per type } i \text{ in period 1,} \]

\[ R_{i,k,p} = \text{Amount of space to reconfigure for excess refurbished components per type } i \text{ in period } p \text{ under scenario } k, \]

\[ O_{i,k,p} = \text{External storage space for excess refurbished components per type } i \text{ in period } p \text{ under scenario } k, \]

\[ U_{i,k,p} = \text{Unused internal storage space per type } i \text{ in period } p \text{ under scenario } k, \]

\[ J_{i,k,p} = \text{Number of disposed components per type } i \text{ in period } p \text{ under scenario } k. \]

The parameters computed by the model while executing are

\[ X_{i,k,p} = \text{Unmet demand per component type } i \text{ in period } p \text{ under scenario } k, \]

\[ E_{i,k,p} = \text{Extra number of refurbished components per type } i \text{ in period } p \text{, max } \left( 0, I_p * Y_{i,k,p} - D_{i,p} \right) \text{, where } E_{i,k,1} = 0. \]

Similarly, additional cost, demand, and capacity variables include

\[ Y_{i,k,p} = \text{Remanufacturing yield percentage } k \text{ per type } i \text{ in period } p, \]

\[ D_{i,p} = \text{Product demand per type } i \text{ in period } p, \]

\[ c_{i,i} = \text{Cost of internally storing each remaining refurbished component of type } i, \]

\[ c_{i,r} = \text{Cost of reconfiguring one unit of storage space for component type } i, \]
\[ c_{i,o} \] = Cost of externally storing each remaining refurbished component of type \( i \),
\[ c_{i,u} \] = Opportunity cost of each unused internal unit of space for component type \( i \),
\[ c_{i,j} \] = Cost of disposing each remaining refurbished component of type \( i \),
\[ c_{i,\infty} \] = Cost of unmet demand for component type \( i \),
\[ C_{i,p}^s \] = Capacity for internal storage space per component type \( i \) in period \( p \), and
\[ C_{i,p}^o \] = Capacity for external storage space per component type \( i \) in period \( p \).

### 3.1 Two-Component and Two-Period SPR Model

To consider multiple components, denote \( i = \) component type and \( t = \) total number of component types. For the \( t = 2 \) component type case, the formulation becomes

\[
\begin{align*}
\text{Minimize} & \sum_{i=1}^{2} c_{i,s} S_{i,1} + \sum_{k=1}^{2} \pi_k \sum_{i=1}^{2} \left( c_{i,r} R_{i,k,2} + c_{i,o} O_{i,k,2} + c_{i,u} U_{i,k,2} + c_{i,j} J_{i,k,2} + c_{i,s} X_{i,k,2} \right) \\
\text{subject to} & \quad I_2 * Y_{i,k,2} - D_{i,2} = E_{i,k,2} - X_{i,k,2} \\
& \quad S_{i,1} + R_{i,k,2} + O_{i,k,2} + J_{i,k,2} - E_{i,k,2} = U_{i,k,2} \\
& \quad S_{i,1} + R_{i,k,2} \leq C_{i,2}^s \\
& \quad O_{i,k,2} \leq C_{i,2}^o \\
& \quad S_{i,1}, J_{i,k,2} X_{i,k,2}, E_{i,k,2}, R_{i,k,2}, O_{i,k,2}, U_{i,k,2}, J_{i,k,2} \geq 0
\end{align*}
\]

where \( k = 1, 2 \) and \( i = 1, 2 \).

### 3.2 Multi-Component and Multi-Period SPR Model

Similarly, the mathematical formulation can be written for multiple components in a multi-period setting as

\[
\begin{align*}
\text{Minimize} & \sum_{i=1}^{m} c_{i,s} S_{i,1} + \sum_{k=1}^{m} \pi_k \sum_{p=1}^{n} \sum_{i=1}^{m} \left( c_{i,r} R_{i,k,p} + c_{i,o} O_{i,k,p} + c_{i,u} U_{i,k,p} + c_{i,j} J_{i,k,p} + c_{i,s} X_{i,k,p} \right) \\
\text{subject to} & \quad I_p * Y_{i,k,p} + E_{i,k,(p-1)} - D_{i,p} = E_{i,k,p} - X_{i,k,p} \\
& \quad S_{i,1} + R_{i,k,p} + O_{i,k,p} + J_{i,k,p} - E_{i,k,p} = U_{i,k,p} \\
& \quad S_{i,1} + R_{i,k,p} \leq C_{i,p}^s \\
& \quad O_{i,k,p} \leq C_{i,p}^o \\
& \quad R_{i,k,p} = R_{i,k+1,p} = \ldots = R_{i,m^p+p-1,p} = R_{i,m^p+p-1+p} = \ldots = R_{i,m^p+p-1+p+1,p} = R_{i,m^p+p+2,p} = \ldots = R_{i,m^p+p+2+p,\infty-1} \\
& \quad O_{i,k,p} = O_{i,k+1,p} = \ldots = O_{i,m^p+p,\infty-1} = O_{i,m^p+p+2+p,\infty-1} = \ldots = O_{i,m^p+p+2+p+1+p,\infty-1} \\
& \quad U_{i,k,p} = U_{i,k+1,p} = \ldots = U_{i,m^p+p,\infty-1} = U_{i,m^p+p+2+p,\infty-1} = \ldots = U_{i,m^p+p+2+p+1+p,\infty-1} \\
& \quad J_{i,k,p} = J_{i,k+1,p} = \ldots = J_{i,m^p+p,\infty-1} = J_{i,m^p+p+2+p,\infty-1} = \ldots = J_{i,m^p+p+2+p+1+p,\infty-1} \\
& \quad X_{i,k,p} = X_{i,k+1,p} = \ldots = X_{i,m^p+p,\infty-1} = X_{i,m^p+p+2+p,\infty-1} = \ldots = X_{i,m^p+p+2+p+1+p,\infty-1}
\end{align*}
\]
where \( k = 1, 2, 3, \ldots, m^{(x)} \), \( p = 2, 3, \ldots n \) and \( l = 1, 2, 3, \ldots t \). As above, the non-anticipativity constraints (36-40) are needed to ensure that all scenarios with a common history have the same set of decisions up to the current time period in the decision tree.

### 3.3 Multiple Products

In a facility where more than one product type is remanufactured that have common components, denote \( \zeta = \) product type and \( w = \) total number of product types. The SPR model then becomes

\[
\text{Minimize} \quad \sum_{i=1}^{w} \sum_{k=1}^{n} c_{i,k} S_{i,1} + \sum_{k=1}^{n} \sum_{p=2}^{w} c_{i,k,p} R_{i,k,p} + c_{i,o} O_{i,k,p} + c_{i,u} U_{i,k,p} + c_{i,x} X_{i,k,p}
\]

subject to

\[
S_{i,1} + R_{i,k,p} \leq C_{i,k}^p
\]

\[
O_{i,k,p} = O_{i,k+1,p} = \ldots = O_{i,m^{n-p},p} \ldots = O_{i,n^{m-p+1},p} = O_{i,m^{n-p+2},p} = \ldots = O_{i,m^{n-1}n^{m-p},n-1}
\]

\[
U_{i,k,p} = U_{i,k+1,p} = \ldots = U_{i,m^{n-p},p} \ldots = U_{i,n^{m-p+1},p} = U_{i,m^{n-p+2},p} = \ldots = U_{i,m^{n-1}n^{m-p},n-1}
\]

\[
J_{i,k,p} = J_{i,k+1,p} = \ldots = J_{i,m^{n-p},p} \ldots = J_{i,n^{m-p+1},p} = J_{i,m^{n-p+2},p} = \ldots = J_{i,m^{n-1}n^{m-p},n-1}
\]

\[
X_{i,k,p} = X_{i,k+1,p} = \ldots = X_{i,m^{n-p},p} \ldots = X_{i,n^{m-p+1},p} = X_{i,m^{n-p+2},p} = \ldots = X_{i,m^{n-1}n^{m-p},n-1}
\]

\[
S_{i,1} + R_{i,k,p} + E_{i,k,p} + R_{i,k,p} + O_{i,k,p} + U_{i,k,p} + J_{i,k,p} \geq 0
\]

where \( k = 1, 2, 3, \ldots, m^{(x)} \), \( p = 2, 3, 4, \ldots n \), \( i = 1, 2, 3, \ldots t \) and \( \zeta = 1, 2, 3, \ldots w \) and where equations (47-51) are the non-anticipativity constraints.

### 4. MODEL PERFORMANCE AND RESULTS

As an example, assume an avionics remanufacturing company collects used airport radar sets, which then are disassembled to salvage three components: a printed circuit mother board, antenna, and chargeable lithium-ion battery. It costs the company \( c_2 = $2 \) to store each antenna on site and excess antennas can be stored by reconfiguring space all period. The opportunity cost of not using the allocated space is \( c_2 = $2 \) per space-period. If there is not enough internal or external space to accommodate all refurbished antennas, they are disposed of at a disposal cost of \( c_2 = $16 \) per item. If demand for the refurbished antennas is not met, the penalty cost is \( c_2 = $20 \) per unit of unmet demand. These values are based on realistic applications the authors are familiar with.

Table 1 compares the results of the optimal SPR versus expected value models for the above case and the following yield and other inputs: \( P(Y_{1,2} = 0.6) = 0.1, P(Y_{2,2} = 0.9) = 0.6, P(Y_{3,2} = 0.75) = 0.3, D_2 = 40, C_{i}^2 = 50, \) and \( C_{i}^3 = 10. \) Note that the expected value formulation produces different values for the recourse variables and an optimal internal storage allocation of 4.8 spaces and $40.96 minimum expected total cost (6.7% higher than the SPR cost of $38.40), due
to the replacement of probabilistic future events with their means. That is, the expected value formulation minimizes the total cost assuming no uncertainty about the future, with all random variables (yield, demand, and so on) set to their expected values. Conversely, the SPR formulation accounts for these uncertainties via probability scenarios such as those depicted in Figures 1 and 2. Table 2 summarizes the results for the earlier 3-period, one-component example based on the following model inputs:

\[
D_p = 40, \ C^i_p = 50, \ C^w_p = 10, \ c_s = 2, \ c_r = 6, \ c_o = 10, \ c_u = 2, \ c_j = 16, \ c_x = 20, \\
P(Y_{1,1,2} = 0.6) = 0.1, \ P(Y_{1,2,2} = 0.9) = 0.6, \ P(Y_{1,3,2} = 0.75) = 0.3, \\
P(Y_{2,1,2} = 0.4) = 0.1, \ P(Y_{2,2,2} = 0.85) = 0.6, \ P(Y_{2,3,2} = 0.65) = 0.3, \\
D_{i,p} = 40, \ C^{i}_{i,p} = 50, \ C^{w}_{i,p} = 10, \\
c_{1,r} = 83, \ c_{1,s} = 6, \ c_{1,o} = 10, \ c_{1,u} = 3, \ c_{1,j} = 16, \ c_{1,x} = 20, \text{ and } \\
c_{2,r} = 8, \ c_{2,s} = 5, \ c_{2,o} = 8, \ c_{2,u} = 2, \ c_{2,j} = 14, \ c_{2,x} = 18.
\]

Table 1: Comparison of two-period SPR versus EV formulations (D = decision variables, R = random variables, blue font indicates recourse variables)

<table>
<thead>
<tr>
<th>Model Variables (D = Decision, R = Random)</th>
<th>Type of variable</th>
<th>Period</th>
<th>Stochastic Programming Scenario k</th>
<th>Expected Value Scenario k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal storage space</td>
<td>D</td>
<td>1, 2</td>
<td>8, 40</td>
<td>4.8, 40</td>
</tr>
<tr>
<td>Number of returned products</td>
<td>D</td>
<td>2</td>
<td>53.3</td>
<td>53.3</td>
</tr>
<tr>
<td>Demand</td>
<td>R</td>
<td>2</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Component yield</td>
<td>R</td>
<td>2</td>
<td>60%, 90%, 75%</td>
<td>60%, 90%, 75%</td>
</tr>
<tr>
<td>Extra number of components</td>
<td>R</td>
<td>2</td>
<td>0, 8, 0</td>
<td>0, 8, 0</td>
</tr>
<tr>
<td>Reconfigured storage space</td>
<td>D</td>
<td>2</td>
<td>0, 0, 0</td>
<td>0, 0, 0</td>
</tr>
<tr>
<td>External storage space</td>
<td>D</td>
<td>2</td>
<td>0, 8, 8</td>
<td>4.8, 4.8</td>
</tr>
<tr>
<td>Unused storage space</td>
<td>D</td>
<td>2</td>
<td>0, 0, 0</td>
<td>0, 0, 0</td>
</tr>
<tr>
<td>Disposed components</td>
<td>D</td>
<td>2</td>
<td>0, 0, 0</td>
<td>0, 0, 0</td>
</tr>
<tr>
<td>Penalty</td>
<td>D</td>
<td>2</td>
<td>8, 0, 0</td>
<td>8, 0, 0</td>
</tr>
<tr>
<td><strong>Total Expected Cost</strong></td>
<td></td>
<td></td>
<td><strong>$38.40</strong></td>
<td><strong>$40.96</strong></td>
</tr>
</tbody>
</table>

Table 2: Comparison of single component three-period SPR versus EV formulations (parenthesis indicate EV formulation)

<table>
<thead>
<tr>
<th>Model Variables (D = Decision, R = Random)</th>
<th>Type of variable</th>
<th>Period</th>
<th>Scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of outcomes</td>
<td>-</td>
<td>-</td>
<td>0.02, 0.05, 0.03, 0.06, 0.24, 0.24, 0.03, 0.03</td>
</tr>
<tr>
<td>Internal storage space</td>
<td>D</td>
<td>1</td>
<td>9.7 (12.9)</td>
</tr>
<tr>
<td>Yield</td>
<td>R</td>
<td>2</td>
<td>60%, 90%, 75%</td>
</tr>
<tr>
<td>Demand</td>
<td>R</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>Number of returned products</td>
<td>D</td>
<td>2</td>
<td>55.2 (56.3)</td>
</tr>
<tr>
<td>Extra number of components</td>
<td>R</td>
<td>2</td>
<td>0 (0)</td>
</tr>
<tr>
<td>Reconfigured storage space</td>
<td>D</td>
<td>2</td>
<td>9.7 (10.7)</td>
</tr>
<tr>
<td>External storage space</td>
<td>D</td>
<td>2</td>
<td>0 (0)</td>
</tr>
<tr>
<td>Unused storage space</td>
<td>D</td>
<td>2</td>
<td>6.9 (6.2)</td>
</tr>
<tr>
<td>Disposed components</td>
<td>D</td>
<td>2</td>
<td>0 (0)</td>
</tr>
<tr>
<td>Penalty</td>
<td>D</td>
<td>2</td>
<td>6.9 (6.2)</td>
</tr>
<tr>
<td>Yield</td>
<td>R</td>
<td>3</td>
<td>75, 60, 90, 90, 65, 55, 95, 75, 50</td>
</tr>
<tr>
<td>Demand</td>
<td>R</td>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>Number of returned products</td>
<td>D</td>
<td>3</td>
<td>55.2 (53.3)</td>
</tr>
<tr>
<td>Extra number of components</td>
<td>R</td>
<td>3</td>
<td>1.4 (0)</td>
</tr>
<tr>
<td>Reconfigured storage space</td>
<td>D</td>
<td>3</td>
<td>0 (0)</td>
</tr>
</tbody>
</table>

Benneyan, Cullinane and Topcu: Stochastic Programming Recourse Models for Reconfigurable Multi-Period Storage Allocation in Remanufacturing Facilities
As seen in Table 2, 55 products are returned for remanufacturing for both the second and third periods, with a total expected cost of $73.02 for the SPR model versus $81.39 for the EV formulation (an 11.46% increase). Table 3 compares the optimal objective values for the single component SPR and EV models based on up to 6 periods, with demand for remanufactured products varying from 20 to 100 in increments of 20. If two components are recaptured instead, Table 4 compares the two-component, two-period SPR and expected value results, with 62 and 53 products returned for remanufacturing respectively, and a total expected cost of $121.85 versus $125.44. Table 5 compares the optimal objective values for the two component SPR and EV models up to 6 periods for the same demand assumptions as above.

Table 3: Summary of objective value for single component SPR and EV models (parenthesis indicate EV formulation)

<table>
<thead>
<tr>
<th>Period</th>
<th>Objective Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>20 (20)</td>
</tr>
<tr>
<td>3</td>
<td>111 (111)</td>
</tr>
<tr>
<td>4</td>
<td>490 (490)</td>
</tr>
<tr>
<td>5</td>
<td>1,949 (1,949)</td>
</tr>
<tr>
<td>6</td>
<td>7,296 (7,296)</td>
</tr>
</tbody>
</table>

Table 4: Comparison of two-component SPR versus EV formulations in a two-period setting (blue font indicates recourse variables)

<table>
<thead>
<tr>
<th>Model Variables (D = Decision, R = Random)</th>
<th>Type of variable</th>
<th>Period</th>
<th>Stochastic Programming</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>D 1</td>
<td>6.2</td>
<td>4.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D 1</td>
<td>12.3</td>
<td>3.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D 2</td>
<td>61.5</td>
<td>53.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 2</td>
<td>40</td>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 2</td>
<td>40</td>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 2</td>
<td>60%</td>
<td>90%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 2</td>
<td>40%</td>
<td>85%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 2</td>
<td>75%</td>
<td>85%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 2</td>
<td>65%</td>
<td>65%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 2</td>
<td>40%</td>
<td>85%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 2</td>
<td>60%</td>
<td>85%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R 2</td>
<td>75%</td>
<td>65%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D 2</td>
<td>3.2</td>
<td>3.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Penalty for component type 3
Penalty for component type 1
Penalty for component type 2
Disposed components of type 3
Disposed components of type 2
Unused storage space, component type 3
Unused storage space, component type 2
Unused storage space, component type 1
External storage space, component type 3
External storage space, component type 2
External storage space, component type 1
Extra number of type 3 components
Extra number of type 2 components
Extra number of type 3 components
Reconfigured storage space for type 1
Reconfigured storage space for type 2
Reconfigured storage space for type 3
External storage space, component type 1
External storage space, component type 2
External storage space, component type 3
Unused storage space, component type 1
Unused storage space, component type 2
Unused storage space, component type 3
Disposed components of type 1
Disposed components of type 2
Disposed components of type 3
Penalty for component type 1
Penalty for component type 2
Penalty for component type 3

Table 6: Comparison of two-component SPR and EV models (values in parenthesis correspond to EV model)

<table>
<thead>
<tr>
<th>Model Variables</th>
<th>Type of variable</th>
<th>Period</th>
<th>Stochastic Programming</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D = Decision, R = Random)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>Scenario k</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(D = Decision, R = Random)</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Internal storage space for component type 1</td>
<td>D</td>
<td>1</td>
<td>27.1</td>
<td>36.8</td>
</tr>
<tr>
<td>Internal storage space for component type 2</td>
<td>D</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Internal storage space for component type 3</td>
<td>D</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Number of returned product type A</td>
<td>D</td>
<td>2</td>
<td>47.1</td>
<td>47.1</td>
</tr>
<tr>
<td>Number of returned product type B</td>
<td>D</td>
<td>2</td>
<td>45.5</td>
<td>45.5</td>
</tr>
<tr>
<td>Demand for component type 1</td>
<td>R</td>
<td>2</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Demand for component type 2</td>
<td>R</td>
<td>2</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Demand for component type 3</td>
<td>R</td>
<td>2</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Yield for component type 1</td>
<td>R</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Yield for component type 2</td>
<td>R</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Yield for component type 3</td>
<td>R</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Extra number of type 1 components</td>
<td>R</td>
<td>2</td>
<td>13.2</td>
<td>45.3</td>
</tr>
<tr>
<td>Extra number of type 2 components</td>
<td>R</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Extra number of type 3 components</td>
<td>R</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Reconfigured storage space for type 1</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>18.4</td>
</tr>
<tr>
<td>Reconfigured storage space for type 2</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Reconfigured storage space for type 3</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>External storage space, component type 1</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>External storage space, component type 2</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>External storage space, component type 3</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Unused storage space, component type 1</td>
<td>D</td>
<td>2</td>
<td>13.9</td>
<td>0</td>
</tr>
<tr>
<td>Unused storage space, component type 2</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Unused storage space, component type 3</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Disposed components of type 1</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Disposed components of type 2</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Disposed components of type 3</td>
<td>D</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Penalty for component type 1</td>
<td>D</td>
<td>2</td>
<td>24.1</td>
<td>0</td>
</tr>
<tr>
<td>Penalty for component type 2</td>
<td>D</td>
<td>2</td>
<td>21.2</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5: Summary of objective value of two-component SPR and EV models (values in parenthesis correspond to EV model)

<table>
<thead>
<tr>
<th>Number of variables</th>
<th>Mean Demand</th>
<th>Objective Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Number of variables</td>
<td>39 (39)</td>
<td>220 (220)</td>
</tr>
<tr>
<td>Number of constraints</td>
<td>25 (27)</td>
<td>205 (207)</td>
</tr>
</tbody>
</table>

Table 6: Comparison of two-product, three-component, overlapping-parts SPR versus EV formulations in a two-period setting (blue font indicates recourse variables)
Alternately, if two types of products (e.g. two different radar sets, A and B) are received that have the same type of circuit board but different types of batteries and antennas, the circuit boards can be stored together whereas the different batteries and antennas are stored in separate locations. Assuming the below inputs where the subscript $P$ denotes the product type, Table 6 compares the results for the SPR and expected value models, with a total expected cost of $367.70 versus $373.50 respectively. As in the case with single and two-components, Table 7 summarizes the optimal objective values for the two product SPR and EV models up to 6 periods, again with demand for remanufactured products varying from 20 to 100 in increments of 20. The inputs in this example are

$$P_A(Y_{1,1,2} = 0.6) = 0.1, P_A(Y_{1,2,2} = 0.9) = 0.6, P_A(Y_{1,3,2} = 0.75) = 0.3,$$

$$P_A(Y_{2,1,2} = 0.4) = 0.1, P_A(Y_{2,2,2} = 0.85) = 0.6, P_A(Y_{2,3,2} = 0.65) = 0.3,$$

$$P_B(Y_{1,1,2} = 0.55) = 0.1, P_B(Y_{1,2,2} = 0.95) = 0.6, P_B(Y_{1,3,2} = 0.7) = 0.3,$$

$$P_B(Y_{2,1,2} = 0.35) = 0.1, P_B(Y_{2,2,2} = 0.88) = 0.6, P_B(Y_{2,3,2} = 0.62) = 0.3,$$

$$D_{i,P} = 40, C_{i,P}^0 = 50, C_{i,P} = 10,$$

$$c_{1,i} = 33, c_{1,o} = 66, c_{1,p} = 10, c_{1,P} = 16, c_{1,N} = 20,$$

$$c_{2,i} = 82, c_{2,o} = 8, c_{2,p} = 8, c_{2,0} = 2, c_{2,j} = 14, c_{2,N} = 18,$$

$$c_{3,i} = 33, c_{3,o} = 27, c_{3,j} = 11, c_{3,N} = 22.$$

Table 7: Summary of objective value for two-product with overlapping parts SPR and EV models (values shown in parenthesis correspond to the EV formulation)

<table>
<thead>
<tr>
<th>Period</th>
<th>Objective Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$367.70</td>
</tr>
<tr>
<td>3</td>
<td>$373.50</td>
</tr>
</tbody>
</table>

Table 5: Summary of objective value for two-product with overlapping parts SPR and EV models (values shown in parenthesis correspond to the EV formulation)

<table>
<thead>
<tr>
<th>Period</th>
<th>Total Expected Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$367.70</td>
</tr>
<tr>
<td>5</td>
<td>$373.50</td>
</tr>
</tbody>
</table>

5. DISCUSSION

Uncertainty in remanufacturing processes can cause significant planning problems to accommodate storage requirements, not knowing what the future will bring. This paper developed stochastic programming recourse models to help decision-makers optimally plan storage space allocation and reconfiguration decisions. These models also illustrate the implications of randomness on inventory space requirements and the potential of SPR models for designing and optimizing effective multi-period reconfigurable storage schedules. As shown, these models can help minimize storage space and operating costs of remanufacturing companies. If probabilistic outcomes are not considered, moreover, expected value formulations produce higher minimum cost solutions, underscoring the value of stochastic programming in this type of application. Expanding these SPR models beyond simple scenarios, however, significantly increases the number of variables and model size due to the explosion of possible future scenarios.

Beyond the above models, each formulation also was implemented for 2 to 6 periods assuming random demand, upper limits for internal and external storage in all periods, component types of $C_{i,P}^0 = 50$ and $C_{i,P}^0 = 10$, and all the other input data as previously. Figures 3 and 4 illustrate the impact of the number of periods on model size and optimal solutions, assuming random demand. Note that the number of constraints and variables in SPR models increases dramatically as more periods are considered. Even a six-period model with a single component and a significantly discretized yield probability space, results in 9,121 constraints and 7,296 variables. When a model with two components and
six periods is considered, the number of constraints and variables increases to 18,241 and 14,587 respectively. The model size increases even more dramatically for a two-product-with-overlapping-parts SPR model, resulting in 27,361 constraints and 21,883 variables.

The expected value formulation always results in more expensive solutions, as illustrated in Figure 4, underscoring the value of SPR approaches. As an example, when the mean demand is 40 per period, the total cost is $3,043 for a five-period, two-product, overlapping parts SPR model, whereas the expected value formulation results in a higher cost of $3,183. On average, the expected value results for the two-product overlapping parts model are roughly 9% higher than for the SPR formulation. For the single and two-component models, the expected value solutions are roughly 34% and 29% higher, respectively.

![Figure 3: Impact of number of periods on the number of constraints and variables in the SPR model](image)

![Figure 4: Comparison of optimal solutions from SPR and EV models as a function of the number of periods](image)
**REFERENCE**


Benneyan, Cullinane and Topcu: Stochastic Programming Recourse Models for Reconfigurable Multi-Period Storage Allocation in Remanufacturing Facilities