Optimal Supply Chain Strategy for Varying Production Cost

Ping-Hui Hsu∗

Department of Business Administration, De Lin Institute of Technology,

No. 1, Lane 380, Ching-Yun Rd, Tu-Cheng

New Taipei, Taiwan, 236, R.O.C.

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Abstract—Changes in production cost always lead to changes in wholesale and retail price. How to adjust the wholesale price, retail price and order quantity in order to derive an optimal strategy for the supplier and the retailer is one of the most perplexing problems. The purpose of this study is to develop a strategy to maximize the expected profit by simultaneously determining the adjustment ratio of wholesale/retail price and the order quantity under customer's uncertain and price-sensitive demand. A coordinated policy is proposed. Numerical examples and sensitivity analysis are provided to illustrate the theory.

Keywords—Production cost change, coordination, newsboy problem, compensation mechanism.

1. INTRODUCTION

The unpredictable price fluctuation of raw materials significantly influences the production cost (Ren, et al. 2009). One typical instance of production cost increase is the impact of weather that reduces the harvest of grains, vegetables, or fruits. When the production cost increases, the supplier has to increase the wholesale price in order to meet the supplier's profit. Adjusting retail price is especially important when customers pay more attention to both quality and price of the products. To adjust the selling price is an important task for the manager. The classical economic production lot size model assumes a predetermined and constant production rate. The unit production cost depends on the production rate. Khouja (1995) extended the economic production lot size model to consider the variable production rate.

Comparative pricing practices are frequently used where actual product prices are accompanied by high external reference prices. All types of stores, regular-price department stores as well as discount stores, use comparative price claims to frame price as an attractive deal (Thaler, 1985; Kogan and Spiegel 2006). Kopalle and Lindsey-Mullikin (2003) used a quadratic model to consider the impact of external reference price on consumer's price expectation. However, the impact of production cost changes has received little attention from previous researches.

Abuo-El-Ata et al. (2003) treated a probabilistic multi-item inventory model with varying order cost and zero lead time. Wu and Chang (2004) assessed an optimal production-planning program in response to varying environmental costs in an uncertain environment. Furthermore, the supplier-retailer coordination which improved the performance of inventory control had received a lot of attention in recent years (Goyal and Gupta, 1989; Fites, 1996; Khouja et.al., 2010; Weng, 1997; Zimmer, 2002; Sucky, 2005; Krichen et al., 2011). Since the last decade, several researchers have studied the integrated inventory models when the suppliers and the retailers coordinate their production and ordering policies in order to achieve a higher joint profit. Information exchange is an important issue for coordination (Schouten et al., 1994). Fiala (2005) addressed the cooperation in supply chain based on formal agreements.

This study considers a newsboy problem in three echelon supply chain. A supply chain considering one supplier and one retailer is assumed. The retailer purchases the product from the supplier and sells to its customers. Due to the change in production cost, the supplier adjusts its retailer's wholesale purchase price. The retailer's selling price is based on the production cost and customer's demand. The well known newsboy problem had received a lot of research (Khouja, 1999; Hsu et al., 2007). Li and Liu (2008) focused on the second order policy, that is: In order to meet a random demand, the

* Corresponding author's email: pinghuihsu@gmail.com

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retailer place a second order at the end of the period if a stock-out occurs. The manufacturer’s reserve capacity for the retailer’s second order is limited to M units. To maximize each individual’s expected profit, the retailer decides his optimal order quantity and the manufacturer decides his optimal reserve capacity. However, little researches had considered production cost change. This study focused on the response of wholesale and retail price as the production cost changes. That is: changes in production cost leads to changes in wholesale and retail price. The purpose is to develop a strategy to maximize the expected profit by simultaneously determining the adjustment ratio of wholesale/retail price and the order quantity under customer's uncertain and price-sensitive demand. We suggest a strategy to determine the retailer’s selling price, order quantity, and supplier’s whole purchase price considering the price change from the supplier. Moreover, if the supplier and retailer coordinate, then we obtain the optimal order quantity.

We suggest three cases and compare the policy with coordination and without coordination. The three cases are: (1) the adjustment ratio for both the retailer’s wholesale purchase price (wholesale price) and the retailer’s selling price (retail price) are fixed; (2) the adjustment ratio for the wholesale price is fixed whereas for the retailer price is variable; and (3) the adjustment ratio for both the wholesale price and the retail price are variable. Practically, the retailer price always depends on the wholesale price. It is not needed to discuss the last case that is the adjustment ratio for the wholesale price is variable and retailer price is fixed.

2. ASSUMPTIONS AND NOTATION

In this study, a supply chains with a retailer and a supplier is assumed. The retailer obtains the products from the supplier for sale to the customers. The retailer has to consider the uncertainty of customers’ demand and the production cost change. Placing an optimal order before the selling period of the product is important to the retailer.

The following notation is used throughout this paper.

The decision variables are

\[ k_b \] adjustment ratio for the retail price without coordination, \( k_b > 0 \)
\[ k_{bj} \] adjustment ratio for the retail price with coordination, \( k_{bj} > 0 \)
\[ k_s \] adjustment ratio for the wholesale price; \( 0 \leq k_s \leq 1, k_s > 0 \)

The parameters related to the supplier are

\[ c_o \] supplier’s original wholesale price per unit
\[ t \] supplier’s original production cost per unit; \( t < c_o \)
\[ \delta \] supplier’s varied production cost per unit
\[ E_s \] supplier’s expected profit
\[ E_{so} \] supplier’s extra profit
\[ S_{jo} \] supplier’s actually expected profit after distribution contract

The parameters related to the retailer are

\[ p \] retail original price per unit; \( p > c_o \)
\[ s \] retailer’s salvage value per unit
\[ r \] retailer’s shortage cost per unit; represents costs of lost goodwill
\[ x \] random demand faced by the retailer
\[ f(x) \] probability density function of \( x \)
\[ Q_j \] retailer’s order quantity without coordination
\[ Q_{jo} \] retailer’s order quantity with coordination
\[ E_b \] retailer’s expected profit
\[ E_{bo} \] retailer’s extra profit
\[ S_{bo} \] retailer’s actually expected profit after distribution contract

The other related parameters are as follows:

\[ \theta \] negotiation factor (\( \theta \geq 0 \))
\[ E \] expected system profit (\( E = E_s + E_b \))
3. MODELING AND ANALYSIS

In this section, we formulate an expected profit model for the retailer and the supplier using newsboy's model (Hadley and Whitin, 1963). When the production cost changes from $t$ to $t+\delta$, the supplier adjusts its wholesale price from $c_o$ to $c_o+k_s \delta$. The retailer taking into consideration of cost and customer's demand, responds to this change, and adjusts the retail price from $p$ to $p+k_r \delta$. With the customer's random demand, $x$, the retailer orders quantity $Q_b$ from the supplier, the retailer's expected profit is

$$E_b = \int_0^{Q_b} \left( (p + k_r \delta - (c_o + k_s \delta - s))Q_b - r(x - Q_b) \right) f(x) dx$$

and for the supplier

$$E_s = \int_{Q_b}^{\infty} \left( (p + k_r \delta - (c_o + k_s \delta + s))Q_b - r(x - Q_b) \right) f(x) dx$$

where $f(x)$ is the probability density function of $x$. The supplier's expected profit is

$$E_s = \int_{Q_b}^{\infty} \left( (p + k_r \delta - (c_o + k_s \delta))Q_b - r(x - Q_b) \right) f(x) dx$$

The expected system profit is

$$E = E_b + E_s$$

Theorem 1. $E = E_b + E_s$ is independent of $k_r$.

Proof: Please refer to Appendix A.

When the retailer derives the optimal order quantity independently, the optimal order quantity is derived from Theorem 2.

Theorem 2. The retailer's optimal order quantity without coordination, $Q_b^*$, satisfies the following expression:

$$F(Q_b^*) = \frac{p + k_r \delta - (c_o + k_s \delta) + r}{p + k_r \delta - s + r}$$

Proof: Please refer to Appendix B.

When $\delta = 0$, Theorem 2 results in a well-known “newsboy” [Hadley and Whitin 1963]. From Theorem 2, since $Q_b^*$ is a function of $k_s$ and $k_r$, the retailer's optimal expected profit, $E_b^*(Q_b^*(k_s, k_r))$, is also a function of $k_s$ and $k_r$. The supplier's expected profit is $E_s^*(Q_b^*(k_s, k_r))$, and the expected system profit is

$$E(Q_b^*(k_s, k_r)) = E_b^*(Q_b^*(k_s, k_r)) + E_s^*(Q_b^*(k_s, k_r))$$

If the retailer and the supplier coordinate by sharing their production and demand information, an order quantity of $Q_J$ results in the expected system profit, $E(Q_J) = E_b(Q_J) + E_s(Q_J)$.

Theorem 3. The retailer's optimal order quantity with coordination, $Q_J^*$, satisfies the following expression:

$$F(Q_J^*) = \frac{p + k_r \delta - (c_o + k_s \delta) + r + k_s \delta - (t + \delta)}{p + k_r \delta - s + r}$$

Proof: Please refer to Appendix C.

From Theorem 3, since $Q_J^*$ is a function of $k_s$ and $k_r$, the retailer's expected profit is $E_b(Q_J^*(k_s, k_r))$, the supplier's expected profit is $E_s(Q_J^*(k_s, k_r))$, and the optimal expected system profit is

$$E(Q_J^*(k_s, k_r)) = E_b(Q_J^*(k_s, k_r)) + E_s(Q_J^*(k_s, k_r))$$

(7)
It is obviously that \( E(Q^*_s) \geq E(Q^*_b) \) (please refer to Appendix D). Some player may lose profit under the order quantity, \( Q^*_s \). However, a win-win strategy can be achieved through a compensation mechanism if they share their production and demand information during the coordination.

**Compensation mechanism**

Although there is an increase in the expected system profit with coordination, the gain is always unilateral. We assume the distribution contract of the optimal expected system profit with coordination as compensating the retailer’s loss \([E_s(Q^*_s) - E_s(Q^*_b)]\). The remaining value \( K = E_s(Q^*_s) - E_s(Q^*_b) \) follows the distribution ratio of \( E_{s\theta} = \theta E_s \), where \( \theta \) is the negotiation factor; \( E_{s\theta} \), is the retailer’s extra profit; \( E_{s\theta} \), is the supplier’s extra profit.

Then \( E_{s\theta} = \frac{\theta K}{\theta + 1} \), the retailer’s actual expected profit after distribution contract is \( S_{sb} = E_b(Q_s) + \frac{\theta K}{\theta + 1} \), the supplier’s actual expected profit after distribution contract is \( S_{sb} = E_b(Q_s) + \frac{\theta K}{\theta + 1} \).

The following three cases consider the system with and without coordination.

3.1. Case 1: when \( k_s \), \( k_b \) are fixed

That is, the adjustment ratio for the wholesale price, \( k_s \), is predetermined by the supplier. The adjustment ratio for the retail price without coordination, \( k_b \), is predetermined by the retailer. In this case, given \( k_s = k_{\theta b}, k_b = k_{\theta b} \), the retailer’s optimal order quantity without coordination, \( Q^*_s(k_{\theta b}, k_{\theta b}) \) is derived as follows:

\[
F\left(Q^*_s(k_{\theta b}, k_{\theta b})\right) = \frac{(p + k_{\theta b} \delta) - (c_s + k_{\theta b} \delta) + \imath}{p + k_{\theta b} \delta - s + r}.
\]

The retailer’s optimal order quantity with coordination, \( Q^*_s(k_{\theta b}, k_{\theta b}) \), is derived as follows:

\[
F\left(Q^*_s(k_{\theta b}, k_{\theta b})\right) = \frac{(p + k_{\theta b} \delta) - (c_s + k_{\theta b} \delta) + r + c_s + k_{\theta b} \delta -(\imath + \delta)}{p + k_{\theta b} \delta - s + r}.
\]

3.2. Case 2: when \( k_s \) is fixed and \( k_b \) is variable

That is, the adjustment ratio for the wholesale price, \( k_s = k_{\theta b} \), is predetermined by the supplier. The adjustment ratio for the retail price without coordination, \( k_b \), is treated as variable by the retailer. (i) Without coordination

By solving the equation \( \frac{d}{dk_b} E_s(Q^*_s(k_s, k_b)) = 0 \), the optimal \( k_b^* \) is derived. The retailer’s optimal order quantity without coordination, \( Q^*_s(k_s^*, k_b^*) \) is derived from Eq. (4), the expected system profit is

\[
E(Q^*_s(k_s^*, k_b^*)) = E_s(Q^*_s(k_s^*, k_b^*)) + E_b(Q^*_s(k_s^*, k_b^*)).
\]

(ii) With coordination

If the retailer and the supplier determine jointly the order quantity \( Q_s \) for the optimal expected system profit, the optimal \( k_b^* \) can be derived by setting \( \frac{d}{dk_b} E_s(Q^*_s(k_s^*, k_b)) = 0 \), then the retailer’s optimal order quantity with coordination, \( Q^*_s(k_s^*, k_b^*) \) is derived from Eq. (6), and the optimal expected system profit is

\[
E(Q^*_s(k_s^*, k_b^*)) = E_s(Q^*_s(k_s^*, k_b^*)) + E_b(Q^*_s(k_s^*, k_b^*)).
\]

It is clear from Theorem 3 that regardless of the \( k_s \) the expected system profit will be the same, so if both sides can agree on an optimal order quantity with coordination, \( Q_s^* \), and adjustment ratio, \( k_b^* \), then both sides can reach more profit through compensation mechanism.

3.3. Case 3: when both \( k_s \) and \( k_b \) are variable

That is, the adjustment ratio for the wholesale price, \( k_s \), is treated as variable by the supplier. The adjustment ratio for
the retail price without coordination, \( k_s \), is treated as variable by the retailer.

(i) Without coordination

When the retailer determines \( Q_b, k_b, \) and \( k_s \) independently, the optimal \( (k_b, k_s) \) can be derived by setting \( \frac{\partial}{\partial k_b} E_i(Q_b^*(k_b, k_s)) = 0 \) and \( \frac{\partial}{\partial k_s} E_i(Q_b^*(k_b, k_s)) = 0 \). Hence the retailer’s optimal expected profit is \( E_b(Q_b^*(k_b^*, k_s^*)) \), and the expected system profit is

\[
E(Q_b^*(k_b^*, k_s^*)) = E_b(Q_b^*(k_b^*, k_s^*)) + E_s(Q_b^*(k_b^*, k_s^*)).
\]

(ii) With coordination

Since \( E = E_b + E_s \) is independent of \( k_s \) by Theorem 3, the optimal expected system profit is \( E^*(Q_J^*(k_b^*, k_s^*)) \), regardless of \( k_s \).

4. NUMERICAL EXAMPLES

The random demand faced by the retailer, \( \xi \), is uniformly distributed over the range \( 0 \) and \( B/(1+ak_b)^2 \), where \( a, B > 0 \) are constant (\( a \) represents the magnitude of the selling price fluctuation), the probability density function of \( \xi \) is

\[
f(\xi) = \frac{(1+ak_b)^2}{B}, \quad \xi \in [0, \frac{B}{(1+ak_b)^2}].
\]

The cumulative distribution function of \( \xi \) is

\[
F(\xi) = \frac{(1+ak_b)^2\xi}{B}, \quad \xi \in [0, \frac{B}{(1+ak_b)^2}].
\]

A simple economic interpretation of the random demand is as follows: When the adjustment ratio for the retail price, \( k_b \), increases, the customer’s demand decreases. From (4), one has

\[
Q_b^*(k_b, k_s) = \frac{B}{(1+ak_b)^2} \left( \frac{p+k_b\delta-(c_s+k_b\delta)+r}{p+k_b\delta-s+r} \right).
\]

From (6), one has

\[
Q_J^*(k_b, k_s) = \frac{B}{(1+ak_b)^2} \left( \frac{p+k_b\delta-(c_s+k_b\delta)+r+c_s+k_b\delta-(t+s)}{p+k_b\delta-s+r} \right).
\]

**Example 1.** Case 1:

\[ t = 400, \delta = 50, c_s = 550, p = 700, \bar{o} = 150, r = 25, B = 1000, a = 1, k_b = 0.7, k_s = 0.9, \text{ and } \theta = 2 \], then

\( Q_b^* = 82,656, E_b(Q_b^*) = \$4183, E_s(Q_b^*) = \$11159, E(Q_b^*) = \$15342, \)

\( Q_J^* = 142,972, E_s(Q_J^*) = \$111.7, E(Q_J^*) = \$19413 \).

% profit increase is \( \left[ (E(Q_J^*) - E(Q_b^*)) / E(Q_b^*) \right] \times 100\% \approx 26.5\% \).

\( E_{\omega} = \$2714, E_{\omega} = \$1357, S_{ja} = \$6897, \text{ and } S_J = \$12516 \).

**Sensitivity Analysis**

Sensitivity analysis with parameters \( a, k_b, \) and \( k_s \) changes are carried out in this section. Table 1 and Figure 1 show the changes of \( Q_b^*, E(Q_b^*), Q_J^* \) and \( E(Q_J^*) \) for parameter \( a \) equals to 0.5, 1, 1.5; for parameter \( k_b \) equals to 0.2, 0.5, 0.8; for parameter \( k_s = 0.3 \) and other fixed parameters. Table 2 and Figure 2 show the change of \( Q_b^*, E(Q_b^*), Q_J^* \) and \( E(Q_J^*) \) for parameter \( k_b = 0.5 \). Table 3 and Figure 3 show the changes of \( Q_i^*, E(Q_i^*), Q_J^* \) and \( E(Q_J^*) \) for parameter \( k_b = 0.7 \). Table 4 shows the changes of \( S_J \) and \( S_J \) for negotiation factor \( \theta \).
(1) When \( a \) increases, which leads to % profit increase unchanged, \( Q^*, Q^*, E_i(Q^*), E_i(Q^*), E_j(Q^*) \) and \( E_j(Q^*) \) decrease. Which means the more magnitude of the selling price fluctuation is, the less profit for both players will be.

(2) When \( k_b \) increases, which leads to a decrease of % profit increase, \( Q^*, Q^*, E_i(Q^*) \) and \( E_j(Q^*) \), that means an increase in retail price will decrease profit for both players.

(3) When \( k_s \), which leads to an increase of % profit increase and \( E_i(Q^*) \), however, \( Q^*, E_i(Q^*) \) and \( E_j(Q^*) \) decrease, both \( Q^* \) and \( E_j(Q^*) \) are still unchanged.

<table>
<thead>
<tr>
<th>( a )</th>
<th>( k_s )</th>
<th>( Q^* )</th>
<th>( E_i(Q^*) )</th>
<th>( E_j(Q^*) )</th>
<th>( E_i(Q^*) )</th>
<th>( E_j(Q^*) )</th>
<th>% profit increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.2</td>
<td>2.26</td>
<td>7752</td>
<td>28255</td>
<td>36007</td>
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</tr>
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<td>30256</td>
<td>338</td>
<td>2760</td>
</tr>
</tbody>
</table>

Note: % profit increase is \([E_i(Q^*)−E_i(Q^*)]/E_i(Q^*)]×100\%

![Figure 1](image-url) The effect of retailer's adjustment ratio \( k_s \) on the expected profit: with vs. without coordination when \( k_s = 0.3 \).

<table>
<thead>
<tr>
<th>( a )</th>
<th>( k_s )</th>
<th>( Q^* )</th>
<th>( E_i(Q^*) )</th>
<th>( E_i(Q^*) )</th>
<th>( E_i(Q^*) )</th>
<th>( E_i(Q^*) )</th>
<th>% profit increase</th>
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<td>23742</td>
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<td>2760</td>
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![Figure 2](image-url) Sensitivity analysis for parameters \( a, k_s \) with \( k_s = 0.3 \).
Figure 2  The effect of retailer's adjustment ratio $k_b$ on the expected profit: with vs. without coordination when $k_v = 0.5$.

Table 3  Sensitivity analysis for parameters $a$, $k_b$ with $k_v = 0.7$

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<th>$r$</th>
<th>$d$</th>
<th>$Q_0^*$</th>
<th>$E_i^<em>(Q_0^</em>)$</th>
<th>$E_i(Q_0^*)$</th>
<th>$Q_i^*$</th>
<th>$E_i(Q_i^*)$</th>
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</tr>
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<td>12195</td>
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<td>21040</td>
<td>27.8%</td>
</tr>
<tr>
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<td>2860</td>
<td>8164</td>
<td>11024</td>
<td>-201</td>
<td>14286</td>
<td>14085</td>
<td>27.8%</td>
</tr>
</tbody>
</table>

Figure 3  The effect of retailer's adjustment ratio $k_v$ on the expected profit: with vs. without coordination when $k_v = 0.7$.

Table 4  Sensitivity analysis for negotiation factor $\theta$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$E_i(Q_r^*)$</th>
<th>$E_i(Q_\gamma)$</th>
<th>$s_p$</th>
<th>$E_i(Q_<em>^</em>)$</th>
<th>$E_i(Q_\gamma)$</th>
<th>$s_p$</th>
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<td>1357</td>
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<td>2</td>
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<td>2721</td>
<td>6897</td>
<td>11159</td>
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</tr>
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</table>

Table 5  Sensitivity analysis for parameters $a$ and $k_v$

<table>
<thead>
<tr>
<th>$a$</th>
<th>$k_v$</th>
<th>$Q_0^*$</th>
<th>$E_i^<em>(Q_0^</em>)$</th>
<th>$E_i(Q_0^*)$</th>
<th>$Q_i^*$</th>
<th>$E_i(Q_i^*)$</th>
<th>$E_i(Q_r^*)$</th>
<th>$%$</th>
<th>$%$</th>
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<td>0</td>
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<td>9760</td>
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<td>9761</td>
<td>32000</td>
<td>41761</td>
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<td>55000</td>
<td>53261</td>
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<td>8682</td>
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<td>53261</td>
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<td>0</td>
<td>260.8</td>
<td>7065</td>
<td>32063</td>
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<td>0.194</td>
<td>180.0</td>
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<tr>
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<td>0.7</td>
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<td>32861</td>
<td>37404</td>
<td>-11304</td>
<td>64565</td>
<td>53261</td>
</tr>
</tbody>
</table>
Example 2. Case 2:

\( \tau = 400, \ \delta = 50, \ \epsilon_i = 550, p = 700, \ \tau = 150, r = 25, B = 1000, a = 1 \) and \( k_p = 0.7 \), then

\( k_s^* = 0.194, (p + k_b^* \delta = 700 + 0.194 \times 50 = 709.7), Q_s^* = 179.567, E_s(Q_s^*(k_s^*, k_p)) = $4674.23, E_s(Q_s^*(k_s^*, k_p)) = $24242, E(Q_s^*(k_s^*, k_p)) = $28916. \)

\( k_s^* = 0, Q_s^* = 478.17, E_s(Q_s^*(k_s^*, k_p)) = -$11292, E_s(Q_s^*(k_s^*, k_p)) = $64553, E_s(Q_s^*(k_s^*, k_p)) = $53261. \)

% profit increase is \( \frac{[(E_s(Q_s^*(k_s^*, k_p)) - E_s(Q_s^*(k_s^*, k_p)))]}{E_s(Q_s^*(k_s^*, k_p))} \times 100\% = 84.2\% \). From the analysis, if the supplier’s production cost per unit is increased by \( \tau + k_p \delta = 550 + 0.7 \times 50 = 585 \), then the retail price per unit is \( p + k_b^* \delta = 700 + 0.194 \times 50 = 709.7 \).

Sensitivity Analysis

Sensitivity analysis with parameters \( a, k_s \) changes and \( k_s \) treated as variable are carried out in this section. Table 5 shows the changes of \( Q_s^* \), \( E(Q_s^*) \), \( Q_J^* \) and \( E(Q_J^*) \), for parameter \( a \) equals to 0.5, 1, 1.5, and for parameter \( k_s \) equals to 0.3, 0.5, 0.7.

1. When \( a \) increases, \( Q_s^* \) increases, but \( k_s^* \) decreases. It means when the magnitude of the selling price fluctuation gets higher, the adjustment ratio for the retailer’s selling price should be reduced in order to boost the order quantity.
2. \( Q_s^* \) is the same since \( E(Q_s^*) \) is independent of \( k_s \). Also, because \( k_{ij}^* = 0 \), so \( Q_s^* \) will not change regardless of \( a \).
3. When \( k_s \) increases, which leads to \( k_s^* \) increases, but \( k_s^* \) decreases, at the same time \( E_s(Q_s^*, k_s^*), E_s(Q_s^*, k_s^*) \) and \( E_s(Q_s^*, k_s^*) \) decrease.

Example 3. Case 3:

\( \tau = 400, \ \delta = 50, \ \epsilon_i = 550, p = 700, \ \tau = 150, r = 25, B = 1000 \) and \( a = 0.5 \), then

\( k_s^* = 0, Q_s^* = 304.348, E_s(Q_s^*(k_s^*, k_s^*)) = $14130, E_s(Q_s^*(k_s^*, k_s^*)) = $30435, \) and \( E_s(Q_s^*(k_s^*, k_s^*)) = $44565 \) is derived.

Since \( E \) is independent of \( k_s \), it coincides with any \( k_s \) and \( k_{ij}^* = 0, Q_J^* = 478.3, E(Q_s^*(k_{ij}, k_b)) = $53261, \) the % profit increase is 19.5%.

5. CONCLUSION

Price fluctuation is a common phenomenon in the market. Both upstream and downstream supply chain need to respond to the production price change appropriately. How to establish a responding policy has become an important topic to the industry lately. This study considers uncertain customer’s demand and price-sensitive model. We conclude that:

1. When \( k_s \) is fixed and \( k_s \) is variable, if the supplier’s production cost per unit, and the supplier’s wholesale price per unit increase, then the optimal retail price per unit increase.
2. When both \( k_s \) and \( k_s \) are variable, if the supplier’s production cost per unit increase, then the optimal supplier’s wholesale price per unit and the optimal retail price per unit still maintain. Therefore, compensation mechanism for the plays is needed.
3. The optimal expected system profit will be better after coordination if both players share their production and demand information.
4. The expected system profit is not affected by the adjustment ratio for the wholesale price. The increase in the retailer’s wholesale purchase price will benefit the supplier but hurts the buyer. When the supplier increases the fixed adjustment ratio of the wholesale price, the retailer may reduce his profit. Therefore, the coordination must consider compensation mechanism.

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REFERENCES


Appendix A: Proof of Theorem 1.

\[ E = \int_{0}^{\infty} \left[ \left( p + k_{0}\delta \right) - c_{0} \right] f(\infty) dx + \int_{0}^{\infty} \left[ -k_{0}\delta Q - c_{0}\delta \right] f(\infty) dx + \int_{0}^{\infty} \left[ \left( p + k_{0}\delta \right) - r \right] f(\infty) dx + \int_{0}^{\infty} -k_{0}\delta f(\infty) dx + Q \left( c_{0} - \delta \right) + \delta k_{0}\delta. \]  

Combine the 2nd, 4th and 6th term of Eq.(A1) that vanish \( k_{0} \). This completes the proof.

Appendix B: Proof of Theorem 2.

\[ \frac{dE}{dQ_{0}} = \left( p + k_{0}\delta \right) - \left( c_{0} + k_{0}\delta \right) + r \left( p + k_{0}\delta - \right) \]
where \( F(x) = \int_{0}^{\infty} f(y) dy \).

\[
\frac{d^2}{dQ_b^2} E_b(Q_b) = -\left( p + k_b \delta - \sigma + r \right) f(Q_b) < 0 .
\] (B2)

Which means \( E_b(Q_b) \) is concave in \( Q_b \), and \( Q_b^* \) is derived by setting \( \frac{d}{dQ_b} E_b(Q_b) = 0 \), this completes the proof.

**Appendix C:** Proof of Theorem 3.

\[
\frac{dE(J_{bb})}{dJ_{bb}} = \left[ (p + k_b \delta - \sigma + r - (p + k_b \delta - \sigma + r) F(J_{bb}) + \left[ r_b + k_b \delta - \sigma + \delta \right] \right] .
\] (C1)

\[
\frac{d^3}{dJ_{bb}^3} E(J_{bb}) = -\left( p + k_b \delta - \sigma + r \right) f(Q_b) < 0 .
\] (C2)

Then \( Q_b^* \) is derived by setting \( \frac{d}{dJ_{bb}} E(J_{bb}) = 0 \), this completes the proof.

**Appendix D:** Proof of \( E(Q_b^*) \geq E(Q_j^*) \).

Since \( E = E_b + E_j \), if \( Q_j^* \) is an optimal solution of \( E(Q_j) \), then \( E(Q_j^*) \geq E(Q_j) \), for any \( Q \). Therefore, \( E(Q_b^*) \geq E(Q_j^*) \).