Efficient Approach for Simulation Optimization – Optimal Computing Budget Allocation (OCBA)

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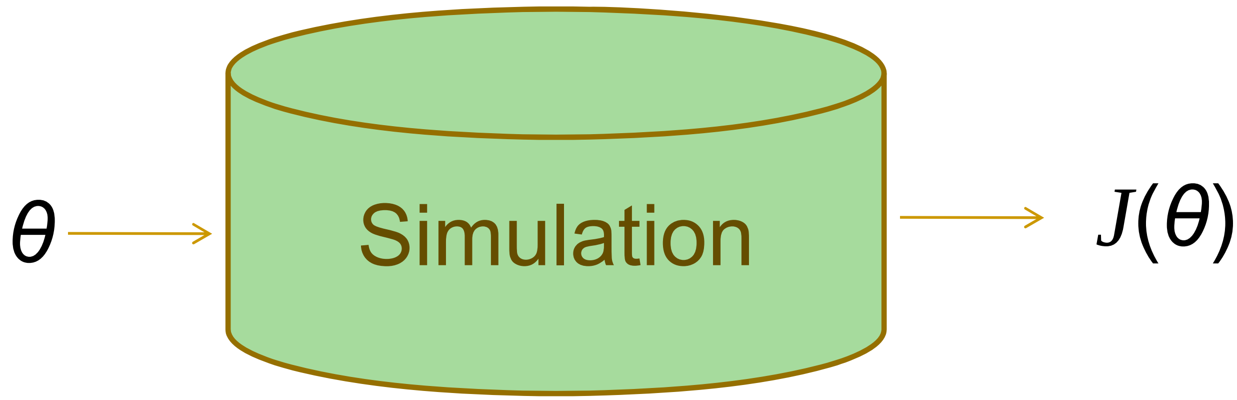
Simulation Optimization Problem

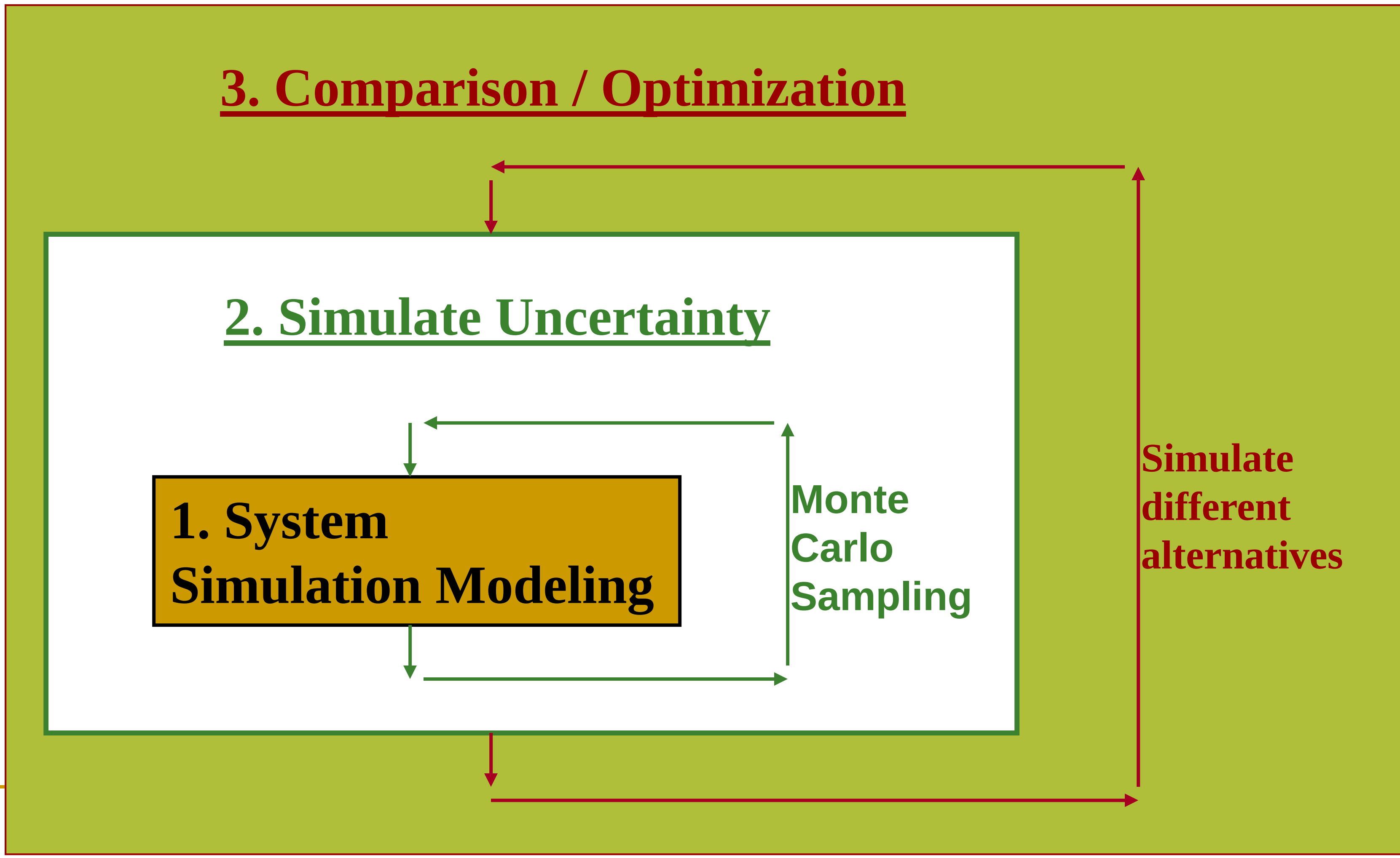


$$\underset{\theta \in \Theta}{Best} \quad J(\theta)$$

Approaches

- Metamodel based method
  - Regression
  - Response Surface Method
- Sample path based method
  - Sample Average Approximation
  - Perturbation Analysis
- Derivative free method (Black Box)
  - Ranking and Selection
  - Integrate with Search method

Simulation Optimization Framework



Challenges

Time consuming simulation

$$\underset{\theta \in \Theta}{Best} \qquad J(\theta) \approx \frac{1}{n}\sum_{i=1}^{n} L(\theta, \omega_i)$$

Huge Search Space

Ordinal Optimization:
Find the "best" answer within limited time frame

Classification

- Finite and relative small Search Space (5 to 1000 designs)
  - Ranking & Selection

- Very large search space (Discrete or Continuous)
  - Incorporate Ranking and Selection in Searching Method

Ranking & Selection

- Among the k designs, pick the best one
- Performance is evaluated using simulation
- Mostly Normality Assumption
- Deal with the replication number of each design
  - To guarantee certain level of probability of correct selection
  - For a fixed budget, how to maximize the probability of correct selection

$$\min \sum_{i=1}^{k} n_i$$

$$PCS \geq P_{sat}$$

$$\max_{n_1,\ldots,n_k} PCS$$

$$n_1 + \ldots + n_k = N$$

Ranking & Selection-Different approaches

- **Equal Simulation**
  - All designs are equally simulated
- **Rinott & Dudewicz** Procedures:
  - Very popular in IE/OR Simulation until ~ 2000
  - Several variations have been developed
  - Based on least favorable configuration
  - Proportional to variance: $N_i = c_i\sigma_i^2$

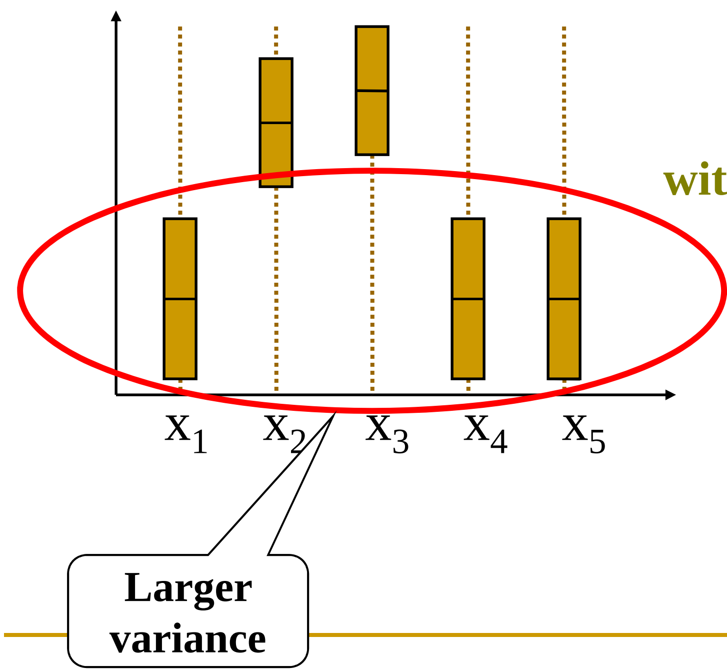
A Much More Efficient Approach

Optimal Computing Budget Allocation (OCBA)-

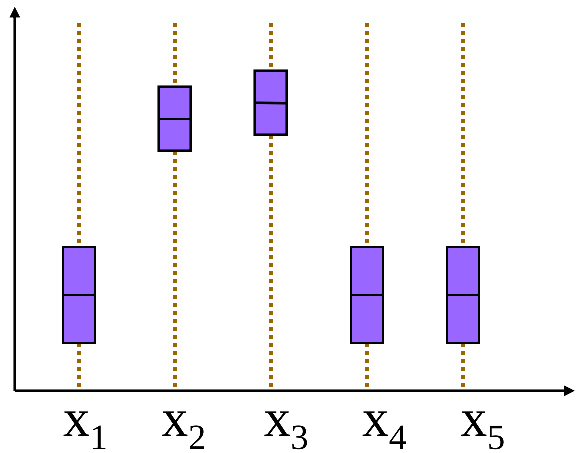
Chen (1995) & Chen (1996)

An Intuitive Example - Maximization

99% Confidence Intervals



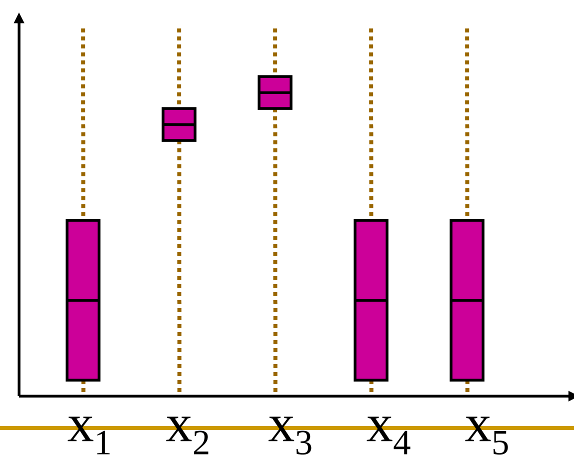
Equal Simulation



with the same number of total runs

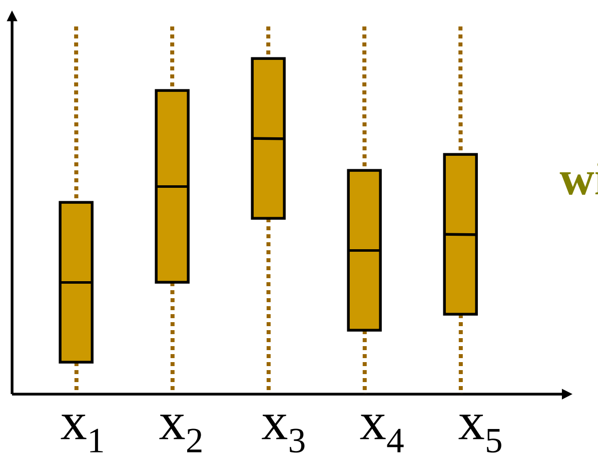
Intelligent

⇒ Option 3 is better isolated

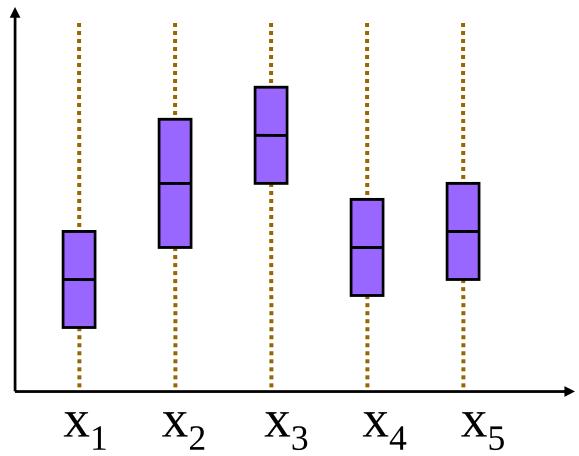


More General Case

99% Confidence Intervals

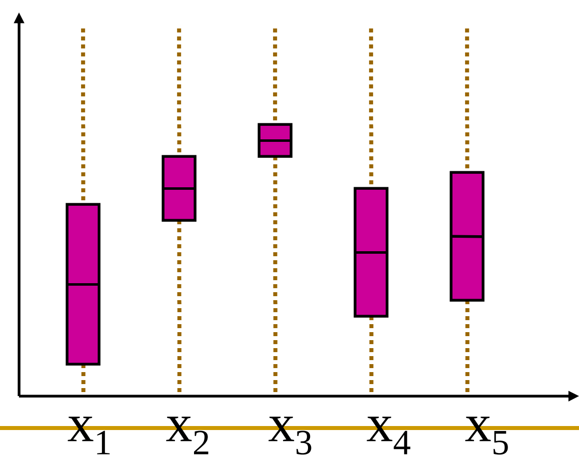


Equal Simulation



with the same number of total runs

Intelligent
⇒ Option 3 is better isolated



Ideas in Deriving the allocation rule

- PCS – no close form
  - Normality assumption
  - Found lower bound by using Bonferroni inequality

- Derive Asymptotic rule
  - Let total budget goes to infinity, and derive the allocation ratio based on KKT conditions, look at allocation ratio α

Asymptotic Rule of OCBA

Given a total number of simulation runs T to be allocated to k competing designs, as $T \to \infty$, $P\{\text{CS}\}$ can be asymptotically maximized when

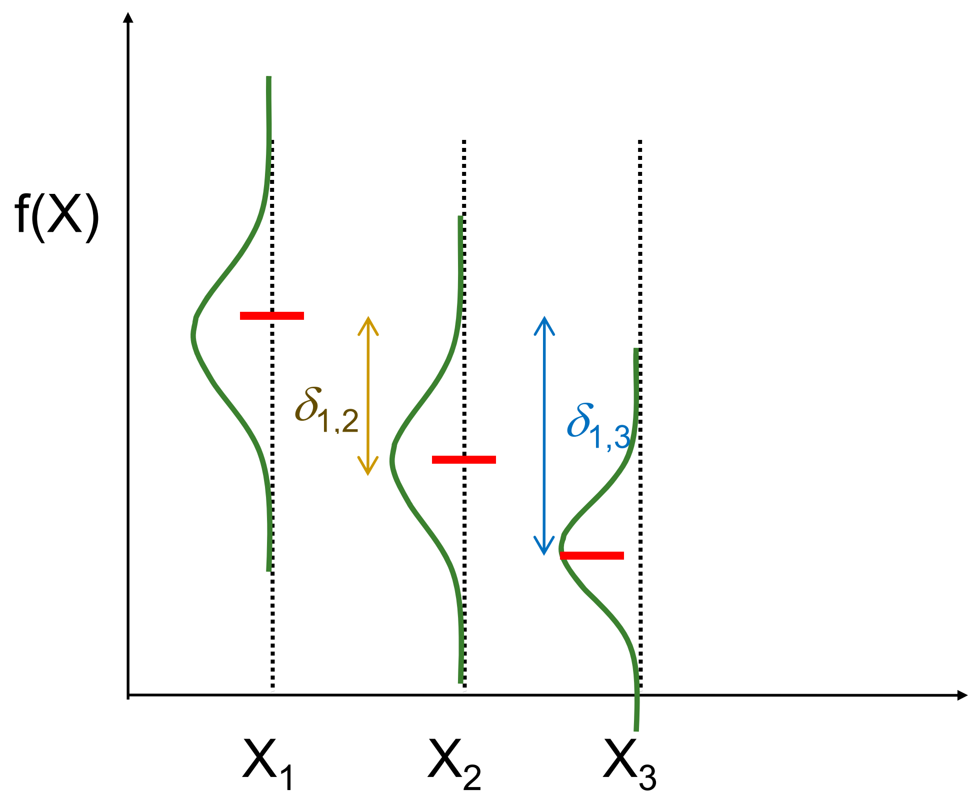
Difference between i and b

$$\frac{\alpha_i}{\alpha_j} = \left(\frac{\sigma_i / \delta_i}{\sigma_j / \delta_j} \right)^2$$

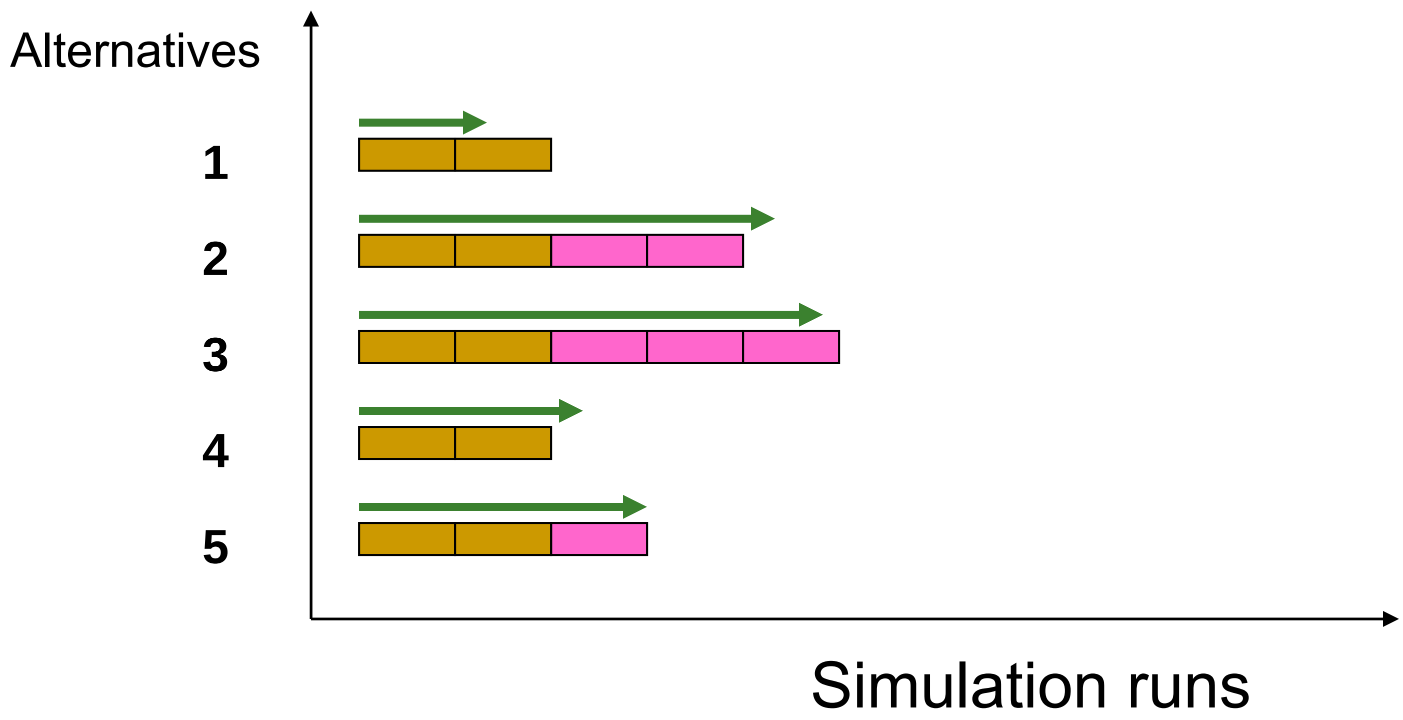
$$\alpha_b = \sigma_b \sqrt{\sum_{i \neq b} \alpha_i^2 / \sigma_i^2}$$

Insight of the rule

- Bad designs
  - Assign according to noise to signal ratio
- Best design
  - Should assign more, and is equal to the square root of weighted sum of square of the allocation for other designs

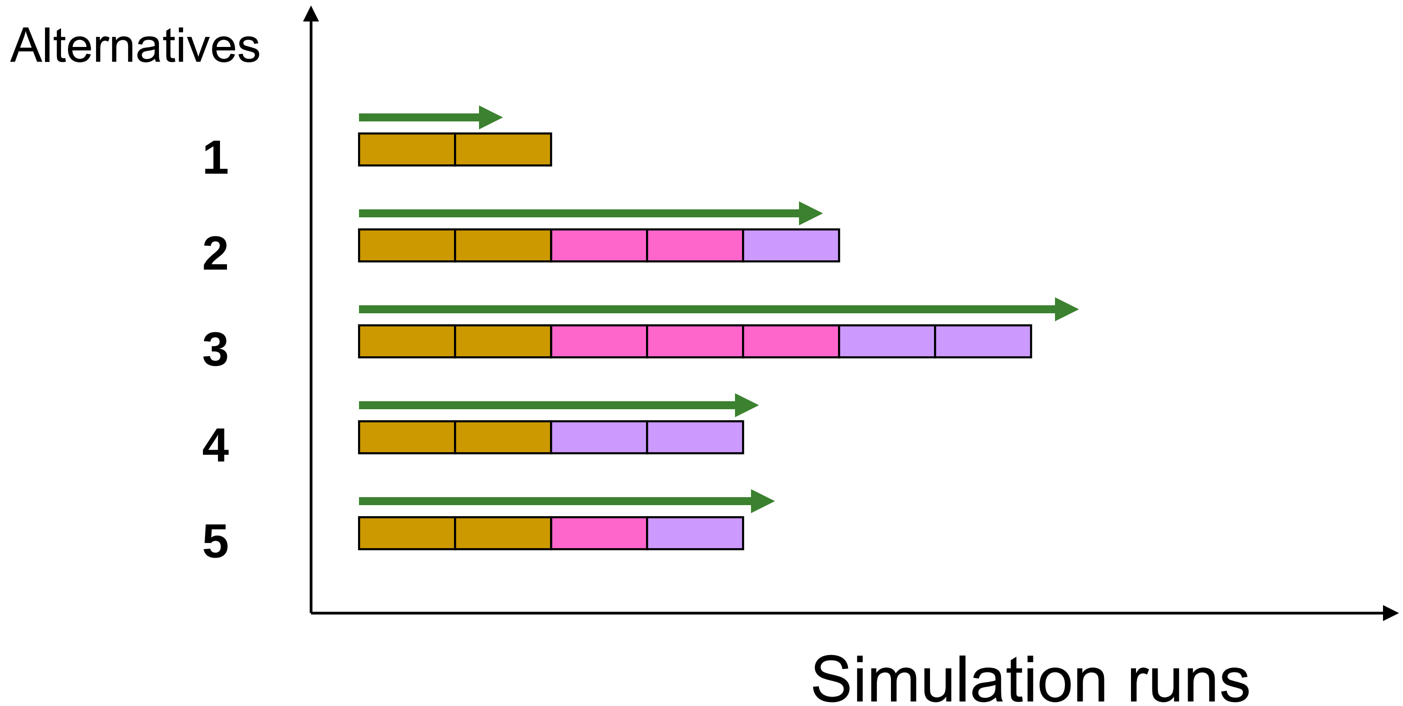


Simulation Run Allocation Using OCBA



Q. If T is increased to 16?

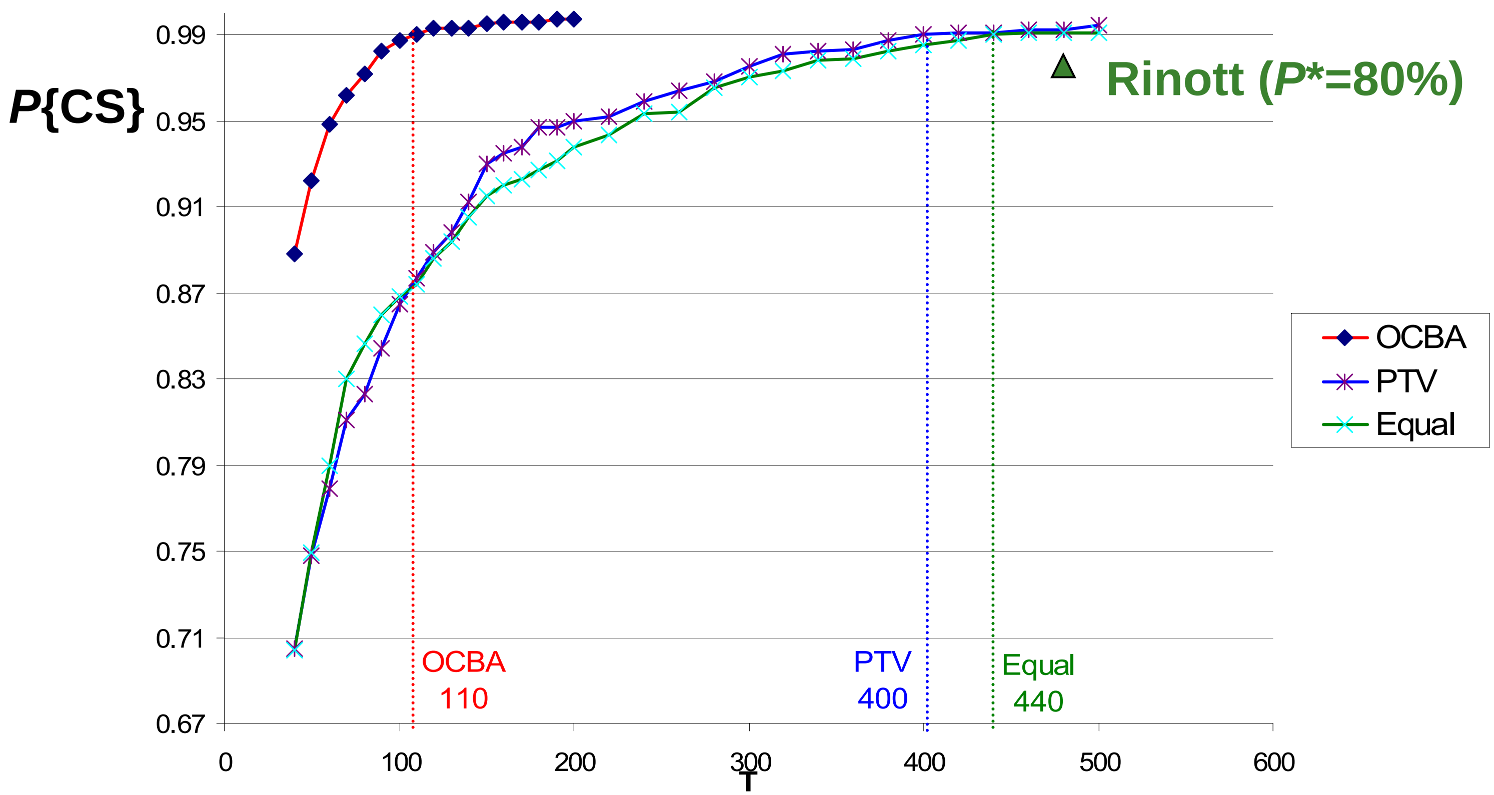
Simulation Run Allocation Using OCBA



Q. If T is increased to 22?

Numerical Testing- G/G/1 - 10 Alternatives

- *P*{CS} ↗ as the computing budget *T* ↗
- OCBA is more than 3 ~ 4 times faster in achieving 99% of *P*{CS}



More Alternatives Higher Speedup

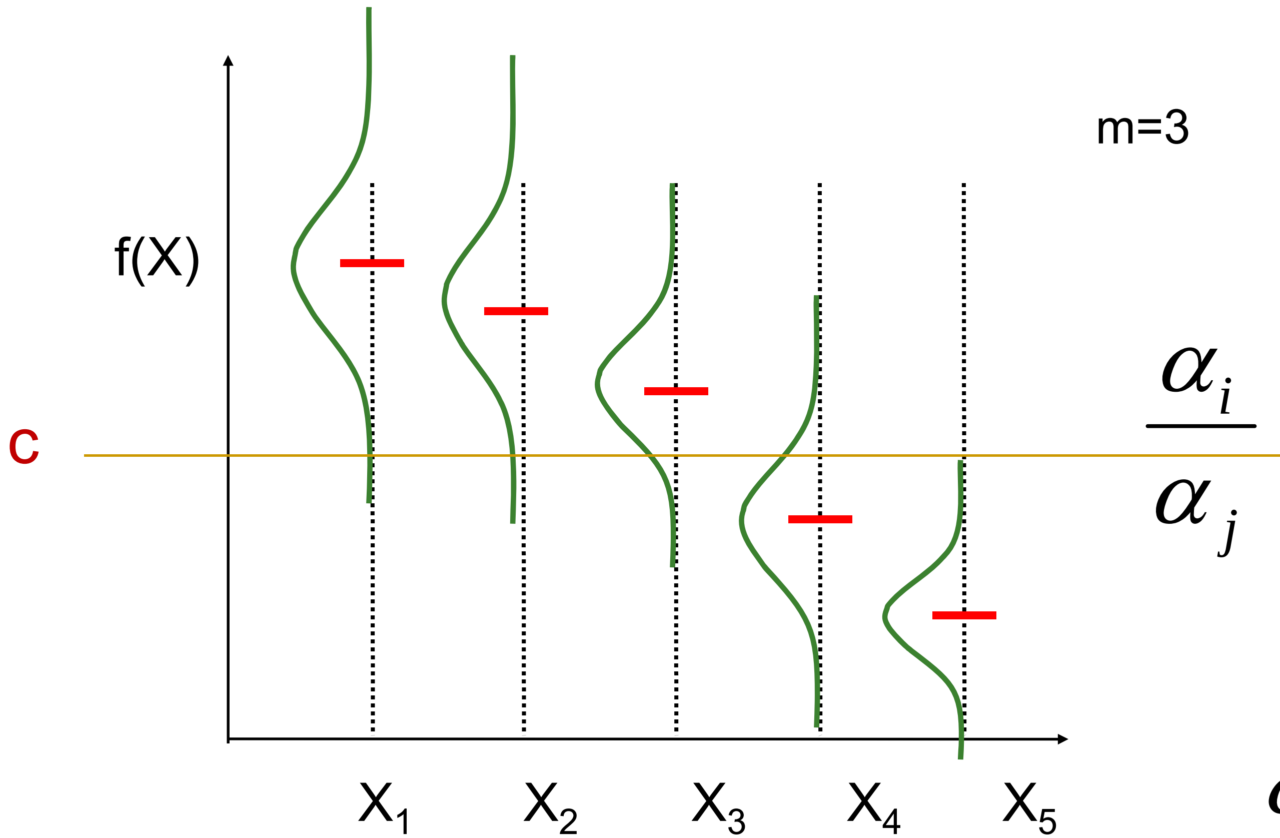
Number of alternatives, *k*	4	10	20	50	75	100
Speedup factor	1.75	3.42	6.45	12.8	16.3	19.8

- **Speedup factor = (*T* using Equal) / (*T* using OCBA)**
  both for achieving *P*{CS} = 99%

Other types of OCBA

- Identifying a Subset: OCBA-m
- Identifying a Pareto Set: MOCBA
- Identifying the best Feasible Design: OCBA-CO
- Identifying the subset for Cross-Entropy Method: OCBA-CE

OCBA-m : Allocation Rule 1

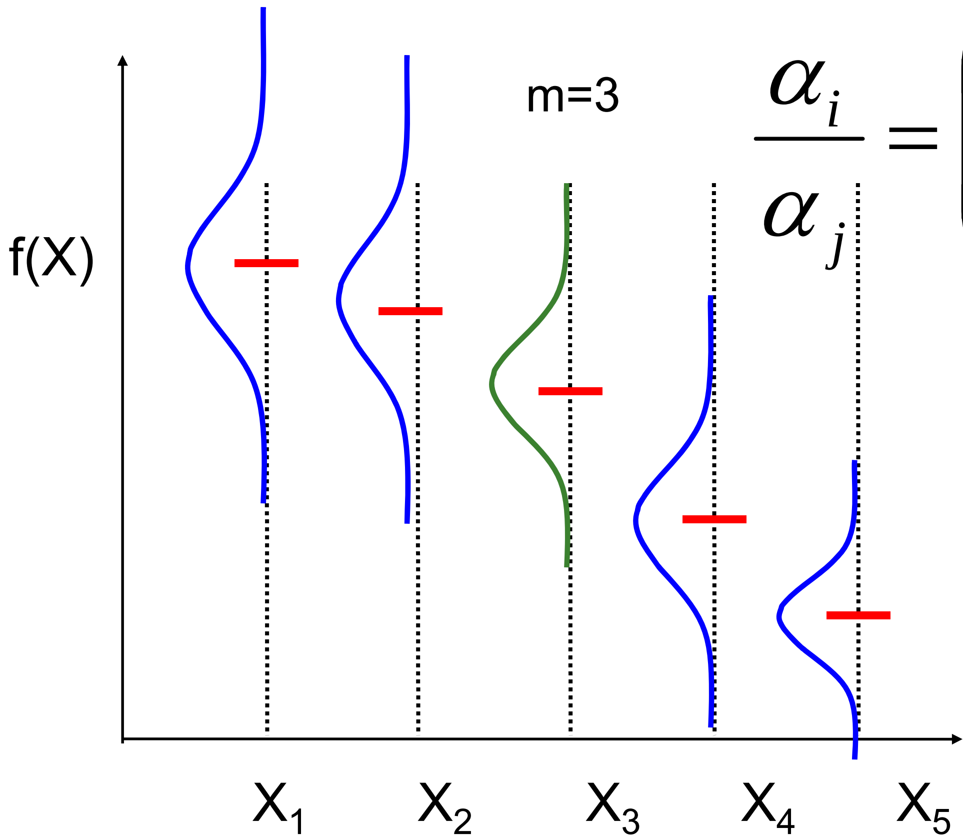


$$\frac{\alpha_i}{\alpha_j} = \left(\frac{\sigma_i / \delta_i}{\sigma_j / \delta_j} \right)^2$$

$$\delta_i = \left| J_i - c \right|$$

- Chen et al. 2008

OCBA-m : Allocation Rule 2



$$\frac{\alpha_i}{\alpha_j} = \left(\frac{\sigma_i / \delta_i}{\sigma_j / \delta_j} \right)^2$$

For $i \neq m$

$$\delta_i = \left| J_m - J_i \right|$$

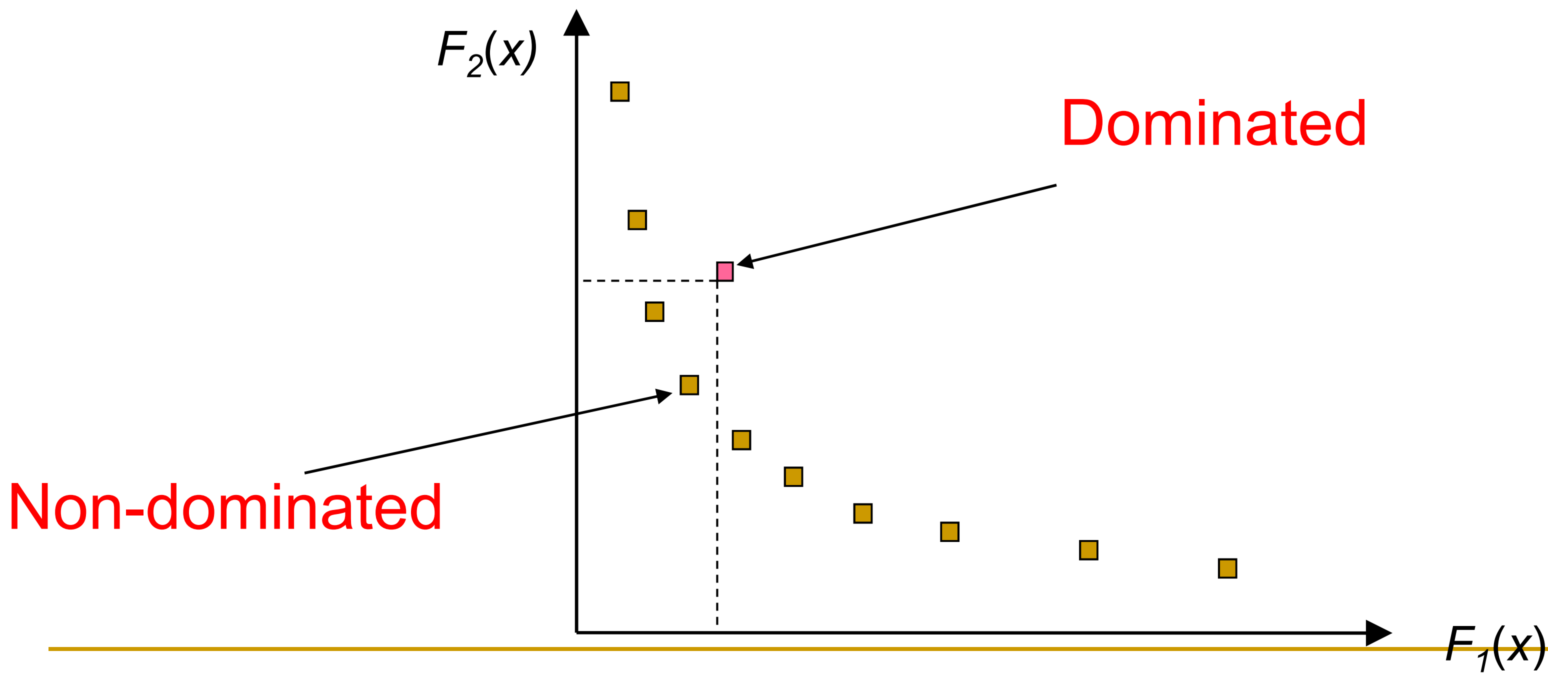
$$\alpha_m = \sigma_m \sqrt{\sum_{i \neq m} \alpha_i^2 / \sigma_i^2}$$

Multi-Objective Computing Budget Allocation (MOCBA)

- Assume a fixed number of designs
  - MOCBA
    - How to identify [2] promising [1] designs.
    - How to allocate [4] simulation runs effectively to improve the quality [3] of the selected designs?

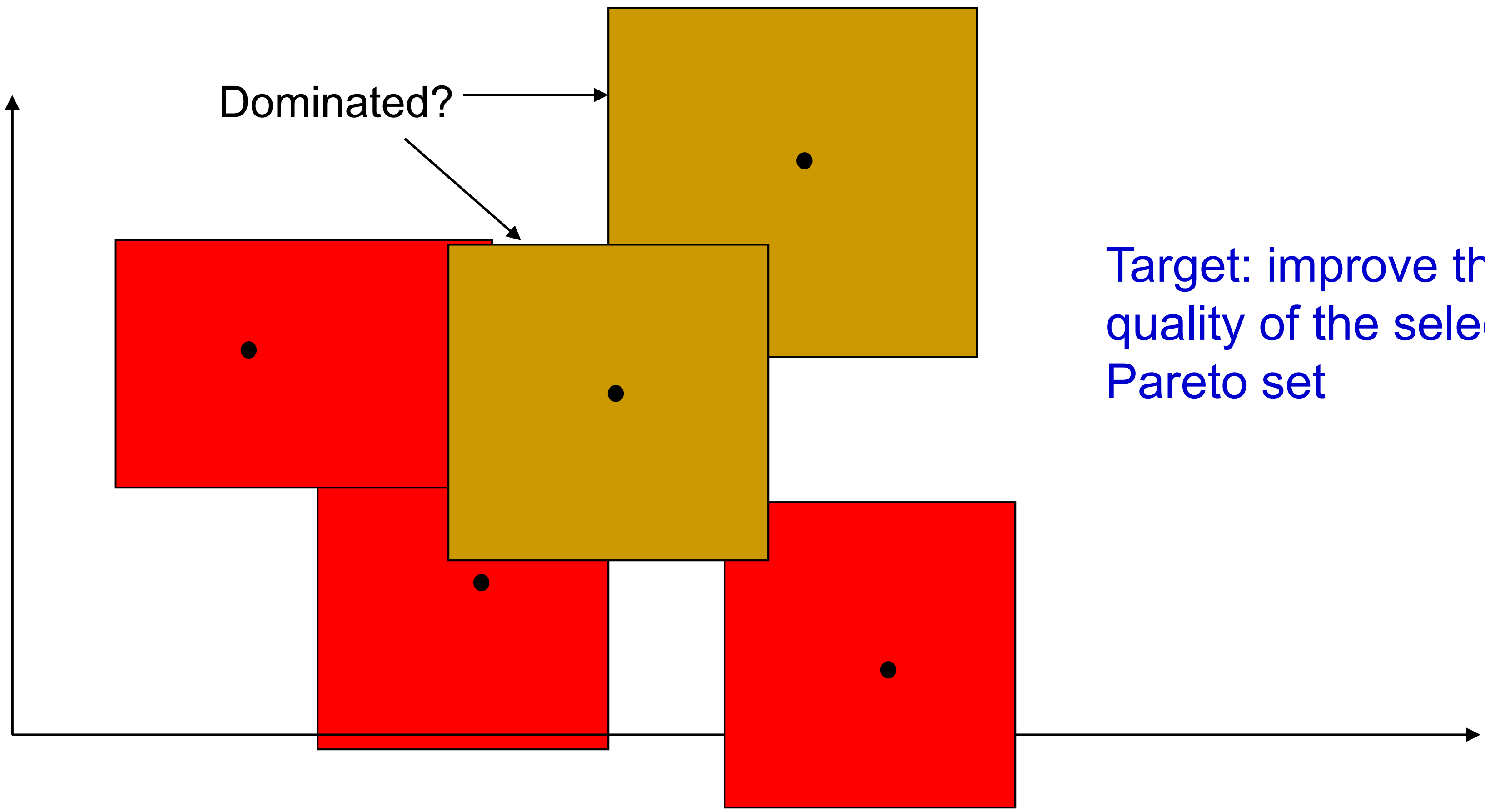
1. Defining Promising -Pareto Optimality

- The solution to a multi-objective problem is not one solution, but a set of non-dominated solutions known as the efficient frontier (Pareto front)



Consider Minimization

1. Dominant Under Stochastic Noise



Target: improve the quality of the selected Pareto set

Not necessary Dominant

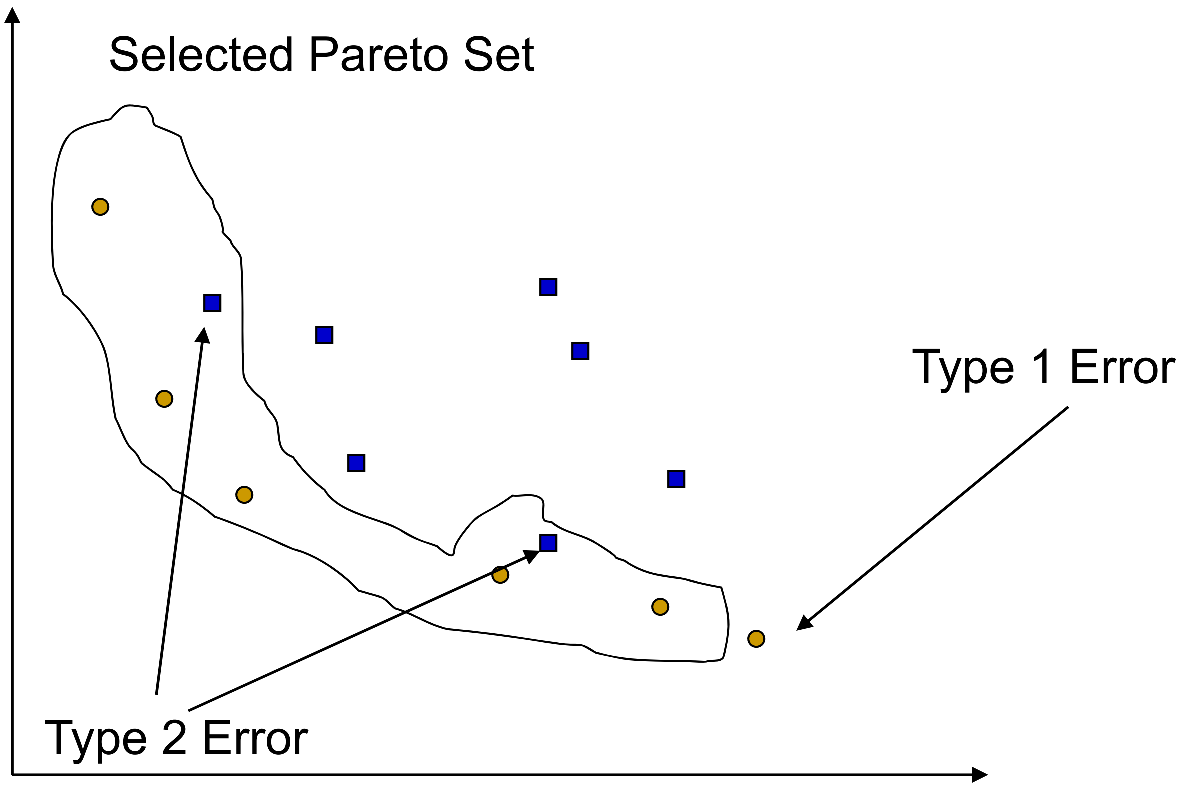
2. Identify Promising-Construction of Pareto Set

- Define Non-dominating Probability
- Select *based on* non-dominated probability

3. Defining Quality – Evaluation of Pareto Set

- A good selected Pareto set:
  - Designs in the selected Pareto set are non-dominated with high probability.
  - Designs outside the selected Pareto set are dominated with high probability.

3. Defining Quality - Quality of the Selected Set



3. Defining Quality – Type I & II Errors

- Type I error: The probability that at least one design outside the selected Pareto set is non-dominated.
- Type II error: The probability that at least one design in the selected Pareto set is dominated by other designs.

When both types of errors approach 0, the true Pareto set is found

4. Allocation Problem

$$\max PCS \leq 1 - error1 - error2$$

$$\sum_{i=1}^{k} n_i = T$$

4. Allocate- Asymptotic Allocation Rule

- For those designs playing dominating role

$$\alpha_d = \sqrt{\sum_{i \in \Omega_d} \frac{\alpha_i^2 \sigma_{dk_d^i}^2}{\sigma_{ik_d^i}^2}} \geq 0$$

Designs that are most dominated by d

- For other designs playing the role of being dominated, l and m,

$$\frac{\alpha_l}{\alpha_m} = \frac{\left(\sigma_{lk_{jl}^l}^2 + \sigma_{j_l k_{jl}^l}^2 \big/ \rho_l\right) \big/ \delta_{lj_l k_{jl}^l}^2}{\left(\sigma_{mk_{jm}^m}^2 + \sigma_{j_m k_{jm}^m}^2 \big/ \rho_m\right) \big/ \delta_{mj_m k_{jm}^m}^2}$$

Noise to signal ratio

OCBA for Constraint Optimization Problem – OCBA-CO

- Stochastic Objective Function
- Stochastic Constraints

$$\min J_0 \qquad \max PCS$$

Need to consider feasibility and optimality

$$J_i \le c_i \qquad \sum_{i=1}^{k} \alpha_i = 1$$

Asymptotic Allocation Rule – OCBA-CO

$$\frac{\alpha_i}{\alpha_j} = \left(\frac{\sigma_i / \delta_i}{\sigma_j / \delta_j} \right)^2$$

For i (feasibility)

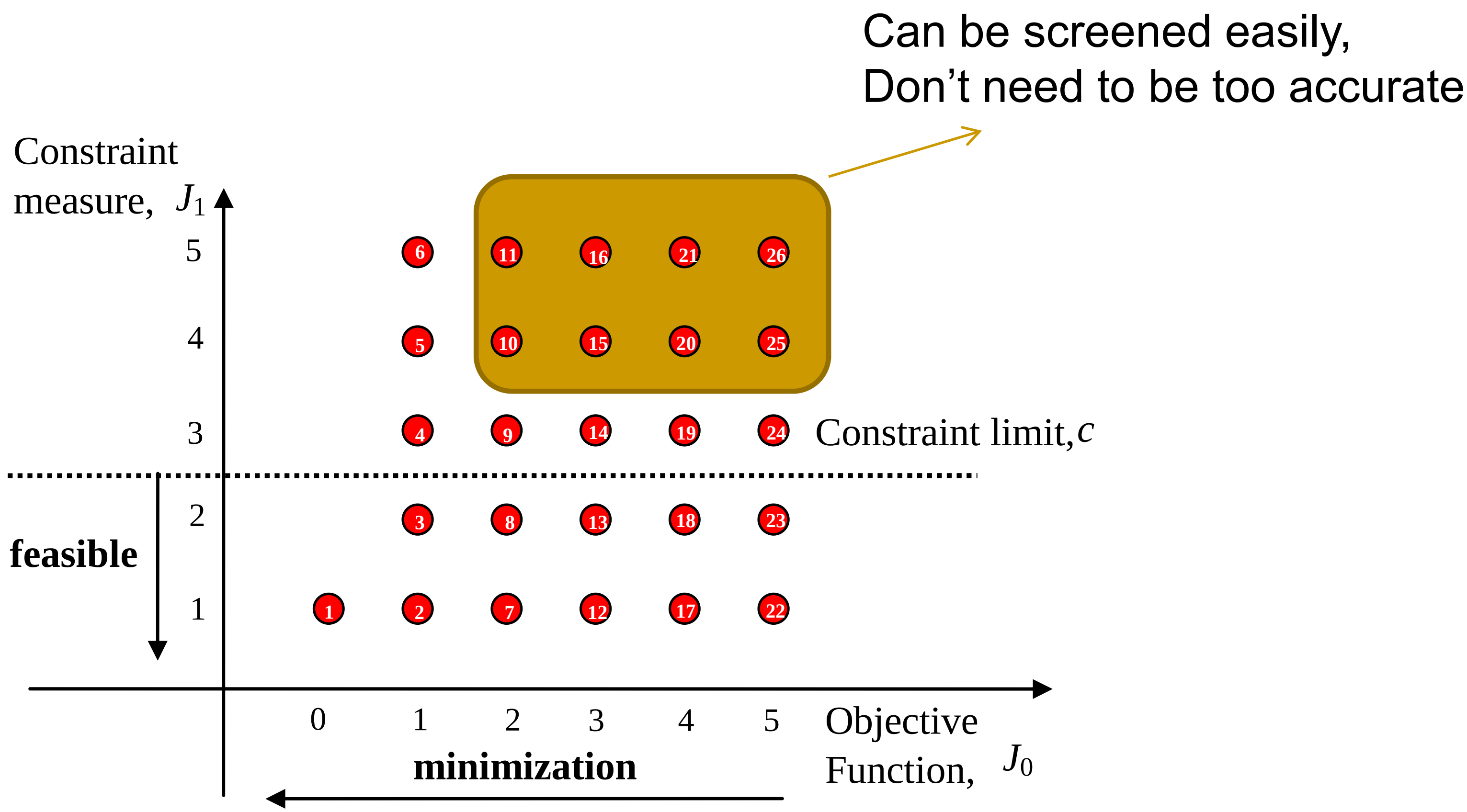
$$\delta_i = \left| J_i - c_i \right|$$

For i (optimality)

$$\delta_i = J_i - J_b$$

$$\alpha_b = \sigma_b \sqrt{\sum_{i \in feasibility} \alpha_i^2 / \sigma_i^2}$$

Intuition on the rule



Summary for all the rules

- Good design using the square root rule
- Bad Designs using the noise to signal ratio rule.

Why?

Large Deviation Theory

Large Deviation Theory

- Let X_1 ,…, X_n be a sequence of bounded, independent and identically distributed random variables each with mean m.
- Let M_n be the sample average for the n samples
- $P(M_n>x) \approx e^{-nI(x)}$

$$I(x) = -\lim_{n\to\infty}\frac{1}{n}\log(P(M_n > x))$$

OCBA-1

$$\max_{i \neq b} P(J_i \leq J_b) \leq PFS \leq \sum_{i \neq b} P(J_i < J_b)$$

- When look at the rates, the lower and upper bound becomes

$$\min_{i \neq b} I_i(x)$$

$$I_i(x) = \frac{\left(J_i - J_b\right)^2}{2(\sigma_i^2 / \alpha_i + \sigma_b^2 / \alpha_b)}$$

Asymptotic Rule Derived under Large Deviation Theory

$$\max z$$

$$I_i(x) \geq z \qquad \text{for } i \neq b$$

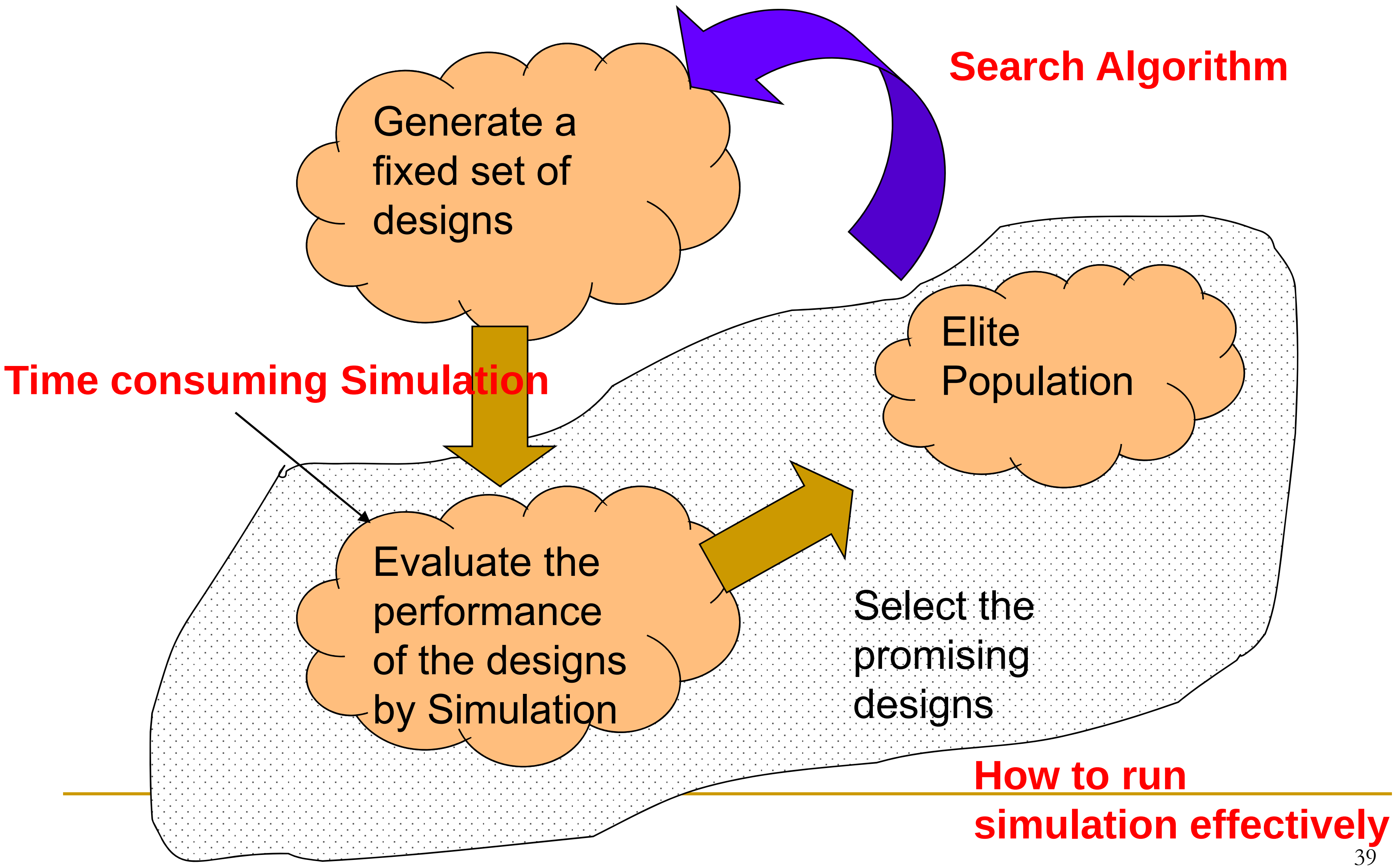
$$I_i(x) = \frac{\left(J_i - J_b\right)^2}{2(\sigma_i^2 / \alpha_i + \sigma_b^2 / \alpha_b)}$$

After some approximation,
same rule with OCBA-1 will be obtained.

Intuition provided by Large Deviation Rule

- OCBA tends to maximize the slowest comparison rate
  - Designs that are closer to the "promising" designs tend to have slowest rate
  - Designs which have high noise

OCBA+ Searching Algorithm



OCBA+Search

- OCBA1 + NP, Shi and Chen (2000)
- OCBA-m + PBIL, Chen et al. (2008)
- OCBA-m + Random Search, Chen et al. (2008)
- OCBA-m + CE standard, Chen et al. (2008)
- OCBA-CE + CE, He et al. (2009)
- MOCBA + NP, Lee et al. (2009)
- MOCBA + GA, Chew et al. (2009)

Some Other OCBA Researches (1)

Non-normal Distributions

- P. Glynn (Stanford Univ.)
- S. Juneja (Columbia Univ.)

Heavy-tailed Distributions

- M. Broadie, M. Han, and A. Zeevi (Columbia University)

Correlated Sampling

- Michael Fu (U. of Maryland at College Park)
- J.Q. Hu (Boston Univ.)

Minimizing Variance Instead of Minimizing Mean

- Lucy Pao & Lidija Trailovic (U. of Colorado, ECE Dept.)

Some Other OCBA Researches (2)

Expected opportunity cost instead of the probability of correct selection

- S. E. Chick (INSEAD, France)
- D. He (CSSI, Inc.)

Industrial Strength Algorithm and Software for Optimization via Simulation

- B. Nelson (Northwest University)
- L. J. Hong (Hong Kong University of Science & Technology)

Empirical Modeling and Information Matrix

- Argon Chen (National Taiwan University)

Selection of One of the Good

- N. Hall (NOBLIS, Inc.)

Consideration of Indifference-zone Selection

- E. Jack Chen (BASF Co.)
- David Kelton (U. of Cincinnati)

Some Other OCBA Researches (3)

Integration with DOE for Semiconductor Scheduling

- S.C. Chang and B.W. Hsieh (National Taiwan University)

Integrating OCBA with Nested Partition for large-scale problems

- L. Shi (U of Wisconsin at Madison)

Integration with Genetic Algorithm

- Z. Li (Xi'an Jiaotong University, China)

Continuous-Variable Probabilistic Optimization

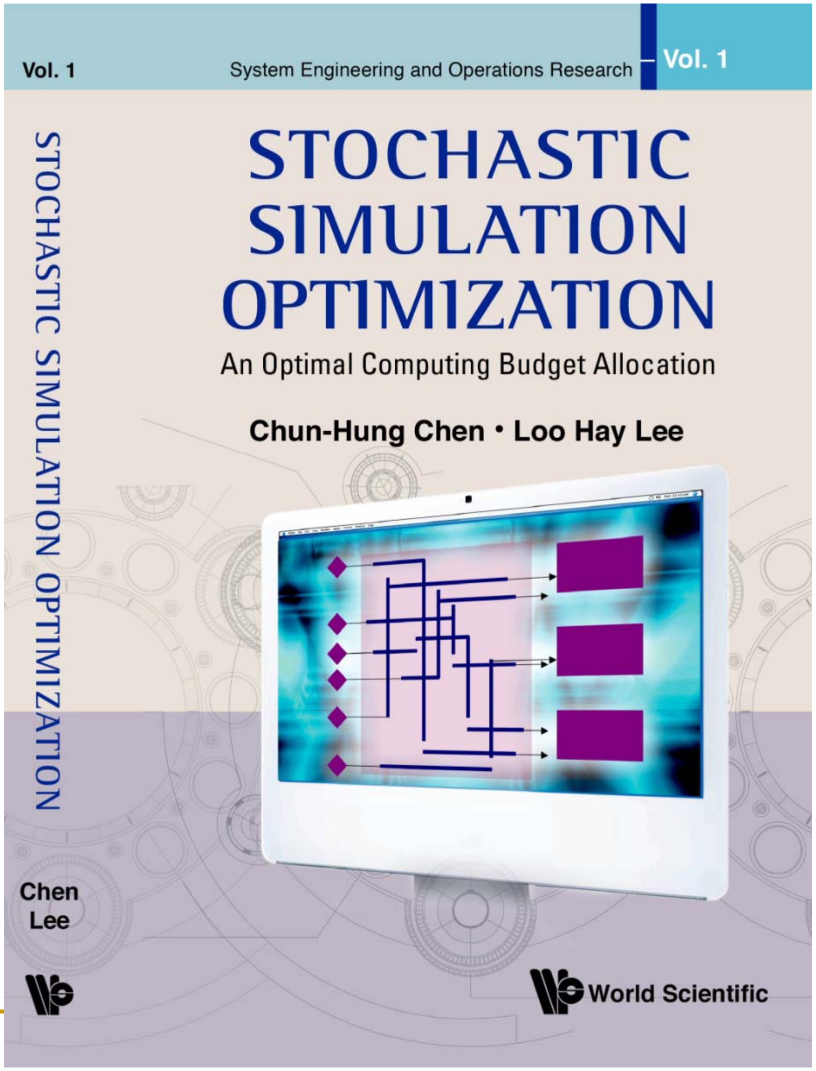
- V. J. Romero (Sandia National Laboratory)

Linear Transient Mean

- D. J. Morrice (University of Texas at Austin)

A book on Optimal Computing Budget Allocation

- Co-author with CH Chen
- Expected to publish in August 2010.
- Can be found at amazon.com



Some future work

- Large Deviation Technique to revisit
  - OCBA-m
  - MOCBA
  - OCBA-CO
  - Correlation issue
- Quantile problem
- Finite Time Performance vs Asymptotic Performance
- Large Scale Problem

Thank You