Price Matching Negotiation in Competitive Channels

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Abstract

In price matching negotiation (PM), a channel matches its price with the resulting wholesale price bargained earlier by the other channel. We investigate this negotiation mechanism and compare it with two benchmarks, simultaneous negotiation (SN) and sequential negotiation (SQ). Through a common-seller two-buyer Bertrand competition model, we find that in PM the seller prefers to negotiate with the less powerful buyer, whereas in SQ the seller prefers to negotiate with the more powerful buyer first. Firms have different preferences for PM and the benchmarks, and their discrepancy is irreconcilable. With side payment or profit sharing coordination, however, PM can emerge as a mutually beneficial choice for all firms as compared to SN and SQ. We also study seller collusion in a bilateral channel model and find that PM incurs fewer collusion incentives than SN and SQ. When the buyers have asymmetric market sizes, *ceteris paribus*, the seller prefers to negotiate with the bigger buyer in PM. We finally demonstrate that our main qualitative results are robust in Cournot competition.

Key words: price matching negotiation; simultaneous negotiation; sequential negotiation; Nash bargaining; channel competition and coordination

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1 Introduction

Price matching (PM) negotiation, a bargaining mechanism where a seller implements the same selling price for all its buyers, has been widely utilized in the industry. For instance, in the US auto industry, United Auto Workers (UAW) utilized it to pick which of the big three to negotiate with first and the resulting wage rate will be accepted by the other two (or UAW will strike against the disagreeable company or companies).[1] In the world iron ore industry, the "benchmark" – price matching – system had been consistently practiced for more than 40 years (Kohler, 2006). Similar pricing mechanisms have also been adopted in a variety of industries, such as the pharmaceutical industry (Marioso et al., 2011) and agriculture (Xia and Sexton, 2004).

In terms of the final sale price, the PM mechanism is similar to the most-favored-customer clause aiming to prevent the seller from offering a lower price to another customer. For instance, as requested by its suppliers, the Canadian International Development Agency imposes a Fair Price Declaration stating that "We certify that the prices charged are not in excess of the lowest price charged to anyone else, including our most favored customer, for like quality and quantity of the products/services." In the ebooks market, according to McCann (2013), "The publishers agreed with Apple that the price of ebooks on Apple platforms would have to be as low as the price of the same ebook on other platforms, principally Amazon."

Bargaining theory was first introduced by Nash (1950) and has since been applied in a wide range of channel structures. Despite its popularity, the extant modeling literature has been surprisingly mute on PM. To explore the PM mechanism, we study a Bertrand competition model with a common seller and two buyers who negotiate on their respective wholesale prices for an identical intermediate good/supply, such as materials, a commodity, or a product component. The buyers then use the intermediate good to manufacture (end) products, either substitutable or complementary. In the game, the firms (i.e., the seller and two buyers) first determine the wholesale price of the intermediate good via negotiation (stage 1), and then the buyers order the intermediate good, manufacture the products, and finally sell the products to end consumers (stage 2). The two buyers are asymmetric in terms of their bargaining powers relative to the seller.

[1]We appreciate the Associate Editor for providing this motivation example.

In PM, the common seller will select a buyer to negotiate on the wholesale price and the resulting wholesale price becomes the industry standard; if the negotiation fails, no trade occurs. We compare PM to two benchmarks: *simultaneous negotiation* (SN) and *sequential negotiation* (SQ). In SN, the seller simultaneously negotiates with two buyers, while in SQ, the seller sequentially negotiates with one buyer at a time. In both SN and SQ, the wholesale prices across channels do not need to be identical, and if a negotiation fails, the corresponding buyer attains zero profit but the common seller can still profit from selling to the other buyer (see Desai and Purohit, 2004; Dukes et al., 2006; Horn and Wolinsky, 1988; O'Brien and Shaffer, 2005).

Our analysis shows that the seller's preference of negotiation sequence is affected by the bargaining power asymmetry. In SQ the seller prefers to negotiate with the more powerful buyer first. This negotiation sequence lowers the wholesale price for the more powerful buyer, which results in more intense horizontal channel competition and subsequently reduces double marginalization in both channels. In PM the same wholesale price will be applied to both buyers, so the seller has incentives to ensure a higher wholesale price by negotiating with the less powerful buyer.

Compared to SN and SQ, PM cannot benefit all firms at the same time. With symmetric bargaining powers, when products are substitutes, the seller prefers PM to SN and SQ, whereas both buyers prefer SN and SQ to PM. This result comes from a trade-off between wholesale prices and demand. When products are substitutes, the wholesale price is higher in PM than in SN and SQ. The seller hence benefits from the higher wholesale price, whereas the buyers suffer from both the higher wholesale price and lower demand caused by worsened double marginalization. As the bargaining power asymmetry grows, the seller's advantage in PM increases for negotiating with the weaker buyer to obtain a higher wholesale price, but at the expense of the buyers. When products are complements, the bargaining power asymmetry result in a lower wholesale price in PM than that of the less powerful buyer in SN/SQ. This price, however, is still higher than that of the more powerful buyer in SN/SQ. Therefore, as long as the bargaining power asymmetry is substantial, the seller can still prefer PM by negotiating with the less powerful buyer.

With side payment coordination, however, PM can emerge as a mutually beneficial mechanism for all firms especially when the bargaining power is symmetric. PM with side payment (PMS) can generate more profits for all firms, as compared to SN and SQ with or without side payment. The side payment can better coordinate both channels by reducing the wholesale price and, thereby, lessening the double marginalization, especially when the products are less substitutable. On the other hand, when products are sufficiently substitutable, the price matching feature in PMS provides an instrument to mitigate intensified horizontal competition caused by side payment. As bargaining powers become substantially asymmetric, the buyer not negotiating with the seller can no longer benefit from PMS, whereas PMS remains the top choice for the seller because of a relatively higher wholesale price in PMS. Our extended discussion demonstrates that these results hold true if we replace the side payment coordination with profit sharing coordination.

We also examine the case where the buyers have different market sizes. With symmetric bargaining power, in PM the seller prefers to negotiate with the bigger buyer to achieve a higher wholesale price throughout the market. In SQ the seller prefers to negotiate with the bigger buyer first, for a higher resulting wholesale price and a bigger market size.

By analyzing a bilateral channel with two sellers, we observe that the two sellers have incentives to collude in SN and SQ, because collusion allows the sellers to command a higher wholesale price. However, the collusion incentives disappear in PM, because the resulting wholesale prices are bound by price matching such that the sellers yield the same profits with and without collusion.

We finally demonstrate that our main qualitative results hold true in Cournot competition, although it is more likely for the seller to prefer PM to SN/SQ in Cournot competition than in Bertrand competition. The likelihood of the seller choosing PM over SN/SQ increases in Cournot competition because the wholesale price in Cournot competition is lower when products are complementary.

This article is closely related to the literature on negotiation in a competitive environment. For example, Zusman and Etgar (1981) applied Nash bargaining theory to analyze a simple three-level channel and examined the interrelations among individual dyadic contracts. O'Brien and Shaffer (1992) studied a model with a seller offering non-linear contracts to buyers. von Ungern-Sternberg (1996) considered a monopoly selling to a number of buyers and studied the impact of reducing the number of buyers on consumer price in both a Cournot model and a perfect competition model. Iyer and Villas-Boas (2003) analyzed how bilateral bargaining affects the degree of channel coordination and overall profit. Inderst and Wey (2003) discussed the merger incentive for a bilaterally oligopolistic case. Desai and Purohit (2004) considered two sellers whose decision is to offer fixed prices or to haggle over prices with customers (i.e., to bargain prices with the customers). In the case of haggling by the seller, a detailed analysis of the disagreement point for customers is given.

In a model with two manufacturers and two multi-product retailers with bilateral channel bargaining, Dukes et al. (2006) showed that the manufacturers can benefit from cost asymmetry between two retailers even though the low cost retailer has a better bargaining position than its rival retailer. Marx and Shaffer (2007) detailed the effect of upfront payments in contracting and pointed out that in any equilibrium the seller trades with one buyer. Chen et al. (2008) developed a simultaneous model of consumer brand choice and negotiated price and showed that their proposed approach fits the data of consumer choice and negotiated price. Feng and Lu (2012) studied a multi-unit bilateral bargaining framework in one-to-one and one-to-two channels and demonstrated that low cost outsourcing may lead to a win-lose outcome such that suppliers gain and manufacturers lose. Cai et al. (2012) provided bargaining solutions for revenue sharing rates in exclusive channels via a model that investigates the firms' channel selection decision among four channel structures. Guo and Iyer (2013) investigated multilateral bargaining in a model with one manufacturer selling through two retailers and compared the impact of negotiation sequence on the firms. The work most related to our study is probably Horn and Wolinsky (1988), who compared simultaneous negotiation and sequential negotiation. They showed that the seller could prefer either mechanism. However, neither Horn and Wolinsky (1988) nor other articles have discussed PM. For more bargaining models, one can refer to Banks et al. (2002), Gurnani and Shi (2006), Iyer and Villas-Boas (2003), Lovejoy (2010), O'Brien and Shaffer (2005), and Wu et al. (2009).

This article is also related to the vast literature on channel competition and coordination. Through a model with two exclusive channels without revenue sharing, McGuire and Staelin (1983) explained why a supplier would want to use an intermediary retailer in a bilateral channel model with one supplier in each channel. In a model with a manufacturer and multiple independent retailers, Ingene and Parry (1995) pointed out that the manufacturer will prefer the second-best two-part tariff to a menu of two-part tariffs maximizing the channel profit. Raju and Zhang (2005) showed that a manufacturer would choose one contract (i.e., quantity discounts or two-part tariffs) over the other in the presence of a dominant retailer. Cui et al. (2008) used a dominant retailer model to demonstrate that trade promotions can benefit manufacturers and the channel owing to the retailers' different inventory-ordering behaviors. Liu and Cui (2010) studied a manufacturer's product line decision when the manufacturer sells through either a centralized channel or a decentralized channel. For more discussion on channel management, one may refer to Coughlan et al. (2006), Desai et al. (2001), Jeuland and Shugan (1983), and the references therein.

Compared to the above literature, our article makes several unique contributions. First, our work is the first attempt to theoretically introduce and analyze PM. The price matching negotiation occurs in a supply market resulting in the same wholesale price for all firms. This feature is different from the price-matching guarantee in consumer markets where manufacturers and retailers, while pricing differently, promise to match the lowest advertising retail price their customers can find (see, e.g., Chen et al., 2001; Coughlan and Shaffer, 2009). Second, we comprehensively compare PM to SN and SQ in terms of firms' performance and demonstrate that PM without coordination is not preferable to all firms at the same time. Third, we point out that, with side payment or profit sharing coordination, all firms can actually benefit from utilizing PM, as compared to SN or SQ. Fourth, our analysis indicates that PM would incur fewer collusion incentives than SQ and SN. We finally demonstrate that our main qualitative results hold true in Cournot competition and when firms have asymmetric market sizes.

2 The Model

We consider a common-seller two-buyer channel model, where the seller sells an identical intermediate good/supply, such as materials, a commodity, or a product component, to the buyers who then manufacture it into end products. For parsimony, we assume manufacturing a unit of the end product requires a unit of the intermediate material. The products can be either substitutable or complementary. This model can be easily adapted to accommodate the scenario where a seller sells an end product through two competing retailers to the market in which only substitutability needs to be considered.

We derive the demand function from the following utility function of a representative consumer, as developed in Shubik and Levitan (1980), Singh and Vives (1984), and Ingene and Parry

(2007).

$$U \equiv \sum_{i=1,2} (ax_i - x_i^2/2) - \gamma x_1 x_2 - \sum_{i=1,2} p_i x_i,$$

where x_i denotes the demand level of the product carried by Buyer i, a denotes the buyers' initial demand base, p_i represents the retail price for Buyer i, and $\gamma \in (-1, 1)$ denotes product substitutability. Maximization of the above utility function in terms of $x_i, i = 1, 2$, yields the following demand function:

$$x_i(p_i, p_j) = \frac{(1-\gamma)a - p_i + \gamma p_j}{1-\gamma^2}, \quad j = 3-i, \; i = 1, 2,$$

The buyers order at the same levels to clear the market. When $\gamma > 0$, the products are substitutes; when $\gamma < 0$, the products are complements. As in Horn and Wolinsky (1988), each seller has sufficient capacity to satisfy the total demand.

The wholesale prices of the intermediate good are negotiated via Nash bargaining. To investigate the impact of firms' bargaining powers, we assume Buyer i's bargaining power relative to the seller is $\theta_i \in (0, 1)$ and the seller's bargaining power relative to Buyer i is $1 - \theta_i$.

In SN, the seller simultaneously negotiates with both buyers on the wholesale price. When a dyad (i.e., the seller and one buyer) negotiates, it is a norm to presume that the other dyad (i.e., the seller and the other buyer) would reach a deal that is in the equilibrium path, although the negotiation results are not revealed until both negotiations end (see Desai and Purohit, 2004; Dukes et al., 2006; Horn and Wolinsky, 1988; O'Brien and Shaffer, 2005). In SQ, the seller-buyer dyads sequentially negotiate on their respective wholesale prices. The first dyad's negotiation result is known to the second dyad, a common assumption in the literature (see, e.g., Guo and Iyer, 2013; Horn and Wolinsky, 1988). In both SN and SQ, if negotiation in Dyad i (i.e., the seller and Buyer i) fails, Buyer i has no gain but the common seller can profit from another negotiation with Buyer j, where $j = 3 - i$.

What sets PM apart from SN and SQ is that the seller will negotiate with only one buyer; once the negotiation succeeds, the seller will announce the resulting wholesale price and then use it for all buyers. If the negotiation fails, no trade occurs. Due to the bargaining power asymmetry, the seller needs to strategically decide which buyer to negotiate with in PM and which buyer to negotiate with first in SQ. In these negotiation schemes, no renegotiation occurs if a negotiation fails, which is in line with Desai and Purohit (2004), Dukes et al. (2006), Horn and Wolinsky (1988), O'Brien and Shaffer (2005), and the references therein.

The timeline of the game is as follows. In the first stage, the seller negotiates with the selected buyer(s) on the wholesale price(s). Naturally, the resulting wholesale price depends on the negotiation scheme, PM, SN, or SQ. In the second stage, the buyers order the intermediate good and use it to manufacture their respective products. Finally, the demand is realized in Bertrand competition between two buyers. The subgame perfect equilibrium is solved using backward induction.

3 Analysis of Bertrand Competition

3.1 Equilibrium Analysis

Since the game is solved backward, we first provide the second-stage results of Bertrand competition, and then analyze first-stage subgame of different negotiation schemes: SN, SQ, and PM.

3.1.1 Second-Stage Results

For any given wholesale prices (w_i, w_j), the outcome of second-stage game is independent of whether (w_i, w_j) is negotiated via PM, SN, or SQ. The profits for the buyers and the seller are

$$\begin{aligned}\pi_{bi} &= (p_i - w_i)\, x_i\,(p_i, p_j)\,,\\ \pi_s &= w_i x_i\,(p_i, p_j) + w_j x_j\,(p_i, p_j)\,.\end{aligned}$$

The subscripts bi and s represent Buyer i and the seller, respectively. The buyers seek to maximize their own profits by choosing respective optimal retail prices p_i in a Bertrand (price) competition. Solving the first-order conditions (FOCs) gives us the equilibrium retail prices as

$$\widehat{p}_i\,(w_i, w_j) = \frac{(2-\gamma-\gamma^2)\,a + 2w_i + \gamma w_j}{(4-\gamma^2)}.$$

The resulting sales quantities are

$$\widehat{x}_i\,(w_i, w_j) = \frac{(2-\gamma-\gamma^2)\,a - (2-\gamma^2)\,w_i + \gamma w_j}{(1-\gamma^2)\,(4-\gamma^2)}.$$

The equilibrium profits are

$$\pi_{bi}(w_i, w_j) = \left(1-\gamma^2\right)\left[\widehat{x}_i(w_i, w_j)\right]^2, \tag{1}$$

$$\pi_s(w_i, w_j) = w_i\widehat{x}_i(w_i, w_j) + w_j\widehat{x}_j(w_j, w_i). \tag{2}$$

Based on the above results, we now proceed to the first stage of the game to compare SN, SQ, and PM.

3.1.2 Simultaneous Negotiation (*SN*)

In SN, the seller negotiates with both buyers simultaneously. The firms' profits are described by (1) and (2). The bargaining solution pair (w_i^{SN}, w_j^{SN}) is an equilibrium, if Buyer i and the seller optimally choose w_i^{SN}, provided that Buyer j and the seller settle on w_j^{SN}. The bargaining solution (w_i^{SN}, w_j^{SN}) satisfies

$$w_i^{SN} = \arg\max_{w_i} \left[\pi_{bi}\left(w_i, w_j^{SN}\right)\right]^{\theta_i}\left[\pi_s\left(w_i, w_j^{SN}\right) - w_j^{SN}x_j\left(w_j^{SN}, w_i^{SN}\right)\right]^{1-\theta_i}.$$

Solving the FOCs yields

$$w_i^{SN} = \frac{(1-\theta_i)(1-\gamma)(2+\gamma)\left(2+\gamma-\gamma^2-\gamma\theta_j\right)a}{2\left(4-5\gamma^2+\gamma^4+\gamma^2\left(\theta_i+\theta_j-\theta_i\theta_j\right)\right)}.$$

The outcomes are affected by product substitutability γ and the firms' bargaining powers θ_i and θ_j. We can obtain that $\frac{\partial w_i^{SN}}{\partial\gamma} < 0$, which suggests that higher product substitutability reduces the wholesale prices. Consequently, retail prices decline, which leads to more intense horizontal competition. Meanwhile, the order sizes also decline, because higher product substitutability forces buyers to order less to avoid overly intense horizontal competition. Accordingly, each firm's profit decreases as product substitutability grows.

We can also obtain $\frac{\partial w_i^{SN}}{\partial\theta_i} < \frac{\partial w_i^{SN}}{\partial\theta_j} < 0$, which indicates that the wholesale prices decrease as the buyers' bargaining powers increase. It is natural that the seller gives up more profit margins to the more powerful buyers.

3.1.3 Sequential Negotiation (SQ)

In SQ, different from SN, the second buyer can observe the first buyer's bargaining outcome (Guo and Iyer, 2013; Horn and Wolinsky, 1988). The firms' profit functions continue to be described by (1) and (2). In the symmetric case where $\theta_i = \theta_j$, the seller is indifferent about which buyer to negotiate with first. Hence, if $\theta_i = \theta_j$, we assume a random tie-breaking rule such that either buyer has the same chance to negotiate first with the seller. If the buyers' bargaining powers are different, however, the seller's choice of negotiation sequence will affect the bargaining outcome.

To compare the two negotiation sequences, we denote Buyer f as the first buyer with whom the seller negotiates, and Buyer s as the second buyer. Thus, the round-2 equilibrium wholesale price is

$$w_s(w_f) = \arg\max_{w_s} \ [\pi_{bs}(w_s, w_f)]^{\theta_i} [\pi_s(w_s, w_f) - w_f x_f(w_f, w_s(w_f))]^{1-\theta_s},$$

which is given by

$$w_s(w_f) = \frac{(1-\theta_s)((2-\gamma-\gamma^2)a + 2\gamma w_f)}{2(2-\gamma^2)}.$$

The equilibrium wholesale price in round 1 is

$$w_f^{SQ} = \arg\max_{w_f} \ [\pi_{bf}(w_f, w_s(w_f))]^{\theta_f} [\pi_s(w_f, w_s(w_f)) - w_s(w_f^{SQ}) x_s(w_s(w_f^{SQ}), w_f^{SQ})]^{1-\theta_f}.$$

Solving the FOC leads to

$$w_f^{SQ} = \frac{(1-\theta_f)(1-\gamma)(2+\gamma)(2+\gamma-\gamma^2-\gamma\theta_s^2)a}{2(4-5\gamma^2+\gamma^4+\gamma^2(\theta_f+\theta_s-\theta_f\theta_s)\theta_s)}.$$

Similar to that in SN, higher product substitutability lowers wholesale prices, retail prices, order sizes, and hence firm profits. The seller will also reduce the wholesale price for the more powerful buyer. Comparing the firms' profits in the two negotiation sequences, $(f, s) = (i, j)$ and $(f, s) = (j, i)$, we observe the following.

Lemma 1 *Suppose $\theta_i \leq \theta_j$.*

1. *The seller always prefers to negotiate with the more powerful buyer (i.e., Buyer j) first.*

2. *Buyer i prefers $(f, s) = (j, i)$ if and only if $\gamma \geq 0$.*

3. *Buyer j prefers $(f, s) = (i, j)$ if $\gamma \geq 0$; otherwise (i.e., $\gamma < 0$),*

- *if* $\theta_i \geq \frac{-2\gamma}{2-\gamma-\gamma^2}$, *she prefers* $(f, s) = (j, i)$*;*
- *if* $\theta_i < \frac{-2\gamma}{2-\gamma-\gamma^2}$, *she prefers* $(f, s) = (j, i)$ *when* $\theta_j \in (\theta_i, \overline{\theta}_j(\theta_i, \gamma)]$ *and* $(f, s) = (i, j)$ *when* $\theta_j \in [\overline{\theta}_j(\theta_i, \gamma), 1)$.

All proofs and threshold values' expressions can be found in the Appendix (Online Supplements).

To better understand Lemma 1, we first explain the symmetric case where $\theta_i = \theta_j$. As we know, in SQ, the sequential negotiation process leads to different bargaining externalities for the firms. This gives the seller an advantage because it takes the second negotiation as the disagreement point while the first buyer gains nothing if the first negotiation fails. Provided $\theta_i = \theta_j$, the seller always commands a higher wholesale price in the first negotiation, although it is indifferent about which buyer to negotiate with first. The asymmetric wholesale prices shift the competition edge from the first buyer to the second one.

When $\theta_i < \theta_j$, negotiating with the more powerful buyer (i.e., Buyer j) as opposed to the less powerful buyer first forces the seller to lower the first wholesale price. This result leads to more intense horizontal channel competition, which subsequently reduces double marginalization in both channels. For the seller, the gain in higher demand and a relatively higher second wholesale price due to negotiating with the less powerful buyer in the second round outweighs the loss in a relatively lower first wholesale price. Therefore, the seller always chooses to negotiate with the more powerful buyer first.

For the less powerful buyer, if the products are substitutable, it prefers to negotiate second, which results in a relatively lower wholesale price caused by the negotiation sequence; however, if the products are complementary, this buyer can benefit from negotiating first for a higher demand complementary effect, because the more powerful buyer gains a higher demand resulting from a lower wholesale price when negotiating second.

For the more powerful buyer, if the products are substitutable, it has no incentive to negotiate first because of the significantly negative impact of a higher wholesale price. Nevertheless, if the products are complementary, as long as the two firms are not substantially different in terms of bargaining powers, the more powerful buyer can benefit from negotiating first because the demand complementary effect compensates for the relatively higher wholesale price.

3.1.4 *Price Matching Negotiation (PM)*

In PM, we first analyze the scenario where the seller chooses to negotiate with Buyer i. The analysis for negotiation with Buyer j is similar. The firms' profit functions continue to be described by (1) and (2). The equilibrium wholesale price w_i^{PM} satisfies

$$w_i^{PM} = \arg\max_{w_i} \ [\pi_{bi}(w_i, w_i)]^{\theta_i} [\pi_s(w_i, w_i)]^{1-\theta_i}.$$

Solving the FOC gives us:

$$w_i^{PM} = \frac{(1-\theta_i)\, a}{2}.$$

It is somewhat surprising that the resulting wholesale price is independent of product substitutability, because both buyers are bound by price matching. PM subdues the channel competition when products are substitutes. At the same time, it blunts the demand complementary effect when products are complements.

Nevertheless, the wholesale price depends on the chosen buyer's negotiation power. The higher the buyer's bargaining power, the lower the wholesale price. Comparing the firms' profits in situations where the seller negotiates with either buyer leads to the following lemma.

Lemma 2 1. *The seller always prefers to negotiate with the less powerful buyer.*

2. *Both buyers always prefer that the seller negotiates with the more powerful buyer.*

A higher wholesale price pushes up retail price and hence brings down demand. The seller will be directly affected by the wholesale prices and the demand levels, while the buyers are also affected by the retail prices. Given that changes in the wholesale price will not be entirely passed onto the retail prices because of the existence of intermediary buyers, the marginal change of demand is in a lower magnitude compared to the marginal change of wholesale prices. Therefore, the firms' preference of which buyer to negotiate with the seller is largely affected by the corresponding wholesale price.

Given that the same wholesale price will be applied to both buyers in PM, the seller has incentives to ensure a higher wholesale price in the negotiation. Naturally, the seller will choose to negotiate with the less powerful buyer and then implement the higher wholesale price in the whole market, at the expense of both buyers.

Lemma 2 demonstrates that the seller's preference to negotiate with the less powerful buyer in PM significantly deviates from that in SQ, as indicated in Lemma 1, where the seller will negotiate first with the more powerful buyer. This is caused by the seller's different incentives in these two negotiation mechanisms. In SQ, due to the asymmetric wholesale prices caused by the negotiation sequence, the seller has incentives to subdue the first buyer's wholesale price to intensify the horizontal channel competition for double marginalization reduction. In contrast, in PM, the seller always implements the same wholesale price. The seller's marginal benefit of increasing the wholesale prices for the whole market surpasses that of double marginalization reduction.

3.2 Comparative Analysis between Negotiation Mechanisms

Without loss of generality, we assume $\theta_i \leq \theta_j$. Following Lemmas 1 and 2, the seller will choose to negotiate with Buyer i in PM and will choose to negotiate with Buyer j first in SQ.

3.2.1 PM versus SN

PM and SN share some similarity in that both buyers obtain the wholesale prices at the same time. However, their best-response functions differ because of different disagreement points. In SN, the wholesale prices decrease as product substitutability grows. By contrast, in PM, the wholesale price is insensitive to product substitutability given that the two buyers are bound by price matching. When products are independent, the wholesale price disparity disappears as the downstream competition vanishes. Comparing firms' profits in PM and SN, we obtain the following proposition.

Proposition 1 *Suppose $\theta_i \leq \theta_j$. There exist two threshold values, $\widehat{\theta}_j^{SN}(\theta_i, \gamma)$ and $\widetilde{\theta}_j^{SN}(\theta_i, \gamma)$, such that*

1. *When products are substitutable, the seller always prefers PM to SN. When products are complementary, she prefers PM to SN iff $\theta_j > \widehat{\theta}_j^{SN}(\theta_i, \gamma)$;*

2. *The less powerful buyer (i.e., Buyer i) prefers SN to PM iff $\gamma \geq 0$;*

3. *When products are substitutable, the more powerful buyer (i.e., Buyer j) always prefers SN to PM. When products are complimentary, Buyer j prefers SN to PM iff $\theta_j > \widetilde{\theta}_j^{SN}(\theta_i, \gamma)$.*

To interpret Proposition 1, we first examine the symmetric case where $\theta_i = \theta_j$. If products are substitutes (i.e., $\gamma > 0$), as shown in the proof of Proposition 1, PM has a higher wholesale price than SN, whereas if products are complements (i.e., $\gamma < 0$), PM results in a lower wholesale price. Consequently, the wholesale price disparity in PM and SN affects firms' preferences for PM or SN. The seller prefers PM to SN whereas buyers prefer SN to PM if $\gamma > 0$ and vice versa otherwise.

When the buyers' bargaining powers differ (i.e., $\theta_i \leq \theta_j$), the seller in PM has an advantage in negotiating with the weaker buyer to obtain a higher wholesale price. This benefit amplifies the seller's advantage in PM as the bargaining power asymmetry grows. When products are substitutable, the seller always prefers PM to SN, which is the same as in the symmetric case. When products are complementary, as long as the bargaining power asymmetry is substantial, the seller can still prefer PM, owing to the higher wholesale price from negotiating with the less powerful buyer.

The seller's gain is the buyers' losses. When products are substitutes, a higher wholesale price in PM makes both buyers worse off. Both buyers become more likely to prefer SN to PM as the bargaining power asymmetry grows. When products are complements, the asymmetric bargaining powers result in a lower wholesale price in PM than that of the less powerful buyer in SN, but higher than that of the more powerful buyer in SN. In general, the lower wholesale price leads to a lower retail price, which could benefit the other buyer by amplifying the demand complementary effect, because higher demand in one channel stimulates more demand for the other, given that $\gamma < 0$. Therefore, there is a trade-off between a lower wholesale price and a higher demand complementary effect. For the less powerful buyer (i.e., Buyer i), it prefers PM to SN when products are complements, owing to the higher demand complementary effect. For the more powerful buyer (i.e., Buyer j), PM becomes less attractive if its bargaining power enables it to command a much lower wholesale price.

To examine whether PM, compared to SN, could result in a higher channel efficiency (i.e., total profit for all firms), we consider the special case where $\theta_i = \theta_j = \theta$. We find that, when $-1 < \gamma < 0$, the impact of higher demand is more significant than the impact of lower retail prices, thanks to the demand complementary effect. Given that PM has a lower wholesale price when $-1 < \gamma < 0$, its channel efficiency is higher. The impact of higher demand continues to be more significant than the impact of lower retail prices as γ becomes positive as long as θ is sufficiently small (i.e., $\theta < \frac{-4+3\gamma+\sqrt{16-8\gamma-15\gamma^2+8\gamma^4}}{2\gamma}$). Due to its lower wholesale prices, SN has higher channel efficiency than PM. As γ becomes larger (i.e., $\theta \geq \frac{-4+3\gamma+\sqrt{16-8\gamma-15\gamma^2+8\gamma^4}}{2\gamma}$ given $\gamma > 0$), however, the impact of higher demand caused by lower wholesale prices slows down because of higher product substitutability. Consequently, for firms in SN, the additional demand cannot compensate for the loss of marginal profits associated with lower retail prices. By contrast, in PM, as product substitutability becomes substantially high, the relatively higher wholesale price softens horizontal channel competition, which prompts PM to generate higher channel efficiency as compared to SN. This result shows that, even though PM cannot benefit all firms at the same time, it can generate higher channel efficiency, which prompts us to explore whether channel coordination can make PM beneficial for all firms in Section 3.3.

3.2.2 PM versus SQ

To compare PM to SQ, we first consider the symmetric case where $\theta_i = \theta_j$. Based on the random tie-breaking rule, the seller and both buyers are indifferent about which buyer negotiates with the seller first. Regardless of the negotiation sequence in SQ, if $\gamma > 0$, $w_j^{PM} > w_j^{SQ}$ and $w_i^{PM} > w_i^{SQ}$; otherwise, $w_j^{PM} \leq w_j^{SQ}$ and $w_i^{PM} \leq w_i^{SQ}$. Thus, when products are substitutes (i.e., $\gamma > 0$), lower wholesale prices in SQ lead to lower retail prices and higher total demand and, therefore, benefit buyers at the expense of the seller. When products are complements (i.e., $\gamma < 0$), the lower wholesale price in PM makes it more attractive to buyers but less so to the seller.

When the buyers' bargaining powers are asymmetric, comparing firms' profits in PM and SQ leads to the following result.

Proposition 2 *Suppose $\theta_i \leq \theta_j$. There exist three threshold values, $\widehat{\theta}_j^{SQ}(\theta_i, \gamma)$, $\widehat{\theta}_i^{SQ}(\gamma)$, and $\widetilde{\theta}_j^{SQ}(\theta_i, \gamma)$, such that*

1. *When products are substitutable, the seller prefers PM to SQ. When products are complementary, she prefers PM to SQ iff $\theta_j > \widehat{\theta}_j^{SQ}(\theta_i, \gamma)$;*

2. *The less powerful buyer (i.e., Buyer i) prefers SQ to PM iff $\gamma > 0$;*

3. *For the more powerful buyer (i.e., Buyer j),*

   (a) *When products are substitutable, Buyer j prefers SQ to PM, if $\theta_i \leq \widehat{\theta}_i^{SQ}(\gamma)$ and $\theta_j > \widetilde{\theta}_j^{SQ}(\theta_i, \gamma)$. If $\theta_i > \widehat{\theta}_i^{SQ}(\gamma)$, Buyer j prefers SQ to PM;*

   (b) *When products are complementary, Buyer j prefers SQ to PM, iff $\theta_j > \widetilde{\theta}_j^{SQ}(\theta_i, \gamma)$.*

The rationale behind Proposition 2 is similar to that for Proposition 1. For the seller, the advantage in PM to negotiate with the weaker buyer results in a higher wholesale price. The advantage expands as the bargaining power asymmetry grows, such that the seller still prefers PM to SQ even if products are complementary as long as the bargaining power asymmetry is substantially large.

Again, the seller's gain is the buyers' losses. Different from that in SN, however, the seller's preferred negotiation sequence in SQ gives an advantage to the less powerful buyer (i.e., Buyer i) who would negotiate second and enjoy a lower wholesale price when products are substitutes (i.e., $\gamma > 0$). Thus, the less powerful buyer (i.e., Buyer i) prefers SQ to PM when products are substitutes (i.e., $\gamma > 0$). When products are complementary, the demand complementary effect in SQ significantly subdues because of the substantially higher wholesale price charged to the more powerful buyer, making PM more preferable to Buyer i.

For the more powerful buyer (i.e., Buyer j), the situation is more complex. As Lemma 1 indicates, in SQ, the more powerful buyer prefers to negotiate second when products are substitutes, which is against the seller's preference. Provided that the seller negotiates with Buyer j first in SQ, it is somewhat non-intuitive that Buyer j may actually prefer SQ to PM in the majority of the domain when products are substitutes. The underlying reason is that when products are substitutes

Buyer j has to pay a higher wholesale price in PM than in SQ. When Buyer i's bargaining power is sufficiently small but close to Buyer j's bargaining power (i.e., $\theta_i \leq \widehat{\theta}_i^{SQ}(\gamma)$ and $\theta_j < \widetilde{\theta}_j^{SQ}(\theta_i, \gamma)$), however, Buyer j's preference shifts to PM, because the wholesale price in PM is relatively more affordable, whereas the adversity of negotiating first with the seller in SQ becomes more significant.

When products are complements, PM will result in a lower wholesale price than that of the more powerful buyer in SQ if the bargaining power asymmetry is small, but higher if the bargaining power asymmetry is substantial. Therefore, when Buyer j's bargaining power becomes sufficiently larger than that of Buyer i's, Buyer j will prefer SQ to PM; otherwise, PM is more preferable to Buyer j.

In terms of channel efficiency, given $\theta_i = \theta_j = \theta$, we find that PM always outperforms SQ if $\gamma \leq 0$; otherwise ($\gamma > 0$), PM outperforms SN if and only if θ is sufficiently large. This result is similar to that between PM and SN. Unfortunately, the advantage of PM in channel efficiency does not automatically benefit all firms as indicated by Propositions 1 and 2. To show that PM can actually outperform SN and SQ for each individual firm simultaneously, we next apply side payment to facilitate better bargaining solutions.

3.3 The Value of Side Payment

As Sebenius (1992) argued, "cooperation and competition cannot be separated in studying negotiated agreements." Side payment, equivalent to the fixed allowance in a two-part tariff contract, has long been utilized as a cooperation tool to compensate the disadvantaged party in negotiation (Feng and Lu, 2013; Harstad, 2007; Weibust, 2009). Side payment has two effects. First, it splits the pie between negotiating parties – the *splitting-pie effect*. Because we can utilize side payments in PM, SN, and SQ, the splitting–pie effect does not automatically warrant an advantage for PM over SN and SQ. Second, side payment can coordinate the negotiated parties and generate a bigger pie – the *making-pie effect*. According to Draganska et al. (2010), in negotiation "the channel members have realized that they can share a larger pie by improving coordination in the channel."

We hereby detail PM, SN, and SQ with side payment. We denote PM with side payment as

PMS, SN with side payment as SNS, and SQ with side payment as SQS. In PMS the wholesale price and the side payment are assumed to be matched in both channels, which is sufficient to demonstrate the benefit of using side payment in PM. We continue to refer to PM, SN, and SQ as cases without side payment. We define T_i as the side payment transferred from Buyer i to the seller, which can be either positive or negative.

3.3.1 SN with Side Payment (SNS)

In SNS, the equilibrium wholesale price and side payment (w_i^{SNS}, T_i^{SNS}) maximize

$$\max_{w_i,T_i} \left[\pi_{bi}\left(w_i, w_j^{SNS}\right) - T_i\right]^{\theta_i}\left[\pi_s\left(w_i, w_j^{SNS}\right) + T_i - w_j^{SNS}x_j\left(w_j^{SNS}, w_i^{SNS}\right)\right]^{1-\theta_i},$$

where the disagreement point $(d_{bi}, d_s) = (0, w_j^{SNS}x_j\left(w_j^{SNS}, w_i^{SNS}\right))$. The optimal side payment satisfies

$$\theta_i\left[\pi_s\left(w_i, w_j^{SNS}\right) + T_i - w_j^{SNS}x_j\left(w_j^{SNS}, w_i^{SNS}\right)\right] = (1-\theta_i)\left[\pi_{bi}\left(w_i, w_j^{SNS}\right) - T_i\right],$$

that is,

$$T_i = (1-\theta_i)\,\pi_{bi}\left(w_i, w_j^{SNS}\right) - \theta_i\left[\pi_s\left(w_i, w_j^{SNS}\right) - w_j^{SNS}x_j\left(w_j^{SNS}, w_i^{SNS}\right)\right].$$

Dyad-i firms choose w_i^{SNS} to maximize $\pi_{bi}\left(w_i, w_j^{SNS}\right) + \pi_s(w_i, w_j^{SNS})$. Solving the FOCs jointly, we obtain the equilibrium wholesale prices

$$w_i^{SNS} = \frac{\gamma^2 a}{4}.$$

It is interesting that the wholesale prices in SNS are independent of the firms' bargaining powers, because the firms shift the influence of bargaining powers to the side payments. The making-pie and splitting pie effects of SNS can be summarized in the following two properties.

Property 1 *(Making-pie effect) In channel i, the Nash bargaining wholesale price can be obtained from maximizing negotiating firms' joint profit $\pi_{bi} + \pi_s$ with respect to w_i.*

Property 2 *(Splitting-pie effect) In channel i, if the negotiation succeeds, the resulting Nash bargaining solution on wholesale price is independent of the disagreement point (d_{bi}, d_s). The negotiation succeeds iff $\pi_{bi} - T_i \geq d_{bi}$ and $\pi_s + T_i \geq d_s$ simultaneously (equivalent to $\pi_{bi} + \pi_s \geq d_{bi} + d_s$);*

that is, the joint profit of negotiating firms is higher compared to the disagreement point. If so, the extra surplus ($\pi_{bi} + \pi_s - d_{bi} - d_s$) will be allocated to the two firms in proportion to their relative bargaining powers.

Because comparison of firms' profits between SNS and SN in the asymmetric case becomes intractable, we focus on the symmetric case where $\theta_i = \theta_j = \theta$.

Lemma 3 *Suppose $\theta_i = \theta_j = \theta$.*

1. *The seller prefers SNS to SN iff either $\theta \leq \min\left(\frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}, \frac{(1-\gamma)\left(4-4\gamma-2\gamma^2+\gamma^3\right)}{\gamma^2(2-\gamma)}\right)$ or $\theta \geq \max\left(\frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}, \frac{(1-\gamma)\left(4-4\gamma-2\gamma^2+\gamma^3\right)}{\gamma^2(2-\gamma)}\right)$.*

2. *When products are substitutable, buyers prefer SNS iff $\theta \geq \frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}$; when products are complementary, the buyers prefer SNS iff $\frac{(2+\gamma)(1-\gamma)}{(2-\gamma)} \leq \theta \leq \overline{\theta}_{SNS-SN}$, where $\overline{\theta}_{SNS-SN} = \frac{(1-\gamma)(4-8\gamma-8\gamma^2+\gamma^4)-\left(2-\gamma^2\right)\sqrt{4-16\gamma-12\gamma^2+\gamma^4}}{2\gamma^2(2+\gamma)(2-\gamma)}$.*

Lemma 3 demonstrates that coordination in competitive channels does not always result in more profits for all firms. Given that the firms' preferences rely on only two parameters (i.e., θ and γ), we can obtain a unique graph showing that in most of the feasible domain (i.e., $\theta \in (0, 1)$ and $\gamma \in (-1, 1)$), the seller prefers SNS to SN, see Figure 1. In contrast, the buyers' preference area of SNS is smaller, which largely overlaps with that of the seller's, as illustrated in Figure 2. In combination, all firms prefer SNS to SN in two regions: (1) $0.58579 < \gamma < 1$ and $\theta \geq \max\left(\frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}, \frac{(1-\gamma)\left(4-4\gamma-2\gamma^2+\gamma^3\right)}{\gamma^2(2-\gamma)}\right)$ when products are substitutable; or (2) $\frac{(2+\gamma)(1-\gamma)}{(2-\gamma)} \leq \theta \leq \overline{\theta}_{SNS-SN}$ when products are complementary.

The above result occurs because, besides the wholesale price, SNS gives the seller one more decision dimension, side payment, to better influence the market, while each buyer has to compete with the other buyer simultaneously. If the seller is less powerful in bargaining (i.e., $\theta \geq \frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}$), the seller can command higher wholesale prices in SNS than in SN to capture a higher profit margin while paying a side payment to the buyers. Otherwise, the wholesale prices would be lower in SNS, so the seller can benefit from double marginalization reduction resulting from more intense horizontal channel competition. When products are sufficiently substitutable (i.e.,

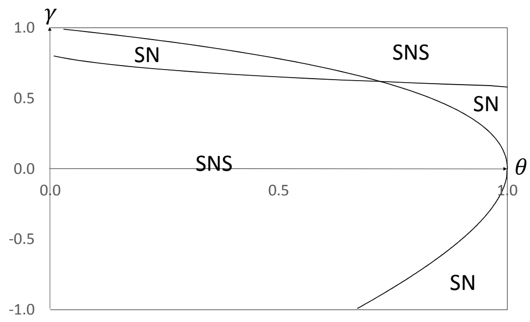

Figure 1: The seller's preference between SNS and SN where $\theta_i = \theta_j = \theta$.

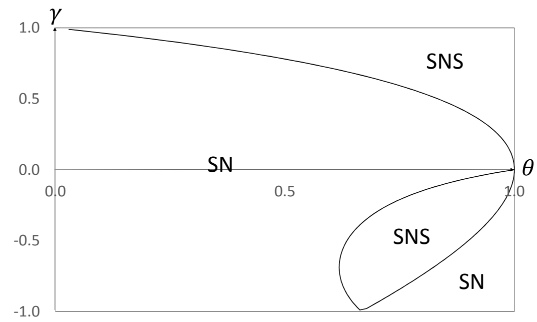

Figure 2: The buyers' preference between SNS and SN where $\theta_i = \theta_j = \theta$.

$0.58579 < \gamma < 1$), the buyers can more significantly benefit from double marginalization reduction. When products are complementary, however, buyers prefer lower wholesale prices to benefit from the demand complementary effect, which occurs when their bargaining powers are at the medium-high level (i.e., $\frac{(2+\gamma)(1-\gamma)}{(2-\gamma)} \leq \theta \leq \overline{\theta}_{SNS-SN}$).

When bargaining powers are asymmetric (i.e., $\theta_i \neq \theta_j$), it becomes more difficult for firms to coordinate the whole system. Because the asymmetric bargaining powers unevenly shift profits from one firm to the other, the common area that all firms prefer SNS to SN shrinks as firms' bargaining powers become more asymmetric. We use Figure 3 to illustrate the firms' preference shift. Each parenthetical notation in Figure 3 indicates the preference of the seller, Buyer i, and Buyer j, respectively. Thus, for example, (SNS, SN, SNS) means that the seller prefers SNS, Buyer i prefers SN, and Buyer j prefers SNS. As Figure 3 indicates, if products are substantially substitutable (i.e., in the (SNS, SNS, SNS) area), all firms prefer SNS to SN, which is consistent with Lemma 3. When products are slightly less substitutable and firms' bargaining powers are close to be symmetric in the area of (SN, SN, SN), all firms suffer from better channel coordination, a phenomenon similar to the prisoner's dilemma. Overall, in line with the symmetric case, the seller is more likely to prefer SNS while buyers are more likely to prefer SN, because the seller is in a better position to command more profits from the coordination. Intuitively, as Figure 3 depicts, the more powerful buyer (i.e., Buyer j) will gain more profits from channel coordination as θ_j grows.

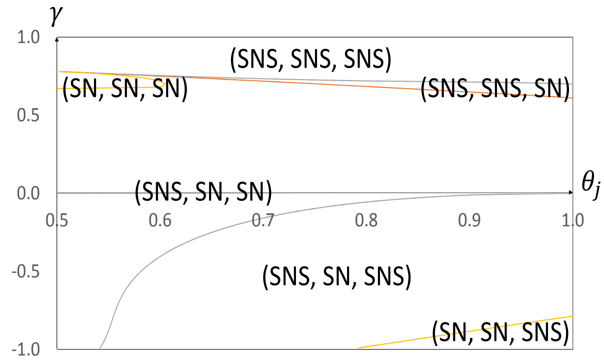

Figure 3: The preference between SNS and SN of (the seller, Buyer i, Buyer j) where $\theta_i = 0.5$.

3.3.2 SQ with Side Payment (SQS)

In SQS, similar to that in SQ, we assume in round 1 the seller negotiates with Buyer f and round 2 with Buyer s. The round-2 equilibrium solution satisfies

$$\max_{w_s, T_s} \ \left[\pi_{bs}\left(w_s, w_f\right) - T_s\right]^{\theta_s} \left[\pi_s(w_s, w_f) + T_s - w_f x_f(w_f, w_s(w_f))\right]^{1-\theta_s}.$$

The round-2 optimal side payment is

$$T_s = \left(1 - \theta_s\right) \pi_{bs}\left(w_s, w_f\right) - \theta_s \left[\pi_s(w_s, w_f) - w_f x_f(w_f, w_s)\right].$$

Then the equilibrium wholesale price that maximizes the joint profit of the two firms is

$$w_s(w_f) = \frac{\gamma\left(\gamma\left(2 - \gamma - \gamma^2\right) a + 4 w_f\right)}{4\left(2 - \gamma^2\right)}.$$

The round-1 side payment is

$$T_f = \left(1 - \theta_f\right) \pi_{bf}\left(w_f, w_s(w_f)\right) - \theta_f \left[\pi_s\left(w_f, w_s(w_f)\right) - w_s(w_f) x_s\left(w_s(w_f), w_f\right)\right].$$

Then the wholesale price w_f^{SQS} maximizing $\pi_{bf}\left(w_f, w_s(w_f)\right) + \pi_s(w_f, w_s(w_f)))$ is given by

$$w_f^{SQS} = \frac{\gamma\left(1 + \gamma\right)\left(2 - \gamma\right) a}{4}.$$

Accordingly, we have

$$w_s^{SQS} = w_s(w_f^{SQS}) = \frac{\gamma^2 a}{2}.$$

Properties 1 and 2 for SNS continue to hold true for SQS. But, it turns out that comparison between SQS and SQ becomes intractable even for the symmetric case where $\theta_i = \theta_j = \theta$. Because

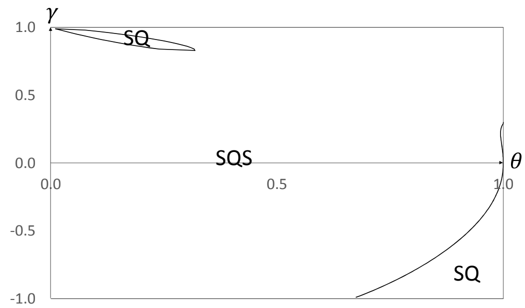

Figure 4: The seller's preference between SQS and SQ where $\theta_i = \theta_j = \theta$.

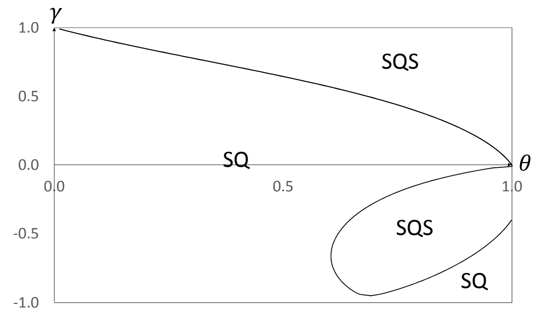

Figure 5: The buyers' preference between SQS and SQ where $\theta_i = \theta_j = \theta$.

there are only two parameters (i.e., θ and γ) in the symmetric case, however, we can obtain unique graphs to depict the firms' preferences. As Figures 4 and 5 illustrate, the seller prefers SQS to SQ in most of the feasible area, while the buyers' preference area of SQS is smaller, which is very similar to what Lemma 3 describes in comparison of SNS and SN. Indeed, the underlying reasons for the firms' preferences are also very similar to that of SNS and SN. Overall, the seller can largely benefit from the side payments, whereas the buyers prefer SQS if and only if their bargaining powers are sufficiently large.

When bargaining powers are asymmetric, we observe very similar results as in the comparison of SNS to SN and thus skip the details here for parsimony. In general, the bargaining power asymmetry reduces the overlapping area where all firms prefer SQS to SQ.

3.3.3 PM with Side Payment (PMS)

In PMS, if the seller chooses to negotiate with Buyer i, due to the price-matching clause, both buyers pay the same (w_i, T_i) to the seller. Therefore, the seller and Buyer i's bargaining solution is to maximize the following equation:

$$\max_{w_i, T_i} \; \left[\pi_{bi}\left(w_i, w_i\right) - T_i - d_{bi}\right]^{\theta_i} \left[\pi_s\left(w_i, w_i\right) + 2T_i - d_s\right]^{1-\theta_i},$$

where (d_{bi}, d_s) is the disagreement point. The FOC with respect to T_i leads to

$$T_i = (1-\theta_i)\left[\pi_{bi}\left(w_i, w_i\right) - d_{bi}\right] - \frac{\theta_i}{2}\left[\pi_s\left(w_i, w_i\right) - d_s\right].$$

Then the wholesale price maximizes $\pi_{bi}\left(w_i, w_i\right) + \frac{1}{2}\pi_s\left(w_i, w_i\right)$, which gives us

$$w_i^{PMS} = \frac{\gamma a}{2}.$$

To demonstrate that PMS can outperform PM, SN, SNS, SQ, and SQS for all firms, we allow (d_{bi}, d_s) of PMS to be the resulting firm profits of PM, SN, SNS, SQ, and SQS. That is, given the current negotiation scheme of PM, SN, SNS, SQ, or SQS, we discuss whether firms have an incentive to move to PMS. The making-pie and splitting pie effects of PMS can be summarized in the following two properties.

Property 3 *(Making-pie effect) In channel i, the Nash bargaining wholesale price can be obtained from maximizing $\pi_{bi} + \frac{1}{2}\pi_s$ with respect to w_i.*

Property 4 *(Splitting-pie effect) In channel i, if the negotiation succeeds, the resulting Nash bargaining solution on wholesale price is independent of the disagreement point (d_{bi}, d_s). The negotiation succeeds iff $\pi_{bi} - T_i \geq d_{bi}$ and $\pi_s + 2T_i \geq d_s$ simultaneously (equivalent to $\pi_{bi} + \frac{1}{2}\pi_s \geq d_{bi} + \frac{1}{2}d_s$). If so, the extra surplus ($\pi_{bi} + \frac{1}{2}\pi_s - d_{bi} - \frac{1}{2}d_s$) will be allocated to the two firms in proportion to their relative bargaining powers.*

As shown in Properties 3 and 4, for PMS the Nash bargaining solution on wholesale price is independent of the disagreement point. Furthermore, the joint profits of all firms in PMS (i.e., $\frac{1}{4(1+\gamma)}$) are independent of the firms' bargaining powers. This might suggest that price matching enables side payment to better coordinate the firms. Indeed, it is the making-pie effect that makes PMS attractive to all firms compared to PM, SN, SNS, SQ, and SQS, which can be supported by the following proposition.

Proposition 3

1. *For all firms, for any (θ_i,θ_j), PMS outperforms PM.*
2. *For the seller, for any (θ_i,θ_j), PMS outperforms SN, SNS, SQ, and SQS. For the buyers, provided $\theta_i = \theta_j = \theta$, PMS outperforms SN, SNS, SQ, and SQS.*

Side payment has two effects: It coordinates each channel (coordination effect), but it intensifies horizontal channel competition by eliminating the intermediary cushion (competition effect). Consider the symmetric case where $\theta_i = \theta_j = \theta$. When products are substitutable, PMS commands a higher average wholesale price than SNS and SQS. Consequently, the average retail price is higher in PMS, which prevents firms from engaging in overly intense horizontal channel competition and, therefore, softens the intensified competition effect caused by side payment. When products are complementary, the wholesale price in PMS is lower than in other negotiation schemes, and accordingly the retail prices are lower, which consequently reduces the double marginalization in both channels. This boosts the competition effect caused by side payment, and enhances the demand complementary effect enabling firms to capture more demand. As a result, in the entire domain, the making-pie effect generates more extra profits for all firms in PMS than in PM, SN, SNS, SQ, and SQS.

Indeed, the making-pie effect in PMS is so significant that the irreconcilable conflict of firms' preferences in PM, as described in Lemma 2, vanishes for any (θ_i,θ_j). At the same time, the splitting-pie effect distributes the extra profits to each firm in proportion to its relative bargaining power such that all firms prefer PMS to PM.

Proposition 3 also demonstrates the superiority of PMS over SNS and SQS for all firms provided $\theta_i = \theta_j = \theta$. The price matching feature in PMS provides an instrument to mitigate the intensified competition caused by side payment when products are substitutes. This is a stark difference from SNS in Lemma 3, where firms may encounter a prisoner's dilemma caused by coordinating competing channels, a phenomenon similar to the disadvantage of channel centralization demonstrated by McGuire and Staelin (1983). Price matching coordinates both channels more effectively by binding them under the same wholesale price, while the making-pie effect of the side payment attenuates horizontal channel competition more significantly and, hence, generates more profits for all firms.

In the asymmetric case where $\theta_i < \theta_j$, to take a glimpse of how the asymmetric bargaining powers impact all firms' preferences, we graphically demonstrate the firms' preferences between PMS and SNS in Figure 6. The comparison between PMS and SN, SQS, and SQ is similar and is omitted for parsimony. We observe that the seller continues to favor PMS over SNS in the entire feasible domain, because the seller can better influence the final wholesale price by selectively negotiating with its preferred buyer. We find that Buyer i also prefers PMS to SNS, because Buyer i can bargain for a better term in wholesale price and side payment when negotiating with the seller

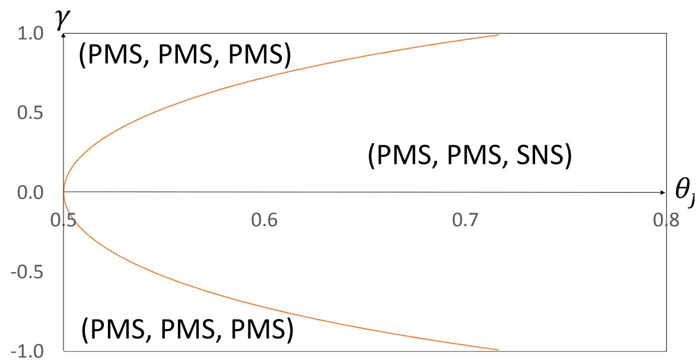

Figure 6: The preference between PMS and SNS of (the seller, Buyer i, Buyer j) where $\theta_i = 0.5$.

in PMS. For Buyer j, PMS is still attractive if products are substantially substitutable so Buyer j can benefit from less intense horizontal competition, or if products are very complementary so it can benefit from significant demand complementary effect. Otherwise, as shown in Figure 6, SNS is better for Buyer j as long as its bargaining power is sufficiently higher than Buyer i's.

4 Extensions

This section extends our baseline model to accommodate a different coordination scheme, asymmetric market sizes, seller collusion in a bilateral channel, Cournot competition, and the impact of a forward market and a cyclical market.

4.1 Profit Sharing Coordination

The extant literature has shown that many coordination mechanisms can perform equivalently (see Cachon, 2003). While one focus of this paper is to demonstrate that for all firms PMS can outperform the other negotiation mechanisms, this section is dedicated to exploring whether or not another coordination mechanism, profit sharing (Goyal, 1976; Joglekar and Tharthare, 1990; Jabera and Osmanb, 2006), can also improve the firms' performance. In profit sharing, let $\phi_i \in [0, 1]$ denote the fraction of Buyer i's profit shared to the seller. The profits for the buyers and the seller in each channel are

$$\begin{aligned}\pi_{bi} &= (1-\phi_i)(p_i - w_i)\, x_i(p_i, p_j)\,,\\ \pi_{si} &= (w_i + \phi_i (p_i - w_i))\, x_i(p_i, p_j)\,.\end{aligned}$$

where π_{si} denotes the seller's profit from selling to Buyer i.

In PM, we assume that the wholesale price and profit sharing ratio are matched in both channels. Suppose the negotiation is in channel i, then we need to solve

$$\max_{w_i,\phi_i} \; [\pi_{bi}(w_i,\phi_i) - d_{bi}]^{\theta_i} [\pi_s(w_i,\phi_i) - d_s]^{1-\theta_i}\,.$$

In SN, the equilibrium decision $(w_1^{SN}, w_2^{SN}, \phi_1^{SN}, \phi_2^{SN})$, (w_i^{SN}, ϕ_i^{SN}) solves

$$\max_{w_i,\phi_i} \; \left[\pi_{bi}\left(w_i, w_j^{SN}, \phi_i, \phi_j^{SN}\right)\right]^{\theta_i} \left[\pi_s\left(w_i, w_j^{SN}, \phi_i, \phi_j^{SN}\right) - \pi_{sj}\left(w_j^{SN}, w_i^{SN}, \phi_j^{SN}, \phi_i^{SN}\right)\right]^{1-\theta_i}\,.$$

In SQ, the equilibrium decisions $(w_s(w_f,\phi_f), \phi_s(w_f,\phi_f))$ solves

$$\max_{w_s,\phi_s} \; [\pi_{bs}(w_s, w_f, \phi_s, \phi_f)]^{\theta_s} [\pi_s(w_f, w_s, \phi_f, \phi_s) - \pi_{sf}(w_f, w_s(w_f,\phi_f), \phi_f, \phi_s(w_f,\phi_f))]^{1-\theta_s}\,,$$

and (w_f^{SQ}, ϕ_f^{SQ}) solves

$$\begin{aligned}\max_{w_f,\phi_f} \; & [\pi_{bf}(w_f, w_s(w_f,\phi_f), \phi_f, \phi_s(w_f,\phi_f))]^{\theta_f} \times\\ & \left[\pi_s(w_f, w_s(w_f,\phi_f), \phi_f, \phi_s(w_f,\phi_f)) - \pi_{ss}\left(w_s^{SQ}, w_f^{SQ}, \phi_s^{SQ}, \phi_f^{SQ}\right)\right]^{1-\theta_f}\,.\end{aligned}$$

Solving for the equilibrium strategies in PM, SN, and SQ with profit sharing and comparing their profits to those with side payment, we observe the following result.

Proposition 4 *For any (θ_i,θ_j), profit sharing is equivalent to side payment in coordinating PM, SN, and SQ in the model with a common seller and two buyers.*

Proposition 4 indicates that with a properly chosen profit sharing rate, firms can obtain the same profits in PM, SN, and SQ as those with side payment. Therefore, similar to the side payment coordination, profit sharing does not always benefit all firms in general, although one can manipulate the profit sharing rate to benefit one firm over the other.

4.2 Bilateral Channel and Seller Collusion

Different negotiation mechanisms lead to different profits for firms. One concern is whether a certain negotiation mechanism would provide more incentives than others for multiple sellers to collude or form coalition. To address this concern, we consider a bilateral channel model consisting of two seller-buyer dyads, also referred to as a dual-exclusive channel or an independent-sellers channel (Ha et al., 2011; Horn and Wolinsky, 1988; McGuire and Staelin, 1983). The sellers sell an identical intermediate good/supply to their respective buyers who then manufacture it into end products. For any given wholesale prices (w_i, w_j), in PM, SN, and SQ, the profits for the buyers and the sellers are

$$\begin{aligned}\pi_{bi} &= [p_i(x_i, x_j) - w_i]\, x_i, \quad i = 1, 2, \\ \pi_{si} &= w_i x_i, \quad i = 1, 2.\end{aligned}$$

The subscripts bi and si represent Buyer i and Seller i, respectively. The remaining model is the same as our baseline model with a common seller.

Our analysis shows that the main qualitative results in the baseline model with a common seller continue to hold true in the bilateral channel model. For example, as compared to SN and SQ, PM without coordination cannot be beneficial for all players at the same time. With side payment, however, PM may emerge as a mutually beneficial mechanism for all firms. This demonstrates that the impact of price matching is consistent in these two popular but different channel structures.

To further compare these two channel structures, for tractability, we limit our following discussion to the case where $\theta_i = \theta_j = \theta$, which is sufficient to demonstrate the disparity of collusion incentives among different negotiation mechanisms. If the two sellers in the bilateral channel have incentives to collude, their joint profit must be larger than the common seller's profit in the common-seller channel. Note that we focus on the collusion incentive rather than how the firms would collude. While explicit collusion is illegal in most markets, a tacit collusion can still garner a portion of aforementioned incentives for engaged sellers. Comparing the firms' profits in the two channel structures, we observe the following.

Proposition 5 *In the symmetric case where $\theta_i = \theta_j = \theta$, there exist two threshold values, $\widetilde{\theta}_{BI}^{SN}(\gamma)$ and $\widetilde{\theta}_{BI}^{SQ}(\gamma)$, such that:*

1. *In SN, the sellers in the bilateral channel have incentives to collude when* $\gamma > 0$*, or when* $\gamma < 0$ *and* $\theta < \widetilde{\theta}_{BI}^{SN}(\gamma)$*;*

2. *In SQ, the sellers in the bilateral channel have incentives to collude when* $\gamma > 0$*, or when* $\gamma < 0$ *and* $\theta < \widetilde{\theta}_{BI}^{SQ}(\gamma)$*;*

3. *In PM, the sellers in the bilateral channel have no incentive to collude.*

Proposition 5 indicates that the upper-stream sellers in the bilateral channel have more incentives to collude when the negotiation mechanism is SN or SQ. When products are substitutes, both sellers will seek coalition or collude in SN and SQ. When products are complements, both sellers have positive collusion incentives if and only if the sellers' negotiation powers are substantially larger than the buyers'. The rationale is that collusion allows the sellers to command a higher wholesale price. Although $\widetilde{\theta}_{BI}^{SN}(\gamma)$ and $\widetilde{\theta}_{BI}^{SQ}(\gamma)$ are very close when $\gamma < 0$, we have $\widetilde{\theta}_{BI}^{SN}(\gamma) < \widetilde{\theta}_{BI}^{SQ}(\gamma)$ in the feasible domain, which indicates that the sellers' collusion incentives in SQ are slightly more than those in SN.

The above collusion incentives in SN and SQ diminish in PM. The reason is that the resulting wholesale prices are bound by price matching and become independent of product substitutability. Therefore, the sellers will obtain the same wholesale price in both channel structures. Given that the downstream competition remains the same, the sellers yield the same profits in both channel structures. This observation suggests that the negotiation mechanism does affect the firms' collusion incentives and that the seller coalition in PM is less attractive than that in SN and SQ.

4.3 Analysis of Cournot Competition

This section compares Bertrand competition to Cournot competition. While in Bertrand competition the buyers compete on price, in Cournot competition firms compete on quantity. To be consistent, we use exactly the same setting in the baseline model of the Bertrand competition, except that the buyers determine their order quantities simultaneously in a Cournot-Nash game.

In PM, we obtain $w_{i-C}^{PM} = \frac{(1-\theta_i)a}{2}$, which is exactly the same as that in Bertrand competition. This occurs because price matching makes the wholesale price independent of product substitutability, and thus independent of the competition format. Therefore, similar to Lemma 2 in Bertrand competition, the seller will prefer to negotiate with the less powerful buyer.

In SN and SQ, we find that the wholesale price is generally higher in Cournot competition than in Bertrand competition if $\gamma > 0$, but it is lower if $\gamma < 0$. This is consistent with the conventional wisdom that quantity competition is less intense than price competition. As a result, the impact of competition formats among negotiation mechanisms lies in the wholesale price disparity in SN and SQ.

Due to the similarity of channel structure and negotiation procedure, however, the results in Cournot competition resemble those in Bertrand competition. For example, in Cournot competition, assuming $\theta_i \leq \theta_j$ and comparing PM to SN in terms of firms' preference of negotiation mechanism, we find the following results where subscript "C" denotes Cournot competition.

1. When products are substitutable, the seller always prefers PM to SN. When products are complementary, she prefers PM to SN iff $\theta_j > \widehat{\theta}^{SN}_{j-C}(\theta_i, \gamma)$.
2. The less powerful buyer (i.e., Buyer i) prefers SN to PM iff $\gamma \geq 0$.
3. When products are substitutable, the more powerful buyer (i.e., Buyer j) always prefers SN to PM. When products are complimentary, Buyer j prefers SN to PM iff $\theta_j \geq \widetilde{\theta}^{SN}_{j-C}(\theta_i, \gamma)$.

One can easily tell that the above results are almost identical to Proposition 1 except for the differences in the two threshold values of θ_j. Similarly, comparing PM to SQ, we have the following.

1. When products are substitutable, the seller prefers PM to SQ. When products are complementary, she prefers PM to SQ iff $\theta_j > \widehat{\theta}^{SQ}_{j-C}(\theta_i, \gamma)$.
2. The less powerful buyer (i.e., Buyer i) prefers SQ to PM iff $\gamma > 0$.
3. For the more powerful buyer (i.e., Buyer j),
   (a) When products are substitutable: i) When $\theta_i \leq \widehat{\theta}^{SQ}_{i-C}$, Buyer j prefers SQ to PM iff $\theta_j > \widetilde{\theta}^{SQ}_{j-C}(\theta_i, \gamma)$; ii) When $\theta_i > \widehat{\theta}^{SQ}_{i-C}$, Buyer j always prefers SQ to PM.

(b) When products are complementary, Buyer j prefers SQ to PM, iff $\theta_j > \widetilde{\theta}^{SQ}_{j-C}(\theta_i, \gamma)$.

The above results are almost identical to Proposition 2 of Bertrand competition except for those threshold values. We then compare the threshold values between Bertrand competition and Cournot competition in SN and SQ and have the following observation.

Proposition 6 *It is more likely for the seller to prefer PM to SN/SQ in Cournot competition than in Bertrand competition. Provided $\theta_i \leq \theta_j$, however, the opposite is true for the Buyer j. Buyer i is indifferent.*

Comparing the threshold values in Bertrand and Cournot competition, we find that in SN, if products are complimentary (i.e., $\gamma < 0$), then $\widehat{\theta}^{SN}_{j}(\theta_i, \gamma) > \widehat{\theta}^{SN}_{j-C}(\theta_i, \gamma)$ and $\widetilde{\theta}^{SN}_{j}(\theta_i, \gamma) > \widetilde{\theta}^{SN}_{j-C}(\theta_i, \gamma)$. In SQ, if products are complimentary (i.e., $\gamma < 0$), then $\widehat{\theta}^{SQ}_{j}(\theta_i, \gamma) > \widehat{\theta}^{SQ}_{j-C}(\theta_i, \gamma)$ and $\widetilde{\theta}^{SQ}_{j}(\theta_i, \gamma) > \widetilde{\theta}^{SQ}_{j-C}(\theta_i, \gamma)$. If products are substitutable (i.e., $\gamma > 0$), $\widehat{\theta}^{SQ}_{i}(\theta_i, \gamma) > \widehat{\theta}^{SQ}_{i-C}(\theta_i, \gamma)$ and, if θ_i is sufficiently small, then $\widetilde{\theta}^{SQ}_{j}(\theta_i, \gamma) > \widetilde{\theta}^{SQ}_{j-C}(\theta_i, \gamma)$. The area where the seller prefers PM to SN/SQ enlarges as the threshold value of θ_j shrinks (e.g., $\widehat{\theta}^{SN}_{j-C}(\theta_i, \gamma)$ is smaller than $\widehat{\theta}^{SN}_{j}(\theta_i, \gamma)$). Therefore, the seller becomes more likely to prefer PM to SN/SQ in Cournot competition than in Bertrand competition. The likelihood increases in Cournot competition because the wholesale price of PM under Cournot competition is lower when products are complementary. The reverse is true for Buyer j. Buyer i is indifferent because its preference is independent of the above threshold values**.**

The comparative results with side payment in Cournot competition are also very similar to those in Bertrand competition. Given that in PMS the wholesale prices are the same in both competition formats, the firms' profits are the same. In both SNS and SQS, however, Cournot competition has a higher channel efficiency than Bertrand competition provided $\theta_i = \theta_j$, because the horizontal channel competition is softened when firms compete in quantity rather than price.

4.4 Impact of Asymmetric Market Size

Oftentimes, buyers have different market sizes (i.e., $a_i \neq a_j$). For tractability, this subsection focuses on the impact of market asymmetry under Bertrand competition assuming $\theta_i = \theta_j = \theta$.

We extend the utility function for a representative consumer in our baseline model to the following where $a_i \neq a_j$:

$$U \equiv \sum_{i=1,2} (a_i x_i - x_i^2/2) - \gamma x_1 x_2 - \sum_{i=1,2} p_i x_i.$$

The game setting is the same as that in the baseline model, except that the parameter "a" will be replaced with "a_i" and "a_j" for Buyer i and Buyer j, respectively. Comparing the firms' profits under different negotiation sequences in PM and SQ results in the following proposition.

Proposition 7 *Suppose $\theta_i = \theta_j = \theta$ and $a_i < a_j$.*

- *In PM, the seller always prefers to negotiate with the bigger buyer (i.e., Buyer j), while the buyers always prefer to let the smaller buyer (i.e., Buyer i) negotiate.*
- *In SQ, the seller always prefers to negotiate with the bigger buyer (i.e., Buyer j) first, while the smaller buyer (i.e., Buyer i) prefers to let the bigger buyer (i.e., Buyer j) negotiate first if and only if products are substitutes (i.e., $\gamma > 0$) and the bigger buyer (i.e., Buyer j) prefers to negotiate first if and only if products are complementary (i.e., $\gamma < 0$) and $a_j < \frac{2-\gamma^2}{-\gamma} a_i$.*

Proposition 7 suggests that firms continue to have different preferences for negotiation sequence, because one's gain is at the expense of the others. In PM, the seller prefers to negotiate with the bigger buyer, because the bigger buyer's stake is higher than the smaller buyer's if the negotiation fails and the bigger buyer has more cushions to absorb a higher wholesale price.

Recall that in the symmetric case where $\theta_i = \theta_j$ and $a_i = a_j$, the seller always commands a higher wholesale price in the first negotiation of SQ. Provided $a_i < a_j$, by negotiating with the bigger buyer (i.e., Buyer j) first, the seller can benefit from not only an even higher wholesale price but also a bigger market size from Buyer j. For Buyer i (i.e., the smaller buyer), if the products are substitutable, it prefers to negotiate second to benefit from a lower wholesale price; however, if the products are complementary, a higher demand complementary effect surpasses the benefit of a lower wholesale price and alters Buyer i's preference to negotiate first. Similarly, Buyer j prefers to negotiate later for a lower wholesale price when products are substitutable. Buyer j would prefer to negotiate first if and only if the demand complementary effect surpasses the disadvantage of a higher wholesale price when its market size is not too substantially bigger than Buyer i's (i.e., $a_i < a_j < \frac{2-\gamma^2}{-\gamma} a_i$ provided $\gamma < 0$).

Similar to our previous discussion of asymmetric bargaining powers, firms continue to have irreconcilably different preferences between PM and SN/SQ as market size asymmetry varies. With coordination (e.g., side payment), PM can again emerge as a mutually beneficial choice for all firms as compared to SN and SQ, especially when buyers' market sizes are close to symmetric.

4.5 Impact of Forward Market and Cyclical Market

For tractability, we have assumed firms earn nothing if all negotiations fail. In reality, firms might have outside options, such as the forward market for commodities, where a price is set for future delivery. If the influence of the forward market is substantial, firms will be less likely to reach a deal via negotiation, because either the seller or the buyer will opt out of the negotiation if the future commodity price is too high or too low, respectively. While a forward market can timely reflect the supply-demand relationship, a long-term negotiated contract as discussed in this paper can better stabilize a firm's operational and financial flows and attract firms averse to volatile markets. In this sense, although our model aims to compare different negotiation mechanism for a wide variety of industries, it is more suitable for industries without an influential forward market.

To obtain analytical comparison, we also assume that there is no demand uncertainty. With demand uncertainty, such as cyclical market trends, firms have to forecast the total demand to estimate the market size (i.e., the parameter a_i in the model). Because it is very challenging to forecast the cyclical trend of most markets, the market size parameters in our model will be largely uncertain. Nevertheless, if firms are of similar market sizes, our main qualitative results are likely to sustain, because the comparative conclusions with symmetric market sizes are independent of the size of the market. If firms are asymmetric in market sizes, as shown in section 4.4, firms' preferences can shift as market sizes change. Notwithstanding, one may submit the average market sizes into the model to approximate the qualitative results.

5 Conclusion and Implications

This paper investigates the price matching negotiation mechanism and compares it to simultaneous negotiation and sequential negotiation in a common-seller two-buyer Bertrand competition model with asymmetric bargaining powers. We first find that in PM, the seller prefers to negotiate with the less powerful buyer, whereas in SQ the seller prefers to negotiate with the more powerful buyer first. Comparing firms' preferences among PM, SN, and SQ reveals that no negotiation mechanism is preferable to all firms at the same time. With side payment or profit sharing coordination, however, PMS can outperform PM, SN, SNS, SQ, and SQS for all firms especially when bargaining powers are symmetric. We also observe that in a bilateral channel with two sellers, the two sellers have more incentives to collude in SN and SQ than in PM. Our main qualitative results hold true in Cournot competition although it is more likely for the seller to prefer PM to SN/SQ in Cournot competition than in Bertrand competition. If the buyers differ in market sizes, the seller prefers to negotiate with the bigger buyer in PM and the seller prefers to negotiate with the bigger buyer first in SQ.

There are three main managerial insights resulted from this investigation. First, in practice, firms have choices of different negotiation mechanisms. Our analysis suggests that, without coordination, the buyers and the sellers may prefer different negotiation mechanisms depending on product substitutability, bargaining power asymmetry, and market size asymmetry. Therefore, it can be beneficial for the firms to form partnership in order to gain a better negotiation position in selecting a specific negotiation mechanism. Second, if firms would coordinate in negotiation, price matching can actually emerge as a mutually beneficial choice for all firms. Managers are thus advised to exert additional endeavor to coordinate the system. Third, from a legal perspective, when there are multiple sellers, compared with SN and SQ, PM can better prevent the sellers from collusion.

This paper is the first attempt to analytically interpret the impact of price matching on negotiation. To characterize PM, we focus on the pros and cons of PM as compared to SN and SQ. Nevertheless, many other issues related to PM remain unsolved and are our future research priority. For example, would firms change their preferences for PM if buyers own stocks of the seller or vice versa? Would other coordination mechanisms beside side payment and profit sharing lead to higher channel efficiency? Or how would production capacity, negotiation cost, renegotiation, and negotiation ending time affect the bargaining solution?

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Appendix: Online Supplements

This Appendix provides supplementary results for the paper "Price Matching Negotiation in Competitive Channels." We present all proofs, in addition to Comparison of SQS and SQ and Cournot Equilibrium Analysis, in the sequence of their appearances in the paper.

Proof of Lemma 1:

First, we discuss the seller's preference on negotiation sequence in SQ. The seller's total profit equals

$$\frac{(1-\gamma)(2+\gamma)\left[\begin{array}{c}\left(2-\theta_f^2-\theta_s^2\right)\left(\gamma^8+16\right)+2\left(1-\theta_f^2\right)\left(1-\theta_s^2\right)\left(-\gamma^7+8\gamma\right)\\ -\left(\left(\theta_s^4-3\theta_s^2+2\theta_s-8\right)\theta_f^2-2\theta_s\left(1-\theta_s\right)\left(2-\theta_s^2\right)\theta_f+\left(2-\theta_s^2\right)\left(9-\theta_s^2\right)\right)\times\\ \left(\gamma^6+4\gamma^2\right)+2\left(1-\theta_f\right)\left(1-\theta_s^2\right)\left(7+7\theta_f-\theta_s^2-2\theta_f\theta_s+\theta_f\theta_s^2\right)\left(\gamma^5-2\gamma^3\right)\\ -\left(\begin{array}{c}\left(23-6\theta_s+12\theta_s^2-2\theta_s^3-3\theta_s^4\right)\theta_f^2\\ +8\theta_s\left(1-\theta_s\right)\left(2-\theta_s^2\right)\theta_f-4\left(2-\theta_s^2\right)\left(7-\theta_s^2\right)\end{array}\right)\gamma^4\end{array}\right]a^2}{4(1+\gamma)(2-\gamma)\left(2-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_s^2+\gamma^2\theta_f\theta_s-\gamma^2\theta_f\theta_s^2\right)^2}.$$

The profit difference between sequence $(f,s)=(i,j)$ and $(f,s)=(j,i)$ equals $R_1\times R_2$, where

$$\begin{aligned}R_1 &= \frac{\gamma^2(1+\gamma)(2-\gamma)(1-\theta_j)(1-\theta_i)(\theta_i-\theta_j)}{\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_j^2+\gamma^2\theta_i\theta_j-\gamma^2\theta_i\theta_j^2\right)^2\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i^2+\gamma^2\theta_i\theta_j-\gamma^2\theta_i^2\theta_j\right)^2}\le 0,\\ R_2 &= u_4\theta_j^4+u_3\theta_j^3+u_2\theta_j^2+u_1\theta_j+u_0,\end{aligned}$$

and

$$\begin{aligned}u_4 &= \gamma^2(1-\theta_i)\left(\begin{array}{c}2\gamma^2(2+\gamma)(1-\gamma)\theta_i^3-2\gamma(2-\gamma)(1+\gamma)\left(2-\gamma^2\right)\theta_i^2\\ +2\gamma^2(1+\gamma)(2-\gamma)\theta_i+(2+\gamma)(1-\gamma)(1+\gamma)^2(2-\gamma)^2\end{array}\right),\\ u_3 &= \gamma^2\left(\begin{array}{c}2\gamma\left(4+4\gamma-5\gamma^2-2\gamma^3+\gamma^4\right)\theta_i^4-2(1+\gamma)^3(2-\gamma)^3\theta_i^3\\ +(2-\gamma)(1+\gamma)\left(12-4\gamma-7\gamma^2+2\gamma^3+3\gamma^4\right)\theta_i^2\\ +(2+\gamma)(1-\gamma)(1+\gamma)^2(2-\gamma)^2\end{array}\right),\\ u_2 &= (1+\gamma)(2-\gamma)\left(\begin{array}{c}-2\gamma^3(1+\gamma)(2-\gamma)\theta_i^4+\gamma^2\left(12-4\gamma-7\gamma^2+2\gamma^3+3\gamma^4\right)\theta_i^3\\ -4\gamma\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(2-\gamma^2\right)\theta_i^2\\ +\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(\left(2+\gamma^2\right)^2-4\gamma^2\right)\theta_i+\left(1-\gamma^2\right)^2\left(4-\gamma^2\right)^2\end{array}\right),\end{aligned}$$

$$\begin{aligned} u_1 &= (1+\gamma)(2-\gamma) \times \left(\begin{array}{c} -\gamma^2 (4-7\gamma^2+\gamma^4)\theta_i^4 + (1-\gamma^2)^2 (4-\gamma^2)^2 \\ +(1-\gamma^2)(4-\gamma^2)(2+2\gamma+\gamma^2)(2-2\gamma+\gamma^2)\theta_i^2 \end{array} \right), \\ u_0 &= \theta_i (1+\theta_i)(2+\gamma)(1-\gamma)(1+\gamma)^2 (2-\gamma)^2 \left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i^2\right). \end{aligned}$$

It is easy to check that $u_4 > 0$, $u_2 > 0$, $u_1 > 0$, $u_0 > 0$. Hence $R_2 \geq (u_3+u_2+u_1+u_0)\theta_j^3 \geq 0$. The last inequality is true because

$$\begin{aligned} & u_3+u_2+u_1+u_0 \\ = \; & 2\gamma^4 (2+\gamma)(1-\gamma)\theta_i^4 + 2\gamma^2 (2-\gamma)(1+\gamma)\left(4-2\gamma-\gamma^2\right)\left(1-\gamma-\gamma^2\right)\theta_i^3 \\ & +2(2-\gamma)(1+\gamma)\left(16-16\gamma-24\gamma^2+26\gamma^3+17\gamma^4-13\gamma^5-6\gamma^6+2\gamma^7+\gamma^8\right)\theta_i^2 \\ & +(1-\gamma)(2+\gamma)(1+\gamma)^2(2-\gamma)^2\left[\left(8-5\gamma^2+2\gamma^4\right)\theta_i + 8-9\gamma^2+2\gamma^4\right], \end{aligned}$$

and can be shown to be positive. Therefore, the profit difference is negative, and the seller always prefers $(f,s)=(j,i)$ to $(f,s)=(i,j)$.

Second, we consider Buyer i's preference. Each buyer's profit difference has the same sign as the sales quantity difference on the two negotiation sequences. Buyer i' sales quantity difference between sequences $(f,s)=(i,j)$ and $(f,s)=(j,i)$ equals

$$\gamma \left[\begin{array}{c} -\gamma\left(-\gamma(1+\gamma)(2-\gamma)\theta_j^2 + 2\gamma^2\theta_j + (1-\gamma^2)(4-\gamma^2)\right)\theta_i^2 \\ -\gamma\left(2\gamma(2-\gamma^2)\theta_j^2 + (1-\gamma^2)(4-\gamma^2)(1-\theta_j)\right)\theta_i - (1-\gamma^2)(4-\gamma^2)(2-\gamma^2)\theta_j \end{array} \right],$$

multiplied by a positive factor. The term in the square brackets is always negative, which can be justified as follows. It is negative at both $\theta_i = 0$ and $\theta_i = \theta_j$. Because $-\gamma(1+\gamma)(2-\gamma)\theta_j^2 + 2\gamma^2\theta_j + (1-\gamma^2)(4-\gamma^2) > 0$, the coefficient of θ_i^2 is positive iff $\gamma < 0$. (i) If $\gamma \leq 0$, the term in the square brackets is a convex function of θ_i, and hence is always negative for $\theta_i \in (0,\theta_j]$; (ii) If $\gamma > 0$, the term in the square brackets is a concave function of θ_i. The first-order derivative with respect to θ_i equals $-\gamma\left(2\gamma(2-\gamma^2)\theta_j^2 + (1-\gamma^2)(4-\gamma^2)(1-\theta_j)\right) < 0$ at $\theta_i = 0$. The term in the square brackets is decreasing in θ_i and hence is always negative. Based on the above discussion, we conclude Buyer i prefers $(f,s)=(j,i)$ iff $\gamma \geq 0$.

Finally, we consider Buyer j's preference. The sales quantity difference between sequences $(f,s)=(i,j)$ and $(f,s)=(j,i)$ equals

$$\gamma \left[\begin{array}{c} \gamma\left(-\gamma(1+\gamma)(2-\gamma)\theta_i^2 + 2\gamma^2\theta_i + (1-\gamma^2)(4-\gamma^2)\right)\theta_j^2 \\ +\gamma\left(2\gamma(2-\gamma^2)\theta_i^2 + (1-\gamma^2)(4-\gamma^2)(1-\theta_i)\right)\theta_j + (1-\gamma^2)(4-\gamma^2)(2-\gamma^2)\theta_i \end{array} \right].$$

Now we consider the term in the square brackets. It is positive at $\theta_j = \theta_i$ and has the same sign as $2\gamma + (2-\gamma-\gamma^2)\,\theta_i$ (which is negative iff $\theta_i < \frac{-2\gamma}{2-\gamma-\gamma^2}$) at $\theta_j = 1$. (i) If $\gamma \leq 0$, the term in the square brackets is a concave function of θ_j. (i.a) When $\theta_i \geq \frac{-2\gamma}{2-\gamma-\gamma^2}$, the term in the square brackets is always positive; (i.b) When $\theta_i < \frac{-2\gamma}{2-\gamma-\gamma^2}$, the term in the square brackets is positive iff $\theta_j \in [\theta_i, \overline{\theta}_j(\theta_i,\gamma)]$ (for some function $\overline{\theta}_j(\theta_i,\gamma)$). (ii) If $\gamma > 0$, the term in the square brackets is increasing in θ_j for $\theta_j \in [\theta_i, 1)$ and hence is always positive. Based on the above discussion, we have: If $\gamma \geq 0$, Buyer j prefers $(f,s) = (i,j)$; If $\gamma < 0$: (a) when $\theta_i \geq \frac{-2\gamma}{2-\gamma-\gamma^2}$, Buyer j always prefers $(f,s) = (j,i)$; (b) Otherwise ($\theta_i < \frac{-2\gamma}{2-\gamma-\gamma^2}$) Buyer j prefers $(f,s) = (j,i)$ when $\theta_j \in [\theta_i, \overline{\theta}_j(\theta_i,\gamma)]$ and $(f,s) = (i,j)$ when $\theta_j \in \left(\overline{\theta}_j(\theta_i,\gamma), 1\right)$. Q.E.D.

Proof of Lemma 2:

In PM, the profit of the common seller who negotiates with Buyer i equals $\pi_s(w_i^{PM}, w_i^{PM}) = \frac{2w_i^{PM}\left(a-w_i^{PM}\right)}{(1+\gamma)(2-\gamma)} = \frac{\left(1-\theta_i^2\right)a^2}{2(1+\gamma)(2-\gamma)}$, greater than the seller's profit when she negotiates with Buyer j, $\pi_s(w_j^{PM}, w_j^{PM}) = \frac{\left(1-\theta_j^2\right)a^2}{2(1+\gamma)(2-\gamma)}$. Each buyer prefers a higher sales quantity (as the profit is in proportion to the square of sales quantity). The sales quantity of each buyer when Buyer i negotiates equals $\frac{a-w_i^{MP}}{(1+\gamma)(2-\gamma)}$, smaller than that when Buyer j negotiates, $\frac{a-w_j^{MP}}{(1+\gamma)(2-\gamma)}$. Hence both buyers prefer a lower wholesale price and to let Buyer j negotiate. Q.E.D.

Proof of Proposition 1:

To isolate the impact of the choice of negotiation sequence or negotiating buyer, we first consider the case with symmetric firms (i.e., $\theta_1 = \theta_2 = \theta$). We conclude that when the products are substitutable (i.e., $\gamma \geq 0$), $w_i^{PM} \geq w_i^{SN}$, w_f^{SQ} and w_s^{SQ}. The conclusions are reversed when products are complementary. Actually, we can verify that

$$
\begin{aligned}
w_i^{SN} &= \frac{(1-\theta)\,(1-\gamma)\,(2+\gamma)\,a}{2\,(2-\gamma-\gamma^2+\theta\gamma)}, \\
w_f^{SQ} &= \frac{(1-\theta)\,(1-\gamma)\,(2+\gamma)\,(2+\gamma-\gamma^2-\gamma\theta^2)\,a}{2\,(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2)}, \\
w_s^{SQ} &= \frac{(1-\theta)\,(1-\gamma)\,(2+\gamma)\,(4+2\gamma-4\gamma^2-\gamma^3+\gamma^4+\theta^2\gamma^2-2\theta\gamma-\theta\gamma^2+\theta\gamma^3)\,a}{2\,(2-\gamma^2)\,(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2)}, \\
w_i^{PM} &= \frac{(1-\theta)\,a}{2}.
\end{aligned}
$$

Then it is easy to check that

$$
\begin{aligned}
w_i^{PM} - w_i^{SN} &= \frac{\gamma\theta\left(1-\theta\right)a}{2\left(2-\gamma-\gamma^2+\theta\gamma\right)}, \\
w_i^{PM} - w_f^{SQ} &= \frac{\gamma\left(1-\theta\right)\theta^2\left(2+\gamma-\gamma^2-\theta\gamma\right)a}{2\left(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2\right)}, \\
w_i^{PM} - w_f^{SQ} &= \frac{\gamma\theta\left(1-\theta\right)\left(4-5\gamma^2+\gamma^4+2\theta\gamma+\theta\gamma^2-\theta\gamma^3+\theta^2\gamma^3-2\theta^2\gamma\right)a}{2\left(2-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2\right)},
\end{aligned}
$$

and the ordering results on wholesale prices follow directly.

Now we proceed to asymmetric case. Under SN: The seller's profit is

$$
\begin{aligned}
&\pi_s(w_i^{SN}, w_j^{SN}) \\
=\;& \frac{\begin{array}{c}\left(1-\theta_i\right)\left(1-\gamma\right)\left(2+\gamma\right)\left(2+\gamma-\gamma\theta_j-\gamma^2\right)\times \\ \left[\gamma\left(\gamma-\theta_i\left(1+\gamma\right)\left(2-\gamma\right)\right)\theta_j+\left(4+2\gamma-3\gamma^2-\gamma^3+\gamma^4\right)\theta_i+\left(1-\gamma^2\right)\left(4-\gamma^2\right)\right]a^2\end{array}}{4\left(1+\gamma\right)\left(2-\gamma\right)\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i+\gamma^2\theta_j-\gamma^2\theta_i\theta_j\right)^2} \\
&+\frac{\begin{array}{c}\left(1-\theta_j\right)\left(1-\gamma\right)\left(2+\gamma\right)\left(2+\gamma-\gamma\theta_i-\gamma^2\right)\times \\ \left[\left(\gamma\left(\gamma^2-\gamma-2\right)\theta_i+\left(4+2\gamma-3\gamma^2-\gamma^3+\gamma^4\right)\right)\theta_j+\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i\right)\right]a^2\end{array}}{4\left(2-\gamma\right)\left(1+\gamma\right)\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i+\gamma^2\theta_j-\gamma^2\theta_i\theta_j\right)^2};
\end{aligned}
$$

Buyer i's profit equals

$$
\frac{\left(1-\gamma\right)\left[\gamma\left(\gamma-2\theta_i-\gamma\theta_i+\gamma^2\theta_i\right)\theta_j+\left(4+2\gamma-3\gamma^2-\gamma^3+\gamma^4\right)\theta_i+\left(1-\gamma^2\right)\left(4-\gamma^2\right)\right]^2a^2}{4\left(1+\gamma\right)\left(2-\gamma\right)^2\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i+\gamma^2\theta_j-\gamma^2\theta_i\theta_j\right)^2};
$$

Buyer j's profit is

$$
\frac{\left(1-\gamma\right)\left[\left(\gamma\left(\gamma^2-\gamma-2\right)\theta_i+4+2\gamma-3\gamma^2-\gamma^3+\gamma^4\right)\theta_j+4-5\gamma^2+\gamma^4+\gamma^2\theta_i\right]^2a^2}{4\left(1+\gamma\right)\left(2-\gamma\right)^2\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i+\gamma^2\theta_j-\gamma^2\theta_i\theta_j\right)^2}.
$$

Under PM: The seller's profit is $\pi_s(w_i^{PM}, w_i^{PM}) = \frac{\left(1-\theta_i^2\right)a^2}{2(1+\gamma)(2-\gamma)}$; Buyer i's profit equals Buyer j's profit, which equals $\frac{(1-\gamma)(1+\theta_i)^2a^2}{4(1+\gamma)(2-\gamma)^2}$.

We consider the seller's preference first. It is easy to verify that

$$
\begin{aligned}
&\pi_s(w_i^{SN}, w_j^{SN}) - \pi_s(w_i^{PM}, w_i^{PM}) \\
=\;& \frac{\left[\begin{array}{c}-\left(\theta_j^2-\theta_i^2\right)\left(\gamma^8+16\right)+\left(\theta_i+\theta_j-2\theta_i\theta_j\right)\left(\theta_i+\theta_j\right)\left(\gamma^7-6\gamma^5+12\gamma^3-8\gamma\right) \\ -2\left(6\theta_i^2-2\theta_i^3-4\theta_j^2+\theta_i^2\theta_j^2+\theta_i\theta_j-\theta_i\theta_j^2-3\theta_i^2\theta_j+2\theta_i^3\theta_j\right)\left(\gamma^6+4\gamma^2\right) \\ +2\left(3\theta_i-2\theta_j-\theta_i^2-\theta_i\theta_j+\theta_i^2\theta_j\right)\left(7\theta_i+6\theta_j-\theta_i^2-\theta_i\theta_j+\theta_i^2\theta_j\right)\gamma^4\end{array}\right]}{\left[4(1+\gamma)\left(2-\gamma\right)\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i+\gamma^2\theta_j-\gamma^2\theta_i\theta_j\right)^2\right]}a^2.
\end{aligned}
$$

We use NR to denote the numerator, and clearly $\pi_s(w_i^{SN}, w_j^{SN}) \geq \pi_s(w_i^{PM}, w_i^{PM})$ iff $NR \geq 0$. It is easy to check that

$$\frac{d^2NR}{d\theta_j^2} = -2\left(\begin{array}{c} 2\gamma^4\left(2\theta_i^3 - \theta_i^4\right) + 2\gamma^2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\theta_i^2 \\ -2\gamma\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)^2\theta_i + \left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)^3 \end{array}\right).$$

The term in the round brackets has the following properties: (i) It is convex in θ_i, as the second-order derivative equals $4\gamma^2\left[\left(1-\gamma^2\right)\left(4-\gamma^2\right) + 6\gamma^2\theta_i\left(1-\theta_i\right)\right] \geq 0$; (ii) It is monotone in θ_i because the first-order derivative equals $-2\gamma\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)^2$ at $\theta_i = 0$ and equals $-2\gamma\left(1-\gamma\right)\left(2+\gamma\right)\left(2-\gamma^2\right)^2$ at $\theta_i = 1$ (the derivatives at the two extreme points have the same sign); (iii) It equals $\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)^3 > 0$ at $\theta_i = 0$ and equals $\left(2+\gamma\right)\left(1-\gamma\right)\left(2-\gamma^2\right)^3 > 0$ at $\theta_i = 1$. Hence the term is the round brackets is positive and we always have $\frac{d^2NR}{d\theta_j^2} < 0$, that is, NR is concave in θ_j. Then

$$NR|_{\theta_j=\theta_i} = -2\gamma\theta_i^2\left(1-\theta_i\right)\left(4-\gamma+\gamma\theta_i-2\gamma^2\right)\left(2+\gamma-\gamma\theta_i-\gamma^2\right)^2,$$

which is positive iff $\gamma < 0$, and $NR|_{\theta_j=1} = -\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)^3\left(1-\theta_i^2\right) < 0$. Hence the results for the seller follow directly for $\gamma < 0$. Now we consider the case $\gamma \geq 0$. We have

$$\begin{aligned} \frac{dNR}{d\theta_j}|_{\theta_j=\theta_i} &= -2\theta_i\left(2+\left(1-\theta_i\right)\gamma-\gamma^2\right) \times \\ & \left(\begin{array}{c} 2\gamma^3\theta_i^3 + 2\gamma^2\left(2-2\gamma-\gamma^2\right)\theta_i^2 \\ -2\gamma\left(4+2\gamma-5\gamma^2-\gamma^3+\gamma^4\right)\theta_i + \left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)^2 \end{array}\right). \end{aligned}$$

The term in the last round brackets is positive because it has the following properties: (i) It is positive for both $\theta_i = 0$ and $\theta_i = 1$; (ii) It is decreasing in θ_i for $\theta_i = 1$; (iii) It is convex in θ_i for $\gamma \leq \sqrt{3}-1$; (iv) For $\sqrt{3}-1 < \gamma < 1$, (iv.a) it is convex in θ_i for $\theta_i \leq \frac{\left(2\gamma+\gamma^2-2\right)}{3\gamma}$ and concave in θ_i otherwise; (iv.b) It is positive when $\theta_i = \frac{\left(2\gamma+\gamma^2-2\right)}{3\gamma}$; (iv.c) It is decreasing in θ_i for $\theta_i = \frac{\left(2\gamma+\gamma^2-2\right)}{3\gamma}$. The above properties imply $\frac{dNR}{d\theta_j}|_{\theta_j=\theta_i} < 0$ when $\gamma \geq 0$. Hence NR is strictly decreasing in θ_j and always negative. Then the results for $\gamma \geq 0$ follow.

Then we discuss Buyer i's preference. $\pi_{bi}(w_i^{SN}, w_j^{SN}) - \pi_{bi}(w_i^{PM}, w_i^{PM})$ equals

$$\frac{\gamma\left(1-\gamma\right)\theta_i\left(1-\theta_j\right)\left(2+\gamma-\gamma\theta_i-\gamma^2\right)a}{2\left(2-\gamma\right)\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i+\gamma^2\theta_j-\gamma^2\theta_i\theta_j\right)},$$

multiplied by a positive factor. Obviously Buyer i prefers SN iff $\gamma \geq 0$.

Finally, we consider Buyer j's preference. $\pi_{bj}(w_j^{SN}, w_i^{SN}) - \pi_{bj}(w_i^{PM}, w_i^{PM})$ equals

$$\left(4+2\gamma-4\gamma^2-\gamma^3+\gamma^4+\gamma^2\theta_i^2-2\gamma\theta_i-\gamma^2\theta_i+\gamma^3\theta_i\right)\theta_j-\theta_i\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i\right),$$

multiplied by a positive factor. This term is linear in θ_j, and equals $\gamma\theta_i\left(1-\theta_i\right)\left(2+\left(1-\theta_i\right)\gamma-\gamma^2\right)$ (which is positive iff $\gamma > 0$) at $\theta_j = \theta_i$ and equals $\left(1-\theta_i\right)\left(2-\gamma\right)\left(1+\gamma\right)\left(2-\gamma^2\right) > 0$ at $\theta_j = 1$. When $\gamma > 0$, this term is always positive, so is $\pi_{bj}(w_j^{SN}, w_i^{SN}) - \pi_{bj}(w_i^{PM}, w_i^{PM})$. When $\gamma < 0$, the term is positive iff $\theta_j \geq \widetilde{\theta}_j^{SN}(\theta_i, \gamma)$, where

$$\widetilde{\theta}_j^{SN}(\theta_i,\gamma)=\frac{\theta_i\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i\right)}{\left(4+2\gamma-4\gamma^2-\gamma^3+\gamma^4+\gamma^2\theta_i^2-2\gamma\theta_i-\gamma^2\theta_i+\gamma^3\theta_i\right)}.$$

Q.E.D.

Proof of Proposition 2:

Under PM: The seller's total profit is $\pi_s(w_i^{PM}, w_i^{PM}) = \frac{2w_i^{MP}\left(a-w_i^{MP}\right)}{(1+\gamma)(2-\gamma)} = \frac{\left(1-\theta_i^2\right)a^2}{2(1+\gamma)(2-\gamma)}$; Buyer i's profit equals Buyer j's profit, which equals $\frac{(1-\gamma)(1+\theta_i)^2a^2}{4(1+\gamma)(2-\gamma)^2}$. Under SQ (assuming the seller chooses the sequence $(f,s) = (j,i)$ following Lemma 1): The seller's total profit is

$$\frac{\begin{array}{l}\left(1-\gamma\right)\left(2+\gamma\right)a^2\times\\ \left[\begin{array}{c}\left(2-\theta_i^2-\theta_j^2\right)\left(\gamma^8+16\right)+2\left(1-\theta_i^2\right)\left(1-\theta_j^2\right)\left(-\gamma^7+8\gamma\right)\\ -\left(\left(\theta_i^4-3\theta_i^2+2\theta_i-8\right)\theta_j^2-2\theta_i\left(1-\theta_i\right)\left(2-\theta_i^2\right)\theta_j+\left(2-\theta_i^2\right)\left(9-\theta_i^2\right)\right)\left(\gamma^6+4\gamma^2\right)\\ +2\left(1-\theta_i^2\right)\left(1-\theta_j\right)\left(7+7\theta_j-\theta_i^2-2\theta_i\theta_j+\theta_i^2\theta_j\right)\left(\gamma^5-2\gamma^3\right)\\ -\left(\left(12\theta_i^2-2\theta_i^3-3\theta_i^4-6\theta_i+23\right)\theta_j^2+8\theta_i\left(1-\theta_i\right)\left(2-\theta_i^2\right)\theta_j-4\left(2-\theta_i^2\right)\left(7-\theta_i^2\right)\right)\gamma^4\end{array}\right]\end{array}}{4\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i^2+\gamma^2\theta_i\theta_j-\gamma^2\theta_i^2\theta_j\right)^2}.$$

Buyer j's profit is

$$\frac{\left(1-\gamma\right)\left[\begin{array}{c}\left(1+\theta_j\right)\left(\gamma^6-8\right)-\left(1-\theta_i\right)\left(\theta_j-\theta_i+\theta_i\theta_j\right)\left(\gamma^5+4\gamma\right)\\ -\left(7+\theta_i+7\theta_j-2\theta_i^2-3\theta_i\theta_j+2\theta_i^2\theta_j\right)\left(\gamma^4-2\gamma^2\right)\\ +\left(1-\theta_i\right)\left(5\theta_j-4\theta_i+3\theta_i\theta_j\right)\gamma^3\end{array}\right]^2a^2}{4\left(1+\gamma\right)\left(2-\gamma\right)^2\left(2-\gamma^2\right)^2\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i^2+\gamma^2\theta_i\theta_j-\gamma^2\theta_i^2\theta_j\right)^2},$$

and Buyer i's profit is

$$\frac{\left(1-\gamma\right)\left(4+4\theta_i-5\gamma^2+\gamma^4+\gamma^2\theta_i^2+2\gamma\theta_i-4\gamma^2\theta_i-\gamma^3\theta_i+\gamma^4\theta_i+\gamma^3\theta_i\theta_j-2\gamma\theta_i\theta_j\right)^2a^2}{4\left(1+\gamma\right)\left(2-\gamma\right)^2\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i^2+\gamma^2\theta_i\theta_j-\gamma^2\theta_i^2\theta_j\right)^2}.$$

The seller's profit difference between PM and SQ equals

$$\begin{pmatrix} 2\gamma^4(2-\gamma^2)(-\theta_i^6+2\theta_i^5)-2\gamma^3(2-\gamma)(1+\gamma)(4-\gamma-2\gamma^2)\theta_i^3 \\ -\gamma(1+\gamma)(2-\gamma)(2-\gamma^2)(8-2\gamma-10\gamma^2+\gamma^3+2\gamma^4)\theta_i^2 \\ +\gamma^2(2+\gamma)(1-\gamma)(1+\gamma)^2(2-\gamma)^2(2\theta_i+\theta_i^4)+(1-\gamma)^2(1+\gamma)^3(2-\gamma)^3(2+\gamma)^2 \end{pmatrix}\theta_j^2$$
$$-2\gamma^2\theta_i^3\left(2-\gamma^2\right)(1-\theta_i)\left(4-2\gamma-6\gamma^2+\gamma^3+\gamma^4+2\gamma^2\theta_i^2\right)\theta_j$$
$$-\theta_i^2\left(2-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i^2\right)\left(4-2\gamma-6\gamma^2+\gamma^3+\gamma^4+2\gamma^2\theta_i^2\right),$$

multiplied by a positive factor**.** The above function has the following properties: (i) It equals $(1+\gamma)(2-\gamma)(1-\theta_i^2)(4-5\gamma^2+\gamma^4+\gamma^2\theta_i)^2>0$ at $\theta_j=1$; (ii) At $\theta_j=\theta_i$, it equals

$$\gamma\theta_i^2(1-\theta_i)\begin{bmatrix} 2\gamma^3(2-\gamma^2)(\theta_i^5-3\theta_i^4)-\gamma(2+\gamma)(1-\gamma)(1+\gamma)^2(2-\gamma)^2\theta_i^3 \\ -\gamma(2-\gamma)(1+\gamma)(12-16\gamma-11\gamma^2+8\gamma^3+3\gamma^4)\theta_i^2 \\ +(1-\gamma)(2+\gamma)(4+\gamma-2\gamma^2)(1+\gamma)^2(2-\gamma)^2(\theta_i+1) \end{bmatrix}.$$

Now we study the term in the square brackets: (ii.a) When $\gamma\le 0$, apparently the term in the square brackets is positive; (ii.b) When $\gamma>0$, we always have

$$\begin{aligned} & -\gamma(2+\gamma)(1-\gamma)(1+\gamma)^2(2-\gamma)^2\theta_i^3+(1-\gamma)(2+\gamma)\left(4-\gamma-2\gamma^2\right)(1+\gamma)^2(2-\gamma)^2 \\ = \;& (1-\gamma)(2+\gamma)(1+\gamma)^2(2-\gamma)^2\left(4-\gamma+\gamma\theta_i^3-2\gamma^2\right) \\ > \;& 0. \end{aligned}$$

We further have

$$\begin{aligned} & 2\gamma^3\left(2-\gamma^2\right)\left(\theta_i^5-3\theta_i^4\right)-\gamma(2-\gamma)(1+\gamma)\left(12-16\gamma-11\gamma^2+8\gamma^3+3\gamma^4\right)\theta_i^2 \\ & +(1-\gamma)(2+\gamma)\left(4+\gamma-2\gamma^2\right)(1+\gamma)^2(2-\gamma)^2\theta_i \\ = \;& \theta_i\begin{bmatrix} 2\gamma^3(2-\gamma^2)(\theta_i^4-3\theta_i^3)-\gamma(2-\gamma)(1+\gamma)(12-16\gamma-11\gamma^2+8\gamma^3+3\gamma^4)\theta_i \\ +(1-\gamma)(2+\gamma)(4+\gamma-2\gamma^2)(1+\gamma)^2(2-\gamma)^2 \end{bmatrix}. \end{aligned}$$

When $\gamma\le 0.63156$, $12-16\gamma-11\gamma^2+8\gamma^3+3\gamma^4\ge 0$; the right-hand side of last equation is decreasing in θ_i and hence is no smaller than

$$\begin{aligned} & \theta_i\begin{bmatrix} 2\gamma^3(2-\gamma^2)(1-3)-\gamma(2-\gamma)(1+\gamma)(12-16\gamma-11\gamma^2+8\gamma^3+3\gamma^4) \\ +(1-\gamma)(2+\gamma)(4+\gamma-2\gamma^2)(1+\gamma)^2(2-\gamma)^2 \end{bmatrix} \\ = \;& 2\theta_i\left(16-24\gamma^2+15\gamma^4-6\gamma^6+\gamma^8\right) \\ > \;& 0. \end{aligned}$$

When $\gamma > 0.63156$, the right-hand side is quasi-concave in θ_i, and is positive at both $\theta_i = 0$ and $\theta_i = 1$. Hence again the term in the square brackets is always positive. Therefore, we conclude at $\theta_j = \theta_i$: The profit difference is positive iff $\gamma > 0$. (iii) The coefficient of θ_j^2 in the profit difference is always positive. This is obviously true when $\gamma \leq 0$. When $\gamma > 0$, the coefficient of θ_j^2 is no smaller than

$$\begin{aligned}
& 2\gamma^4 \left(2-\gamma^2\right)\left(-\theta_i^6+2\theta_i^5\right) - 2\gamma^3 (2-\gamma)(1+\gamma)\left(4-\gamma-2\gamma^2\right)\theta_i^3 \\
& -\gamma (1+\gamma)(2-\gamma)\left(2-\gamma^2\right)\left(8-2\gamma-10\gamma^2+\gamma^3+2\gamma^4\right)\theta_i^2 \\
& +3\gamma^2 (2+\gamma)(1-\gamma)(1+\gamma)^2 (2-\gamma)^2 \theta_i^2 + (1-\gamma)^2 (1+\gamma)^3 (2-\gamma)^3 (2+\gamma)^2 \theta_i^2 \\
= \; & \theta_i^2 \left(\begin{array}{c} 2\gamma^4 (2-\gamma^2)(-\theta_i^4+2\theta_i^3) - 2\gamma^3 (2-\gamma)(1+\gamma)(4-\gamma-2\gamma^2)\theta_i \\ +(2-\gamma)(1+\gamma)(16-16\gamma-24\gamma^2+28\gamma^3+14\gamma^4-14\gamma^5-6\gamma^6+2\gamma^7+\gamma^8) \end{array} \right).
\end{aligned}$$

The term in the square bracket is decreasing in θ_i, hence the above term is no smaller than

$$\begin{aligned}
& \theta_i^2 \left(\begin{array}{c} 2\gamma^4 (2-\gamma^2)(-1+2) - 2\gamma^3 (2-\gamma)(1+\gamma)(4-\gamma-2\gamma^2) \\ +(2-\gamma)(1+\gamma)(16-16\gamma-24\gamma^2+28\gamma^3+14\gamma^4-14\gamma^5-6\gamma^6+2\gamma^7+\gamma^8) \end{array} \right) \\
= \; & \theta_i^2 (2+\gamma)(1-\gamma)\left(2-\gamma^2\right)^4 \\
> \; & 0.
\end{aligned}$$

Based on the above three properties: (a) When $\gamma \leq 0$, obviously the profit difference is positive (i.e., the seller prefers PM to SQ) iff $\theta_j \in (\widehat{\theta}_j^{SQ}(\theta_i,\gamma), 1)$, and is negative iff $\theta_j \in (\theta_i, \widehat{\theta}_j^{SQ}(\theta_i,\gamma))$. (b) When $\gamma > 0$, the derivative of profit difference with respect to θ_j at $\theta_j = \theta_i$ equals

$$2\theta_i \left[\begin{array}{c} 2\gamma^4 (2-\gamma^2)(3\theta_i^5-\theta_i^6) + \gamma^2 (8+4\gamma-18\gamma^2-5\gamma^3+9\gamma^4+\gamma^5-\gamma^6)\theta_i^4 \\ +\gamma^2 (2-\gamma)(1+\gamma)(4-12\gamma-2\gamma^2+6\gamma^3+\gamma^4)\theta_i^3 \\ -2\gamma (2-\gamma)(1+\gamma)(2-\gamma^2)(4-6\gamma^2+\gamma^4)\theta_i^2 \\ +2\gamma^2 (2+\gamma)(1-\gamma)(1+\gamma)^2 (2-\gamma)^2 \theta_i + (2+\gamma)^2 (1-\gamma)^2 (1+\gamma)^3 (2-\gamma)^3 \end{array} \right].$$

The above term is always positive because

$$\begin{aligned}
&2\gamma^4\left(2-\gamma^2\right)\left(3\theta_i^5-\theta_i^6\right)+\gamma^2\left(8+4\gamma-18\gamma^2-5\gamma^3+9\gamma^4+\gamma^5-\gamma^6\right)\theta_i^4\\
&+\gamma^2\left(2-\gamma\right)\left(1+\gamma\right)\left(4-12\gamma-2\gamma^2+6\gamma^3+\gamma^4\right)\theta_i^3\\
&-2\gamma\left(2-\gamma\right)\left(1+\gamma\right)\left(2-\gamma^2\right)\left(4-6\gamma^2+\gamma^4\right)\theta_i^2\\
&+2\gamma^2\left(2+\gamma\right)\left(1-\gamma\right)\left(1+\gamma\right)^2\left(2-\gamma\right)^2\theta_i+\left(2+\gamma\right)^2\left(1-\gamma\right)^2\left(1+\gamma\right)^3\left(2-\gamma\right)^3\\
\geq\ &\theta_i^6\left[\begin{array}{c}
2\gamma^4\left(2-\gamma^2\right)\left(3-1\right)+\gamma^2\left(8+4\gamma-18\gamma^2-5\gamma^3+9\gamma^4+\gamma^5-\gamma^6\right)\\
+\gamma^2\left(2-\gamma\right)\left(1+\gamma\right)\left(4-12\gamma-2\gamma^2+6\gamma^3+\gamma^4\right)\\
-2\gamma\left(2-\gamma\right)\left(1+\gamma\right)\left(2-\gamma^2\right)\left(4-6\gamma^2+\gamma^4\right)\\
+2\gamma^2\left(2+\gamma\right)\left(1-\gamma\right)\left(1+\gamma\right)^2\left(2-\gamma\right)^2+\left(2+\gamma\right)^2\left(1-\gamma\right)^2\left(1+\gamma\right)^3\left(2-\gamma\right)^3
\end{array}\right]\\
=\ &\theta_i^6\left(2+\gamma\right)\left(1-\gamma\right)\left(2-\gamma^2\right)^4>0.
\end{aligned}$$

Hence the profit difference is increasing in θ_j and the seller always prefers PM to SQ.

Buyer j's profit difference between PM and SQ equals

$$\begin{aligned}
NR\ =\ &-\left(\begin{array}{c}
\gamma^2\left(2-\gamma^2\right)\theta_i^3-\gamma\left(1+\gamma\right)^2\left(2-\gamma\right)^2\theta_i^2+2\gamma^2\left(1+\gamma\right)\left(2-\gamma\right)\theta_i\\
+\left(1-\gamma\right)\left(2+\gamma\right)\left(1+\gamma\right)^2\left(2-\gamma\right)^2
\end{array}\right)\theta_j\\
&+\theta_i\left(2-\gamma^2\right)\left(\gamma^2\theta_i^2-\gamma\left(\gamma+1\right)\left(2-\gamma\right)\theta_i+\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)\right),
\end{aligned}$$

multiplied by a positive factor. Notice that

$$NR|_{\theta_j=1}=-\left(1+\gamma\right)\left(2-\gamma\right)\left(1-\theta_i\right)\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i\right)<0,$$

and

$$NR|_{\theta_j=\theta_i}=\gamma\theta_i\left(1-\theta_i\right)\left(\gamma\left(2-\gamma^2\right)\theta_i^2-\left(1+\gamma\right)^2\left(2-\gamma\right)^2\theta_i+\gamma\left(1+\gamma\right)\left(2-\gamma\right)\right).$$

(i) If $\gamma\leq 0$, then $NR|_{\theta_j=\theta_i}>0$: (i.a) when $\theta_j\in[\theta_i,\widetilde{\theta}_j^{SQ}\left(\theta_i,\gamma\right)]$ with

$$\widetilde{\theta}_j^{SQ}\left(\theta_i,\gamma\right)=\frac{\theta_i\left(2-\gamma^2\right)\left(\gamma^2\theta_i^2-\gamma\left(\gamma+1\right)\left(2-\gamma\right)\theta_i+\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)\right)}{\left[\begin{array}{c}
\gamma^2\left(2-\gamma^2\right)\theta_i^3-\gamma\left(1+\gamma\right)^2\left(2-\gamma\right)^2\theta_i^2\\
+2\gamma^2\left(1+\gamma\right)\left(2-\gamma\right)\theta_i+\left(1-\gamma\right)\left(2+\gamma\right)\left(1+\gamma\right)^2\left(2-\gamma\right)^2
\end{array}\right]},$$

$NR>0$ and hence Buyer j prefers PM; (i.b) when $\theta_j\in\left(\widetilde{\theta}_j^{SQ}\left(\theta_i,\gamma\right),1\right)$, $NR<0$ and hence

Buyer j prefers SQ. (ii) If $\gamma > 0$, $NR|_{\theta_j=\theta_i} > 0$ iff $\theta_i \in \left(0, \widehat{\theta}_i^{SQ}(\gamma)\right)$ with

$$\widehat{\theta}_i^{SQ}(\gamma) = \frac{\left[\begin{array}{c} 4+4\gamma-3\gamma^2-2\gamma^3+\gamma^4 \\ -\sqrt{16+32\gamma-24\gamma^2-48\gamma^3+17\gamma^4+24\gamma^5-6\gamma^6-4\gamma^7+\gamma^8} \end{array}\right]}{2\gamma\left(2-\gamma^2\right)},$$

and $NR|_{\theta_j=\theta_i} < 0$ iff $\theta_i \in \left(\widehat{\theta}_i^{SQ}(\gamma), 1\right)$ (we can show that $\widehat{\theta}_i^{SQ}(\gamma) \in (0,1)$). If $\gamma > 0$, (ii.a) for $\theta_i \in \left(0, \widehat{\theta}_i^{SQ}(\gamma)\right)$: When $\theta_j \in [\theta_i, \widetilde{\theta}_j^{SQ}(\theta_i, \gamma))$, we have $NR > 0$ and hence Buyer j prefers PM; when $\theta_j \in \left(\widetilde{\theta}_j^{SQ}(\theta_i, \gamma), 1\right)$, we have $NR < 0$ and hence Buyer j prefers SQ. (ii.b) for $\theta_i \in \left(\widehat{\theta}_i^{SQ}(\gamma), 1\right)$, we always have $NR < 0$ and Buyer j prefers SQ.

Buyer i's profit difference between PM and SQ equals

$$\frac{\begin{array}{l}\gamma(1-\gamma)\theta_i\left(2+\left(1-\theta_i^2\right)\gamma-\gamma^2\right)(1-\theta_j)a^2 \times \\ \left[\begin{array}{c}\gamma\theta_i\left(2-\left(1-\theta_i^2\right)\gamma-\gamma^2\right)\theta_j \\ -\left(\gamma^2\left(\theta_i^3+2\theta_i^2\right)+(1+\gamma)(2-\gamma)\left(4-\gamma-2\gamma^2\right)\theta_i+2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\right)\end{array}\right]\end{array}}{4(1+\gamma)(2-\gamma)^2\left(4-5\gamma^2+\gamma^4+\gamma^2\theta_i^2+\gamma^2\theta_i\theta_j-\gamma^2\theta_i^2\theta_j\right)^2}.$$

We can show that the numerator is negative for all $\theta_j \in [\theta_i, 1)$. Hence the profit difference is positive iff $\gamma < 0$. That is, Buyer i prefers PM iff $\gamma < 0$. Q.E.D.

Proof of Lemma 3:

In *SNS*, the equilibrium wholesale price and side payment (w_i^{SNS}, T_i^{SNS}) maximize

$$\max_{w_i, T_i} \left[\pi_{bi}\left(w_i, w_j^{SNS}\right) - T_i\right]^{\theta_i}\left[\pi_s\left(w_i, w_j^{SNS}\right) + T_i - w_j^{SNS} x_j\left(w_j^{SNS}, w_i^{SNS}\right)\right]^{1-\theta_i}.$$

It is clear that under the optimal side payment

$$\theta_i\left[\pi_s\left(w_i, w_j^{SNS}\right) + T_i - w_j^{SNS} x_j\left(w_j^{SNS}, w_i^{SNS}\right)\right] = (1-\theta_i)\left[\pi_{bi}\left(w_i, w_j^{SNS}\right) - T_i\right].$$

That is,

$$T_i = (1-\theta_i)\pi_{bi}\left(w_i, w_j^{SNS}\right) - \theta_i\left[\pi_s\left(w_i, w_j^{SNS}\right) - w_j^{SNS} x_j\left(w_j^{SNS}, w_i^{SNS}\right)\right].$$

Firms choose w_i^{SNS} to maximize $\pi_{bi}\left(w_i, w_j^{SNS}\right) + \pi_s(w_i, w_j^{SNS})$. The FOC with respect to w_i^{SNS} is

$$\frac{\gamma^2(2+\gamma)(1-\gamma)a - 4\left(2-\gamma^2\right)w_i + 4\gamma w_j}{\left(1-\gamma^2\right)\left(4-\gamma^2\right)^2} = 0.$$

Solving the FOCs jointly, we obtain the equilibrium wholesale prices, equilibrium price and sales quantity

$$\begin{aligned} w_i^{SNS} &= \frac{\gamma^2 a}{4}, \\ p_i^{SNS} &= \frac{(2-\gamma)\,a}{4}, \\ x_i^{SNS} &= \frac{(2+\gamma)\,a}{4\,(1+\gamma)}. \end{aligned}$$

Each channel's profit equals $\frac{\left(4-\gamma^2\right)a^2}{16(1+\gamma)}$. Buyer i's profit (after accounting for side payment) equals $\frac{\theta_i\left(4-\gamma^2\right)a^2}{16(1+\gamma)}$; Buyer j's profit equals $\frac{\theta_j\left(4-\gamma^2\right)a^2}{16(1+\gamma)}$; The seller's total profit equals $\frac{(2-\theta_i-\theta_j)\left(4-\gamma^2\right)a^2}{16(1+\gamma)}$. In the following we restrict to symmetric case ($\theta_i = \theta_j = \theta$) for ease of comparison between SNS and SN.

Now we consider the seller's preference. The seller's profit difference between SNS and SN equals a positive factor multiplying

$$\begin{aligned} &2\,(1-\theta)\left(-2+\gamma+2\theta+\gamma^2-\gamma\theta\right)\times \\ &\left(-4+8\gamma-2\gamma^2-3\gamma^3+\gamma^4+2\gamma^2\theta-\gamma^3\theta\right)\left(2+\gamma-\gamma^2-\gamma\theta\right)^2. \end{aligned}$$

Notice that: (i) $(-2+\gamma+2\theta+\gamma^2-\gamma\theta)$ is positive iff $\theta \geq \frac{(2+\gamma)(1-\gamma)}{(2-\gamma)} \in (0,1)$; (ii) Define $V \equiv (-4+8\gamma-2\gamma^2-3\gamma^3+\gamma^4+2\gamma^2\theta-\gamma^3\theta)$. If $\gamma \leq 0.58579$, V is always negative; If $0.58579 < \gamma < 0.80606$, V is positive iff $\theta > \frac{(1-\gamma)\left(4-4\gamma-2\gamma^2+\gamma^3\right)}{\gamma^2(2-\gamma)}$ (the two threshold levels $\frac{(1-\gamma)\left(4-4\gamma-2\gamma^2+\gamma^3\right)}{\gamma^2(2-\gamma)} > \frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}$ iff $\gamma < 0.618$); If $\gamma \geq 0.80606$, V is always positive. Hence the seller prefers SNS to SN in the following regions: (a) $\gamma \leq 0.58579$ and $\theta_i \leq \frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}$; (b) $0.58579 < \gamma < 0.80606$ and either $\theta_i \leq \min\left(\frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}, \frac{(1-\gamma)\left(4-4\gamma-2\gamma^2+\gamma^3\right)}{\gamma^2(2-\gamma)}\right)$ or $\theta_i \geq \max\left(\frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}, \frac{(1-\gamma)\left(4-4\gamma-2\gamma^2+\gamma^3\right)}{\gamma^2(2-\gamma)}\right)$; (c) $\gamma \geq 0.80606$ and $\theta_i \geq \frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}$. The above regions are the same as that described in the lemma.

Each buyer's profit difference between SNS and SN equals

$$\begin{aligned} &\left(2-\gamma-2\theta-\gamma^2+\gamma\theta\right)\left(2+\gamma-\gamma^2-\gamma\theta\right)^2\times \\ &\left(-\gamma^2\,(2+\gamma)\,(2-\gamma)^2\,\theta^2+(1-\gamma)\,(2-\gamma)\left(4-8\gamma-8\gamma^2+\gamma^4\right)\theta-4\,(2+\gamma)\,(1-\gamma)^2\right), \end{aligned}$$

multiplied by a positive factor. The first part of the above equation $(2-\gamma-2\theta-\gamma^2+\gamma\theta) \geq 0$ iff $\theta \leq \frac{(2+\gamma)(1-\gamma)}{(2-\gamma)} \in (0,1)$. When $\gamma > 0.21535$, the second part of the above equation,

$-\gamma^2(2+\gamma)(2-\gamma)^2\theta^2 + (1-\gamma)(2-\gamma)(4-8\gamma-8\gamma^2+\gamma^4)\theta - 4(2+\gamma)(1-\gamma)^2$, is always negative; otherwise when $\gamma \leq 0.21535$, the quadratic function is increasing in θ_i in the range $\theta_i \in (0,1)$, and equals $-\gamma(4-\gamma)(2-\gamma^2)^2$ at $\theta = 1$. For $0 \leq \gamma \leq 0.21535$, again the quadratic function is always negative. For $\gamma < 0$, the quadratic function is positive iff θ_i is greater than $(1-\gamma)\frac{4-8\gamma-8\gamma^2+\gamma^4-(2-\gamma^2)\sqrt{4-16\gamma-12\gamma^2+\gamma^4}}{2\gamma^2(2+\gamma)(2-\gamma)}$, which is larger than $\frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}$ when $\gamma < 0$. To summarize, (a) when $\gamma \leq 0$, the buyer prefers SNS iff

$$\frac{(2+\gamma)(1-\gamma)}{(2-\gamma)} \leq \theta_i \leq (1-\gamma)\frac{4-8\gamma-8\gamma^2+\gamma^4-(2-\gamma^2)\sqrt{4-16\gamma-12\gamma^2+\gamma^4}}{2\gamma^2(2+\gamma)(2-\gamma)};$$

(b) When $\gamma \geq 0$, buyer prefers SNS iff $\theta_i \geq \frac{(2+\gamma)(1-\gamma)}{(2-\gamma)}$. Q.E.D

Comparison of SQS and SQ:

In *SQS*, assume round 1 is with buyer *f* and round 2 is with buyer *s*. Clearly, the round-2 equilibrium decisions satisfy

$$\max_{w_s,T_s} \ [\pi_{bs}(w_s,w_f) - T_s]^{\theta_s}[\pi_s(w_s,w_f) + T_s - w_f x_f(w_f, w_s(w_f))]^{1-\theta_s}.$$

Again we have

$$T_s = (1-\theta_s)\pi_{bs}(w_s,w_f) - \theta_s[\pi_s(w_s,w_f) - w_f x_f(w_f, w_s(w_f))].$$

Then the equilibrium wholesale price (that maximizes the joint profit of the two firms) is

$$w_s(w_f) = \gamma\frac{\gamma(1-\gamma)(2+\gamma)a + 4w_f}{4(2-\gamma^2)}.$$

The side payment in round 1 is

$$T_f = (1-\theta_f)\pi_{bf}(w_f, w_s(w_f)) - \theta_f[\pi_s(w_f, w_s(w_f)) - w_s(w_f)x_s(w_s(w_f), w_f)].$$

Then the wholesale price w_f^{SQS} maximizes $\pi_{bf}(w_f, w_s(w_f)) + \pi_s(w_f, w_s(w_f)))$ and the FOC with respect to w_f leads to

$$w_f^{SQS} = \frac{\gamma(2-\gamma)(1+\gamma)a}{4}.$$

The round-2 wholesale price becomes

$$w_s^{SQS} = w_s(w_f^{SQS}) = \frac{\gamma^2 a}{2}.$$

The equilibrium sales quantities are

$$x_f^{SQS} = \frac{\left(2-\gamma^2\right)a}{4\left(1+\gamma\right)}, \quad x_s^{SQS} = \frac{\left(2+\gamma\right)a}{4\left(1+\gamma\right)}.$$

Channel f's total profit equals $\frac{\left(2-\gamma^2\right)a^2}{8(\gamma+1)}$; Channel s' total profit equals $\frac{(2+\gamma)\left(2-\gamma+\gamma^2\right)a^2}{16(1+\gamma)}$. Buyer f's profit equals $\frac{\theta_f\left(2-\gamma^2\right)a^2}{8(1+\gamma)}$; Buyer s' profit equals $\frac{\theta_s(2+\gamma)\left(2-\gamma+\gamma^2\right)a^2}{16(1+\gamma)}$; The seller's total profit is $\frac{\left(1-\theta_f\right)\left(2-\gamma^2\right)a^2}{8(1+\gamma)} + \frac{(1-\theta_s)(2+\gamma)\left(2-\gamma+\gamma^2\right)a^2}{16(1+\gamma)}$. The above results are used in the comparison in Section 3.3.2. Q.E.D.

Proof of Proposition 3:

Under PMS, assume the seller negotiates with Buyer i. The equilibrium wholesale price is $w_i^{PMS} = \frac{\gamma a}{2}$ and sales quantities are $x_i^{PMS} = \frac{a}{2(1+\gamma)}$. PMS is successful iff $\pi_{bi}\left(w_i^{PMS}, w_i^{PMS}\right) + \frac{1}{2}\pi_s\left(w_i^{PMS}, w_i^{PMS}\right) \geq d_{bi} + \frac{d_s}{2}$, i.e., $\frac{a^2}{4(1+\gamma)} \geq d_{bi} + \frac{d_s}{2}$. Then Buyer i's profit equals Buyer j's profit which equals $d_{bi} + \theta_i\left[\frac{a^2}{4(1+\gamma)} - d_{bi} - \frac{d_s}{2}\right]$; The seller's profit equals $d_s + 2\left(1-\theta_i\right)\left[\frac{a^2}{4(1+\gamma)} - d_{bi} - \frac{d_s}{2}\right]$. The analysis for PMS in which the seller negotiates with Buyer j is similar. We observe that: If $d_{bi} \leq d_{bj}$, then in PMS the seller always prefers to negotiate with Buyer i. Actually, the seller's profit when negotiating with Buyer i is $d_s + 2\left(1-\theta_i\right)\left[\frac{a^2}{4(1+\gamma)} - d_{bi} - \frac{d_s}{2}\right]$, greater than the profit when negotiating with Buyer j, $d_s + 2\left(1-\theta_j\right)\left[\frac{a^2}{4(1+\gamma)} - d_{bj} - \frac{d_s}{2}\right]$. That is, the weaker Buyer i gains a lower profit than the stronger Buyer j. Under PMS, the joint pie of the seller and the buyer is a constant $\frac{a^2}{4(1+\gamma)}$, hence, (a) the incremental value to the joint profit of the seller with the weaker Buyer i is larger (the additional pie is bigger with the weaker Buyer i); (b) the seller gains a larger portion of the additional pie with the weaker Buyer i.

Suppose PM is the disagreement point. In PM, Buyer i (the weaker buyer) will be picked to negotiate. Then the seller's profit $\pi_s(w_i^{PM}, w_i^{PM}) = \frac{\left(1-\theta_i^2\right)a^2}{2(1+\gamma)(2-\gamma)}$; Buyer i's profit equals Buyer j's profit, which equals $\frac{(1+\theta_i)^2(1-\gamma)a^2}{4(1+\gamma)(2-\gamma)^2}$. Based on the above discussion, the seller prefers to negotiate with buyer i under PMS. Notice that $\frac{a^2}{4(1+\gamma)} - d_{bi} - \frac{d_s}{2} = \frac{a^2}{4(1+\gamma)} - \frac{(1+\theta_i)^2(1-\gamma)a^2}{4(1+\gamma)(2-\gamma)^2} - \frac{\left(1-\theta_i^2\right)a^2}{4(1+\gamma)(2-\gamma)} = \frac{(1-\gamma-\theta_i)^2a^2}{4(1+\gamma)(2-\gamma)^2} \geq 0$, PMS will be successful. Because the two buyers' profits equal each other under either PM, or PMS, both of them get better off with PMS. So does the seller.

For general case (in which we may have $\theta_i \neq \theta_j$) with either SN, SQ, PM, SNS, SQS as disagreement point, we conclude that PMS will be successful and both the seller and the negotiating buyer will get better off. Actually in PMS the wholesale price is always $\frac{\gamma a}{2}$ regardless of which buyer negotiates. We can verify that the system profit under PMS with $(w_i, w_j) = \left(\frac{\gamma a}{2}, \frac{\gamma a}{2}\right)$ equals $\frac{a^2}{2(1+\gamma)}$, and is always greater than the system profit under SN, SQ, PM, SNS and SQS. Hence we must have either $\frac{a^2}{4(1+\gamma)} \geq d_{bi} + \frac{d_s}{2}$ or $\frac{a^2}{4(1+\gamma)} \geq d_{bj} + \frac{d_s}{2}$ (because $\frac{a^2}{2(1+\gamma)} \geq d_{bi} + d_{bj} + d_s$), and the seller and negotiating buyer will get better off.

For symmetric case with $\theta_i = \theta_j = \theta$ (with either SN, SQ, PM, SNS or SQS as disagreement point), the two buyers are identical under SN, PM, SNS. In SQ and SQS, we assume random tie-breaking such that the disagreement point for each buyer is the average profit of the two buyers (and hence these two buyers are identical under this assumption). Notice that (a) $d_{bi} + \frac{d_s}{2}$ equals half of the system profit under disagreement point; (b) $(w_i, w_j) = \left(w_i^{PMS}, w_i^{PMS}\right) = \left(\frac{\gamma a}{2}, \frac{\gamma a}{2}\right)$ maximize the system profit $\pi_b (w_i, w_j) + \pi_{si} (w_i, w_j) + \pi_{sj} (w_j, w_i)$. Obviously PMS will succeed and PMS makes all firms better off (compared with any aforementioned disagreement point). Q.E.D.

Proof of Proposition 4:

In profit sharing, let $\phi_i \in [0, 1]$ denote the fraction of Buyer i's profit shared to Seller. Given (ϕ_i, ϕ_j) and (w_i, w_j), the profits for the buyers and the sellers are

$$\begin{aligned} \pi_{bi} &= (1 - \phi_i) (p_i - w_i) x_i (p_i, p_j), \\ \pi_s &= \pi_{si} + \pi_{sj}, \end{aligned}$$

where

$$\pi_{si} = (w_i + \phi_i (p_i - w_i)) x_i (p_i, p_j)$$

is the seller's profit derived from sales to Buyer i. From the FOCs we obtain

$$\widehat{p}_i (w_i, w_j) = \frac{(1 - \gamma)(2 + \gamma) a + 2w_i + \gamma w_j}{4 - \gamma^2},$$

and the equilibrium outcomes are

$$\begin{aligned} \widehat{x}_i (w_i, w_j) &= \frac{\widehat{p}_i (w_i, w_j) - w_i}{1 - \gamma^2} \\ &= \frac{(1 - \gamma)(2 + \gamma) a - (2 - \gamma^2) w_i + \gamma w_j}{(1 - \gamma^2)(4 - \gamma^2)}. \end{aligned}$$

Then

$$\pi_{bi} (w_i, w_j) = (1 - \phi_i) \left(1 - \gamma^2\right) \left[\widehat{x}_i (w_i, w_j)\right]^2, \quad i = 1, 2, \tag{B-1}$$

$$\pi_{si} (w_i, w_j) = \left[w_i + \phi_i \left(1 - \gamma^2\right) \widehat{x}_i (w_i, w_j)\right] \widehat{x}_i (w_i, w_j), \quad i = 1, 2. \tag{B-2}$$

In *SN* with profit sharing, the equilibrium decisions $(w_1^{SN}, w_2^{SN}, \phi_1^{SN}, \phi_2^{SN})$ solve the problems of

$$\max_{w_i, \phi_i} \left[\pi_{bi}\left(w_i, w_j^{SN}, \phi_i, \phi_j^{SN}\right)\right]^{\theta_i} \left[\pi_s\left(w_i, w_j^{SN}, \phi_i, \phi_j^{SN}\right) - \pi_{sj}\left(w_j^{SN}, w_i^{SN}, \phi_j^{SN}, \phi_i^{SN}\right)\right]^{1-\theta_i}.$$

We can use a sequential procedure to determine these decisions. (a) We first fix w_i and find the optimal ϕ_i as a function of w_i. We denote

$$\Delta_j = \left(w_j^{SN} + \phi_j^{SN}\left(p_j\left(w_j^{SN}, w_i^{SN}\right) - w_j^{SN}\right)\right) x_j\left(w_j^{SN}, w_i^{SN}\right).$$

The FOC with respect to ϕ_i is

$$\theta_i\left(\pi_s - \Delta_j\right)\frac{\partial \pi_{bi}}{\partial \phi_i} + \left(1-\theta_i\right)\pi_{bi}\frac{\partial \pi_s}{\partial \phi_i} = 0,$$

i.e.,

$$-\theta_i\left(1-\gamma^2\right)\left(\pi_s - \Delta_j\right)\left[\widehat{x}_i\left(w_i, w_j\right)\right]^2 + \left(1-\theta_i\right)\left(1-\gamma^2\right)\pi_{bi}\left[\widehat{x}_i\left(w_i, w_j\right)\right]^2 = 0.$$

It is clear that in equilibrium ϕ_i is such that

$$\frac{\pi_{bi}}{\left(\pi_s - \Delta_j\right)} = \frac{\theta_i}{\left(1-\theta_i\right)},$$

which yields

$$\phi_i = 1 - \theta_i - \frac{\theta_i w_i}{\left(1-\gamma^2\right)\widehat{x}_i\left(w_i, w_j\right)}.$$

(b) We treat ϕ_i as a function of w_i (which is given above). Then the FOC with respect to w_i becomes

$$\theta_i\left(\pi_s - \Delta_j\right)\left(\frac{\partial \pi_{bi}}{\partial w_i} + \frac{\partial \pi_{bi}}{\partial \phi_i}\frac{\partial \phi_i}{\partial w_i}\right) + \left(1-\theta_i\right)\pi_{bi}\left(\frac{\partial \pi_s}{\partial w_i} + \frac{\partial \pi_s}{\partial \phi_i}\frac{\partial \phi_i}{\partial w_i}\right) = 0.$$

Notice that

$$\theta_i\left(\pi_s - \Delta_j\right)\frac{\partial \pi_{bi}}{\partial \phi_i} + \left(1-\theta_i\right)\pi_{bi}\frac{\partial \pi_s}{\partial \phi_i} = 0,$$

we have

$$\theta_i\left(\pi_s - \Delta_j\right)\frac{\partial \pi_{bi}}{\partial w_i} + \left(1-\theta_i\right)\pi_{bi}\frac{\partial \pi_s}{\partial w_i} = 0,$$

i.e.,

$$\frac{\partial \pi_{bi}}{\partial w_i} + \frac{\partial \pi_s}{\partial w_i} = 0.$$

The wholesale price maximizes the joint profit of the two firms, as it does in SNS. Obviously the wholesale prices under profit sharing are the same as that under side payment.

The analysis for *SQ* with profit sharing is similar, and results are also similar.

In *PM* with profit sharing, we assume both channels match both wholesale price and profit sharing ratio. Suppose the negotiation is in dyad i. Then we need to solve

$$\max_{w_i,\phi_i} \left[\pi_{bi}\left(w_i,\phi_i\right)-d_{bi}\right]^{\theta_i}\left[\pi_s\left(w_i,\phi_i\right)-d_s\right]^{1-\theta_i}.$$

The FOC with respect to ϕ_i leads to

$$\theta_i\left[\pi_s\left(w_i,\phi_i\right)-d_s\right]\frac{d\pi_{bi}\left(w_i,\phi_i\right)}{d\phi_i}+\left(1-\theta_i\right)\left[\pi_{bi}\left(w_i,\phi_i\right)-d_{bi}\right]\frac{d\pi_s\left(w_i,\phi_i\right)}{d\phi_i}=0,$$

i.e.,

$$\begin{aligned}&-\theta_i\left(1-\gamma^2\right)\left[\pi_s\left(w_i,\phi_i\right)-d_s\right]\left[\widehat{x}_i\left(w_i,w_i\right)\right]^2\\&+\left(1-\theta_i\right)\left(1-\gamma^2\right)\left[\pi_{bi}\left(w_i,\phi_i\right)-d_{bi}\right]\left(\left[\widehat{x}_i\left(w_i,w_i\right)\right]^2+\left[\widehat{x}_j\left(w_i,w_i\right)\right]^2\right)\\=\;&0.\end{aligned}$$

Notice that $\widehat{x}_i\left(w_i,w_i\right)=\widehat{x}_j\left(w_i,w_i\right)$, we have

$$\frac{\pi_{bi}\left(w_i,\phi_i\right)-d_{bi}}{\pi_s\left(w_i,\phi_i\right)-d_s}=\frac{\theta_i}{2\left(1-\theta_i\right)}.$$

Then the FOC with respect to w_i leads to

$$\theta_i\left[\pi_s\left(w_i,\phi_i\right)-d_s\right]\frac{d\pi_{bi}\left(w_i,w_i\right)}{dw_i}+\left(1-\theta_i\right)\left[\pi_{bi}\left(w_i,\phi_i\right)-d_{bi}\right]\frac{d\pi_s\left(w_i,w_i\right)}{dw_i}=0,$$

i.e.,

$$\frac{d\pi_{bi}\left(w_i,w_i\right)}{dw_i}+\frac{1}{2}\frac{d\pi_s\left(w_i,w_i\right)}{dw_i}=0.$$

Hence, similar to PMS, the wholesale price maximizes $\pi_{bi}\left(w_i,w_i\right)+\frac{\pi_s\left(w_i,w_i\right)}{2}$. We conclude that both wholesale price and profit allocation in PM with profit sharing are the same as that under PMS. Q.E.D.

Proof of Proposition 5:

We consider Bertrand competition where $\theta_i=\theta_j=\theta$. We use π_s^Y to denote the common seller's total profit under $Y=SN,SQ,PM$. From our previous analysis, in the common-seller channel model, we have

$$\begin{aligned}
\pi_s^{SN} &= \frac{(1-\theta)(1-\gamma)(2+\gamma)(2+2\theta-\gamma+\theta\gamma-\gamma^2-\theta\gamma^2)a^2}{2(1+\gamma)(2-\gamma)(2-\gamma-\gamma^2+\theta\gamma)^2}, \\
\pi_s^{SQ} &= \frac{(1-\theta)(2+\gamma)(1-\gamma)a^2}{4(1+\gamma)(2-\gamma)(2-\gamma^2)(4-5\gamma^2+2\theta^2\gamma^2-\theta^3\gamma^2+\gamma^4)^2}\times \\
&\quad \left(\begin{array}{c} \gamma^2(1+\gamma)^2(2-\gamma)^2\theta^5-\gamma^2(4+12\gamma-3\gamma^2-6\gamma^3+\gamma^4)\theta^4 \\ -\gamma(2-\gamma)(1+\gamma)(8-2\gamma-7\gamma^2+\gamma^3+2\gamma^4)\theta^3 \\ -\gamma(2-\gamma)(1+\gamma)(8-14\gamma-9\gamma^2+7\gamma^3+2\gamma^4)\theta^2 \\ +2(1-\gamma)(2+\gamma)(2-\gamma^2)(1+\gamma)^2(2-\gamma)^2(\theta+1) \end{array}\right), \\
\pi_s^{PM} &= \frac{(1-\theta^2)a^2}{2(1+\gamma)(2-\gamma)}.
\end{aligned}$$

We use notations with "$\overline{\cdot}$" for the bilateral channel model. Following the same procedure as in the baseline model, we can obtain the wholesale prices in different negotiation mechanisms as follows:

$$\begin{aligned}
\overline{w}_i^{SN} &= \frac{(1-\theta)(1-\gamma)(2+\gamma)a}{4-\gamma+\theta\gamma-2\gamma^2}, \\
\overline{w}_f^{SQ} &= \frac{(1-\theta)(1-\gamma)(2+\gamma)(4+\gamma-\theta\gamma-2\gamma^2)a}{2(8-9\gamma^2+2\gamma^4+\theta\gamma^2)}, \\
\overline{w}_s^{SQ} &= \frac{(1-\theta)(1-\gamma)(2+\gamma)(16+4\gamma-17\gamma^2-2\gamma^3+4\gamma^4+\theta^2\gamma^2-4\theta\gamma+2\theta\gamma^3)a}{4(2-\gamma^2)(8-9\gamma^2+2\gamma^4+\theta\gamma^2)}, \\
\overline{w}_i^{PM} &= \frac{(1-\theta)a}{2}.
\end{aligned}$$

The corresponding demands are given by:

$$\begin{aligned}
\overline{x}_i^{SN} &= \frac{(1+\theta)(2-\gamma^2)a}{(1+\gamma)(2-\gamma)(4-\gamma-2\gamma^2+\theta\gamma)}, \\
\overline{x}_f^{SQ} &= \frac{(1+\theta)(4+\gamma-2\gamma^2-\theta\gamma)a}{4(1+\gamma)(2-\gamma)(2-\gamma^2)}, \\
\overline{x}_s^{SQ} &= \frac{(1+\theta)(16+4\gamma-17\gamma^2-2\gamma^3+4\gamma^4+\theta^2\gamma^2-4\theta\gamma+2\theta\gamma^3)a}{4(1+\gamma)(2-\gamma)(8-9\gamma^2+\gamma^2\theta+2\gamma^4)}, \\
\overline{x}_i^{PM} &= \frac{(1+\theta)a}{2(1+\gamma)(2-\gamma)}.
\end{aligned}$$

We use $\overline{\pi}_s^Y$ to denote the sellers' total profit under $Y = SN, SQ, PM$, respectively for this bilateral channel. We can verify that

$$\begin{aligned}
\overline{\pi}_s^{SN} &= \frac{2\left(1-\theta^2\right)\left(1-\gamma\right)\left(2+\gamma\right)\left(2-\gamma^2\right)a^2}{\left(1+\gamma\right)\left(2-\gamma\right)\left(4-\gamma+\theta\gamma-2\gamma^2\right)^2}, \\
\overline{\pi}_s^{SQ} &= \frac{\left(1-\theta^2\right)\left(1-\gamma\right)\left(2+\gamma\right)\left(4+\gamma-\theta\gamma-2\gamma^2\right)^2 a^2}{8\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)\left(8-9\gamma^2+\theta\gamma^2+2\gamma^4\right)} \\
&\quad + \frac{\left(1-\theta^2\right)\left(1-\gamma\right)\left(2+\gamma\right)\left(16+4\gamma-17\gamma^2-2\gamma^3+4\gamma^4+\theta^2\gamma^2-4\theta\gamma+2\theta\gamma^3\right)^2 a^2}{16\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)\left(8-9\gamma^2+\theta\gamma^2+2\gamma^4\right)^2}, \\
\overline{\pi}_s^{PM} &= \frac{\left(1-\theta^2\right)a^2}{2\left(1+\gamma\right)\left(2-\gamma\right)}.
\end{aligned}$$

For SN, we can verify that

$$\begin{aligned}
\pi_s^{SN}-\overline{\pi}_s^{SN} &= \gamma\left(1-\gamma\right)\left(2+\gamma\right)\left(1-\theta\right)^2 a^2 \times \\
&\quad \frac{\gamma\left(6-\gamma-3\gamma^2\right)\theta^2+2\left(8-4\gamma-7\gamma^2+2\gamma^3+2\gamma^4\right)\theta+\gamma\left(2+\gamma\right)\left(1-\gamma\right)}{2\left(1+\gamma\right)\left(2-\gamma\right)\left(4-\gamma+\theta\gamma-2\gamma^2\right)^2\left(2-\gamma-\gamma^2+\theta\gamma\right)^2}.
\end{aligned}$$

When $\gamma > 0$, obviously the profit difference is positive. When $\gamma < 0$, notice that the concave quadratic numerator is negative at $\theta = 0$ and positive at $\theta = 1$, the profit difference is positive iff

$$\theta < \widetilde{\theta}_{BI}^{SN}(\gamma) = \frac{-2\left(8-4\gamma-7\gamma^2+2\gamma^3+2\gamma^4\right)+2\left(2-\gamma^2\right)\sqrt{16-16\gamma-11\gamma^2+8\gamma^3+4\gamma^4}}{2\gamma\left(6-\gamma-3\gamma^2\right)}.$$

For SQ, we can similarly prove the conclusion. For PM, $\pi_s^{PM} = \overline{\pi}_s^{PM}$ and hence the seller has no incentive to collude. Q.E.D.

Cournot Equilibrium Analysis

We analyze the game backward to obtain the subgame perfect equilibrium.

Second-Stage Game

For any given wholesale price (w_i, w_j), the outcome of second-stage game is independent of whether (w_i, w_j) is negotiated via PM, SN, or SQ. Given (w_i, w_j), the profits for the buyers and the seller are

$$\begin{aligned}
\pi_{bi} &= \left(p_i\left(x_i, x_j\right)-w_i\right)x_i, \\
\pi_s &= w_i x_i + w_j x_j,
\end{aligned}$$

where

$$p_i\left(x_i, x_j\right) = a - x_i - \gamma x_j.$$

The buyers seek to maximize their own profits by choosing respective optimal sales quantity x_i in a Cournot competition. Solving the FOCs gives us the equilibrium sales quantities as

$$\widehat{x}_i\left(w_i, w_j\right) = \frac{\left(2-\gamma\right)a - 2w_i + \gamma w_j}{4-\gamma^2},$$

and the resulting firm profits are

$$\pi_{bi}\left(w_i, w_j\right) = \left[\widehat{x}_i\left(w_i, w_j\right)\right]^2, \tag{B-3}$$

$$\pi_s(w_i, w_j) = w_i \widehat{x}_i\left(w_i, w_j\right) + w_j \widehat{x}_j\left(w_j, w_i\right). \tag{B-4}$$

Based on the above results, we now proceed to the first stage of the game.

First-Stage Results

SN: Because the procedure to find the bargaining solution in Cournot competition is exactly the same as that in Bertrand competition, for parsimony, we skip the computation details but list the results here. To differ from Bertrand competition in notation, we use subscript "-C" to denote Cournot competition. In SN, we have

$$w_{i-C}^{SN} = \left(1-\theta_i\right)\frac{4-\gamma^2+\gamma^2\theta_j-2\gamma\theta_j}{2\left(4-\gamma^2\left(1-\theta_i\right)\left(1-\theta_j\right)\right)}a, \quad i=1,2.$$

The equilibrium sales quantity

$$\begin{aligned} x_{i-C}^{SN} &= \widehat{x}_i\left(w_{i-C}^{SN}, w_{j-C}^{SN}\right) \\ &= \frac{\left(4+4\theta_i-\gamma^2+2\gamma\theta_i+\gamma^2\theta_i+\gamma^2\theta_j-\gamma^2\theta_i\theta_j-2\gamma\theta_i\theta_j\right)a}{2\left(2+\gamma\right)\left(4-\gamma^2\left(1-\theta_i\right)\left(1-\theta_j\right)\right)}. \end{aligned}$$

SQ: In SQ, we again assume in round 1 the seller negotiates with Buyer *f* and in round 2 with Buyer *s*. Therefore, $(f,s)=(j,i)$ or (i,j). The round-1 and round-2 equilibrium wholesale prices are, respectively,

$$\begin{aligned} w_{f-C}^{SQ} &= \frac{\left(1-\theta_f\right)\left(2-\gamma\right)\left(2+\gamma-\gamma\theta_s^2\right)a}{2\left(4-\gamma^2\left(1-\theta_s\right)\left(1+\theta_s-\theta_f\theta_s\right)\right)}; \\ w_{s-C}^{SQ} &= \frac{\left(1-\theta_s\right)\left(2-\gamma\right)\left(4+2\gamma-2\gamma\theta_f-\gamma^2\theta_f+\gamma^2\theta_f\theta_s\right)a}{4\left(4-\gamma^2\left(1-\theta_s\right)\left(1+\theta_s-\theta_f\theta_s\right)\right)}. \end{aligned}$$

The equilibrium sales quantities are

$$\begin{aligned} x_{f-C}^{SQ} &= \frac{\left(\begin{array}{c} \left(-\gamma\left(\gamma^2+4\gamma+4\right)\theta_s^2+2\gamma^2\left(\gamma+3\right)\theta_s+\left(2-\gamma\right)\left(2+\gamma\right)^2\right)\theta_f \\ +2\left(4-\gamma^2-2\gamma\theta_s-\gamma^2\theta_s+2\gamma^2\theta_s^2+2\gamma\theta_s^2\right) \end{array}\right)a}{4\left(2+\gamma\right)\left(4-\gamma^2\left(1-\theta_s\right)\left(1+\theta_s-\theta_f\theta_s\right)\right)}, \\ x_{s-C}^{SQ} &= \frac{\left(4-\gamma^2+\gamma^2\theta_s^2+4\theta_s+2\gamma\theta_s-2\gamma\theta_f\theta_s\right)a}{2\left(2+\gamma\right)\left(4-\gamma^2\left(1-\theta_s\right)\left(1+\theta_s-\theta_f\theta_s\right)\right)}. \end{aligned}$$

Comparing the firms' profits in $(f,s)=(i,j)$ and $(f,s)=(j,i)$ conditional on $\theta_i \leq \theta_j$, we can obtain the following firms' preferences.

1. The seller always prefers to negotiate with the more powerful buyer (i.e., Buyer j) first.

2. Buyer i prefers $(f,s)=(j,i)$ if and only if $\gamma \geq 0$.

3. Buyer j prefers $(f,s)=(i,j)$ if $\gamma \geq 0$; otherwise (i.e., $\gamma < 0$),

   - if $\theta_i \geq \frac{-2\gamma}{2-\gamma-\gamma^2}$, she prefers $(f,s)=(j,i)$;
   - if $\theta_i < \frac{-2\gamma}{2-\gamma-\gamma^2}$, she prefers $(f,s)=(j,i)$ when $\theta_j \in (\theta_i, \overline{\theta}_{j-C}(\theta_i,\gamma)]$ and $(f,s)=(i,j)$ when $\theta_j \in [\overline{\theta}_{j-C}(\theta_i,\gamma),1)$.

The seller's total profit is

$$\frac{(2-\gamma)\left[\begin{array}{c} -\left((2+\gamma)^2\left(\gamma^2\theta_s^4+2\gamma^2\theta_s+4-\gamma^2\right)-2\gamma^3(\gamma+4)\theta_s^3-4\gamma(4+3\gamma)\theta_s^2\right)\theta_f^2 \\ +8\gamma^2\theta_s(1-\theta_s)\left(2+\gamma-\gamma\theta_s^2-\theta_s^2\right)\theta_f+4\left(2+\gamma-\theta_s^2-\gamma\theta_s^2\right)\left(4-\gamma^2+\gamma^2\theta_s^2\right) \end{array}\right]a^2}{8(2+\gamma)\left(4-\gamma^2(1-\theta_s)(1+\theta_s-\theta_f\theta_s)\right)^2}.$$

The profit difference between sequence $(f,s)=(i,j)$ and $(f,s)=(j,i)$ equals $NR_1\times NR_2$, where

$$NR_1=\frac{\gamma^2(1-\theta_j)(1-\theta_i)(\theta_i-\theta_j)(2+\gamma)a^2}{\left(4-\gamma^2(1-\theta_j)(1+\theta_j-\theta_i\theta_j)\right)^2\left(4-\gamma^2(1-\theta_i)(1+\theta_i-\theta_j\theta_i)\right)^2}\leq 0$$

and

$$\begin{aligned} NR_2 \;=\; & \gamma^2(1-\theta_i)\left(8+4\gamma-2\gamma^2-\gamma^3-4\gamma^2\theta_i^2+4\gamma^2\theta_i^3-2\gamma^3\theta_i^3-8\gamma\theta_i^2+4\gamma^2\theta_i+2\gamma^3\theta_i\right)\theta_j^4 \\ & +\gamma^2\left(\begin{array}{c} 2\gamma\left(4+4\gamma-\gamma^2\right)\theta_i^4-2(2+\gamma)^3\theta_i^3 \\ +(2+\gamma)\left(12-4\gamma+5\gamma^2\right)\theta_i^2+(2+\gamma)\left(4-\gamma^2\right) \end{array}\right)\theta_j^3 \\ & +(2+\gamma)\left(\begin{array}{c} -2\gamma^3(2+\gamma)\theta_i^4+\gamma^2\left(12-4\gamma+5\gamma^2\right)\theta_i^3 \\ +4\left(4-\gamma^2\right)\left(\left(1+\gamma^2\right)\theta_i-2\gamma\theta_i^2\right)+\left(4-\gamma^2\right)^2 \end{array}\right)\theta_j^2 \\ & +(2+\gamma)\left(16-8\gamma^2+\gamma^4+16\theta_i^2+12\gamma^2\theta_i^2-4\gamma^2\theta_i^4-4\gamma^4\theta_i^2+3\gamma^4\theta_i^4\right)\theta_j \\ & +\theta_i(1+\theta_i)(2-\gamma)(2+\gamma)^2\left(4-\gamma^2+\gamma^2\theta_i^2\right). \end{aligned}$$

We can show that the coefficients of θ_j and the constant term are all positive. Follow the procedure in Bertrand competition, we can show that $NR_2 \geq 0$ and the profit difference is negative. Therefore, the seller always prefers $(f,s) = (j,i)$ to $(f,s) = (i,j)$.

Each buyer's profit difference has the same sign as sales quantity difference. Buyer i's sales quantity difference between sequences $(f,s) = (i,j)$ and $(f,s) = (j,i)$ equals

$$\frac{\begin{array}{l}\gamma\left(1-\theta_j\right)\left(1-\theta_i\right)a^2 \times \\ -\gamma^2\theta_i\left(4-2\theta_i-\gamma\theta_i\right)\theta_j^2+\left(2\gamma^2-\gamma^3\theta_i-2\gamma^3\theta_i^2+4\gamma\theta_i-8\right)\theta_j-\gamma\theta_i\left(4-\gamma^2\right)\left(1+\theta_i\right)\end{array}}{4\left(4-\gamma^2\left(1-\theta_j\right)\left(1+\theta_j-\theta_i\theta_j\right)\right)\left(4-\gamma^2\left(1-\theta_i\right)\left(1+\theta_i-\theta_j\theta_i\right)\right)}.$$

The numerator is always negative, which can be justified as follows. It is negative at both $\theta_j = \theta_i$ and $\theta_j = 1$. The numerator is concave in θ_j. The first-order derivative of the numerator with respect to θ_j at $\theta_j = \theta_i$ equals $-2\gamma^2\theta_i^2\left(4+\gamma-2\theta_i-\gamma\theta_i\right) - \left(4-\gamma^2\right)\left(2-\gamma\theta_i\right)$ and is always negative. Hence the numerator is decreasing in θ_j and always negative, and we conclude Buyer i prefers $(f,s) = (j,i)$ iff $\gamma \geq 0$.

Buyer j's sales quantity difference between sequence $(f,s) = (i,j)$ and $(f,s) = (j,i)$ equals

$$\frac{\begin{array}{l}\gamma\left(1-\theta_j\right)\left(1-\theta_i\right)a^2 \times \\ \left[\gamma\left(4-\gamma^2\theta_i^2+2\gamma^2\theta_i-\gamma^2-2\gamma\theta_i^2\right)\theta_j^2+\gamma\left(4-4\theta_i-\gamma^2+4\gamma\theta_i^2+\gamma^2\theta_i\right)\theta_j+2\theta_i\left(4-\gamma^2\right)\right]\end{array}}{4\left(4-\gamma^2\left(1-\theta_j\right)\left(1+\theta_j-\theta_j\theta_i\right)\right)\left(4-\gamma^2\left(1-\theta_i\right)\left(1+\theta_i-\theta_i\theta_j\right)\right)}.$$

Now we consider the numerator: It is positive at $\theta_j = \theta_i$; it has the same sign as $2\gamma + (2-\gamma)\theta_i$ (which is negative only when $\theta_i < \frac{-2\gamma}{2-\gamma}$) at $\theta_j = 1$. (i) If $\gamma \leq 0$, the numerator is a concave function of θ_j. When $\theta_i \geq \frac{-2\gamma}{2-\gamma}$, the numerator is always positive; when $\theta_i < \frac{-2\gamma}{2-\gamma}$, it is positive for $\theta_j \in [\theta_i, \overline{\theta}_{j-C}(\theta_i,\gamma)]$ and negative for $\theta_j \in \left(\overline{\theta}_{j-C}(\theta_i,\gamma), 1\right)$ (for some function $\overline{\theta}_{j-C}(\theta_i,\gamma)$). (ii) If $\gamma > 0$, the term in the numerator is increasing in θ_j for $\theta_j \in [\theta_i, 1)$ and hence is always positive. Based on the above: If $\gamma \geq 0$, Buyer j always prefers $(f,s) = (i,j)$; If $\gamma < 0$: (a) when $\theta_i \geq \frac{-2\gamma}{2-\gamma}$, Buyer j always prefers $(f,s) = (j,i)$; (b) Otherwise ($\theta_i < \frac{-2\gamma}{2-\gamma}$) Buyer j prefers $(f,s) = (j,i)$ when $\theta_j \in [\theta_i, \overline{\theta}_{j-C}(\theta_i,\gamma)]$ and $(f,s) = (i,j)$ when $\theta_j \in \left(\overline{\theta}_{j-C}(\theta_i,\gamma), 1\right)$.

PM: In PM, given that the negotiation is between the seller and Buyer i, we obtain $w_{i-C}^{PM} = \frac{(1-\theta_i)a}{2}$ with sales quantities $x_{i-C}^{PM} = x_{j-C}^{PM} = \frac{a-w_{i-C}^{PM}}{2+\gamma}$. We can verify that the seller prefers to negotiate with the weaker Buyer i given $\theta_j \geq \theta_i$, while the buyers' preferences are reversed. Q.E.D.

Proof of Proposition 6:

First, we discuss firms' preferences between PM and SN in Cournot competition, and have the following conclusions: (i) (The sellers' preference) When products are substitutable, the seller always prefers PM to SN; When products are complementary, she prefers SN to PM iff $\theta_j \in \left[\theta_i, \widehat{\theta}_{j-C}^{SN}(\theta_i,\gamma)\right]$ for some $\widehat{\theta}_{j-C}^{SN}(\theta_i,\gamma) < 1$. (ii) (Buyer i's preference) Buyer i prefers SN to PM iff $\gamma \geq 0$. (iii) (Buyer j's Preference) When products are substitutable, Buyer j always prefers SN to PM; When products are complimentary, Buyer j prefers SN to PM iff $\theta_j \geq \widetilde{\theta}_{j-C}^{SN}(\theta_i,\gamma) = \frac{\theta_i\left(4-\gamma^2+\gamma^2\theta_i\right)}{\left(4+2\gamma+\gamma^2\theta_i^2-2\gamma\theta_i-\gamma^2\theta_i\right)}$.

The above statements are true due to the following facts. Under SN: The seller's profit $\pi_{s-C}(w_{i-C}^{SN}, w_{j-C}^{SN})$ is

$$\begin{aligned}&\frac{(1-\theta_i)(2-\gamma)(2+\gamma-\gamma\theta_j)\left((4+2\gamma+\gamma^2-\gamma\theta_j(2+\gamma))\theta_i+(4-\gamma^2+\gamma^2\theta_j)\right)}{4(2+\gamma)\left(4-\gamma^2(1-\theta_i)(1-\theta_j)\right)^2}a^2\\ &+\frac{(1-\theta_j)(2-\gamma)(2+\gamma-\gamma\theta_i)\left((4+2\gamma+\gamma^2-\gamma(2+\gamma)\theta_i)\theta_j+(4-\gamma^2+\gamma^2\theta_i)\right)}{4(2+\gamma)\left(4-\gamma^2(1-\theta_i)(1-\theta_j)\right)^2}a^2;\end{aligned}$$

Buyer i's profit is

$$\left(\frac{(4+4\theta_i-\gamma^2+2\gamma\theta_i+\gamma^2\theta_i+\gamma^2\theta_j-\gamma^2\theta_i\theta_j-2\gamma\theta_i\theta_j)}{2(2+\gamma)\left(4-\gamma^2(1-\theta_i)(1-\theta_j)\right)}a\right)^2;$$

Buyer j's profit is

$$\left(\frac{(4+4\theta_j-\gamma^2+2\gamma\theta_j+\gamma^2\theta_j+\gamma^2\theta_i-\gamma^2\theta_i\theta_j-2\gamma\theta_i\theta_j)}{2(2+\gamma)\left(4-\gamma^2(1-\theta_i)(1-\theta_j)\right)}a\right)^2.$$

Under PM: The seller's profit is $\pi_{s-C}(w_{i-C}^{PM}, w_{i-C}^{PM}) = \frac{(1-\theta_i^2)}{2(2+\gamma)}a^2$; Buyer i's profit equals Buyer j's profit, which equals $\left(\frac{(1+\theta_i)}{2(2+\gamma)}a\right)^2$.

The seller's preference: We can verify that $\pi_{s-C}(w_{i-C}^{SN}, w_{j-C}^{SN}) - \pi_{s-C}(w_{i-C}^{PM}, w_{i-C}^{PM})$ equals

$$\begin{aligned}NR &= \left(-8-4\gamma-4\gamma^2\theta_i^2+\gamma^4\theta_i^2-2\gamma^4\theta_i^3+\gamma^4\theta_i^4+8\gamma\theta_i+4\gamma^2\theta_i\right)\theta_j^2\\ &\quad -2\gamma\theta_i(1-\theta_i)\left(4+2\gamma-\gamma^3\theta_i^2-4\gamma\theta_i+\gamma^3\theta_i\right)\theta_j\\ &\quad +\theta_i^2\left(8-4\gamma-8\gamma^2+\gamma^4+\gamma^4\theta_i^2+8\gamma^2\theta_i-2\gamma^4\theta_i\right),\end{aligned}$$

multiplied by a positive factor. It is easy to check that

$$\frac{d^2NR}{d\theta_j^2} = 2\left(-8-4\gamma-4\gamma^2\theta_i^2+\gamma^4\theta_i^2-2\gamma^4\theta_i^3+\gamma^4\theta_i^4+8\gamma\theta_i+4\gamma^2\theta_i\right).$$

The term in the round brackets has the following properties: (i) It is concave in θ_i, as

$$\begin{aligned}
& \frac{d^2}{d\theta_i^2}\left(-8-4\gamma-4\gamma^2\theta_i^2+\gamma^4\theta_i^2-2\gamma^4\theta_i^3+\gamma^4\theta_i^4+8\gamma\theta_i+4\gamma^2\theta_i\right) \\
= & -2\gamma^2\left(4-\gamma^2+6\gamma^2\theta_i-6\gamma^2\theta_i^2\right) \\
\leq & \ 0;
\end{aligned}$$

(ii) It is monotone in θ_i because the first derivative equals $4\gamma(2+\gamma)$ at $\theta_i = 0$ and equals $4\gamma(2-\gamma)$ at $\theta_i = 1$ (the derivatives at the two extreme points have the same sign); (iii) It equals $-4(2+\gamma) < 0$ at $\theta_i = 0$ and equals $-4(2-\gamma) < 0$ at $\theta_i = 1$. Hence we have $\frac{d^2 NR}{d\theta_j^2} < 0$, that is, NR is concave in θ_j.

$$NR|_{\theta_j=\theta_i} = -\gamma\theta_i^2(1-\theta_i)(4-\gamma+\gamma\theta_i)(2+\gamma-\gamma\theta_i)^2$$

is positive iff $\gamma < 0$, and

$$NR|_{\theta_j=1} = -4\left(1-\theta_i^2\right)(2+\gamma) < 0.$$

$\pi_{s-C}(w_{i-C}^{SN}, w_{j-C}^{SN}) \geq \pi_{s-C}(w_{i-C}^{PM}, w_{i-C}^{PM})$ iff $NR \geq 0$, hence the results for the seller follow directly for $\gamma < 0$. Now we consider the case $\gamma \geq 0$. We have

$$\frac{dNR}{d\theta_j}|_{\theta_j=\theta_i} = -2\theta_i(2+\gamma-\gamma\theta_i)\left(4+2\gamma-4\gamma\theta_i-2\gamma^2\theta_i+\gamma^3\theta_i+2\gamma^2\theta_i^2-2\gamma^3\theta_i^2+\gamma^3\theta_i^3\right) < 0,$$

where the term in the second round brackets is positive because it has the following properties: (i) It is positive for both $\theta_i = 0$ and $\theta_i = 1$; (ii) It is decreasing in θ_i for $\theta_i = 1$; (iii) It is convex in θ_i as the second-order derivative equals $2\gamma^2(2-2\gamma+3\gamma\theta_i) > 0$. Hence $NR < 0$, and the results for the seller when $\gamma \geq 0$ follow.

Buyer i's preference: $\pi_{bi-C}(w_{i-C}^{SN}, w_{j-C}^{SN}) - \pi_{bi}(w_{i-C}^{PM}, w_{i-C}^{PM})$ equals

$$\frac{\gamma\theta_i(1-\theta_j)(2+\gamma-\gamma\theta_i)a}{2(2+\gamma)(4-\gamma^2(1-\theta_i)(1-\theta_j))},$$

multiplied by a positive factor. The profit difference is positive iff $\gamma > 0$.

Buyer j's preference: $\pi_{bj-C}(w_{j-C}^{SN}, w_{i-C}^{SN}) - \pi_{bj-C}(w_{i-C}^{PM}, w_{i-C}^{PM})$ equals

$$NR = \frac{(4+2\gamma+\gamma^2\theta_i^2-2\gamma\theta_i-\gamma^2\theta_i)\theta_j-\theta_i(4-\gamma^2+\gamma^2\theta_i)}{2(2+\gamma)(4-\gamma^2(1-\theta_i)(1-\theta_j))}a,$$

multiplied by a positive factor. The numerator increases with θ_j, $NR|_{\theta_j=\theta_i} = \gamma\theta_i(1-\theta_i)(2+\gamma-\gamma\theta_i)$ and $NR|_{\theta_j=1} = 2(1-\theta_i)(2+\gamma) > 0$. When $\gamma > 0$, NR is always positive. Therefore,

$\pi_{bj-C}(w_{j-C}^{SN}, w_{i-C}^{SN}) - \pi_{bj-C}(w_{i-C}^{PM}, w_{i-C}^{PM})$ is positive when $\gamma > 0$. When $\gamma < 0$, NR is positive (i.e., $\pi_{bj-C}(w_{j-C}^{SN}, w_{i-C}^{SN}) \geq \pi_{bj-C}(w_{i-C}^{PM}, w_{i-C}^{PM})$) iff $\theta_j \geq \frac{\theta_i\left(4-\gamma^2+\gamma^2\theta_i\right)}{\left(4+2\gamma+\gamma^2\theta_i^2-2\gamma\theta_i-\gamma^2\theta_i\right)}$.

Threshold comparison: The two threshold levels in Bertrand competition are

$$\widehat{\theta}_j^{SN}(\theta_i,\gamma) = \theta_i \frac{\left(\begin{array}{c} \gamma(1-\theta_i)\times \\ \left(-2\gamma^3\theta_i^2 - 2\gamma(1-\gamma^2)(4-\gamma^2)\theta_i + 8 + 4\gamma - 12\gamma^2 - 4\gamma^3 + 6\gamma^4 + \gamma^5 - \gamma^6\right) \\ -(2-\gamma^2)(2+\gamma-\gamma\theta_i-\gamma^2)\times \\ \sqrt{(2+\gamma)(1-\gamma)(2-\gamma^2)(4-2\gamma-6\gamma^2+\gamma^3+\gamma^4+2\gamma^2\theta_i^2)} \end{array}\right)}{-\left(\begin{array}{c} 2\gamma^4(2\theta_i^3-\theta_i^4) + 2\gamma^2(1-\gamma^2)(4-\gamma^2)\theta_i^2 \\ -2\gamma(1+\gamma)(2-\gamma)(2-\gamma^2)^2\theta_i + (1+\gamma)(2-\gamma)(2-\gamma^2)^3 \end{array}\right)}$$

and

$$\widetilde{\theta}_j^{SN}(\theta_i,\gamma) = \frac{\theta_i(4-5\gamma^2+\gamma^4+\gamma^2\theta_i)}{4+2\gamma-4\gamma^2-\gamma^3+\gamma^4+\gamma^2\theta_i^2-2\gamma\theta_i-\gamma^2\theta_i+\gamma^3\theta_i},$$

while the threshold levels in Cournot competition are

$$\widehat{\theta}_{j-C}^{SN}(\theta_i,\gamma) = \theta_i \frac{\left(\begin{array}{c} -\gamma(1-\theta_i)(4+2\gamma-4\gamma\theta_i+\gamma^3\theta_i-\gamma^3\theta_i^2) \\ +2(2+\gamma-\gamma\theta_i)\sqrt{(2-\gamma)(2-\gamma-\gamma^2+\gamma^2\theta_i^2)} \end{array}\right)}{(8+4\gamma-8\gamma\theta_i-4\gamma^2\theta_i+4\gamma^2\theta_i^2-\gamma^4\theta_i^2+2\gamma^4\theta_i^3-\gamma^4\theta_i^4)}$$

and

$$\widetilde{\theta}_{j-C}^{SN}(\theta_i,\gamma) = \frac{\theta_i(4-\gamma^2+\gamma^2\theta_i)}{4+2\gamma+\gamma^2\theta_i^2-2\gamma\theta_i-\gamma^2\theta_i}.$$

We have

$$\begin{aligned} & \widetilde{\theta}_{j-C}^{SN}(\theta_i,\gamma) - \widetilde{\theta}_j^{SN}(\theta_i,\gamma) \\ = \; & \gamma^3\theta_i(1-\theta_i)\frac{(4+4\gamma-\gamma^2-\gamma^3-4\gamma\theta_i-\gamma^2\theta_i+\gamma^3\theta_i)}{(4+2\gamma+\gamma^2\theta_i^2-2\gamma\theta_i-\gamma^2\theta_i)((1+\gamma)(2-\gamma)(2-\gamma^2-\gamma\theta_i)+\gamma^2\theta_i^2)}, \end{aligned}$$

which is positive in the feasible domain $\theta_i \in (0,1)$ and $\gamma \in (-1,1)$ iff $\gamma > 0$. Similarly, $\widehat{\theta}_{j-C}^{SN}(\theta_i,\gamma) - \widehat{\theta}_j^{SN}(\theta_i,\gamma)$ is positive iff $\gamma > 0$. Hence, it is more likely for the seller to prefer PM to SN, Buyer i is indifferent, while Buyer j is less likely to prefer PM to SN.

Second, we discuss firms' preferences between PM and SQ in Cournot competition, and have the following conclusions: Assuming the seller chooses her preferred sequence, that is, $(f,s) = (j,i)$ in SQ and Buyer i is picked in PM, (i) (The sellers' preference) When products are substitutable, the seller always prefers PM to SQ; When products are complementary, she prefers SQ to PM iff $\theta_j \in \left[\theta_i, \widehat{\theta}^{SQ}_{j-C}(\theta_i,\gamma)\right]$ for some $\widehat{\theta}^{SQ}_{j-C}(\theta_i,\gamma) < 1$. (ii) (Buyer i's preference) Buyer i prefers SQ to PM iff $\gamma \geq 0$. (iii) (Buyer j's Preference) When products are substitutable: (a) When $\theta_i \leq \widehat{\theta}^{SQ}_{i-C}(\gamma)$, Buyer j prefers SQ to PM iff $\theta_j > \widetilde{\theta}^{SQ}_{j-C}(\theta_i,\gamma)$; (b) When $\theta_i > \widehat{\theta}^{SQ}_{i-C}(\gamma)$, Buyer j always prefers SQ to PM. When products are complimentary, Buyer j prefers SQ to PM, iff $\theta_j > \widetilde{\theta}^{SQ}_{j-C}(\theta_i,\gamma)$.

Under SQ ($(f,s)=(j,i)$): The seller's total profit is

$$(2-\gamma)\frac{\left[\begin{array}{c} -\left((2+\gamma)^2\left(\gamma^2\theta_i^4+2\gamma^2\theta_i+(4-\gamma^2)\right)-2\gamma^3(\gamma+4)\theta_i^3-4\gamma(3\gamma+4)\theta_i^2\right)\theta_j^2 \\ +8\gamma^2\theta_i(1-\theta_i)\left(2+\gamma-\gamma\theta_i^2-\theta_i^2\right)\theta_j+4\left(2+\gamma-\theta_i^2-\gamma\theta_i^2\right)\left(4-\gamma^2+\gamma^2\theta_i^2\right)\end{array}\right]}{8(2+\gamma)\left(4-\gamma^2(1-\theta_i)(1+\theta_i-\theta_j\theta_i)\right)^2}a^2;$$

Buyer j's profit is

$$\left[\frac{\left((2+\gamma)^2\left(-\gamma\theta_i^2+2-\gamma\right)+2\gamma^2(3+\gamma)\theta_i\right)\theta_j+2\left(2\gamma(1+\gamma)\theta_i^2-\gamma(2+\gamma)\theta_i+(4-\gamma^2)\right)}{4(2+\gamma)\left(4-\gamma^2(1-\theta_i)(1+\theta_i-\theta_i\theta_j)\right)}a\right]^2,$$

and Buyer i's profit is

$$\left[\frac{\left(4+4\theta_i-\gamma^2+\gamma^2\theta_i^2+2\gamma\theta_i-2\gamma\theta_j\theta_i\right)}{2(2+\gamma)\left(4-\gamma^2(1-\theta_i)(1+\theta_i-\theta_j\theta_i)\right)}a\right]^2.$$

The seller's profit difference between PM and SQ equals

$$\frac{\left[\begin{array}{c}\left(\begin{array}{c}4\gamma^4\left(-\theta_i^6+2\theta_i^5\right)+\gamma^2(2-\gamma)(2+\gamma)^2\left(\theta_i^4+2\theta_i\right)-2\gamma^3(2+\gamma)(4-\gamma)\theta_i^3 \\ -4\gamma(\gamma+2)\left(4-\gamma-\gamma^2\right)\theta_i^2+(2-\gamma)^2(2+\gamma)^3\end{array}\right)\theta_j^2 \\ -8\gamma^2\theta_i^3(1-\theta_i)\left(2-\gamma-\gamma^2+\gamma^2\theta_i^2\right)\theta_j-4\theta_i^2\left(2-\gamma-\gamma^2+\gamma^2\theta_i^2\right)\left(4-\gamma^2+\gamma^2\theta_i^2\right)\end{array}\right]}{8(2+\gamma)\left(4-\gamma^2(1-\theta_i)(1+\theta_i-\theta_j\theta_i)\right)^2}a^2.$$

The numerator has the following properties: (i) It equals $(2+\gamma)\left(1-\theta_i^2\right)\left(4-\gamma^2+\gamma^2\theta_i\right)^2 > 0$ at $\theta_j = 1$; (ii) At $\theta_j=\theta_i$, it equals

$$\gamma\theta_i^2(1-\theta_i)\left[\begin{array}{c}4\gamma^3\left(\theta_i^5-3\theta_i^4\right)-\gamma(2-\gamma)(2+\gamma)^2\theta_i^3-\gamma(2+\gamma)\left(12-16\gamma+\gamma^2\right)\theta_i^2 \\ +(4+\gamma)(2-\gamma)(2+\gamma)^2\theta_i+(2-\gamma)(4-\gamma)(2+\gamma)^2\end{array}\right].$$

Now we study the term in the square brackets: (a) When $\gamma \leq 0$, apparently the term in the square brackets is positive; (b) When $\gamma > 0$, we always have

$$-\gamma(2-\gamma)(2+\gamma)^2\theta_i^3+(2-\gamma)(4-\gamma)(2+\gamma)^2=(2-\gamma)(2+\gamma)^2\left(4-\gamma-\gamma\theta_i^3\right)>0.$$

We further have

$$
\begin{aligned}
& 4\gamma^3\left(\theta_i^5-3\theta_i^4\right)-\gamma\left(2+\gamma\right)\left(12-16\gamma+\gamma^2\right)\theta_i^2+\left(4+\gamma\right)\left(2-\gamma\right)\left(2+\gamma\right)^2\theta_i \\
= \; & \theta_i\left(4\gamma^3\left(\theta_i^4-3\theta_i^3\right)-\gamma\left(\gamma+2\right)\left(12-16\gamma+\gamma^2\right)\theta_i+\left(4+\gamma\right)\left(2-\gamma\right)\left(2+\gamma\right)^2\right).
\end{aligned}
$$

When $\gamma \leq 0.78890$, $12-16\gamma+\gamma^2 \geq 0$; the term in the round brackets is decreasing in θ_i and hence is no smaller than $4\gamma^3\left(1-3\right)-\gamma\left(\gamma+2\right)\left(12-16\gamma+\gamma^2\right)+\left(4+\gamma\right)\left(2-\gamma\right)\left(2+\gamma\right)^2 = 2\left(16+8\gamma^2-\gamma^4\right) > 0$. When $\gamma > 0.78890$, the term in the round brackets is quasi-concave in θ_i; the term in the round brackets is positive at both $\theta_i = 0$ and $\theta_i = 1$, hence again the term in the square brackets is always positive. Therefore, we conclude at $\theta_j = \theta_i$: The profit difference is positive iff $\gamma > 0$. (iii) The coefficient of θ_j^2 in the profit difference is always positive. This is obviously true when $\gamma \leq 0$. When $\gamma > 0$, we have

$$
\begin{aligned}
& 4\gamma^4\left(-\theta_i^6+2\theta_i^5\right)+\gamma^2\left(2-\gamma\right)\left(2+\gamma\right)^2\left(\theta_i^4+2\theta_i\right)-2\gamma^3\left(2+\gamma\right)\left(4-\gamma\right)\theta_i^3 \\
& -4\gamma\left(\gamma+2\right)\left(4-\gamma-\gamma^2\right)\theta_i^2+\left(2-\gamma\right)^2\left(2+\gamma\right)^3 \\
\geq \; & -2\gamma^3\left(2+\gamma\right)\left(4-\gamma\right)\theta_i^3-4\gamma\left(\gamma+2\right)\left(4-\gamma-\gamma^2\right)\theta_i^2+2\gamma^2\left(2-\gamma\right)\left(2+\gamma\right)^2\theta_i \\
& +\left(2-\gamma\right)^2\left(2+\gamma\right)^3 \\
> \; & 0.
\end{aligned}
$$

(iv) (a) When $\gamma \leq 0$, obviously the profit difference is positive (i.e., the seller prefers PM to SQ) iff $\theta_j \in (\widehat{\theta}_{j-C}(\theta_i,\gamma),1)$, and is negative iff $\theta_j \in (\theta_i, \widehat{\theta}_{j-C}(\theta_i,\gamma))$. (b) When $\gamma > 0$, the derivative of profit difference with respect to θ_j at $\theta_j = \theta_i$ equals

$$
2\theta_i\left(\begin{array}{c} 4\gamma^4\left(-\theta_i^6+3\theta_i^5\right)+\gamma^2\left(8+4\gamma-6\gamma^2-\gamma^3\right)\theta_i^4+2\gamma^2\left(2+\gamma\right)\left(2-6\gamma+\gamma^2\right)\theta_i^3 \\ -8\gamma\left(2+\gamma\right)\left(2-\gamma^2\right)\theta_i^2+2\gamma^2\left(2-\gamma\right)\left(2+\gamma\right)^2\theta_i+\left(2-\gamma\right)^2\left(2+\gamma\right)^3 \end{array}\right) > 0.
$$

Hence the profit difference is increasing in θ_j and always positive, and the seller always prefers PM to SQ.

Buyer i's profit difference between PM and SQ equals

$$
\frac{-\gamma\theta_i\left(1-\theta_j\right)\left(2+\gamma-\gamma\theta_i^2\right)}{2\left(2+\gamma\right)\left(4-\gamma^2\left(1-\theta_i\right)\left(1+\theta_i-\theta_j\theta_i\right)\right)},
$$

multiplied by a positive factor. Clearly, Buyer i prefers PM iff $\gamma < 0$.

Buyer j's profit difference between PM and SQ equals

$$\frac{-\left(2+\gamma-\gamma\theta_i\right)\left(4-\gamma^2+2\gamma\theta_i+\gamma^2\theta_i-2\gamma\theta_i^2\right)\theta_j+2\theta_i\left(4+2\gamma+\gamma^2\theta_i^2-2\gamma\theta_i-\gamma^2\theta_i\right)}{4\left(2+\gamma\right)\left(4-\gamma^2\left(1-\theta_i\right)\left(1+\theta_i-\theta_i\theta_j\right)\right)},$$

multiplied by a positive factor. Clearly the sign depends on the sign of numerator NR. Notice that

$$NR|_{\theta_j=1}=-\left(2+\gamma\right)\left(1-\theta_i\right)\left(4-\gamma^2+\gamma^2\theta_i\right)<0,$$

and

$$NR|_{\theta_j=\theta_i}=\gamma\theta_i\left(1-\theta_i\right)\left(2\gamma\theta_i^2-\left(2+\gamma\right)^2\theta_i+\gamma\left(2+\gamma\right)\right).$$

(i) If $\gamma\leq 0$, then $NR|_{\theta_j=\theta_i}>0$: (i.a) when $\theta_j\in\left[\theta_i,\widetilde{\theta}_{j-C}^{SQ}(\theta_i,\gamma)\right]$, $NR>0$ and hence Buyer j prefers PM; (i.b) when $\theta_j\in\left(\widetilde{\theta}_{j-C}^{SQ}(\theta_i,\gamma),1\right)$, $NR<0$ and hence Buyer j prefers SQ. (ii) If $\gamma>0$, $NR|_{\theta_j=\theta_i}>0$ iff $\theta_i\in\left(0,\widehat{\theta}_{i-C}^{SQ}\right)$, and $NR|_{\theta_j=\theta_i}<0$ iff $\theta_i\in\left(\widehat{\theta}_{i-C}^{SQ},1\right)$ (we can show that $\widehat{\theta}_{i-C}^{SQ}\in(0,1)$). If $\gamma>0$, (ii.a) for $\theta_i\in\left(0,\widehat{\theta}_{i-C}^{SQ}\right)$: When $\theta_j\in\left[\theta_i,\widetilde{\theta}_{j-C}^{SQ}(\theta_i,\gamma)\right)$, we have $NR>0$ and hence Buyer j prefers PM; when $\theta_j\in\left(\widetilde{\theta}_{j-C}^{SQ}(\theta_i,\gamma),1\right)$, we have $NR<0$ and hence Buyer j prefers SQ. (ii.b) for $\theta_i\in\left(\widehat{\theta}_{i-C}^{SQ},1\right)$, we always have $NR<0$ and Buyer j prefers SQ.

We have the following threshold comparison. The three threshold levels in Bertrand competition are

$$\begin{aligned}\widehat{\theta}_j^{SQ}\left(\theta_i,\gamma\right) &= \theta_i\frac{\left(\begin{array}{c}\gamma^2\theta_i^2\left(2-\gamma^2\right)\left(1-\theta_i\right)\left(4-2\gamma-6\gamma^2+\gamma^3+\gamma^4+2\gamma^2\theta_i^2\right)\\ +\left(4-5\gamma^2+\gamma^2\theta_i+\gamma^4\right)\left(2+\gamma-\gamma\theta_i^2-\gamma^2\right)\times\\ \sqrt{\left(1-\gamma\right)\left(2+\gamma\right)\left(2-\gamma^2\right)\left(4-2\gamma-6\gamma^2+\gamma^3+\gamma^4+2\gamma^2\theta_i^2\right)}\end{array}\right)}{\left(\begin{array}{c}2\gamma^4\left(2-\gamma^2\right)\left(-\theta_i^6+2\theta_i^5\right)-2\gamma^3\left(2-\gamma\right)\left(1+\gamma\right)\left(4-\gamma-2\gamma^2\right)\theta_i^3\\ -\gamma\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)\left(8-2\gamma-10\gamma^2+\gamma^3+2\gamma^4\right)\theta_i^2\\ +\gamma^2\left(2+\gamma\right)\left(1-\gamma\right)\left(1+\gamma\right)^2\left(2-\gamma\right)^2\left(2\theta_i+\theta_i^4\right)\\ +\left(1-\gamma\right)^2\left(1+\gamma\right)^3\left(2-\gamma\right)^3\left(2+\gamma\right)^2\end{array}\right)},\\ \widehat{\theta}_i^{SQ}\left(\gamma\right) &= \frac{\left(\begin{array}{c}\left(1+\gamma\right)^2\left(2-\gamma\right)^2\\ -\sqrt{16+32\gamma-24\gamma^2-48\gamma^3+17\gamma^4+24\gamma^5-6\gamma^6-4\gamma^7+\gamma^8}\end{array}\right)}{2\gamma\left(2-\gamma^2\right)},\end{aligned}$$

and

$$\widetilde{\theta}_j^{SQ}(\theta_i,\gamma)=\frac{\theta_i\left(2-\gamma^2\right)\left(\gamma^2\theta_i^2-\gamma\left(\gamma+1\right)\left(2-\gamma\right)\theta_i+\left(1+\gamma\right)\left(2-\gamma\right)\left(2-\gamma^2\right)\right)}{\left(\begin{array}{c}\gamma^2\left(2-\gamma^2\right)\theta_i^3-\gamma\left(1+\gamma\right)^2\left(2-\gamma\right)^2\theta_i^2\\ +2\gamma^2\left(1+\gamma\right)\left(2-\gamma\right)\theta_i+\left(1-\gamma\right)\left(2+\gamma\right)\left(1+\gamma\right)^2\left(2-\gamma\right)^2\end{array}\right)}.$$

The three threshold levels in Cournot competition are

$$\widehat{\theta}_{j-C}^{SQ}(\theta_i,\gamma) = \theta_i \frac{\begin{pmatrix} 4\gamma^2\theta_i^2(1-\theta_i)(2-\gamma-\gamma^2+\gamma^2\theta_i^2) \\ +2(4-\gamma^2+\gamma^2\theta_i)(2+\gamma-\gamma\theta_i^2)\sqrt{(2-\gamma)(2-\gamma-\gamma^2+\gamma^2\theta_i^2)} \end{pmatrix}}{\begin{pmatrix} 4\gamma^4(-\theta_i^6+2\theta_i^5)+\gamma^2(2-\gamma)(2+\gamma)^2(\theta_i^4+2\theta_i)-2\gamma^3(2+\gamma)(4-\gamma)\theta_i^3 \\ -4\gamma(\gamma+2)(4-\gamma-\gamma^2)\theta_i^2+(2-\gamma)^2(2+\gamma)^3 \end{pmatrix}},$$

$$\widehat{\theta}_{i-C}^{SQ} = \frac{4+4\gamma+\gamma^2-\sqrt{16+32\gamma+8\gamma^2+\gamma^4}}{4\gamma},$$

and

$$\widetilde{\theta}_{j-C}^{SQ}(\theta_i,\gamma) = \frac{2\theta_i(4+2\gamma+\gamma^2\theta_i^2-2\gamma\theta_i-\gamma^2\theta_i)}{(2+\gamma-\gamma\theta_i)(4-\gamma^2+2\gamma\theta_i+\gamma^2\theta_i-2\gamma\theta_i^2)}.$$

We can verify that, in the feasible domain $\theta_i \in (0,1)$ and $\gamma \in (-1,1)$, $\widehat{\theta}_j^{SQ}(\theta_i,\gamma) - \widehat{\theta}_{j-C}^{SQ}(\theta_i,\gamma)$ is positive when $\gamma < 0$. We further have

$$\begin{aligned} & \widehat{\theta}_i^{SQ}(\gamma) - \widehat{\theta}_{i-C}^{SQ}(\gamma) \\ = & \frac{\begin{bmatrix} -4\gamma^2+3\gamma^4-2\sqrt{16+32\gamma-24\gamma^2-48\gamma^3+17\gamma^4+24\gamma^5-6\gamma^6-4\gamma^7+\gamma^8} \\ +(2-\gamma^2)\sqrt{16+32\gamma+8\gamma^2+\gamma^4} \end{bmatrix}}{4\gamma(2-\gamma^2)}, \end{aligned}$$

which is positive iff $\gamma > 0$.

$$\begin{aligned} & \widetilde{\theta}_j^{SQ}(\theta_i,\gamma) - \widetilde{\theta}_{j-C}^{SQ}(\theta_i,\gamma) \\ = & -\gamma^3\theta_i(1-\theta_i)\frac{\begin{pmatrix} \gamma^4\theta_i^3-\gamma(4-\gamma^2)(4+2\gamma-\gamma^2)\theta_i^2 \\ +(1+\gamma)^2(4-\gamma^2)^2\theta_i-\gamma(1+\gamma)(4-\gamma^2)^2 \end{pmatrix}}{\begin{pmatrix} (2+\gamma-\gamma\theta_i)(2+\gamma-\gamma^2-\gamma\theta_i)(4-\gamma^2+2\gamma\theta_i-2\gamma\theta_i^2+\gamma^2\theta_i) \\ (4-5\gamma^2+\gamma^4+\gamma^3\theta_i^2+2\gamma\theta_i-2\gamma\theta_i^2+\gamma^2\theta_i-\gamma^3\theta_i) \end{pmatrix}}, \end{aligned}$$

which is positive when $\gamma < 0$ or when $\gamma > 0$ as long as θ_i is sufficiently small. Hence the results follow. Q.E.D.

Proof of Proposition 7:

In *PM*, suppose the seller negotiates with Buyer i. The equilibrium wholesale price w_i^{PM} satisfies

$$w_i^{PM} = \arg\max_{w_i} \ [\pi_{bi}(w_i,w_i)]^{\theta}[\pi_s(w_i,w_i)]^{1-\theta}.$$

The FOC is

$$\left[\theta(w_i x_i + w_j x_j)\left(\frac{\partial \pi_{bi}}{\partial w_j} + \frac{\partial \pi_{bi}}{\partial w_i}\right) + (1-\theta)\,\pi_{bi}\left(x_j + w_j\frac{dx_j}{dw_i} + x_i + w_i\frac{dx_i}{dw_i}\right)\right]\Bigg|_{w_j = w_i} = 0.$$

We obtain $w_i^{PM} = \frac{-u_1 - \sqrt{u_1^2 - 4u_2 u_0}}{2u_2}$ which is the smaller root to equation

$$u_2 w^2 + u_1 w + u_0 = 0,$$

where

$$\begin{aligned}
u_2 &= 4\,(2+\gamma)\,(1-\gamma) > 0,\\
u_1 &= -\left(10 - 6\theta - \gamma - 5\gamma^2 - \theta\gamma + 3\theta\gamma^2\right) a_i + \left(2 + 2\theta - 5\gamma - \gamma^2 + 3\theta\gamma - \theta\gamma^2\right) a_j < 0,\\
u_0 &= (1-\theta)\,(a_i + a_j)\left(2a_i - \gamma^2 a_i - \gamma a_j\right) > 0.
\end{aligned}$$

The equilibrium sales quantities are

$$\begin{aligned}
x_i^{PM} &= \frac{(2-\gamma^2)\,a_i - \gamma a_j - (1-\gamma)\,(2+\gamma)\,w_i^{MP}}{(1-\gamma^2)\,(4-\gamma^2)},\\
x_j{}^{PM} &= \frac{(2-\gamma^2)\,a_j - \gamma a_i - (1-\gamma)\,(2+\gamma)\,w_i^{MP}}{(1-\gamma^2)\,(4-\gamma^2)}.
\end{aligned}$$

The seller's total profit equals $w_i^{PM}\left(x_i^{PM} + x_j{}^{PM}\right) = w_i^{PM}\frac{a_i + a_j - 2w_i^{MP}}{(1+\gamma)(2-\gamma)}$. As $w_i^{PM} \leq \frac{a_i + a_j}{4}$ (this is true because the above quadratic function equals $-\frac{\theta(2+\gamma)(a_i+a_j)^2}{4(1-\gamma)(1+\gamma)^3} < 0$ when $w = \frac{a_i + a_j}{4}$), the seller prefers a higher wholesale price and chooses the buyer that leads to a higher wholesale price. Obviously, each buyer prefers a higher demand, equivalent to a lower wholesale price (based on the above equilibrium demands as functions of the wholesale price). Actually we can verify that

$$\begin{aligned}
w_j^{PM} &= \frac{\left(\begin{array}{c}(10 - 6\theta - \gamma - 5\gamma^2 - \theta\gamma + 3\theta\gamma^2)\,a_j - (2 + 2\theta - 5\gamma - \gamma^2 + 3\theta\gamma - \theta\gamma^2)\,a_i\\ -\sqrt{\begin{array}{c}((2 + 2\theta - 5\gamma - \gamma^2 + 3\theta\gamma - \theta\gamma^2)\,a_i - (10 - 6\theta - \gamma - 5\gamma^2 - \theta\gamma + 3\theta\gamma^2)\,a_j)^2\\ -16\,(2+\gamma)\,(1-\gamma)\,(1-\theta)\,(a_i + a_j)\,(2a_j - \gamma^2 a_j - \gamma a_i)\end{array}}\end{array}\right)}{8\,(2+\gamma)\,(1-\gamma)}\\
&\geq\\
w_i^{PM} &= \frac{\left(\begin{array}{c}(10 - 6\theta - \gamma - 5\gamma^2 - \theta\gamma + 3\theta\gamma^2)\,a_i - (2 + 2\theta - 5\gamma - \gamma^2 + 3\theta\gamma - \theta\gamma^2)\,a_j\\ -\sqrt{\begin{array}{c}((2 + 2\theta - 5\gamma - \gamma^2 + 3\theta\gamma - \theta\gamma^2)\,a_j - (10 - 6\theta - \gamma - 5\gamma^2 - \theta\gamma + 3\theta\gamma^2)\,a_i)^2\\ -16\,(2+\gamma)\,(1-\gamma)\,(1-\theta)\,(a_i + a_j)\,(2a_i - \gamma^2 a_i - \gamma a_j)\end{array}}\end{array}\right)}{8\,(2+\gamma)\,(1-\gamma)}
\end{aligned}$$

when $a_j \geq a_i$, thus, the results for PM follow.

In *SQ*, assume round 1 is with Buyer *f* and round 2 is with Buyer *s*. Clearly, the round-2 equilibrium wholesale price is

$$w_s(w_f) = \arg\max_{w_s} \ \left[\pi_{bs}\left(w_s, w_f\right)\right]^{\theta}\left[\pi_s(w_s, w_f) - w_f x_f(w_f, w_s(w_f))\right]^{1-\theta}.$$

We obtain

$$w_s(w_f) = \frac{(1-\theta)\left(2a_s - \gamma^2 a_s - \gamma a_f + 2\gamma w_f\right)}{2\left(2-\gamma^2\right)}.$$

The equilibrium wholesale price in round 1 is

$$w_f^{SQ} = \arg\max_{w_f} \ \left[\pi_{bf}\left(w_f, w_s(w_f)\right)\right]^{\theta}\left[\pi_s(w_f, w_s(w_f)) - w_s(w_f^{SQ}) x_s(w_s(w_f^{SQ}), w_f^{SQ})\right]^{1-\theta}.$$

The FOC is

$$(1-\theta)\left(x_f + x_s \frac{dw_s}{dw_f} + w_s \frac{dx_s}{dw_f}\right) + (1+\theta)\, w_f \frac{dx_f}{dw_f} = 0,$$

which leads to

$$w_f^{SQ} = (1-\theta)\, \frac{\left(4 - 5\gamma^2 + \gamma^4 + \theta^2\gamma^2\right) a_f - \gamma\left(2-\gamma^2\right)\theta^2 a_s}{2\left(4 - 5\gamma^2 + \gamma^4 + 2\theta^2\gamma^2 - \theta^3\gamma^2\right)}.$$

The round-2 wholesale price becomes

$$\begin{aligned} w_s^{SQ} &= w_s(w_f^{SQ}) \\ &= (1-\theta)\, \frac{\left(2-\gamma^2\right)\left(4 - 5\gamma^2 + \gamma^4 + \theta^2\gamma^2\right) a_s - \gamma\theta\left(4 - 5\gamma^2 + \gamma^4 + \theta\gamma^2\right) a_f}{2\left(2-\gamma^2\right)\left(4 - 5\gamma^2 + \gamma^4 + 2\theta^2\gamma^2 - \theta^3\gamma^2\right)}. \end{aligned}$$

The equilibrium sales quantities are

$$x_f^{SQ} = \frac{\left[\begin{array}{c} \left(\begin{array}{c} \left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(-\gamma^2\theta^3 + \left(2-\gamma^2\right)^2\right) \\ +2\gamma^2\left(8 - 9\gamma^2 + 2\gamma^4\right)\theta^2 + \left(1-\gamma^2\right)^2\left(4-\gamma^2\right)^2\theta \end{array}\right) a_f \\ -\gamma\left(2-\gamma^2\right)\left(\left(7\gamma^2 - \gamma^4 - 4\right)\theta^2 + \left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(1 + \theta + \theta^3\right)\right) a_s \end{array}\right]}{2\left(1-\gamma^2\right)\left(2-\gamma^2\right)\left(4-\gamma^2\right)\left(4 - 5\gamma^2 + \gamma^4 + 2\theta^2\gamma^2 - \theta^3\gamma^2\right)}$$

and

$$x_s^{SQ} = \frac{\left(\begin{array}{c} \left(2-\gamma^2\right)\left(2\gamma^2\theta^2 + \left(1-\gamma^2\right)\left(4-\gamma^2\right)(1+\theta)\right) a_s \\ -\gamma\left(\left(\gamma^4 - 3\gamma^2 + 4\right)\theta^2 + \left(1-\gamma^2\right)\left(4-\gamma^2\right)\right) a_f \end{array}\right)}{2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(4 - 5\gamma^2 + \gamma^4 + 2\theta^2\gamma^2 - \theta^3\gamma^2\right)}.$$

The seller's total profit is $w_f^{SQ} x_f^{SQ} + w_s^{SQ} x_s^{SQ}$. Suppose $a_j \geq a_i$. In SQ, there are two negotiating sequences $(f,s) = (j,i)$ and $(f,s) = (i,j)$. The seller's profit difference between $(f,s) = (j,i)$ and $(f,s) = (i,j)$ equals

$$\frac{\gamma^2\theta^2\left(1-\theta\right)^2\left(4+4\theta^2-5\gamma^2+\gamma^4-3\theta^2\gamma^2+\theta^2\gamma^4\right)\left(a_j^2-a_i^2\right)}{4\left(2-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2\right)^2} \geq 0.$$

Hence the seller prefers the sequence $(f,s) = (j,i)$ in SQ. Notice that each buyer's equilibrium profit $\pi_{bi} = \left(1-\gamma^2\right)\left[\widehat{x}_i\left(w_i, w_j\right)\right]^2$, hence each buyer prefers a higher demand.

We consider Buyer j's preference first. Under $(f,s) = (j,i)$, Buyer j's demand equals

$$\frac{\begin{pmatrix} \left(2\gamma^2\left(8-9\gamma^2+2\gamma^4\right)\theta^2+\left(1-\gamma^2\right)^2\left(4-\gamma^2\right)^2\theta+\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(\left(2-\gamma^2\right)^2-\gamma^2\theta^3\right)\right)a_j \\ -\gamma\left(2-\gamma^2\right)\left(\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(\theta^3+\theta+1\right)+\left(7\gamma^2-\gamma^4-4\right)\theta^2\right)a_i \end{pmatrix}}{2\left(1-\gamma^2\right)\left(2-\gamma^2\right)\left(4-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2\right)};$$

Under $(f,s) = (i,j)$, the demand equals

$$\frac{\left(2-\gamma^2\right)\left(2\gamma^2\theta^2+\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(\theta+1\right)\right)a_j-\gamma\left(4-5\gamma^2+\gamma^4+4\theta^2-3\theta^2\gamma^2+\theta^2\gamma^4\right)a_i}{2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2\right)}.$$

The demand difference between $(f,s) = (j,i)$ and $(f,s) = (i,j)$ equals

$$-\gamma\theta\left(1-\theta\right)^2\frac{\left(2-\gamma^2\right)a_i+\gamma a_j}{2\left(2-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2\right)},$$

hence Buyer j prefers $(f,s) = (j,i)$ iff $\gamma < 0$ and $a_j < \frac{2-\gamma^2}{-\gamma}a_i$.

Finally we consider Buyer i. Under $(f,s) = (j,i)$, Buyer i's demand equals

$$\frac{\left(2-\gamma^2\right)\left(2\gamma^2\theta^2+\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(\theta+1\right)\right)a_i-\gamma\left(4-5\gamma^2+\gamma^4+4\theta^2-3\theta^2\gamma^2+\theta^2\gamma^4\right)a_j}{2\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2\right)};$$

and under $(f,s) = (i,j)$, the demand equals

$$\frac{\begin{pmatrix} \left(2\gamma^2\left(8-9\gamma^2+2\gamma^4\right)\theta^2+\left(1-\gamma^2\right)^2\left(4-\gamma^2\right)^2\theta+\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(\left(2-\gamma^2\right)^2-\gamma^2\theta^3\right)\right)a_i \\ -\gamma\left(2-\gamma^2\right)\left(\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(\theta^3+\theta+1\right)+\left(7\gamma^2-\gamma^4-4\right)\theta^2\right)a_j \end{pmatrix}}{2\left(1-\gamma^2\right)\left(2-\gamma^2\right)\left(4-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2\right)}.$$

The demand difference between $(f,s) = (j,i)$ and $(f,s) = (i,j)$ equals

$$\gamma\theta\left(1-\theta\right)^2\frac{\left(2-\gamma^2\right)a_j+\gamma a_i}{2\left(2-\gamma^2\right)\left(4-5\gamma^2+\gamma^4+2\theta^2\gamma^2-\theta^3\gamma^2\right)},$$

which is positive iff $\gamma > 0$. Hence Buyer i prefers $(f,s) = (j,i)$ iff $\gamma > 0$. Q.E.D.