

Online Manufacturer Referral to Heterogeneous Retailers

Hao Wu ^{*} Gangshu (George) Cai [†] Jian Chen [‡] Chwen Sheu [§]

Abstract

Since the development of the Internet, thousands of manufacturers have been referring consumers visiting their websites to some or all of their retailers. Through a model with one manufacturer and two heterogeneous retailers, we investigate whether it is an equilibrium for the manufacturer to refer consumers exclusively to a retailer or nonexclusively to both retailers. Our analysis indicates that nonexclusive referral is the manufacturer's equilibrium choice, if the referral segment market size is sufficiently large; otherwise, exclusive referral is the equilibrium choice. In exclusive referral, the manufacturer would refer consumers to the more cost-efficient and smaller retailer. In the presence of infomediary referral, it is less likely for both exclusive and nonexclusive referrals to be an equilibrium, as the infomediary referral segment grows. We also show our qualitative results are robust even if there were price discrimination among consumers, referral position disparity, local consumers, and asymmetric referral market sizes.

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1 Introduction

More and more consumers are shopping online. According to Interactive Media in Retail Group, “Global business-to-consumer e-commerce sales will pass the 1 trillion euro (\$1.25 trillion) mark by 2013, and the total number of Internet users will increase to approximately 3.5 billion from around 2.2 billion at the end

^{*}School of Economics and Management, Tsinghua University, Beijing, 100084, China, wuh.04@sem.tsinghua.edu.cn

[†]OMIS Department, Santa Clara University, Santa Clara, CA 95053, USA, gcai@scu.edu.

[‡]School of Economics and Management, Tsinghua University, Beijing, 100084, China, Chenj@em.tsinghua.edu.cn.

[§]Department of Management, Kansas State University, Manhattan, KS 66506, USA, csheu@ksu.edu

of 2011” (Montaqim, 2012). Owing to the overwhelming volume of information, however, it is almost impossible for any consumer to digest all available information and find all retailers of their particular interest. As the consultancy firm, Interbrand, puts it, “In a world where consumers are oftentimes overwhelmed with information, the role a brand plays in people’s lives has become all the more important to ensuring a business’ overarching success” (Interbrand, 2012). When searching for certain brand names, consumers are oftentimes led to manufacturer websites. For example, when searching “Acer monitor” or “Thinkpad” or “iPhone” on Google, the first resulting item is the link to the relevant manufacturer’s homepage.

To grasp a share of the lucrative online retailing, many manufacturers bypass retailers and sell directly to consumers. While online direct channels can yield more profits for manufacturers (Chiang et al., 2003), their addition likely causes channel conflict (Cai, 2010). As Home Depot claimed in a letter to more than 1000 of its suppliers, if they add direct channels, Home Depot has “the right to be selective in regard to the vendors we select ... a company may be hesitant to do business with its competitors” (Brooker, 1999). As a matter of fact, Home Depot is not alone. According to Grondin (2011), “in a recent survey conducted by Shopatron, 64 percent of retailers confirmed they would reduce or stop buying from brands that decided to sell direct on their website[s].”

To avoid direct channel conflict, numerous manufacturers opt for *manufacturer referral*, where manufacturers refer customers visiting their websites to some specific retailers. For example, PlayStation website refers its consumers to GameStop, Target, and a couple of other specific retailers for purchases; and Xbox 360 online consumers are referred to retailers such as Sears and Amazon. Even though some manufacturers have launched their own direct channels, they maintain their referral services, such as the “where to buy” highlighted on the homepage of Acer.com, the “Service Locations” on Samsung.com, and the “store-locator” on Nike.com. In fact, thousands of seemingly “direct channels,” such as those of Alpine Electronics, Bosch Home Appliances, Outdoor Gear, and Suzuki Motorcycle, are actually operated as manufacturer referral because their orders are fulfilled by so called “retail-integrated e-commerce.” As Grondin (2011) described, “Shoppers come to your branded website, research products and make their purchases. ... Once the sale is completed, the transaction is handed off to a retailer in the buyer’s area who fulfills the sale.” In some industries, manufacturers have to fully rely on manufacturer referral because of legal regulations (Ghose et al., 2007). For instance, manufacturers in the auto industry are prohibited by franchise law from selling directly to consumers. Therefore, car manufacturers like Ford and GM have built their own referral websites such as

FordDirect.com and GMBuyPower.com.

There are two types of manufacturer referral. In *exclusive referral*, the manufacturer refers consumers to only one retailer. When there are more than two retailers, the sense of exclusiveness can be extended to more than one, but not all, retailers. This type of referral is exemplified by direct “product-to-product” links from the manufacturers to their online retailers and some retail-integrated e-commerce where the manufacturers authorize an exclusive retailer or a selected few to ship all orders (Grondin, 2011). Many manufacturers, like Xbox360 and PlayStation, provide a list of some, but not all, of their retailers. In *nonexclusive referral*, the manufacturer refers consumers to all retailers. For instance, auto manufacturers usually provide an exhaustive list of dealers online. Acer also provides a nonexclusive list of its retailers on its Acerstore homepage. Other nonexclusive referrals include the “Service Locations” on Samsung.com and the “store-locator” on Nike.com.

Obviously, both exclusive and nonexclusive referrals expose a manufacturer’s product to new consumers and, hence, can benefit both the manufacturer and the referred retailers. However, few have discussed which referral type the manufacturer should implement. It is strategically relevant for the manufacturer to understand under what conditions to adopt exclusive referral or nonexclusive referral, in order to optimize its profit.

To address these concerns, we investigate a supply chain model with a manufacturer and two heterogeneous retailers, differing in terms of their market sizes and operational costs. Both retailers compete in a traditional market segment, where consumers already know both retailers and may choose to purchase from either retailer. In addition, new consumers visiting the manufacturer’s website – the referral segment – are referred to either one or both retailers. In the first stage of the game, the manufacturer first chooses the referral type, either exclusive or nonexclusive, and the referred retailer(s) decide whether to accept the referral proposal. The resulting referral type constructs an equilibrium choice, if both the manufacturer and the referred retailers agree on the referral deal. In stage two, the manufacturer determines the wholesale price, and finally the retailers determine their respective retail prices. The whole game is solved backward and characterized in subgame perfect equilibrium.

Our analysis shows that the nonexclusive referral can be the equilibrium choice as long as the referral segment is sufficiently high. The nonexclusive referral leads to bigger realized demand than exclusive

referral, because consumers are exposed to both retailers and may choose where to buy. The bigger demand, however, stimulates the manufacturer to increase the wholesale price, which subsequently worsens double marginalization in both channels. In a trade-off, if the referral segment is sufficiently high, the benefit of a bigger demand surpasses the loss of double marginalization deterioration, such that the manufacturer prefers nonexclusive referral; otherwise, the exclusive referral is more profitable than the nonexclusive referral for the manufacturer and becomes the equilibrium choice. In exclusive referral, the manufacturer's referral preference for a specific retailer is more sensitive to both retailers' operational costs than their initial market sizes. The exclusive selection of a specific retailer also depends on channel substitutability level. If channel substitutability is low, it is more likely that the more cost-efficient retailer would obtain the referral offer; otherwise, the smaller retailer would have the edge. Meanwhile, exclusive referral to the bigger retailer may result in less efficiency for the supply chain that includes the manufacturer and two retailers, because of relatively lessened horizontal competition compared with exclusive referral to the smaller retailer.

Our extended analysis demonstrates that our above qualitative results hold when we change the equal pricing in the baseline model to unequal pricing, when consumers have different evaluations of the positions on the referral list, or when the referral market sizes are asymmetric to the retailers. When there exists a group of local consumers who are aware of only one retailer before the manufacturer referral, it is more likely for the manufacturer to choose exclusive referral over nonexclusive and to choose no referral over exclusive referral as the market size of local consumers grows, because a bigger segment of local consumers downplays the significance of manufacturer referral. The main insights are also sustained in the presence of a referral infomediary where a third-party search engine refers its consumers to retailers. In general, the presence of a bigger infomediary referral segment more significantly shadows the importance of manufacturer referral.

The literature about online referral is relatively recent and small. [Chen et al. \(2002\)](#) focused on infomediary referral and, theoretically, showed a price discrimination effect generated by the referral services. They suggested that exclusive referral outperforms nonexclusive referral for the infomediary. They further pointed out that, if the referral market (reach) is so big that competition between retailers becomes too intense, no retailer can benefit from the referral. Based on an extensive secondary data set of about 27900 samples, [Viswanathan et al. \(2007\)](#) suggested that, with an infomediary referral, "a traditional (auto) dealer can benefit from using these different categories of infomediaries as complementary referral mechanisms."

They developed an analytic model based on Hotelling competition, and identified three different kinds of infomediary referrals in terms of price, product, and portal clusters of consumer usage patterns. They further utilized extensive data to justify their results, and showed that consumers obtaining price information tend to pay less while those obtaining product information tend to pay more. However, [Viswanathan et al. \(2007\)](#) did not provide analytic results regarding whether a nonexclusive referral outperforms an exclusive referral for manufacturer referral. Although infomediary referral and manufacturer referral share some similarities (e.g., the competition at the retailer level is the same), the revenue flow is apparently different at the manufacturer level.

Another type of referral is in-store referral, where a retailer exposes its consumers to its rivals by displaying the links to the competing retailers directly (direct referral), or display the referral link provided by a third-party advertising agency (third-party referral) ([Cai and Chen, 2011](#)). Through a model with two competing retailers, [Cai and Chen \(2011\)](#) demonstrated that both retailers can be better off in either one-way or two-way in-store referral, but possibly at the expense of the consumers, because the referrals may align the retailers' incentives and facilitate implicit collusion. Different from in-store referral, our paper focuses on referral from a manufacturer to two competing retailers.

To the best of our knowledge, the paper by [Ghose et al. \(2007\)](#) is the sole modeling work on manufacturer referral. Through a model with a manufacturer and two retailers, [Ghose et al. \(2007\)](#) compared infomediary referral, manufacturer referral, and a mixture of both. They suggested that “the manufacturer is equally well off enrolling only one retailer as it is enrolling both retailers.” They explained that the manufacturer might want to keep all retailers in the referral because some retailers can be better off. It is worth noting that their work is based on symmetric retailers with identical initial market size and operation costs. In reality, asymmetric settings have been widely seen, where initial market sizes and operational costs are not identical, and retailers are not perfectly substitutable because of their different store features (see [Brynjolfsson and Smith \(2000\)](#) and [Brynjolfsson et al. \(2003\)](#)). As [Brynjolfsson and Smith \(2000\)](#) put it, “While there are a variety of potential unobserved retailer characteristics, one promising candidate is heterogeneity in the “trust” consumers have for the various Internet retailers and the associated value of branding.” We demonstrate the retailer heterogeneousness in Table 1, which shows several products sold by three major retailers who are frequently referred by manufacturers and differ in online market sizes and prices. Theoretically, the cost heterogeneousness is correlated with the price heterogeneousness because a

higher operational cost typically leads to a higher retail price.

Table 1: Heterogenous retailer online market sizes (online revenue in 2013) and retail prices.

Product	Company (market size)	Amazon.com (\$74.5B)	WalMart.com (\$9-10B)	Staples.com (\$11.5B)
Acer C720-2420 Chromebook		229.00	229.00	252.49
Ipad mini(16GB)		279.95	269.00	299.00
Sony Playstation 4		399.00	399.00	399.00
Canon PowerShot SX170		149.00	149.00	173.00
Netgear WNDR3400 N600 Wireless Router		72.80	79.99	79.99
Microsoft Office Home and Business 2013		219.00	219.00	219.00

Source: The data is from Amazon.com, Walmart.com and Staples.com; accessed on 05/04/2014.

Our results differ from those of [Ghose et al. \(2007\)](#), by suggesting that exclusive referral and nonexclusive referral are not equivalent for the manufacturer with asymmetric retailers. In their model, retailers tend to price the same in order to equally share the market under perfect competition. In our model, retailers are heterogeneous and not perfectly substitutable. As a result, retailers can price differently to maximize their profits. Our study complements [Ghose et al. \(2007\)](#) from a different perspective by showing that the interaction between heterogeneous retailers does affect the manufacturer's referral selection and, thus, the manufacturer's referral strategy diverts accordingly.

The literature related to generic retailing and Internet commerce is enormous. Researchers have explored both from different perspectives, such as product development ([Chen and Seshadri, 2007](#)), keyword auctions ([Chen et al., 2009](#)), supply chain models ([Swaminathan and Tayur, 2003](#)), and pricing ([Gaur and Fisher, 2005](#)), among many others. Since our paper has focused on two retailing channels and a referral channel, in particular, multichannel competition is a related area. Researchers have studied the impact of a direct channel ([Chiang et al., 2003](#); [Hsiao and Chen, 2013](#)), advertising ([Liu et al., 2014](#)), the role of an intermediary retailer ([McGuire and Staelin, 1983](#)), effectiveness of equal pricing ([Cattani et al., 2006](#)), channel coordination ([Desai et al., 2004](#); [Ryan et al., 2012](#)), and optimal control of selling channels ([Chen and Seshadri, 2007](#)), among others. We refer interested readers to comprehensive reviews by [Cattani et al. \(2004\)](#) and

[Tsay and Agrawal \(2004\)](#) and a monograph by [Ingene and Parry \(2004\)](#). Due to their special focus, these papers do not directly address manufacturer referral.

Our work contributes to the literature in two main aspects. First, our work is the first to provide a comprehensive comparison of exclusive and nonexclusive manufacturer referral with heterogeneous retailers. Our results are consistent with the industry practice that both exclusive referral and nonexclusive referral coexist. We further provide conditions where a certain referral type will generate more profits for involved firms. This observation could be useful for manufacturers when selecting a certain referral type. Second, we also study the impact of referral position priority, the presence of local consumers, asymmetric referral market sizes, unequal pricing, and infomediary referral. These extensions demonstrate the robustness of our main qualitative insights and provide subtle directions for manufacturers to adjust their referral strategies.

The remainder of this paper is organized as follows. We present the model in Section 2 and compare exclusive referral to nonexclusive referral in Section 3. Extended discussion on the impact of unequal pricing, referral position priority, local consumers, infomediary referral, and asymmetric referral market sizes is included in Section 4. We conclude in Section 5 and all proofs are relegated to the Appendix.

2 The Model

We consider an e-commerce environment where a manufacturer sells its product through two retailers. The retailers are heterogeneous in the sense that they are not perfectly substitutable because of their unique brand names, service levels, shipping policies, return policies, and/or other store features. Accordingly, their initial market sizes and operational costs are not identical in general. The manufacturer owns a website attracting a special group of consumers seeking product information and purchasing outlets. Because of regulation, concern over channel conflict, avoidance of business distraction, and/or location issues, the manufacturer does not sell the product on its own website, or, even if selling directly online, it will rely on the retail-integrated e-commerce. It now faces a choice of referring the visiting consumers to two (i.e., nonexclusive referral), one (i.e., exclusive referral), or none of its retailers. The referral decision is based upon whether or not a certain referral type is more profitable than the others.

In line with [Balachander et al. \(2010\)](#), [Chen et al. \(2002\)](#), and [Ghose et al. \(2007\)](#), we assume that con-

sumers are grouped into two independent market segments: traditional and manufacturer referral. In the *traditional segment*, consumers already know both retailers and may choose to purchase from either retailer. In the *manufacturer referral segment*, consumers visit the manufacturer’s website and will purchase from the retailer(s) referred by the manufacturer. These referred consumers can also be deemed to be manufacturer-loyal consumers who take the manufacturer’s referral seriously and would purchase only from the referred retailer(s). The assumption of segment independence allows us to obtain tractability, and is consistent with the informed/uninformed consumers in Ghose et al. (2007) and the comparison/non-comparison shoppers in Chen et al. (2002). In practice, some consumers belong to both segments. In that situation, we treat these customers as part of the traditional segment because they already know the retailers. A relaxed assumption of mixed market segments does not alter our qualitative insights. In Section 4.3, we show that our qualitative results hold if some consumers know only one retailer prior to the manufacturer referral.

To characterize demand for both retailers, we adopt a utility function of a representative consumer from an aggregate demand perspective in each market segment. In the traditional segment, the representative consumer’s utility minus the purchase cost is given by

$$U_t \equiv \sum_{i=1,2} (\alpha_{ti} D_{ti} - D_{ti}^2/2) - \theta D_{t1} D_{t2} - \sum_{i=1,2} p_i D_{ti}. \quad (1)$$

We use subscript “ ti ” to represent Retailer i in the traditional segment, and “ tj ” the other retailer, where $j = 3 - i, i = 1, 2$. The term α_{ti} represents the initial market size for Retailer i in the traditional segment given that both retail prices $p_i, i = 1, 2$ equal zero. α_{ti} also reflects the representative consumer’s preference for purchasing from Retailer i and captures consumers’ loyalty and value-adding service from Retailer i . The term D_{ti} denotes realized demand for Retailer $i, i = 1, 2$, in the traditional segment. The term θ ($0 \leq \theta < 1$) denotes channel/store substitutability. When $\theta = 0$, the channels are purely monopolistic; while θ approaches 1, the channels converge to purely substitutable.

The utility function (i.e., $\sum_{i=1,2} (\alpha_{ti} D_{ti} - D_{ti}^2/2) - \theta D_{t1} D_{t2}$) was first introduced by Spence (1976), Dixit (1979), and Shubik and Levitan (1980), and has been widely utilized in the literature (see Cai et al., 2012; Ingene and Parry, 2007; Lus and Muriel, 2009; Singh and Vives, 1984). The term “representative consumer” is drawn from the economic notion of “a fictional individual” (Mas-Colell et al., 1995, Chapter 4) and can be considered as a “theoretically average consumer” (Ingene and Parry, 2004, Chapter 11). The utility function implies that the value of using multiple substitutable packages is less than the sum of the

separate values of using each package by itself (Samuelson, 1974). The consumer utility decreases as products become more substitutable. The utility function also encompasses the classical economic features of diminishing marginal rates of substitution and diminishing marginal utility. The representative consumer pays $\sum_{i=1,2} p_i D_{ti}$ supposing it has sufficient budget.

Maximization of U_t yields the demand for each retailer in the traditional segment as follows:

$$D_{ti} = \frac{\alpha_{ti} - \theta \alpha_{tj} - p_i + \theta p_j}{1 - \theta^2}, j = 3 - i, i = 1, 2.$$

This demand function resembles the typical linear demand functions commonly used in the literature (see, e.g., Choi 1991 and McGuire and Staelin 1983). Different from that in Chen et al. (2002) and Ghose et al. (2007), the lower-price retailer does not always capture all demand.

Similarly, in the referral segment, the second representative consumer's utility minus the purchase cost is given by

$$U_r \equiv \sum_{i=1,2} (\alpha_{ri} D_{ri} - D_{ri}^2/2) - \theta D_{r1} D_{r2} - \sum_{i=1,2} p_{ri} D_{ri}, \quad (2)$$

where α_{ri} is the initial market size for Retailer $i = 1, 2$, in the referral segment and D_{r1} and D_{r2} are the corresponding realized demand. We use subscript “ ri ” to represent Retailer i in the referral segment, and “ rj ” the other retailer, where $j = 3 - i, i = 1, 2$. For conciseness and tractability, we adopt a symmetric setting that $\alpha_{ri} = \alpha_m, i = 1, 2$, where α_m denotes the maximum potential referral market size supposing only one retailer is present. The reason for adopting the symmetric assumption is mainly because the consumers do not know the retailers in advance and are assumed to hold no presumed preference toward either retailer. We consider an asymmetric setting in Section 4.5. We let p_{ri} denote the price in the referral segment by Retailer i , which equals p_i under equal pricing. As in Chen et al. (2002) and Ghose et al. (2007), each retailer's retail price is the same for all segments, that is $p_{ri} = p_i$. To avoid channel conflict, more and more retailers choose to honor channel price consistency by using equal pricing. Because of market segmentation and equal pricing, α_m must be sufficiently large to warrant profitability of manufacturer referral for involved firms; otherwise, firms may opt for non-referral.

Maximization of U_r yields the demand for each retailer from the referral segment. More specifically, in the exclusive referral to Retailer i , we have

$$D_{ri} = \alpha_m - p_{ri} \quad \text{and} \quad D_{rj} = 0.$$

Demand under nonexclusive referral is given by

$$D_{ri} = \frac{(1 - \theta)\alpha_m - p_{ri} + \theta p_j}{1 - \theta^2}.$$

Therefore, the total demand for Retailer i is $D_i = D_{ti} + D_{ri}$. For meaningful discussion, each retailer's demand must be nonnegative.

To characterize profit functions, we assume each retailer incurs an operational cost, $c_i, i = 1, 2$ per item. The manufacturer charges the same wholesale price, w , to both retailers, which is consistent with the Robinson-Patman Act. We assume no manufacturer referral fee, which has been commonly seen in practice, such as in the auto industry (Ghose et al., 2007), mainly because the manufacturer earns more profits from wholesaling. Therefore, the profit functions of the retailers and the manufacturer can be described by

$$\begin{aligned}\Pi_i &= (p_i - w - c_i)(D_{ti} + D_{ri}), \\ \Pi_m &= w \sum_{i=1}^2 (D_{ti} + D_{ri}).\end{aligned}\tag{3}$$

In the first stage of the game, the manufacturer chooses the referral type, either exclusive or nonexclusive, and the referred retailer(s) decides whether to accept the referral proposal. If the referred retailer(s) agree upon a certain referral type chosen by the manufacturer, this referral type becomes an equilibrium choice. In the second stage, the manufacturer determines the wholesale price as a Stackelberg leader and, in the third stage, the retailers determine their respective retail prices in a Nash subgame. The game is solved backward resulting in a subgame perfect equilibrium.

3 Equilibrium Analysis

We start with the case without referral, then nonexclusive referral, and then exclusive referral. Finally we study the equilibrium referral decision by comparing the firms' profits among these referral types.

3.1 No Referral

In this benchmark, the manufacturer refers no customers to either retailer. As assumed, in line with Balachander et al. (2010), Chen et al. (2002), and Ghose et al. (2007), the referral segment is independent

of the traditional segment. Therefore, the retailers see only the traditional consumer provided that there is no referral. We use superscript “nr” to denote this no-referral case. The following lemma describes the players’ equilibrium strategies in this subgame.

Lemma 1 *In the no-referral case, the equilibrium is given by*

$$\begin{aligned} p_i^{nr} &= \frac{(10+\theta(1-4\theta))\alpha_{ti} + (6-\theta)c_i + (2-3\theta)(\alpha_{tj}-c_j)}{4(4-\theta^2)}, \\ w^{nr} &= \frac{1}{4}(\alpha_{t1} + \alpha_{t2} - c_1 - c_2), \\ \Pi_m^{nr} &= \frac{(\alpha_{t1} + \alpha_{t2} - c_1 - c_2)^2}{8(2-\theta)(1+\theta)}. \end{aligned}$$

Without referral, the retailer with a larger initial market size (α_{ti}) charges a higher price. The more cost-efficient retailer (i.e., the one with a lower operational cost c_i) prices lower to catch a larger market share. The manufacturer’s wholesale price and profit increase with the retailers’ initial market sizes but decrease with the retailers’ operational costs.

3.2 Nonexclusive Referral

Consider the nonexclusive referral where the manufacturer refers consumers to both retailers. In the game, the manufacturer first offers the referral, and then both retailers decide whether or not to accept the offer. Nonexclusive referral, superscripted by “ne,” is formed only if both retailers accept the referral offer. After that, the manufacturer determines the wholesale price and finally the retailers determine the retail prices. Solving the game gives us the following lemma.

Lemma 2 *In nonexclusive referral, the equilibrium is given by*

$$\begin{aligned} p_i^{ne} &= \frac{(10+\theta(1-4\theta))\alpha_{ti} + 2(6-\theta)c_i + (2-3\theta)(\alpha_{tj}-2c_j) + 2(6-\theta-2\theta^2)\alpha_m}{8(4-\theta^2)}, \quad i = 1, 2, \\ w^{ne} &= \frac{1}{8}(\alpha_{t1} + \alpha_{t2} + 2\alpha_m - 2c_1 - 2c_2), \\ \Pi_m^{ne} &= \frac{(2\alpha_m + \alpha_{t1} + \alpha_{t2} - 2c_1 - 2c_2)^2}{16(2-\theta)(1+\theta)}. \end{aligned}$$

In nonexclusive referral, both retail prices increase with referral segment market size (α_m) because of an extra premium endowed by the referral segment. The wholesale price also increases with the referral segment market size, as does the manufacturer’s profit. Given that $0 \leq \theta < 1$, we find that

$\frac{\partial p_i^{ne}}{\partial \alpha_m} = \frac{2(6-\theta-2\theta^2)}{8(4-\theta^2)} > \frac{\partial w^{ne}}{\partial \alpha_m} = \frac{1}{4}$. Similarly, we have $\frac{\partial p_i^{ne}}{\partial \alpha_{ti}} = \frac{(10+\theta(1-4\theta))}{8(4-\theta^2)} > \frac{\partial w^{ne}}{\partial \alpha_{ti}} = \frac{1}{8}$. These results show that the retail price increases more quickly than the whole sale price as α_m and α_{ti} increase, that is, the double marginalization is worsened as the market sizes grow. Because $\frac{2(6-\theta-2\theta^2)}{8(4-\theta^2)}$ and $\frac{(10+\theta(1-4\theta))}{8(4-\theta^2)}$ decrease with $\theta \in [0, 1)$, however, the double marginalization reduces as the horizontal competition intensifies.

The referral segment creates a new competition front for the retailers. If the referral segment market size (α_m) is sufficiently large, the retail prices and the wholesale price will exceed those without referral. Otherwise, the retailers may reduce their retail prices to compete for the referred consumers; subsequently, the double marginalization can be lessened. This is not necessarily beneficial for the manufacturer, since the wholesale price is subdued. Therefore, the referral segment market size must be sufficiently large to allow the manufacturer to benefit from the nonexclusive referral.

3.3 Exclusive Referral

In exclusive referral, the manufacturer refers consumers to only one retailer, either Retailer 1 or Retailer 2. In either case, the manufacturer first determines which retailer to refer, and then the referred retailer decides whether or not to accept the offer. After that, the manufacturer determines the wholesale price and finally the retailers determine the retail prices. Similar to the nonexclusive referral, the retail prices and the wholesale price increase with the referral segment market size and the traditional market sizes.

Comparing the manufacturer's profits between exclusive referral and the non-referral results in the following observation.

Lemma 3 *In the exclusive referral to Retailer i , there exists a lower-bound threshold, $\hat{\alpha}_m^{Li}$, such that exclusive referral to Retailer i is the equilibrium choice if and only if $\alpha_m \geq \hat{\alpha}_m^{Li}$, as compared to no referral.*

To establish an exclusive referral, both the manufacturer and the referred retailer must be more profitable than no referral; otherwise, the disadvantaged party will back off. Similar to the nonexclusive referral, the referral segment market size (α_m) must be sufficiently large for an exclusive referral. If the referral market size is too small, the referred retailer can be hurt either by a reduced retail price to appeal to the new exclusively referred consumers or by a higher wholesale price caused by additional demand from the referral segment. In this situation, the benefit of additional demand cannot compensate for the loss of

marginal profit. The situation could be worse if the other retailer charges an even lower retail price. Due to heterogeneousness, the thresholds of the referral market size are different for the two retailers.

Two comparative concerns arise: 1. Should the manufacturer refer customers to the more cost-efficient retailer or the less cost-efficient retailer? 2. Should the manufacturer refer customers to the bigger retailer or the smaller retailer? We compare the profits of the manufacturer in two different cases where either Retailer 1 or Retailer 2 is chosen for the exclusive referral, and obtain the following result.

Proposition 1 *Suppose $\alpha_m \geq \max\{\hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}\}$. For the manufacturer, exclusive referral to Retailer 1 outperforms exclusive referral to Retailer 2 if and only if $(\alpha_{t2} - \alpha_{t1}) + \frac{4-3\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$.*

Because retailers are heterogeneous, the selection of an exclusive retailer depends on both retailers' traditional market sizes (α_{ti}) and operational costs (c_i). To single out the impact of initial market sizes and operational costs, we consider two special cases. If the retailers have the same initial market size (i.e., $\alpha_{t1} = \alpha_{t2}$), referring to the more cost-efficient retailer (i.e., lower operational cost) is more profitable for the manufacturer. The reason is that referring consumers to the more cost efficient retailer yields a larger realized demand, since the more cost-efficient retailer charges a relatively lower retail price.

If the retailers are of the same cost efficiency (i.e., $c_1 = c_2$), referring to the smaller retailer is more profitable for the manufacturer. This result is somewhat counterintuitive, since one might argue that the bigger retailer has the edge. The rationale is that referring to the smaller retailer leads to a relatively lower retail price than referring to the bigger retailer, which gives rise to a bigger overall realized demand. While a bigger market size typically earns the advantage in channel competition, our finding indicates that being smaller is not always a disadvantage, especially considering the potential for earning an exclusive referral position from the manufacturer.

In reality, we do observe that some manufacturers refer consumers to small retailers. For example, Acer refers its customers to a small retailer, DR globalTech Inc, “the authorized reseller and merchant of the products and services offered within this [Acer] store.” Although to avoid conflict with big retailers or for easier implementation, manufacturers may refer consumers to big retailers, our finding suggests that doing so is not necessarily in the best interest of the manufacturers.

When both initial market sizes and operational costs become unequal, there is a tradeoff between them

if they vary in opposite directions (e.g., $\alpha_{t1} > \alpha_{t2}$ but $c_1 < c_2$). If the manufacturer would select the bigger retailer, its operational cost must be sufficiently low to compensate for the disadvantage rendered by its bigger initial market size. Note that $\frac{4-3\theta^2}{\theta(1+\theta)}$ strictly decreases with θ and crosses the unity line (i.e., $\frac{4-3\theta^2}{\theta(1+\theta)} = 1$) at $\theta = 0.88$. This implies that when the channels/retailers are more monopolistic (e.g., $\theta < 0.88$), the operational cost carries a higher weight than the initial market size in the tradeoff.

Proposition 1 is based on $\alpha_m \geq \max\{\hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}\}$ such that all participated firms benefit from the exclusive referral. If $\min\{\hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}\} \leq \alpha_m < \max\{\hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}\}$, however, exclusive referral to Retailer 2 could emerge as a better choice for the manufacturer even if $(\alpha_{t2} - \alpha_{t1}) + \frac{4-3\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$, because Retailer 1 would turn down the referral offer.

3.4 The Equilibrium Referral Choice

We are now in a position to explore whether a certain referral type is the equilibrium choice. The manufacturer may choose nonexclusive referral, exclusive referral to a specific retailer, or no referral. By comparing the firms' profits in nonexclusive referral, exclusive referral to Retailer 1, exclusive referral to Retailer 2, and no referral, we obtain the following result.

Proposition 2 Suppose $(\alpha_{t2} - \alpha_{t1}) + \frac{4-3\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$. There exist threshold values, $\hat{\alpha}_m^H$ and $\hat{\alpha}_m^{Li}$, such that

1. If $\alpha_m \geq \hat{\alpha}_m^H$, the equilibrium referral type is nonexclusive referral;
2. If $\min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}] \leq \alpha_m < \hat{\alpha}_m^H$, the equilibrium referral type is exclusive referral to Retailer 1;
3. If $\min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}] \leq \alpha_m < \min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}]$, the equilibrium referral type is exclusive referral to Retailer 2;
4. If $\alpha_m < \min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}]$, the equilibrium referral type is no referral.

We use Figure 1 to illustrate Proposition 2. Both exclusive and nonexclusive referral generate extra demand, but they also justify a higher wholesale price from the manufacturer resulting in higher retail

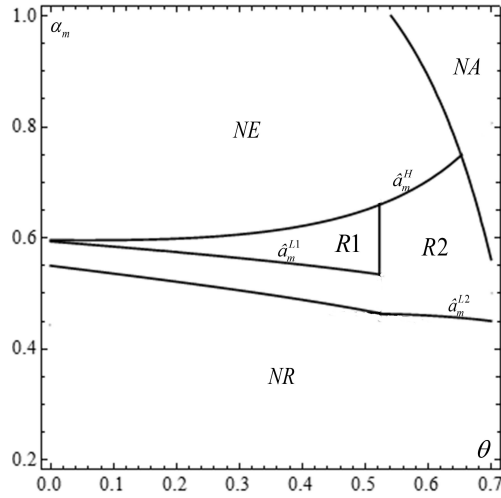


Figure 1: The equilibrium referral type conditional on $\alpha_{t1} = 1$, $\alpha_{t2} = 0.8$, $c_1 = 0$, and $c_2 = 0.05$.

prices. The magnitude of demand increase differs between referral types. In general, the realized demand is bigger in nonexclusive referral than in exclusive referral, because consumers are referred to more retailers providing more choices of where to purchase. Nevertheless, because of higher demand, the manufacturer commands a higher wholesale price and consequently the retailers drive up the retail prices. This worsens the double marginalization in nonexclusive referral more significantly than in exclusive referral. Therefore, the manufacturer faces a trade-off. When α_m is substantially large, the benefit of a higher demand is more significant in nonexclusive referral; thus, nonexclusive referral stands out. In this situation, the referred retailers also benefit from the nonexclusive referral.

On the other hand, when the referral market size is not sufficiently large (i.e., $\alpha_m < \hat{\alpha}_m^H$), the additional demand in nonexclusive referral can be less than that in exclusive referral, especially when store substitutability is high. As a result, the manufacturer has to reduce wholesale price to compensate retailers for decreased retail prices caused by intensified horizontal competition. Therefore, the manufacturer can benefit more from exclusive referral.

A nuance arises between exclusive referral to Retailer 1 and exclusive referral to Retailer 2. As supported by $(\alpha_{t2} - \alpha_{t1}) + \frac{4-3\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$, it is more profitable for the manufacturer to choose exclusive referral to Retailer 1 as long as $\alpha_m > \hat{\alpha}_m^{L1}$. If $\alpha_m < \hat{\alpha}_m^{L1}$, however, the realized referral demand is zero for Retailer 1, such that it is not profitable for the manufacturer to refer consumers to Retailer 1. At this point, exclusive referral to Retailer 2 becomes the only viable choice for the manufacturer. When $\alpha_m < \min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}]$,

all realized referral demand is non-positive; thus, the referral case degenerates into no referral.

The equilibrium referral choice also depends on the retailers' initial market sizes and their operational costs. If $(\alpha_{t2} - \alpha_{t1}) + \frac{4-3\theta^2}{\theta(1+\theta)}(c_2 - c_1) < 0$, exclusive referral to Retailer 2 becomes more favorable than exclusive referral to Retailer 1 when α_m is sufficiently large. Therefore, the equilibrium area of exclusive referral to Retailer 2 encroaches on the equilibrium area of exclusive referral to Retailer 1 as α_{t1} and c_1 increase or as α_{t2} and c_2 decrease. Because the manufacturer gains more profits for exclusively referring consumers to Retailer 2 as α_{t1} and c_1 increase or as α_{t2} and c_2 decrease, the equilibrium area of nonexclusive referral shrinks upward accordingly.

The results in Proposition 2 deviate from those of Ghose et al. (2007), who suggested “the manufacturer is equally well off enrolling only one retailer as it is enrolling both retailers.” The analysis in Ghose et al. (2007) is based on perfect competition assuming retailers are identical. Our work explicitly assumes that retailers are not identical because of differences in brand name, service level, return policy, location, and so on. In the Internet era, consumers still choose one retailer over the other even if they carry the same item. Therefore, a retailer does not win all market with a slightly lower retail price. Based on this more generic assumption, Proposition 2 indicates that the choice of either nonexclusive referral or exclusive referral depends on the referral segment market size, the store substitutability, the retailers' initial market sizes, and their operational costs. Our analytic result is consistent with the fact that some manufacturers use exclusive referral while others use nonexclusive referral.

Would the chosen referral type always result in higher overall supply chain efficiency – the total profit of the manufacturer and two retailers? The answer is no. Intuitively, supply chain efficiency grows with the referral segment market size for both exclusive and nonexclusive referrals. In the equilibrium area as illustrated in Figure 1, no firm would benefit from unilaterally deviating from that area. However, this is not equivalent to optimal performance of the supply chain. As Figure 2 indicates, exclusive referral to Retailer 2 generates more profits for the supply chain even if exclusive referral to Retailer 1 is the equilibrium choice for the manufacturer. This is because exclusive referral to the smaller retailer (i.e., Retailer 2) more significantly intensifies the horizontal competition, hence more soundly reducing the double marginalization than exclusive referral to the bigger retailer (i.e., Retailer 1).

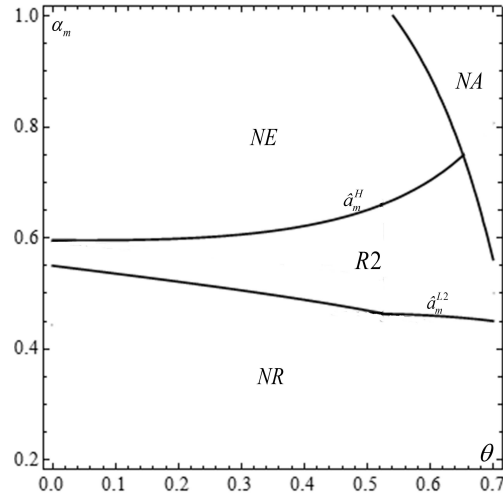


Figure 2: The dominant referral type in terms of supply chain efficiency conditional on $\alpha_{t1} = 1, \alpha_{t2} = 0.8, c_1 = 0, c_2 = 0.05$

4 Extended Discussions

This section discusses the impact of unequal pricing, referral position, local consumers, infomediary referral, and asymmetric referral market size.

4.1 Impact of Unequal Pricing

The preceding analysis is built on the prevailing concept that consumers are treated equally through different channels in terms of pricing, which is widely practiced by retailers to avoid channel conflicts (Cattani et al., 2006). Nevertheless, unequal pricing has also been seen in practice as a tool to discriminate among consumers (see Chen et al., 2002; Ghose et al., 2007). As shown by Morton et al. (2001), prices in online car referral services could be different from regular retail prices. Therefore, we dedicate this section to discussing the potential impact of unequal pricing.

Proposition 3

1. In exclusive referral with unequal pricing, for the manufacturer, referring consumers to the more cost-efficient retailer dominates referring to the less cost-efficient retailer.
2. In nonexclusive referral, the manufacturer is indifferent between equal pricing and unequal pricing.

In exclusive referral, Proposition 3 suggests a slightly different result from Proposition 1, in that the manufacturer is insensitive to the retailers' initial market sizes when selecting an exclusive retailer. Unequal pricing enables the referred retailer to discriminate among consumers by pricing differently in two separate market segments. Therefore, the capability to attract more consumers in the referral segment relies on the retail price in the referral segment, which is mainly determined by the retailer's operational cost rather than its initial market size in the traditional segment. As a result, the manufacturer would refer consumers to the more cost-efficient retailer.

In nonexclusive referral, the wholesale price is the same in both cases of equal and unequal pricing. The retail prices differ though. A closer examination indicates that, for the same retailer, the retail price under equal pricing is in between the two retail prices in two market segments under unequal pricing. Hence, an increase of demand in one market segment is traded off with a decrease of demand in the other market segment. As a result, overall demand remains unchanged for both market segments. Therefore, the manufacturer is indifferent to both pricing formats.

Comparing all firms' profits between equal pricing and unequal pricing, we find that unequal pricing will shift the equilibrium area of each referral type, but the direction of the impact is inconclusive. In most cases, numerically we find the equilibrium area of nonexclusive referral expands while that of exclusive referral shrinks under unequal pricing compared with equal pricing. In some special cases, however, the equilibrium area of exclusive referral encroaches on that of nonexclusive referral when the store substitutability is sufficiently high. Regardless, the qualitative result based on equal pricing as demonstrated in Proposition 2 holds true in the case of unequal pricing.

4.2 Nonexclusive Referral with Position Priority

We have so far assumed that referral positions are symmetric and indifferent in nonexclusive referral. This concept has been supported by the fact that many manufacturers list referred retailers alphabetically. However, as demonstrated by Internet giants like Google and Yahoo, referral position affects the click-through rate. Similarly, in manufacturer referral, consumers might be more inclined to click on one link than another, which could become more apparent if retailers are displayed on different referral pages.

To explore the impact of the referral position, we assume that one position on the referral list has an

advantage over the other. Without loss of generality, we assume the manufacturer would place Retailer 1 in a better position and Retailer 2 in a worse position. Therefore, we have $\alpha_{r1} = (\alpha_m + \delta)$ and $\alpha_{r2} = (\alpha_m - \delta)$, where δ reflects the fact that the discrepancy in referral position alters consumers' valuation of the retailers. For instance, the top position on the referral list might possess advantages over the bottom one and those on the next pages. The utility function follows that of Eq. (2).

Proposition 4 *In nonexclusive referral, the manufacturer is indifferent in assigning the better referral position to either retailer. However, the supply chain is more efficient when the manufacturer assigns the better referral position to Retailer 1 if and only if $\alpha_{t1} - \alpha_{t2} > 2(c_1 - c_2)$.*

One might guess that assigning the better referral position to different retailers would affect the manufacturer's performance; however, Proposition 4 suggests the opposite. The intuition behind Proposition 4 is consistent with current Internet referral practices by auto manufacturers. As shown in the proof, the manufacturer's wholesale price remains constant with respect to the referral position disparity and total demand remains unchanged, although each retailer's demand changes in the opposite direction on account of asymmetric referral positions. Hence, the manufacturer's profit is unchanged in regard to different referral sequences.

However, retailers' profits are not immune to the referral position priority. Retailers change their prices in response to the referral position disparity. Although this does not change overall demand, it redistributes the channel profit from one retailer to the other and eventually alters the supply chain efficiency. Proposition 4 points out that the supply chain can become more efficient if the manufacturer assigns the better referral position to the more powerful retailer (i.e., $(\alpha_{t1} - \alpha_{t2})/(c_1 - c_2) > 2$). Specifically, if both retailers are of the same size, referring to the more cost-efficient retailer renders higher supply chain efficiency; if both retailers are the same in cost efficiency, referring to the bigger retailer is more profitable for the supply chain. In general, supply chain efficiency is more sensitive to cost efficiency than initial market size, because every unit of cost efficiency change requires two units of initial market size. This is because costs have direct impact on retail prices whereas a retailer's initial market size would be offset by the other one's. Thus, it is socially responsible to promote the more cost efficient retailer, unless the other retailer substantially dominates the market.

4.3 The Impact of Local Consumers

For tractability our baseline model does not consider local consumers. One may argue that some consumers in the traditional segment might initially know only one retailer before manufacturer referral but be exposed to the other retailer via manufacturer referral. This section fills the void by investigating the impact of local consumers on the manufacturer's referral decision. In the traditional segment, we now have three subsegments. The first subsegment is the same as our original one who know both retailers. The other two subsegments are local consumers who know only one retailer, respectively, before manufacturer referral and are exposed to manufacturer referral afterward. We use β_i to denote the market size of local consumers who initially know only Retailer $i = 1, 2$.

Without manufacturer referral, the demand to Retailer i becomes

$$D_i = \frac{\alpha_{ti} - \theta\alpha_{tj} - p_i + \theta p_j}{1 - \theta^2} + \beta_i - p_{ri}.$$

If the manufacturer exclusively refers consumers to Retailer i , the consumers of β_i continue to shop in Retailer i , whereas the consumers of β_j , $j = 3 - i$, get to know Retailer i via manufacturer referral. Therefore, the demand to Retailer i consists of the original first subsegment (the local consumers β_i), new consumers from β_j , and new consumers from manufacturer referral, which is described as follows.

$$D_i = \frac{\alpha_{ti} - \theta\alpha_{tj} - p_i + \theta p_j}{1 - \theta^2} + \beta_i - p_i + \frac{(1 - \theta)\beta_j - p_i + \theta p_j}{1 - \theta^2} + \alpha_m - p_i.$$

For Retailer j , the demand becomes

$$D_j = \frac{\alpha_{tj} - \theta\alpha_{ti} - p_j + \theta p_i}{1 - \theta^2} + \frac{(1 - \theta)\beta_j - p_j + \theta p_i}{1 - \theta^2}.$$

If the manufacturer nonexclusively refers consumers to both retailers, the demand to Retailer i becomes

$$D_i = \frac{\alpha_{ti} - \theta\alpha_{tj} + (1 - \theta)(\beta_i + \beta_j) + (1 - \theta)\alpha_m - 3p_i + 3\theta p_j}{1 - \theta^2}.$$

Because of computational complexity, we numerically demonstrate the impact of the existence of local consumers. Numerically, we find that the qualitative result in Proposition 2 holds true as long as the local market sizes $\beta_i, i = 1, 2$, are not substantially different. For any given β_i , we can find the closed-form but lengthy upper bound and lower bound for β_j satisfying the nonnegative demand constraints. Numerically, supposing $\alpha_{t1} = \alpha_{t2} = 1$ and $c_1 = c_2 = 0$, for example, we obtain the upper bound and lower bound of

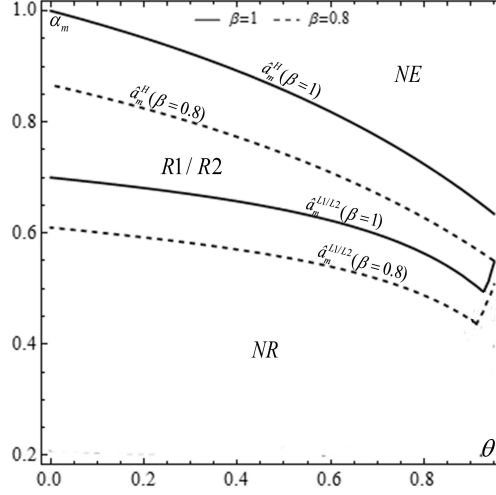


Figure 3: The equilibrium referral type with local consumers given $\alpha_{t1} = \alpha_{t2} = 1$, $c_1 = c_2 = 0$.

β_2 given β_1 , as shown in Table 2. In other words, our main qualitative results sustain if β_i and β_j are in the bounded area.

Table 2: Boundary values of β_i given $\alpha_{t1} = \alpha_{t2} = 1$ and $c_1 = c_2 = 0$.

	θ	0.1	0.3	0.5	0.7	0.9
$\beta_1 = 0.8$	β_2 up-bound	1.77	1.66	1.56	1.48	1.44
	β_2 low-bound	0.60	0.58	0.55	0.51	0.44
$\beta_1 = 1$	β_2 up-bound	1.75	1.66	1.59	1.54	1.54
	β_2 low-bound	0.63	0.60	0.58	0.55	0.48

To demonstrate how the local market size impacts the equilibrium result, we consider the special case where $\beta_i = \beta_j = \beta$. As Figure 3 illustrates, for both values $\beta = 0.8$ and $\beta = 1.0$, the boundary line between nonexclusive referral and exclusive referral moves up when the market is less competitive, but moves down when the market is substantially competitive. When the market is less competitive, a bigger local consumer segment allows the manufacturer to charge a higher wholesale price when implementing exclusive referral. Nevertheless, when the market is substantially competitive and the referral market segment is mid-sized, the manufacturer would more likely opt for nonexclusive referral to attract more consumers because of lessened double marginalization.

Comparing the boundary lines of cases $\beta = 0.8$ and $\beta = 1.0$ as shown in Figure 3, we observe that the boundary lines between no referral and exclusive referral, and between exclusive referral and nonexclusive referral, move up as the local consumer segment grows (i.e., β increases from 0.8 to 1.0). Intuitively, the wholesale price, retail price, and total demand increase with β . The manufacturer becomes more reluctant to implement referral if the referral segment is too small to avoid reducing the wholesale price. Therefore, a bigger presence of local consumers downplays the significance of implementing manufacturer referral.

4.4 The Impact of Infomediary Referral

For brevity our baseline model assumes away the infomediary referral, where a third-party refers consumers to certain retailers, such as autobytel.com, avviva.com, and pricegrabber.com. This decision not only enables us to focus on the main insights of manufacturer referral but also enables us to single out the impact of infomediary referral on manufacturer referral. Referral infomediary is typically powered by search engines that also provide price information and consumer feedback, reflecting the retailer heterogeneousness. To characterize the impact of infomediary referral, we include an independent *infomediary referral market segment*. The infomediary referral representative consumer's utility minus the purchase cost is given by

$$U_h \equiv \sum_{i=1,2} (\alpha_{hi} D_{hi} - D_{hi}^2/2) - \theta D_{h1} D_{h2} - \sum_{i=1,2} p_i D_{hi}$$

where α_{hi} is the initial market size for Retailer $i = 1, 2$, and is assumed to be sufficiently large to warrant nonnegative infomediary referral demand. The term D_{hi} denotes the corresponding realized demand. Since referral infomediary usually refers consumers to a long list of retailers, we adopt a nonexclusive infomediary referral, where consumers are referred to both retailers. Similarly, we assume $\alpha_{hi} = \alpha_h$, $i = 1, 2$. For tractability, we assume the decision of infomediary referral is external and the infomediary referral fee is normalized to zero. Therefore, demand in nonexclusive infomediary referral is given by

$$D_{hi} = \frac{(1 - \theta)\alpha_h - p_i + \theta p_j}{1 - \theta^2}.$$

Total demand for Retailer i is $D_i = D_{ti} + D_{ri} + D_{hi}$. The profit functions of the retailers and the manufacturer are, respectively,

$$\Pi_i = (p_i - w - c_i)(D_{ti} + D_{ri} + D_{hi}), \quad (4)$$

$$\Pi_m = w \sum_{i=1}^2 (D_{ti} + D_{ri} + D_{hi}). \quad (5)$$

The game setting with referral infomediary is similar to our baseline model except that the referral infomediary is exogenously given in this extended discussion. For parsimony, we here reexamine only Propositions 1 and 2 in the presence of a referral infomediary. We start with the exclusive manufacturer referral.

Proposition 5 *Suppose $\alpha_m \geq \max\{\hat{\alpha}_{m(h)}^{L1}, \hat{\alpha}_{m(h)}^{L2}\}$ in the exclusive referral in the presence of infomediary referral. For the manufacturer, exclusive referral to Retailer 1 outperforms exclusive referral to Retailer 2 if and only if $(\alpha_{t2} - \alpha_{t1}) + \frac{6-4\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$.*

Compared to Proposition 1, Proposition 5 demonstrates that the manufacturer's preference for a specific retailer in exclusive referral is more sensitive to the operational cost difference (i.e., $c_2 - c_1$) against the initial market size difference (i.e., $\alpha_{t2} - \alpha_{t1}$), because $\frac{6-4\theta^2}{\theta(1+\theta)} > \frac{4-3\theta^2}{\theta(1+\theta)}$, recalling that without infomediary referral the manufacturer prefers exclusive referral to Retailer 1 as long as $(\alpha_{t2} - \alpha_{t1}) + \frac{4-3\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$. As a result, in the presence of referral infomediary, the manufacturer is more likely to select a retailer with a lower operational cost rather than a retailer with a smaller initial base demand. The presence of the referral infomediary equips the manufacturer with a higher overall demand and a higher wholesale price, which push up the retail price and worsen double marginalization particularly when store substitutability is relatively low. Therefore, the retailer with a lower operational cost becomes more attractive to the manufacturer who seeks to enlarge the overall demand. Given that $\frac{6-4\theta^2}{\theta(1+\theta)}$ decreases with θ , the attractiveness of the retailer with a lower operational cost decreases as store substitutability grows.

Comparing the firms' profits in nonexclusive referral, exclusive referral, and no referral in the presence of infomediary referral, we can obtain a similar result to Proposition 2. That is, in the presence of infomediary referral conditional on $(\alpha_{t2} - \alpha_{t1}) + \frac{6-4\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$, there exist $\hat{\alpha}_{m(h)}^H$ and $\hat{\alpha}_{m(h)}^{Li}$ where $i = 1, 2$, such that

1. If $\alpha_m \geq \hat{\alpha}_{m(h)}^H$, the equilibrium referral type is nonexclusive referral;
2. If $\min[\hat{\alpha}_{m(h)}^H, \hat{\alpha}_{m(h)}^{L1}] \leq \alpha_m < \hat{\alpha}_{m(h)}^H$, the equilibrium referral type is exclusive referral to Retailer 1;
3. If $\min[\hat{\alpha}_{m(h)}^H, \hat{\alpha}_{m(h)}^{L1}, \hat{\alpha}_{m(h)}^{L2}] \leq \alpha_m < \min[\hat{\alpha}_{m(h)}^H, \hat{\alpha}_{m(h)}^{L1}]$, the equilibrium referral type is exclusive referral to Retailer 2;

4. If $\alpha_m < \min[\hat{\alpha}_{m(h)}^H, \hat{\alpha}_{m(h)}^{L1}, \hat{\alpha}_{m(h)}^{L2}]$, the equilibrium referral type is no referral.

Comparing $\hat{\alpha}_{m(h)}^H$ and $\hat{\alpha}_{m(h)}^{Li}$ to $\hat{\alpha}_m^H$ and $\hat{\alpha}_m^{Li}$, respectively, we have the following result.

Proposition 6 *With infomediary referral, $\hat{\alpha}_{m(h)}^H$ and $\hat{\alpha}_{m(h)}^{Li}$ increase with α_h . There exists a threshold, $\tilde{\alpha}_h$, such that, if $\alpha_h \geq \tilde{\alpha}_h$, then $\hat{\alpha}_{m(h)}^H \geq \hat{\alpha}_m^H$ and $\min[\hat{\alpha}_{m(h)}^H, \hat{\alpha}_{m(h)}^{L1}, \hat{\alpha}_{m(h)}^{L2}] \geq \min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}]$.*

Proposition 6 suggests that the equilibrium area of nonexclusive referral and that of exclusive referral shift upward as the infomediary referral segment market size increases. When the infomediary referral segment grows larger, the impact of manufacturer referral on total demand relatively reduces. The incentives for introducing the nonexclusive manufacturer referral, rather than exclusive referral, to intensify the horizontal competition also subdue. Overall, the presence of a big infomediary referral segment shadows the importance of manufacturer referral.

The above observation is valid only if the infomediary referral segment is substantially large. The mere existence of a very small infomediary referral segment can actually make manufacturer referral more preferable because of the retailer competition. Without the retailer competition, for example in a monopoly setting, the manufacturer and the monopoly retailer welcome both manufacturer referral and infomediary referral. With the retailer competition, however, a retailer would reduce the retail price in response to the introduction of the infomediary referral. If the infomediary referral segment size is too small, the additional demand is not sufficient to compensate for the retailer's loss of marginal profit. In this circumstance, manufacturer referral can bring in the desired additional demand to make firms more profitable. Thus, the equilibrium area of manufacturer referral – both nonexclusive referral and exclusive referral – is bigger in the presence of a small infomediary referral segment than without. Particularly, both nonexclusive referral and exclusive referral equilibrium areas, as shown in Figure 1, shift downward.

4.5 Asymmetric Referral Market Sizes

Our baseline model assumes that the two retailers have the same initial referral market size, a symmetric assumption in line with Chen et al. (2002) and Ghose et al. (2007) for conciseness and tractability. In reality, consumers could have different preferences toward the retailers after viewing the retailers' names on the

referral list. To be more comprehensive, we assume an asymmetric referral market size, that is, $\alpha_{ri} = \rho_i \alpha_m$, $i = 1, 2$, where α_m denotes the base potential referral market size and ρ_i is Retailer i 's non-negative relative attraction scale reflecting the referred consumers' preference. The utility function remains the same as Eq. (2). Similarly, in the exclusive referral to Retailer i , we have

$$D_{ri} = \rho_i \alpha_m - p_{ri} \quad \text{and} \quad D_{rj} = 0.$$

Demand under nonexclusive referral is given by

$$D_{ri} = \frac{(\rho_i - \theta \rho_j) \alpha_m - p_{ri} + \theta p_j}{1 - \theta^2}.$$

The analysis is similar to that under symmetric referral market. To be comparable with the baseline model, we extend the original Proposition 1 and Proposition 2. First consider the exclusive referral.

Proposition 7 *When referral market sizes are asymmetric, suppose $\alpha_m \geq \max\{\tilde{\alpha}_m^{L1}, \tilde{\alpha}_m^{L2}\}$. For the manufacturer, exclusive referral to Retailer 1 outperforms exclusive referral to Retailer 2 if and only if $(\alpha_{t2} - \alpha_{t1}) + \frac{(4+\theta(1-2\theta))\alpha_m}{\theta(\theta+1)}(\rho_1 - \rho_2) + \frac{4-3\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$.*

Proposition 7 is based on $\alpha_m \geq \max\{\tilde{\alpha}_m^{L1}, \tilde{\alpha}_m^{L2}\}$, such that all participated firms benefit from the exclusive referral. We find that $\tilde{\alpha}_m^{Li}$ decreases with ρ_i , which means that, as Retailer i 's relative attraction scale (ρ_i) grows, it is easier for the manufacturer and the referred Retailer i to benefit from the exclusive referral.

Obviously, if the retailers' relative attraction scales in the referral market are the same (i.e., $\rho_1 = \rho_2 = 1$), Proposition 7 degenerates into Proposition 1, that is, for the manufacturer, exclusive referral to Retailer 1 outperforms exclusive referral to Retailer 2 if and only if $(\alpha_{t2} - \alpha_{t1}) + \frac{4-3\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$. If $\rho_1 > \rho_2$, Proposition 7 indicates that it is more likely for the manufacturer to refer consumers to Retailer 1, because the additional term of $\frac{(4+\theta(1-2\theta))\alpha_m}{\theta(\theta+1)}(\rho_1 - \rho_2)$ is positive. This result suggests it is more likely for the manufacturer to refer consumers to the retailer that is more appealing to the referred consumers (i.e., the retailer with a higher ρ_i).

Note that the above discussion implicitly assumes that ρ_i is independent of α_{ti} . What if ρ_i is correlated with α_{ti} ? It is easy to infer that the manufacturer would refer consumers to Retailer i as long as ρ_i is substantially big. To demonstrate this point, we now consider the special case where $\rho_i = \alpha_{ti}$. If so, for

the manufacturer, exclusive referral to Retailer 1 outperforms exclusive referral to Retailer 2 if and only if $(\theta(1 + \theta) - (4 + \theta(1 - 2\theta))\alpha_m)(\alpha_{t2} - \alpha_{t1}) + (4 - 3\theta^2)(c_2 - c_1) \geq 0$. In this case, supposing the retailers are equally cost-efficient (i.e., $c_2 = c_1$) and the referral market size is sufficiently small such that $\theta(1 + \theta) - (4 + \theta(1 - 2\theta))\alpha_m > 0$, the manufacturer will still refer consumers to the smaller retailer, a result qualitatively equivalent to Proposition 1 where $\rho_1 = \rho_2 = 1$. However, if the referral market size is sufficiently large that $\theta(1 + \theta) - (4 + \theta(1 - 2\theta))\alpha_m < 0$, the manufacturer will instead refer consumers to the bigger retailer, because the referral market becomes more lucrative thanks to Retailer 2's higher attraction rate.

The analysis of nonexclusive referral is similar to that in Section 3.2 and, thus is skipped here. We now compare the results between nonexclusive referral and exclusive referral.

Proposition 8 *Given $\frac{(4+\theta(1-2\theta))\alpha_m}{\theta(\theta+1)}(\rho_1 - \rho_2) + (\alpha_{t2} - \alpha_{t1}) + \frac{4-3\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$, there exists threshold values, $\tilde{\alpha}_m^{Hi}$ and $\tilde{\alpha}_m^{Li}$, such that*

1. *If $\alpha_m \geq \max[\tilde{\alpha}_m^{H1}, \tilde{\alpha}_m^{H2}]$, the equilibrium referral type is exclusive referral to Retailer 1;*
2. *If $\tilde{\alpha}_m^{H2} \leq \alpha_m < \max[\tilde{\alpha}_m^{H1}, \tilde{\alpha}_m^{H2}]$, the equilibrium referral type is nonexclusive referral;*
3. *If $\min[\tilde{\alpha}_m^{H2}, \tilde{\alpha}_m^{L1}] \leq \alpha_m < \tilde{\alpha}_m^{H2}$, the equilibrium referral type is exclusive referral to Retailer 1;*
4. *If $\min[\tilde{\alpha}_m^{H2}, \tilde{\alpha}_m^{L1}, \tilde{\alpha}_m^{L2}] \leq \alpha_m < \min[\tilde{\alpha}_m^{H2}, \tilde{\alpha}_m^{L1}]$, the equilibrium referral type is exclusive referral to Retailer 2;*
5. *If $\alpha_m < \min[\tilde{\alpha}_m^{H2}, \tilde{\alpha}_m^{L1}, \tilde{\alpha}_m^{L2}]$, the equilibrium referral type is no referral.*

Proposition 2 is a special case of Proposition 8. Specifically, when $\rho_1 = \rho_2 = 1$, Proposition 8 degenerates into Proposition 2. In fact, the first item of Proposition 8 disappears, as long as $\frac{\rho_1}{\rho_2}$ is in a reasonable range (i.e., $\frac{1}{\hat{\rho}_K(\theta)} < \frac{\rho_1}{\rho_2} < \hat{\rho}_K(\theta)$ as defined in the proof).

Following Proposition 7, we know that the manufacturer has more incentives to refer to the retailer with a bigger ρ_i . Therefore, the equilibrium area of referring to Retailer 2 is bigger if $\rho_1 < \rho_2$ (see Figure 4); otherwise if $\rho_1 > \rho_2$, the equilibrium area of referring to Retailer 1 is bigger (see Figure 5). In fact, if ρ_1 is sufficiently big, the whole supply chain can also benefit from exclusively referring to Retailer 1, because the benefit of a bigger market size outpaces the drawback of more intense horizontal competition.

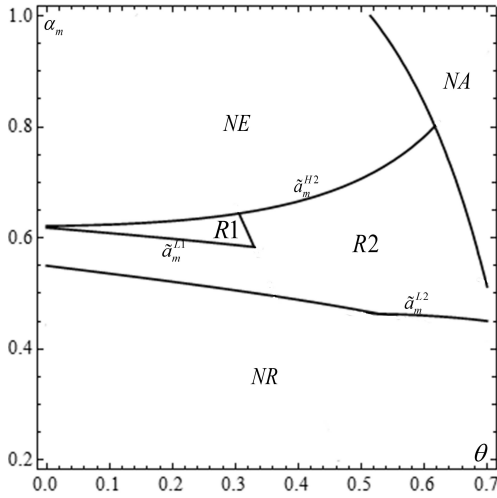


Figure 4: The equilibrium referral type conditional on $\alpha_{t1} = 1$, $\alpha_{t2} = 0.8$, $c_1 = 0$, $c_2 = 0.05$, $\rho_1 = 0.96$, and $\rho_2 = 1$.

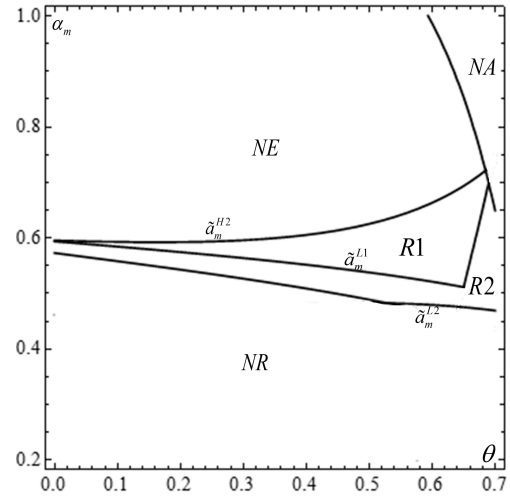


Figure 5: The equilibrium referral type conditional on $\alpha_{t1} = 1$, $\alpha_{t2} = 0.8$, $c_1 = 0$, $c_2 = 0.05$, $\rho_1 = 1$, and $\rho_2 = 0.96$.

When $\frac{\rho_1}{\rho_2}$ goes to extreme (e.g., $\frac{\rho_1}{\rho_2} = 10$), we find that the manufacturer would refer to Retailer 1 as long as the referral market size (α_m) is sufficiently large. This is because referring to the much bigger retailer (i.e., Retailer 1) allows the manufacturer to price higher and correspondingly leads to more profits.

5 Conclusion

This paper investigates manufacturer referral where the manufacturer refers its visiting consumers to certain retailer(s). As e-tailing grows more and more competitive, manufacturer referral prevails in the current e-commerce environment as a tool to avoid overly intensive channel conflict with retailers. In a model with two heterogeneous retailers, we study both exclusive referral and nonexclusive referral. In exclusive referral, the manufacturer prefers referring consumers to the more cost-efficient retailer if the retailers are of the same size, or the smaller retailer if the retailers are equally cost efficient. However, the equilibrium choice of exclusive referral to the bigger retailer can lead to supply chain inefficiency. The nonexclusive referral emerges as the equilibrium choice for all firms as long as the manufacturer referral market size is sufficiently large.

Our extended analysis shows that, first, if retailers use unequal pricing to discriminate among their

consumers, the selection of an exclusively referred retailer solely depends on their cost efficiencies, while the manufacturer is indifferent between equal and unequal pricing in nonexclusive referral. Second, if referral positions differ in nonexclusive referral, the supply chain is better off if the manufacturer assigns the better position to the sufficiently bigger or more cost-efficient retailer. Third, a bigger market size of local consumers in the traditional segment downplays the significance of implementing manufacturer referral. Fourth, in the presence of an infomediary referral, it is more likely for firms to prefer manufacturer referral when the infomediary referral segment is sufficiently small; otherwise, the benefits of manufacturer referral become less attractive as the infomediary referral segment grows. Finally, with asymmetric referral market sizes, although the manufacturer will more likely refer consumers to the retailer with a higher attraction rate, our main qualitative results hold as long as the retailers' relative attraction rates are not too skewed.

Several aspects of the model warrant further comments. First, as shown previously, different referral types (i.e., exclusive referral and nonexclusive referral) and referral positions yield more profits to the advantaged retailer. Naturally, retailers would have incentives to compete for an exclusive referral slot or a better referral position by paying a premium. To that end, the manufacturer can set up a competition mechanism, such as an online auction, to select a winner. In order to avoid channel conflict, however, the manufacturer might resort to nonexclusive referral or randomly assigning referral positions to retailers, which ratifies the popularity of nonexclusive referral.

Second, the existing literature and our model have assumed a single manufacturer due to tractability; however, multiple competitive manufacturers may provide referral service at the same time, such as GM and Ford in the auto industry. Naturally, the horizontal competition intensifies as more manufacturers compete. In this situation, we speculate that manufacturers would have fewer incentives to refer their consumers to all retailers, because the manufacturers' incentives to bring down the double marginalization has been subdued by the increased horizontal competition. As a result, the manufacturers might have more incentives to use exclusive referral. This speculation is consistent with the realization that many manufacturers do not refer their consumers to all retailers. Another potential direction is to study how the manufacturer refers multiple products through different retailers. Given that the computational complexity will be quite intense in a game that includes multiple manufacturers and multiple retailers, some simulation approaches can be applied for this task.

Third, the firms' bargaining power will certainly affect their decisions on many issues, such as the prices

and the manufacturer's referral decision. If a retailer's bargaining power on the wholesale price increases, we expect to see more manufacturer referral to the retailer with less bargaining power, because referring more consumers to the more powerful retailers undercuts the manufacturer's marginal profit. In Walmart and Home Depot cases, the manufacturer could more likely refer its visiting consumers to a smaller competing retailer, unless Walmart and Home Depot enforce a referral clause in their contracts.

Finally, demand has been certain in our model. Although this assumption is consistent with the related literature for tractability, it would be intriguing to investigate how demand uncertainty affects the manufacturer's choice.

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APPENDIX: Online Supplements

Proof of Lemma 1: We solve the no-referral game backward. We first consider retailers' price competition given the wholesale price, and then solve the wholesale price. The same computation sequence is followed in games with referral.

The original profit functions without referral are given by

$$\begin{aligned}\Pi_1^{nr} &= (p_1 - w - c_1) \left(\frac{\alpha_{t1} - \theta\alpha_{t2} - p_1 + \theta p_2}{1 - \theta^2} \right), \\ \Pi_2^{nr} &= (p_2 - w - c_2) \left(\frac{\alpha_{t2} - \theta\alpha_{t1} - p_2 + \theta p_1}{1 - \theta^2} \right), \\ \Pi_m^{nr} &= w \left(\frac{\alpha_{t1} - \theta\alpha_{t2} - p_1 + \theta p_2}{1 - \theta^2} + \frac{\alpha_{t2} - \theta\alpha_{t1} - p_2 + \theta p_1}{1 - \theta^2} \right).\end{aligned}$$

The second order conditions are given by

$$\frac{\partial^2 \Pi_1^{nr}}{\partial p_1^2} = -\frac{2}{1 - \theta^2} < 0, \quad \text{and} \quad \frac{\partial^2 \Pi_2^{nr}}{\partial p_2^2} = -\frac{2}{1 - \theta^2} < 0.$$

The following first order conditions (FOCs) suffice to assure maximization of profits for both retailers.

$$\frac{\partial \Pi_1^{nr}}{\partial p_1} = 0 \quad \text{and} \quad \frac{\partial \Pi_2^{nr}}{\partial p_2} = 0.$$

Solving the FOCs gives us

$$\begin{aligned}p_1^{nr}(w^{nr}) &= \frac{(2 - \theta^2) \alpha_{t1} - \theta\alpha_{t2} + 2c_1 + \theta c_2 + w(2 + \theta)}{4 - \theta^2}, \\ p_2^{nr}(w^{nr}) &= \frac{(2 - \theta^2) \alpha_{t2} - \theta\alpha_{t1} + 2c_2 + \theta c_1 + w(2 + \theta)}{4 - \theta^2}.\end{aligned}$$

Substituting the best response retail prices in terms of the wholesale price into the manufacturer's profit function, from the FOC, $\frac{\partial \Pi_m^{nr}}{\partial w} = 0$, we obtain the optimal wholesale price that optimizes the manufacturer's profit:

$$w^{nr} = \frac{1}{4}(\alpha_{t1} + \alpha_{t2} - c_1 - c_2).$$

The above wholesale price is unique, since

$$\frac{\partial^2 \Pi_m^{nr}}{\partial w^2} = -\frac{4}{2 + \theta - \theta^2} < 0.$$

Substituting w^{nr} into $p_1^{nr}(w^{nr})$ and $p_2^{nr}(w^{nr})$, we obtain

$$p_1^{nr} = \frac{(10 + \theta(1 - 4\theta)) \alpha_{t1} + (6 - \theta) c_1 + (2 - 3\theta)(\alpha_{t2} - c_2)}{4(4 - \theta^2)},$$

$$p_2^{nr} = \frac{(10+\theta(1-4\theta))\alpha_{t2} + (6-\theta)c_2 + (2-3\theta)(\alpha_{t1}-c_1)}{4(4-\theta^2)}.$$

The firms' optimal profits are

$$\begin{aligned}\Pi_m^{nr} &= \frac{(\alpha_{t1} + \alpha_{t2} - c_1 - c_2)^2}{8(2-\theta)(1+\theta)}, \\ \Pi_1^{nr} &= \frac{((6+\theta-3\theta^2)(\alpha_{t1}-c_1) - (2+3\theta-\theta^2)(\alpha_{t2}-c_2))^2}{16(4-\theta^2)^2(1-\theta^2)}, \\ \Pi_2^{nr} &= \frac{((6+\theta-3\theta^2)(\alpha_{t2}-c_2) - (2+3\theta-\theta^2)(\alpha_{t1}-c_1))^2}{16(4-\theta^2)^2(1-\theta^2)}.\end{aligned}$$

To make sure that all firms will be in the game, we must have nonnegative profits, which are ensured by

$$\begin{aligned}D_1^{nr} &= \frac{(6+\theta-3\theta^2)(\alpha_{t1}-c_1) - (2+3\theta-\theta^2)(\alpha_{t2}-c_2)}{4(4-\theta^2)(1-\theta^2)} \geq 0, \\ D_2^{nr} &= \frac{(6+\theta-3\theta^2)(\alpha_{t2}-c_2) - (2+3\theta-\theta^2)(\alpha_{t1}-c_1)}{4(4-\theta^2)(1-\theta^2)} \geq 0.\end{aligned}$$

Since $c_1 < \alpha_{t1}$, $c_2 < \alpha_{t2}$, and

$$(6+\theta-3\theta^2) - (2+3\theta-\theta^2) = 2(2+\theta)(1-\theta) > 0.$$

The above nonnegative demand constraint is equivalent to

$$\frac{2+3\theta-\theta^2}{6+\theta-3\theta^2} \leq \frac{\alpha_{t1}-c_1}{\alpha_{t2}-c_2} \leq \frac{6+\theta-3\theta^2}{2+3\theta-\theta^2}.$$

The nonnegative demand condition is also equivalent to that of nonnegative marginal profits as follows:

$$\begin{aligned}p_1^{nr} - w^{nr} - c_1 &= \frac{(6+\theta-3\theta^2)(\alpha_{t1}-c_1) - (2+3\theta-\theta^2)(\alpha_{t2}-c_2)}{4(4-\theta^2)} \geq 0, \\ p_2^{nr} - w^{nr} - c_2 &= \frac{(6+\theta-3\theta^2)(\alpha_{t2}-c_2) - (2+3\theta-\theta^2)(\alpha_{t1}-c_1)}{4(4-\theta^2)} \geq 0.\end{aligned}$$

Q.E.D.

Proof of Lemma 2: Consider the nonexclusive referral. To avoid trivial case, we assume the referral segment market size (α_m) is sufficiently large so nonnegative demand is satisfied in both the traditional and referral segments. The profit functions are thus given by

$$\begin{aligned}\Pi_1^{ne} &= (p_1 - w - c_1) \left(\frac{\alpha_{t1} - \theta\alpha_{t2} - p_1 + \theta p_2}{1 - \theta^2} + \frac{\alpha_m - \theta\alpha_m - p_1 + \theta p_2}{1 - \theta^2} \right), \\ \Pi_2^{ne} &= (p_2 - w - c_2) \left(\frac{\alpha_{t2} - \theta\alpha_{t1} - p_2 + \theta p_1}{1 - \theta^2} + \frac{\alpha_m - \theta\alpha_m - p_2 + \theta p_1}{1 - \theta^2} \right),\end{aligned}$$

$$\Pi_m^{ne} = w \sum_{i=1,2} \left(\frac{\alpha_{ti} - \theta \alpha_{tj} - p_i + \theta p_j}{1 - \theta^2} + \frac{\alpha_m - \theta \alpha_m - p_i + \theta p_j}{1 - \theta^2} \right).$$

Solving the FOCs for both retailers, we have

$$p_i^{ne}(w^{ne}) = \frac{(4 + 2\theta)w^{ne} + (2 - \theta^2)\alpha_{ti} - \theta \alpha_{tj} + (2 - \theta - \theta^2)\alpha_m + 4c_i + 2\theta c_j}{8 - 2\theta^2}, j = 3 - i, i = 1, 2.$$

The second order conditions are given by

$$\frac{\partial^2 \Pi_1^{ne}}{\partial p_1^2} = -\frac{4}{1 - \theta^2} < 0 \text{ and } \frac{\partial^2 \Pi_2^{ne}}{\partial p_2^2} = -\frac{4}{1 - \theta^2} < 0.$$

Substituting the best response retail prices in terms of the wholesale price into the manufacturer's profit function, from the FOC, we obtain the wholesale price that optimizes the manufacturer's profit:

$$w^{ne} = \frac{1}{8}(\alpha_{t1} + \alpha_{t2} + 2\alpha_m - 2c_1 - 2c_2).$$

The above wholesale price is unique, because

$$\frac{\partial^2 \Pi_m^{ne}}{\partial w^2} = -\frac{8}{2 + \theta - \theta^2} < 0.$$

Substituting w^{ne} into $p_1^{ne}(w^{ne})$ and $p_2^{ne}(w^{ne})$, we obtain the optimal retail prices:

$$p_i^{ne} = \frac{(10 + \theta(1 - 4\theta))\alpha_{ti} + 2(6 - \theta)c_i + (2 - 3\theta)(\alpha_{tj} - 2c_j) + 2(6 - \theta - 2\theta^2)\alpha_m}{8(4 - \theta^2)}, j = 3 - i, i = 1, 2.$$

Furthermore, we obtain the optimal profit as follows:

$$\Pi_m^{ne} = \frac{(2\alpha_m + \alpha_{t1} + \alpha_{t2} - 2c_1 - 2c_2)^2}{16(2 - \theta)(1 + \theta)}.$$

Q.E.D.

Proof of Lemma 3: Without loss of generality, we consider exclusive referral to Retailer 1. We use superscript “ ri ” to represent the case of exclusive referral to Retailer i . For exclusive referral to Retailer 1 to be the equilibrium choice, both the manufacturer and Retailer 1 must be better off than in no referral. The discussion is also conditional on nonnegative demand. We enumerate on them sequentially.

(i) That the manufacturer is better off in the exclusive referral to Retailer 1 than in no referral is equivalent to

$$\sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{nr}} = \frac{2(4 + \theta(5 - \theta - 2\theta^2))\alpha_m + 2((4 + \theta(1 - \theta(3 + \theta)))\alpha_{t1} + (4 + (2 - \theta)\theta)\alpha_{t2} - (4 + 3\theta)(2 - \theta^2)c_1 - (4 + (2 - \theta)\theta)c_2)}{4\sqrt{(1 + \theta)(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))}} - \frac{\alpha_{t1} + \alpha_{t2} - c_1 - c_2}{2\sqrt{2}\sqrt{2 + \theta - \theta^2}}$$

$$\geq 0.$$

Hence, $\sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{nr}}$ is a linear function of α_m . Meanwhile,

$$\frac{\partial \left(\sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{nr}} \right)}{\partial \alpha_m} = \frac{4 + \theta (5 - \theta - 2\theta^2)}{2\sqrt{(1 + \theta)(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))}} > 0.$$

Thus, there exists a single crossing point, defined as $\alpha_{m(e)}^{m1}$, such that $\sqrt{\Pi_m^{r1}} = \sqrt{\Pi_m^{nr}}$ when $\alpha_m = \alpha_{m(e)}^{m1}$.

(ii) That Retailer 1 is better off in the exclusive referral to Retailer 1 than in no referral is equivalent to

$$\begin{aligned} & \sqrt{\Pi_1^{r1}} - \sqrt{\Pi_1^{nr}} = \\ & \frac{\sqrt{2 - \theta^2}((1 - \theta)(1 + \theta)(32 + \theta(16 - 3\theta(5 + 2\theta)))\alpha_m + (32 + \theta(32 - \theta(19 + 3\theta(8 - \theta^2))))\alpha_{t1} \\ & - (16 + \theta(28 - \theta(2 + 3\theta(5 + \theta))))\alpha_{t2} - (64 + \theta(48 - \theta(66 + \theta(46 - 3\theta(5 + 3\theta))))c_1 \\ & + (16 + \theta(28 - \theta(2 + 3\theta(5 + \theta))))c_2)}{2\sqrt{1 - \theta^2}(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))} \\ & - \frac{(6 + \theta - 3\theta^2)\alpha_{t1} - (2 + (3 - \theta)\theta)\alpha_{t2} - (6 + \theta(1 - 3\theta))c_1 + (2 + (3 - \theta)\theta)c_2}{4\sqrt{1 - \theta^2}(4 - \theta^2)} \geq 0. \end{aligned}$$

Similarly, $\sqrt{\Pi_1^{r1}} - \sqrt{\Pi_1^{nr}}$ is a linear function of α_m , and

$$\frac{\partial \left(\sqrt{\Pi_1^{r1}} - \sqrt{\Pi_1^{nr}} \right)}{\partial \alpha_m} = \frac{\sqrt{(1 - \theta^2)(2 - \theta^2)}(32 + \theta(16 - 3\theta(5 + 2\theta)))}{2(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))} > 0.$$

Thus, there exists a single crossing point, defined as $\alpha_{m(e)}^{p1}$, such that $\sqrt{\Pi_1^{r1}} = \sqrt{\Pi_1^{nr}}$ when $\alpha_m = \alpha_{m(e)}^{p1}$.

(iii) We also ensure that the realized demand from the referral market is nonnegative. The demand is a linear function of α_m as below:

$$\begin{aligned} & ((128 + \theta(72 - \theta(125 + \theta(65 - 2\theta(15 + 7\theta))))\alpha_m - (64 + \theta(40 - \theta(63 + \theta(37 - \theta(15 + 8\theta))))\alpha_{t1} \\ & - (16 - \theta(12 + \theta(26 - \theta(5 + 8\theta))))\alpha_{t2} - (2 - \theta^2)(32 + \theta(16 - 3\theta(5 + 2\theta)))c_1 \\ & + (16 - \theta(12 + \theta(26 - \theta(5 + 8\theta))))c_2) \\ & \frac{}{2(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))} \geq 0. \end{aligned}$$

The corresponding slope with respect to α_m is

$$\frac{1}{240} \left(112 + \frac{45}{2 + \theta} + \frac{144}{8 - 5\theta^2} + \frac{45 - 25\theta}{6 + \theta - 3\theta^2} \right) > 0.$$

Define $\alpha_{m(e)}^{d1}$ such that the referral market demand is nonnegative as long as $\alpha_m \geq \alpha_{m(e)}^{d1}$. If $\alpha_m < \alpha_{m(e)}^{d1}$, in our analysis, Retailer 1 receives zero demand rather than a negative demand.

Let $\hat{\alpha}_m^{L1} \equiv \max[\alpha_{m(e)}^{m1}, \alpha_{m(e)}^{p1}, \alpha_{m(e)}^{d1}]$. Therefore, if $\alpha_m \geq \hat{\alpha}_m^{L1}$, exclusive referral to Retailer 1 is the equilibrium choice as compared to no referral. Otherwise, no referral is the equilibrium choice. Similar analysis applies to Retailer 2, and $\hat{\alpha}_m^{L2} \equiv \max[\alpha_{m(e)}^{m2}, \alpha_{m(e)}^{p2}, \alpha_{m(e)}^{d2}]$. Q.E.D.

Proof of Proposition 1: Compare two different exclusive referral scenarios where either Retailer 1 or Retailer 2 is selected in the referral. Our following discussion is based on the assumption that the referral segment market size is sufficiently large. More specifically, this assumption ensures nonnegative demand in both traditional and referral markets. Furthermore, an exclusive referral will not occur if either the manufacturer or the referred retailer cannot benefit from such a referral. Therefore, if an exclusive referral does occur, the referring and referred firms must be no worse off from the referral. We examine this assumption throughout our discussion.

Without loss of generality, here we mainly consider the case of referring to Retailer 1 and the results can be easily extended to referring to Retailer 2. The profit functions are given by

$$\begin{aligned}\Pi_1^{r1} &= (p_1 - w - c_1) \left(\frac{\alpha_{t1} - \theta\alpha_{t2} - p_1 + \theta p_2}{1 - \theta^2} + \alpha_m - p_1 \right), \\ \Pi_2^{r1} &= (p_2 - w - c_2) \left(\frac{\alpha_{t2} - \theta\alpha_{t1} - p_2 + \theta p_1}{1 - \theta^2} \right), \\ \Pi_m^{r1} &= w \left(\frac{\alpha_{t1} - \theta\alpha_{t2} - p_1 + \theta p_2}{1 - \theta^2} + \alpha_m - p_1 + \frac{\alpha_{t2} - \theta\alpha_{t1} - p_2 + \theta p_1}{1 - \theta^2} \right).\end{aligned}$$

The second order conditions are given by

$$\frac{\partial^2 \Pi_1^{r1}}{\partial p_1^2} = -\frac{2(2 - \theta^2)}{1 - \theta^2} < 0 \quad \text{and} \quad \frac{\partial^2 \Pi_2^{r1}}{\partial p_2^2} = -\frac{2}{1 - \theta^2} < 0.$$

Solving the FOCs, we have

$$\begin{aligned}p_1^{r1}(w^{r1}) &= \frac{(4 + \theta - 2\theta^2)w^{r1} + (2 - \theta^2)\alpha_{t1} - \theta\alpha_{t2} + 2(1 - \theta^2)\alpha_m + 2(2 - \theta^2)c_1 + \theta c_2}{8 - 5\theta^2}, \\ p_2^{r1}(w^{r1}) &= \frac{(4 + 2\theta - 2\theta^2 - \theta^3)w^{r1} - (3\theta - 2\theta^3)\alpha_{t1} + (4 - 3\theta^2)\alpha_{t2} + (\theta - \theta^3)\alpha_m + \theta(2 - \theta^2)c_1 + 2(2 - \theta^2)c_2}{8 - 5\theta^2}.\end{aligned}$$

Substituting the best response retail prices in terms of the wholesale price into the manufacturer's profit function, from the FOC, we obtain the optimal wholesale price that optimizes the manufacturer's profit:

$$w^{r1} = \frac{(4 + \theta(5 - \theta - 2\theta^2))\alpha_m + (4 + \theta(1 - \theta(3 + \theta)))\alpha_{t1} + (4 + (2 - \theta)\theta)\alpha_{t2} - (4 + 3\theta)(2 - \theta^2)c_1 - (4 + 2\theta - \theta^2)c_2}{2(2 + \theta)(6 + \theta(1 - 3\theta))}.$$

Substituting w^{r1} into $p_1^{r1}(w^{r1})$ and $p_2^{r1}(w^{r1})$, we obtain the corresponding optimal retail prices. Similarly, we can obtain the results for the case where the manufacturer exclusively refers to Retailer 2. Denote Π_m^{ri} as the manufacturer's profit when the manufacturer exclusively refers to Retailer i . Due to the structure symmetry, we obtain the optimal profits as follows:

$$\Pi_m^{ri} = \frac{((4+\theta(5-\theta-2\theta^2))\alpha_m + (4+\theta(1-\theta(3+\theta)))\alpha_{ti} + (4+(2-\theta)\theta)\alpha_{tj} - (4+3\theta)(2-\theta^2)c_i - (4+2\theta-\theta^2)c_j)^2}{4(1+\theta)(2+\theta)(8-5\theta^2)(6+\theta-3\theta^2)}, j = 3-i, i = 1, 2.$$

Moreover,

$$\sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{r2}} = \frac{(\theta+1)((\alpha_{t2} - \alpha_{t1})\theta(\theta+1) + (c_2 - c_1)(4-3\theta^2))}{2\sqrt{(\theta+1)(\theta+2)(8-5\theta^2)(\theta(1-3\theta)+6)}}.$$

Given $\sqrt{\Pi_m^{r1}} + \sqrt{\Pi_m^{r2}}$ is positive, the result in Proposition 1 is concluded.

Our model implicitly assumes that there is an adequate number of consumers who find exclusively referred retailers' information from the manufacturer's website but won't search other retailers' information. This assumption is supported by the fact that 1) many consumers' search costs are not trivial because of time constraint; 2) consumers generally cannot confirm all retailers from the manufacturer; and 3) manufacturers can disguise themselves as referred retailers via retail-integrated e-commerce. If the assumption does not hold and all consumers will find all retailers after seeing some referred retailers on the manufacturer's website, the manufacturer's exclusive referral will degenerate into a nonexclusive referral. In the extreme case, if all consumers will find all retailers regardless of whether or not they will visit the manufacturer's website, the referral segment will degenerate into part of the traditional consumer segment, where consumers are informed of all retailers, and both referral types will give way to non-referral. Q.E.D.

Proof of Proposition 2: For nonexclusive referral to be the equilibrium referral type, it must be the choice of the manufacturer and both retailers. For the manufacturer, it is equivalent to prove that

$$\sqrt{\Pi_m^{ne}} - \max[\sqrt{\Pi_m^j}] \geq 0, j = nr, r1, r2.$$

From previous proofs, it is easy to know that $\sqrt{\Pi_m^{ne}} - \max[\sqrt{\Pi_m^j}]$ is a linear function of α_m . For example,

$$\begin{aligned} \sqrt{\Pi_m^{ne}} - \sqrt{\Pi_m^{r1}} &= \frac{2\alpha_m + \alpha_{t1} + \alpha_{t2} - 2c_1 - 2c_2}{4\sqrt{2+\theta-\theta^2}} \\ &\quad - \frac{(4+\theta(5-\theta-2\theta^2))\alpha_m + (4+\theta(1-\theta(3+\theta)))\alpha_{t1} + (4+(2-\theta)\theta)\alpha_{t2} - (4+3\theta)(2-\theta^2)c_1 - (4+(2-\theta)\theta)c_2}{2\sqrt{(1+\theta)(2+\theta)(8-5\theta^2)(6+\theta(1-3\theta))}}, \end{aligned}$$

$$\sqrt{\Pi_m^{ne}} - \sqrt{\Pi_m^{nr}} = \frac{\alpha_m - (\sqrt{2} - 1)(\alpha_{t1} + \alpha_{t2} + \sqrt{2}(c_1 + c_2))}{4\sqrt{(2 - \theta)(1 + \theta)}}.$$

We also obtain

$$\frac{\partial (\sqrt{\Pi_m^{ne}} - \sqrt{\Pi_m^{nr}})}{\partial \alpha_m} = \frac{1}{2\sqrt{2 + \theta - \theta^2}} > 0,$$

and

$$\frac{\partial (\sqrt{\Pi_m^{ne}} - \sqrt{\Pi_m^{ri}})}{\partial \alpha_m} = \frac{1}{2\sqrt{2 + \theta - \theta^2}} - \frac{4 + \theta(5 - \theta - 2\theta^2)}{2\sqrt{(1 + \theta)(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))}} > 0, j = 3 - i, i = 1, 2.$$

Therefore, there exists a single crossing point, $\alpha_{m(ne)}^m$, such that $\sqrt{\Pi_m^{ne}} \geq \max[\sqrt{\Pi_m^j}]$ as long as $\alpha_m \geq \alpha_{m(ne)}^m$.

For both retailers, $\sqrt{\Pi_i^{ne}} - \sqrt{\Pi_i^{rj}}$ is a linear function of α_m . For example, for Retailer 1, we have

$$\begin{aligned} \sqrt{\Pi_1^{ne}} - \sqrt{\Pi_1^{r2}} = & \frac{2(2 - \theta(1 + \theta))\alpha_m + (6 + \theta - 3\theta^2)\alpha_{t1} - (2 + (3 - \theta)\theta)\alpha_{t2} - 2(6 + \theta(1 - 3\theta))c_1 + 2(2 + (3 - \theta)\theta)c_2}{4\sqrt{2(1 - \theta^2)(4 - \theta^2)}} \\ & (80 + \theta(64 - \theta(2 + \theta)(46 + \theta(13 - 19\theta))))\alpha_{t1} - (16 + \theta(68 + \theta(22 - \theta(75 + \theta(38 - \theta(20 + 11\theta))))))\alpha_{t2} \\ & - (1 - \theta)(1 + \theta)(16 + \theta(12 + \theta(26 - \theta(5 + 8\theta))))\alpha_m - (80 + \theta(64 - \theta(2 + \theta)(46 + \theta(13 - 19\theta))))c_1 \\ & + (2 - \theta^2)(16 + \theta(28 - \theta(2 + 3\theta(5 + \theta))))c_2 \\ & - \frac{2\sqrt{1 - \theta^2}(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))}{2\sqrt{1 - \theta^2}(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))}. \end{aligned}$$

For Retailer 2,

$$\begin{aligned} \sqrt{\Pi_2^{ne}} - \sqrt{\Pi_2^{r1}} = & \frac{2(2 - \theta(1 + \theta))\alpha_m + (6 + \theta - 3\theta^2)\alpha_{t2} - (2 + (3 - \theta)\theta)\alpha_{t1} - 2(6 + \theta(1 - 3\theta))c_2 + 2(2 + (3 - \theta)\theta)c_1}{4\sqrt{2(1 - \theta^2)(4 - \theta^2)}} \\ & (80 + \theta(64 - \theta(2 + \theta)(46 + \theta(13 - 19\theta))))\alpha_{t2} - (16 + \theta(68 + \theta(22 - \theta(75 + \theta(38 - \theta(20 + 11\theta))))))\alpha_{t1} \\ & - (1 - \theta)(1 + \theta)(16 + \theta(12 + \theta(26 - \theta(5 + 8\theta))))\alpha_m - (80 + \theta(64 - \theta(2 + \theta)(46 + \theta(13 - 19\theta))))c_2 \\ & + (2 - \theta^2)(16 + \theta(28 - \theta(2 + 3\theta(5 + \theta))))c_1 \\ & - \frac{2\sqrt{1 - \theta^2}(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))}{2\sqrt{1 - \theta^2}(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))}. \end{aligned}$$

We also have

$$\frac{\partial (\sqrt{\Pi_i^{ne}} - \sqrt{\Pi_i^{rj}})}{\partial \alpha_m} = \frac{2 - \theta - \theta^2}{2\sqrt{2}(4 - \theta^2)\sqrt{(1 - \theta^2)}} + \frac{\sqrt{1 - \theta^2}(16 - \theta(12 + \theta(26 - \theta(5 + 8\theta))))}{2(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))} > 0.$$

Therefore, there exists a single crossing point, $\alpha_{m(ne)}^{pi}$, such that $\sqrt{\Pi_i^{ne}} \geq \sqrt{\Pi_i^{rj}}, i = 1, 2$, as long as $\alpha_m \geq \alpha_{m(ne)}^{pi}$.

We also ensure that the realized referral demand in nonexclusive referral is nonnegative, that is

$$\frac{2(5\alpha_m - 5\alpha_{t1} - \alpha_{t2}) + \theta((1 + \theta)\alpha_{t1} + (13 + \theta - 4\theta^2)\alpha_{t2} - 2(3 - \theta)(3 + 2\theta)\alpha_m) - 2(6 + \theta(1 - 3\theta))c_1 + (4 + 2(3 - \theta)\theta)c_2}{8(4 - 5\theta^2 + \theta^4)} \geq 0,$$

$$\frac{2(5\alpha_m - 5\alpha_{t2} - \alpha_{t1}) + \theta((1 + \theta)\alpha_{t2} + (13 + \theta - 4\theta^2)\alpha_{t1} - 2(3 - \theta)(3 + 2\theta)\alpha_m) - 2(6 + \theta(1 - 3\theta))c_2 + (4 + 2(3 - \theta)\theta)c_1}{8(4 - 5\theta^2 + \theta^4)} \geq 0.$$

The demand is a linear function of α_m , and the slope of α_m is

$$\frac{5 - 2\theta}{8 + 4\theta - 4\theta^2} > 0.$$

Therefore, there exists a single crossing point, $\alpha_{m(ne)}^{di}$, such that the realized referral demand in nonexclusive referral is nonnegative as long as $\alpha_m \geq \alpha_{m(ne)}^{di}$.

Define $\hat{\alpha}_m^H \equiv \max[\alpha_{m(ne)}^m, \alpha_{m(ne)}^{pi}, \alpha_{m(ne)}^{di}]$. Thus, nonexclusive referral is the equilibrium referral type for the manufacturer and both retailers as long as $\alpha_m \geq \hat{\alpha}_m^H$. When $\alpha_m < \hat{\alpha}_m^H$, the exclusive referral emerges as the equilibrium referral type. Based on Proposition 1, if $\min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}] \leq \alpha_m < \hat{\alpha}_m^H$, the manufacturer will exclusively refer to Retailer 1, since $\theta(\theta + 1)(\alpha_{t2} - \alpha_{t1}) + (4 - 3\theta^2)(c_2 - c_1) \geq 0$. If $\min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}] \leq \alpha_m < \min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}]$, the manufacturer will exclusively refer to Retailer 2, because Retailer 1 declines the exclusive referral offer. When $\alpha_m < \min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}]$, the referral segment market size is too small such that no referral is the equilibrium choice. Q.E.D.

Proof of Proposition 3: First consider exclusive referral. Under unequal pricing, without loss of generality, we solve the case of referring to Retailer 1 and then generalize the result to exclusive referral to Retailer 2.

$$\begin{aligned} \Pi_1^{r1U} &= (p_1 - w - c_1) \left(\frac{\alpha_{t1} - \theta\alpha_{t2} - p_1 + \theta p_2}{1 - \theta^2} \right) + (p_{r1} - w - c_1)(\alpha_m - p_{r1}), \\ \Pi_2^{r1U} &= (p_2 - w - c_2) \left(\frac{\alpha_{t2} - \theta\alpha_{t1} - p_2 + \theta p_1}{1 - \theta^2} \right), \\ \Pi_m^{r1U} &= w \left(\frac{\alpha_{t1} - \theta\alpha_{t2} - p_1 + \theta p_2}{1 - \theta^2} + \alpha_m - p_{r1} + \frac{\alpha_{t2} - \theta\alpha_{t1} - p_2 + \theta p_1}{1 - \theta^2} \right). \end{aligned}$$

We let superscript “U” denote unequal pricing. For Retailer 1, we have

$$\frac{\partial^2 \Pi_1^{r1U}}{\partial p_1^2} = -\frac{2}{1 - \theta^2} < 0,$$

$$\frac{\partial^2 \Pi_1^{r1U}}{\partial p_{r1}^2} = -2 < 0,$$

and the Hessian matrix is negative definite. Therefore, Retailer 1's profit is jointly concave in p_1 and p_{r1} for any given w . Similar to the proof in Proposition 1, it is easy to see that the concavity condition for Retailer 2 is satisfied, thus, the details are skipped. Consider retailers' price competition when wholesale price w is given. The following FOCs suffice to assure maximization.

$$\frac{\partial \Pi_1^{r1U}}{\partial p_1} = 0, \quad \frac{\partial \Pi_2^{r1U}}{\partial p_2} = 0, \quad \text{and} \quad \frac{\partial \Pi_1^{r1U}}{\partial p_{r1}} = 0.$$

By solving the FOCs, we have

$$\begin{aligned} p_1^{r1U}(w) &= \frac{w(2+\theta) + 2c_1 + \theta c_2 + (2-\theta^2)\alpha_{t1} - \theta\alpha_{t2}}{4-\theta^2}, \\ p_2^{r1U}(w) &= \frac{w(2+\theta) + \theta c_1 + 2c_2 - \theta\alpha_{t1} + (2-\theta^2)\alpha_{t2}}{4-\theta^2}, \\ p_{r1}^{r1U}(w) &= \frac{1}{2}(w + c_1 + \alpha_m). \end{aligned}$$

Furthermore, we obtain the concave condition for Retailer 2 and jointly concave condition for Retailer 1.

Substituting them into the manufacturer's profit function, from the FOC, we obtain

$$w^{r1U} = \frac{2(\alpha_{t1} + \alpha_{t2}) + (2-\theta)(1+\theta)\alpha_m - (4+\theta-\theta^2)c_1 - 2c_2}{2(3-\theta)(2+\theta)}.$$

The manufacturer's profit is concave in w , because

$$\frac{\partial^2 \Pi_m^{riU}}{\partial w^2} = -\frac{6+\theta-\theta^2}{2+\theta-\theta^2} < 0,$$

which confirms the above solution is optimal and unique. Therefore, we can obtain the generalized profit for the manufacturer:

$$\Pi_m^{riU} = \frac{((2+\theta-\theta^2)\alpha_m + 2(\alpha_{t1} + \alpha_{t2}) - (4+(1-\theta)\theta)c_i - 2c_j)^2}{8(3-\theta)(2-\theta)(1+\theta)(2+\theta)}, j = 3-i, i = 1, 2.$$

We have $(2+\theta-\theta^2)\alpha_m + 2(\alpha_{t1} + \alpha_{t2}) - (4+(1-\theta)\theta)c_i - 2c_j > 0$, $i = 1, 2$, because $p_i - w - c_i > 0$, $i = 1, 2$. Given that $4+(1-\theta)\theta > 2$, the condition that $\Pi_m^{r1U} - \Pi_m^{r2U} > 0$ is equivalent to $c_2 - c_1 > 0$. Therefore, it is more profitable for the manufacturer to refer consumers to the more cost-efficient retailer.

Now consider nonexclusive referral. Under unequal pricing, the retailers' profit functions are given by

$$\Pi_1^{neU} = (p_1 - w - c_1) \left(\frac{\alpha_{t1} - \theta\alpha_{t2} - p_1 + \theta p_2}{1 - \theta^2} \right) + (p_{r1} - w - c_1) \left(\frac{\alpha_m - \theta\alpha_m - p_{r1} + \theta p_{r2}}{1 - \theta^2} \right),$$

$$\Pi_2^{neU} = (p_2 - w - c_2) \left(\frac{\alpha_{t2} - \theta\alpha_{t1} - p_2 + \theta p_1}{1 - \theta^2} \right) + (p_{r2} - w - c_2) \left(\frac{\alpha_m - \theta\alpha_m - p_{r2} + \theta p_{r1}}{1 - \theta^2} \right).$$

We have

$$\begin{aligned} \frac{\partial^2 \Pi_i^{neU}}{\partial p_i^2} &= -\frac{2}{1 - \theta^2} < 0, \\ \frac{\partial^2 \Pi_i^{neU}}{\partial p_{ri}^2} &= -\frac{2}{1 - \theta^2} < 0, \end{aligned}$$

and the Hessian matrix is negative definite. Therefore, Retailer i 's profit is jointly concave in p_i and p_{ri} for any given w . For the manufacturer, the profit function is

$$\Pi_m^{neU} = w \sum_{i=1,2} \left(\frac{\alpha_{ti} - \theta\alpha_{tj} - p_i + \theta p_j}{1 - \theta^2} + \frac{\alpha_m - \theta\alpha_m - p_{ri} + \theta p_{rj}}{1 - \theta^2} \right),$$

which is concave in w after replacing the optimal solution into it. Similar to the proof of Lemma 2, solving the first order conditions jointly results in the optimal retail prices as follows.

$$\begin{aligned} p_i^{ne} &= \frac{2c_i + \theta c_j + (2 - \theta^2)\alpha_{ti} - \theta\alpha_{tj} + (\frac{1}{4} + \frac{1}{8}\theta)(\alpha_i + \alpha_{tj} + 2\alpha_m - 2c_i - 2c_j)}{4 - \theta^2}, \\ p_{ir}^{ne} &= \frac{4c_1 + 2\theta c_2 + (2 + \theta)(\frac{1}{4}(\alpha_{t1} + \alpha_{t2} + 2\alpha_m - 2c_1 - 2c_2) + 2(1 - \theta)\alpha_m)}{2(4 - \theta^2)}, j = 3 - i, i = 1, 2. \end{aligned}$$

The optimal wholesale price is

$$w^{ne} = \frac{1}{8}(\alpha_{t1} + \alpha_{t2} + 2\alpha_m - 2c_1 - 2c_2).$$

The manufacturer's profit is given by

$$\Pi_m^{neU} = \frac{(\alpha_{t1} + \alpha_{t2} + 2\alpha_m - 2c_1 - 2c_2)^2}{16(2 - \theta)(1 + \theta)}.$$

This profit is the same as that with equal pricing. Therefore, the manufacturer is indifferent between equal and unequal pricing. Q.E.D.

Proof of Proposition 4: Consider the nonexclusive referral assigning the better referral position to Retailer 1. We solve the game backward. We first consider the retailers' price competition given the wholesale price, and then solve the wholesale price for the manufacturer. The following FOCs suffice to assure maximization of profits for both retailers.

$$\frac{\partial \Pi_1^{ne1}}{\partial p_1^1} = 0 \quad \text{and} \quad \frac{\partial \Pi_2^{ne1}}{\partial p_2^1} = 0.$$

By solving the FOCs, we have

$$p_1^{ne1}(w^{ne1}) = \frac{2w(2+\theta) + (2-\theta^2)\alpha_{t1} - \theta\alpha_{t2} + 2(2-\theta-\theta^2)\alpha_m + 8c_1 + 4\theta c_2 + 2\delta(2-\theta)(1+\theta)}{4(4-\theta^2)},$$

$$p_2^{ne1}(w^{ne1}) = \frac{2w(2+\theta) + (2-\theta^2)\alpha_{t2} - \theta\alpha_{t1} + 2(2-\theta-\theta^2)\alpha_m + 8c_2 + 4\theta c_1 - 2\delta(2-\theta)(1+\theta)}{4(4-\theta^2)}.$$

The second order conditions are given by

$$\frac{\partial^2 \Pi_1^{ne1}}{\partial p_1^2} = -\frac{4}{1-\theta^2} < 0 \quad \text{and} \quad \frac{\partial^2 \Pi_2^{ne1}}{\partial p_2^2} = -\frac{4}{1-\theta^2} < 0.$$

Substituting the best response retail prices in terms of the wholesale price into the manufacturer's profit function, from the FOC, $\frac{\partial \Pi_m^{ne1}}{\partial w} = 0$, we obtain the optimal wholesale price that optimizes the manufacturer's profit:

$$w^{ne1} = \frac{1}{8}(\alpha_{t1} + \alpha_{t2} + 2\alpha_m - 2c_1 - 2c_2).$$

The above wholesale price is unique, because

$$\frac{\partial^2 \Pi_m^{ne1}}{\partial w^2} = -\frac{8}{2+\theta-\theta^2} < 0.$$

Substituting w^{ne1} into $p_1^{ne1}(w^{ne1})$, $p_2^{ne1}(w^{ne1})$, we obtain the optimal retail prices, and furthermore, the optimal profit is as follows:

$$\Pi_m^{ne1} = \frac{(2\alpha_m + \alpha_{t1} + \alpha_{t2} - 2c_1 - 2c_2)^2}{16(2-\theta)(1+\theta)},$$

which is the same as that in nonexclusive referral without referral position priority. Therefore, the manufacturer is indifferent regarding whether to assign the better referral position to either retailer.

Continuing from the above proof, we obtain the sum of two retailers' profits in both cases where either Retailer 1 or Retailer 2 gets the better referral position and then compare their accumulative profits as follows:

$$(\Pi_1^{ne1} + \Pi_2^{ne1}) - (\Pi_1^{ne2} + \Pi_2^{ne2}) = \frac{2\delta(1+\theta)(\alpha_{t1} - \alpha_{t2} - 2(c_1 - c_2))}{(1-\theta)(2+\theta)^2},$$

where the superscript $nei, i = 1, 2$, represents that the better referral position is assigned to Retailer i in nonexclusive referral. Therefore, we can infer the supply chain is more efficient when the manufacturer assigns the better referral position to Retailer 1 if and only if $\alpha_1 - \alpha_{t2} > 2(c_1 - c_2)$. Q.E.D.

Proof of Proposition 5: Compare two different exclusive referral scenarios where either Retailer 1 or Retailer 2 is selected in the manufacturer referral. Without loss of generality, here we mainly consider

the case of exclusive referring to Retailer 1 and the results can be easily extended to exclusive referring to Retailer 2. Let subscript (h) denote the case with infomediary referral. The profit functions are given by

$$\begin{aligned}\Pi_{1(h)}^{r1} &= (p_1 - w - c_1) \left(\frac{\alpha_{t1} - \theta * \alpha_{t2} - p_1 + \theta * p_2}{1 - \theta^2} + \alpha_m - p_1 + \frac{\alpha_h - \theta * \alpha_h - p_1 + \theta * p_2}{1 - \theta^2} \right), \\ \Pi_{2(h)}^{r1} &= (p_2 - w - c_2) \left(\frac{\alpha_{t2} - \theta * \alpha_{t1} - p_2 + \theta * p_1}{1 - \theta^2} + \frac{\alpha_h - \theta * \alpha_h - p_2 + \theta * p_1}{1 - \theta^2} \right), \\ \Pi_{m(h)}^{r1} &= w \left(\frac{\alpha_{t1} - \theta * \alpha_{t2} - p_1 + \theta * p_2}{1 - \theta^2} + \alpha_m - p_1 + \frac{\alpha_{t2} - \theta * \alpha_{t1} - p_2 + \theta * p_1}{1 - \theta^2} + \frac{2\alpha_h - p_1 - p_2}{1 + \theta} \right).\end{aligned}$$

The second order conditions are given by

$$\frac{\partial^2 \Pi_{1(h)}^{r1}}{\partial p_1^2} = -\frac{2(3 - \theta^2)}{1 - \theta^2} < 0 \quad \text{and} \quad \frac{\partial^2 \Pi_{2(h)}^{r1}}{\partial p_2^2} = -\frac{4}{1 - \theta^2} < 0.$$

By solving the FOCs, $\frac{\partial \Pi_{1(h)}^{r1}}{\partial p_1} = 0$ and $\frac{\partial \Pi_{2(h)}^{r1}}{\partial p_2} = 0$, we have

$$\begin{aligned}p_{1(h)}^{r1}(w_h^{r1}) &= \frac{4w(3 + \theta - \theta^2) + 2(2 - \theta^2)\alpha_{t1} - 2\theta\alpha_{t2} + 4(3 - \theta^2)c_1 + 4\theta c_2 + 4(1 - \theta^2)\alpha_m + 2(2 - \theta - \theta^2)\alpha_h}{12(2 - \theta^2)}, \\ p_{2(h)}^{r1}(w_h^{r1}) &= \frac{2(2 + \theta)(3 - \theta^2)w - 2\theta(2 - \theta^2)\alpha_{t1} + 2(3 - 2\theta^2)\alpha_{t2} + 2\theta(3 - \theta^2)c_1 + 4(3 - \theta^2)c_2 + 4(1 - \theta^2)\alpha_m + 2(3 - 2\theta - 2\theta^2)\alpha_h}{12(2 - \theta^2)}.\end{aligned}$$

Substituting the best response retail prices in terms of the wholesale price into the manufacturer's profit function, from the FOC, $\frac{\partial \Pi_{m(h)}^{r1}}{\partial w} = 0$, we obtain the optimal wholesale price that optimizes the manufacturer's profit:

$$w_h^{r1} = \frac{(12 + \theta(1 - \theta)(5 + \theta))\alpha_h + 2(3 + \theta(4 - \theta^2))\alpha_m + (6 + \theta(2 - \theta(3 + \theta)))\alpha_{t1} + (6 + \theta(3 - \theta))\alpha_{t2} - 2(3 + 2\theta)(3 - \theta^2)c_1 - 2(6 + (3 - \theta)\theta)c_2}{4(15 + \theta(9 - 2\theta(2 + \theta)))}.$$

The wholesale price is positively related to the retailers' market size, negatively related to the operation cost, and positively related to manufacturer and infomediary referral segment market sizes. We can show that the above wholesale price is unique, because

$$\frac{\partial^2 \Pi_{m(h)}^{r1}}{\partial w^2} = -\frac{2}{3} \left(2 + \frac{4}{1 + \theta} + \frac{3 + 2\theta}{2 - \theta^2} \right) < 0.$$

Substituting w_h^{r1} into $p_{1(h)}^{r1}(w_h^{r1})$, $p_{2(h)}^{r1}(w_h^{r1})$, we obtain the corresponding optimal retail prices.

$$\begin{aligned}p_{i(h)}^{r1} &= \frac{(96 + \theta(33 - \theta(83 + 5\theta(6 - \theta(3 + \theta))))\alpha_h + (78 + 2\theta(33 - \theta(37 + \theta(29 - \theta(7 + 5\theta))))\alpha_m + (78 + \theta(48 - \theta(59 + \theta(34 - 5\theta(2 + \theta))))\alpha_{ti} + (18 - \theta(15 + \theta(24 - \theta(4 + 5\theta))))\alpha_{tj} + 2(3 - \theta^2)(21 + (1 - \theta)\theta(9 + 2\theta))c_i - 2(2 + \theta)(9 - \theta(12 + \theta(6 - 5\theta)))c_j}{12(2 - \theta^2)(15 + \theta(9 - 2\theta(2 + \theta)))}, \\ &, j = 3 - i, i = 1, 2.\end{aligned}$$

The enrolled retailer's retail price is positively related to its base demand, operation cost and intermediary referral segment market size.

Similarly, we can obtain the results for the case where the manufacturer refers to Retailer 2.

For exclusive referral to Retailer 1 to be the equilibrium choice, both the manufacturer and Retailer 1 must be better off than in no referral. The discussion is also conditional on nonnegative demand. We enumerate on them sequentially.

(i) That the manufacturer is better off in the exclusive referral to Retailer 1 than in no referral is equivalent to

$$\begin{aligned} & (6 + 2\theta(4 - \theta^2))\alpha_m + (12 + \theta(1 - \theta)(5 + \theta))\alpha_h + (6 + \theta(2 - \theta(3 + \theta)))\alpha_{t1} \\ \sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{nr}} = & \frac{+ (6 + \theta(3 - \theta))\alpha_{t2} - 2(3 + 2\theta)(3 - \theta^2)c_1 - 2(6 + (3 - \theta)\theta)c_2}{4\sqrt{3(1+\theta)(2-\theta^2)(15+\theta(9-2\theta(2+\theta)))}} \\ & - \frac{(2\alpha_h + \alpha_{t1} + \alpha_{t2} - 2c_1 - 2c_2)}{4\sqrt{(2-\theta)(1+\theta)}} \geq 0 \end{aligned}$$

Hence, $\sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{nr}}$ is a linear function of α_m . Meanwhile,

$$\frac{\partial (\sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{nr}})}{\partial \alpha_m} = \frac{6 + 2\theta(4 - \theta^2)}{4\sqrt{3(1 + \theta)(2 - \theta^2)(15 + \theta(9 - 2\theta(2 + \theta)))}} > 0.$$

Thus, there exists a single crossing point, defined as $\alpha_{m(e)}^{m1}$, such that $\sqrt{\Pi_m^{r1}} = \sqrt{\Pi_m^{nr}}$ when $\alpha_m = \alpha_{m(e)}^{m1}$.

(ii) That Retailer 1 is better off in the exclusive referral to Retailer 1 than in no referral is equivalent to

$$\begin{aligned} & \sqrt{3 - \theta^2}((42 - 2(9 - \theta(4 + \theta)(7 + \theta(1 - 2\theta))))\alpha_m \\ & + (24 + \theta(3 - \theta(23 + \theta(3 + \theta)(3 - 2\theta))))\alpha_h \\ \sqrt{\Pi_1^{r1}} - \sqrt{\Pi_1^{nr}} = & + (42 + \theta(36 - \theta(23 + \theta(22 - \theta - 2\theta^2))))\alpha_1 - (18 - \theta(33 - \theta^2(13 + 2\theta)))\alpha_2 \\ & - 2(63 + \theta(45 - \theta(51 + \theta(33 - 4\theta(2 + \theta))))c_1 + (36 + 66\theta - 26\theta^3 - 4\theta^4)c_2) \\ & \frac{12(2 - \theta^2)(15 + \theta(9 - 2\theta(2 + \theta)))\sqrt{1 - \theta^2}}{4(4 - \theta^2)\sqrt{2(1 - \theta^2)}} \\ & - \frac{(4 - 2\theta(1 + \theta))\alpha_h + (6 + \theta(1 - 3\theta))\alpha_{t1} - (2 + \theta(3 - \theta))\alpha_{t2} - 2(6 + \theta(1 - 3\theta))c_1 + 2(2 + \theta(3 - \theta))c_2}{4(4 - \theta^2)\sqrt{2(1 - \theta^2)}} \geq 0. \end{aligned}$$

Similarly, $\sqrt{\Pi_1^{r1}} - \sqrt{\Pi_1^{nr}}$ is a linear function of α_m , and

$$\frac{\partial (\sqrt{\Pi_1^{r1}} - \sqrt{\Pi_1^{nr}})}{\partial \alpha_m} = \frac{\sqrt{3 - \theta^2}(42 - 2(9 - \theta(4 + \theta)(7 + \theta(1 - 2\theta))))}{12(2 - \theta^2)(15 + \theta(9 - 2\theta(2 + \theta)))\sqrt{1 - \theta^2}} > 0.$$

Thus, there exists a single crossing point, defined as $\alpha_{m(e)}^{p1}$, such that $\sqrt{\Pi_1^{r1}} = \sqrt{\Pi_1^{nr}}$ when $\alpha_m = \alpha_{m(e)}^{p1}$.

(iii) We also ensure that the realized demand from the referral market is nonnegative. The demand is a linear function of α_m as below:

$$\begin{aligned} & \frac{1}{12} \left(7 - \frac{2}{2-\theta^2} + \frac{21+3\theta-6\theta^2}{15+\theta(9-2\theta(2+\theta))} \right) \alpha_m - \frac{1}{24} \left(5 + \frac{2\theta}{2-\theta^2} - \frac{3(7+\theta-2\theta^2)}{15+\theta(9-2\theta(2+\theta))} \right) \alpha_h \\ & - \left(\frac{5}{24} + \frac{1+\theta}{8(15+\theta(9-2\theta(2+\theta)))} \right) \alpha_{t1} + \left(\frac{\theta}{24-12\theta^2} - \frac{3-\theta^2}{4(15+\theta(9-2\theta(2+\theta)))} \right) \alpha_{t2} \\ & - \frac{1}{6} \left(1 + \frac{1}{2-\theta^2} + \frac{9-3\theta^2}{15+\theta(9-2\theta(2+\theta))} \right) c_1 - \frac{1}{6} \left(\frac{\theta}{2-\theta^2} - \frac{3(3-\theta^2)}{15+\theta(9-2\theta(2+\theta))} \right) c_2 \geq 0. \end{aligned}$$

The corresponding slope with respect to α_m is

$$\frac{1}{12} \left(7 - \frac{2}{2-\theta^2} + \frac{21+3\theta-6\theta^2}{15+\theta(9-2\theta(2+\theta))} \right) > 0.$$

Define $\alpha_{m(e)}^{d1}$ such that the referral market demand is nonnegative as long as $\alpha_m \geq \alpha_{m(e)}^{d1}$. If $\alpha_m < \alpha_{m(e)}^{d1}$, in our analysis, Retailer 1 receives zero demand rather than a negative demand.

Let $\hat{\alpha}_{m(h)}^{L1} \equiv \max[\alpha_{m(e)}^{m1}, \alpha_{m(e)}^{p1}, \alpha_{m(e)}^{d1}]$. Therefore, if $\alpha_m \geq \hat{\alpha}_{m(h)}^{L1}$, exclusive referral to Retailer 1 is the equilibrium choice as compared to no referral. Otherwise, no referral is the equilibrium choice. Similar analysis applies to Retailer 2, and $\hat{\alpha}_{m(h)}^{L2} \equiv \max[\alpha_{m(e)}^{m2}, \alpha_{m(e)}^{p2}, \alpha_{m(e)}^{d2}]$.

Denote $\Pi_{m(t)}^{ri}$ as the manufacturer's profit when the manufacturer refers to Retailer i . Due to the structure symmetry, we obtain the optimal profits as follows:

$$\begin{aligned} & ((12 + \theta(1 - \theta)(5 + \theta))\alpha_h + 2(3 + \theta(4 - \theta^2))\alpha_m + (6 + \theta(2 - \theta(3 + \theta)))\alpha_{ti} \\ & + (6 + \theta(3 - \theta))\alpha_{tj} - 2(3 + 2\theta)(3 - \theta^2)c_i - 2(6 + (3 - \theta)\theta)c_j)^2 \\ \Pi_{m(h)}^{ri} = & \frac{48(1 + \theta)(2 - \theta^2)(15 + \theta(9 - 2\theta(2 + \theta)))}{}, j = 3-i, i = 1, 2. \end{aligned}$$

Furthermore,

$$\sqrt{\Pi_{m(h)}^{r1}} - \sqrt{\Pi_{m(h)}^{r2}} = \frac{\theta(1 + \theta)^2((\alpha_{t2} - \alpha_{t1}) + \frac{6-4\theta^2}{\theta(1+\theta)}(c_2 - c_1))}{\sqrt{48(1 + \theta)(2 - \theta^2)(15 + \theta(9 - 2\theta(2 + \theta)))}}.$$

Therefore, the manufacturer prefers exclusive referral to Retailer 1 to exclusive referral to Retailer 2 if and only if $(\alpha_{t2} - \alpha_{t1}) + \frac{6-4\theta^2}{\theta(1+\theta)}(c_2 - c_1) \geq 0$. Q.E.D.

Proof of Proposition 6: We first prove that with infomediary referral, $\hat{\alpha}_{m(h)}^H$ and $\hat{\alpha}_{m(h)}^{Li}$ increase with α_h . Compare the manufacturer's profits in exclusive referral and no referral. We have

$$\begin{aligned} \frac{\partial(\sqrt{\Pi_{m(h)}^{ri}} - \sqrt{\Pi_{m(h)}^{nr}})}{\partial \alpha_h} &= \frac{1}{12} \left(-\frac{6}{\sqrt{(2 - \theta)(1 + \theta)}} + \frac{\sqrt{3}(12 + (1 - \theta)\theta(5 + \theta))}{\sqrt{(1 + \theta)(2 - \theta^2)(15 + \theta(9 - 2\theta(2 + \theta)))}} \right) \\ &< 0. \end{aligned}$$

Define $\alpha_{m(he)}^{mi}$ as the threshold value such that when $\alpha_m = \alpha_{m(he)}^{mi}$, $\sqrt{\Pi_{m(h)}^{ri}} = \sqrt{\Pi_{m(h)}^{nr}}$. Based on Proposition 2, we can thus infer that $\alpha_{m(he)}^{mi}$ increases as α_h increases.

Compare the manufacturer's profits in nonexclusive referral and exclusive referral. We have,

$$\frac{\partial(\sqrt{\Pi_{m(h)}^{ne}} - \sqrt{\Pi_{m(h)}^{ri}})}{\partial\alpha_h} = \frac{\sqrt{2}}{2\sqrt{3(2-\theta)(1+\theta)}} - \frac{12 + (1-\theta)\theta(5+\theta)}{4\sqrt{3(1+\theta)(2-\theta^2)(15+\theta(9-2\theta(2+\theta)))}} < 0,$$

as long as $\theta < 0.73$. Define $\alpha_{m(hne)}^{mi}$ as the threshold value such that when $\alpha_m = \alpha_{m(hne)}^{mi}$, $\sqrt{\Pi_{m(h)}^{ne}} = \sqrt{\Pi_{m(h)}^{ri}}$. Therefore, $\alpha_{m(hne)}^{mi}$ increases with α_h if $\theta < 0.73$; otherwise, $\alpha_{m(hne)}^{mi}$ decreases with α_h . Note that when

$$\frac{\partial(\sqrt{\Pi_{m(h)}^{ne}} - \sqrt{\Pi_{m(h)}^{ri}})}{\partial\alpha_h} < 0, \text{ we may still have } \sqrt{\Pi_{m(h)}^{ne}} > \sqrt{\Pi_{m(h)}^{ri}}.$$

Compare the retailers' profits in exclusive referral and no referral. We have

$$\frac{\partial(\sqrt{\Pi_{i(h)}^{ri}} - \sqrt{\Pi_{i(h)}^{nr}})}{\partial\alpha_h} = -\frac{(2+\theta)(1-\theta)}{2(4-\theta^2)\sqrt{2(1-\theta^2)}} + \frac{\sqrt{3-\theta^2}(24+\theta(3-\theta(23+\theta(3+\theta)(3-2\theta))))}{12(2-\theta^2)(15+\theta(9-2\theta(2+\theta)))\sqrt{(1-\theta^2)}} < 0.$$

Define $\alpha_{m(he)}^{pi}$ as when $\alpha_m = \alpha_{m(he)}^{pi}$, $\sqrt{\Pi_{i(h)}^{ri}} = \sqrt{\Pi_{i(h)}^{nr}}$. So $\alpha_{m(he)}^{pi}$ increases with α_h .

Compare the retailers' profits in nonexclusive referral and exclusive referral. We have

$$\frac{\partial(\sqrt{\Pi_{i(h)}^{ne}} - \sqrt{\Pi_{i(h)}^{rj}})}{\partial\alpha_h} = \frac{(2+\theta)(1-\theta)}{2(4-\theta^2)\sqrt{3(1-\theta^2)}} - \frac{108 - \theta(6 + \theta(153 + \theta(2 - 60\theta + 7\theta^3)))}{12(2-\theta^2)(15+\theta(9-2\theta(2+\theta)))\sqrt{2(1-\theta^2)}} < 0.$$

Define $\alpha_{m(hne)}^{pi}$ as when $\alpha_m = \alpha_{m(hne)}^{pi}$, $\sqrt{\Pi_{i(h)}^{ne}} = \sqrt{\Pi_{i(h)}^{rj}}$. So, $\alpha_{m(hne)}^{pi}$ increases with α_h .

Consider the demand nonnegative boundary. Define $\alpha_{m(he)}^{di}$ as the threshold point where $D_{ri(t)}^{ri} = 0$ when $\alpha_m = \alpha_{m(he)}^{di}$. We have

$$\frac{\partial D_{ri(t)}^{ri}}{\partial\alpha_h} = \frac{1}{24}(-5 + \frac{2\theta}{2-\theta^2} - \frac{3(7+\theta-2\theta^2)}{15+\theta(9-2\theta(2+\theta))}) < 0.$$

So when α_h increases, $\alpha_{m(he)}^{di}$ increases. Define $\alpha_{m(hne)}^{di}$ as the threshold point where $D_{ri(t)}^{ne} = 0$ when $\alpha_m = \alpha_{m(hne)}^{di}$. We have

$$\frac{\partial D_{ri(t)}^{ne}}{\partial\alpha_h} = -\frac{3-2\theta}{6(2-\theta)(1+\theta)} < 0.$$

So when α_h increases, $\alpha_{m(hne)}^{di}$ increases. We further obtain

$$\frac{\alpha_{m(hne)}^{p2} - \alpha_{m(hne)}^m}{\alpha_{t2} - c_2} = 2(A_0 \frac{(1+\theta)(\alpha_{t2} - \alpha_{t1}) + 2(c_2 - c_1)}{\alpha_2 - c_2} + C_0(\frac{\alpha_{t1} - c_1}{\alpha_{t2} - c_2} + B_0)),$$

where A_0 , B_0 , and C_0 are very lengthy and thus skipped here. After some nontrivial algebra, we can show that $A_0 > 0$ and $C_0(\frac{\alpha_{t1} - c_1}{\alpha_{t2} - c_2} + B_0) > 0$ when $\theta > 0.73$. Thus, $\alpha_{m(hne)}^{p2} - \alpha_{m(hne)}^m > 0$ when $\theta > 0.73$.

Let $\hat{\alpha}_{m(h)}^H \equiv \max[\alpha_{m(hne)}^{mi}, \alpha_{m(hne)}^{pi}, \alpha_{m(hne)}^{di}]$ and $\hat{\alpha}_{m(h)}^{Li} \equiv \max[\alpha_{m(hne)}^{mi}, \alpha_{m(hne)}^{pi}, \alpha_{m(hne)}^{di}]$. Therefore, $\hat{\alpha}_{m(h)}^H$ and $\hat{\alpha}_{m(h)}^{Li}$ increases with α_h .

We now show that $\hat{\alpha}_{m(h)}^H$ can be lower than $\hat{\alpha}_m^H$, and $\hat{\alpha}_{m(h)}^{Li}$ can be lower than $\hat{\alpha}_m^{Li}$ if α_h is sufficiently low. By definition, we first have

$$\hat{\alpha}_m^H = \max[\alpha_{m(ne)}^{p1}, \alpha_{m(ne)}^{p2}, \alpha_{m(ne)}^{d1}, \alpha_{m(ne)}^{d2}, \alpha_{m(ne)}^m] \geq \max[\alpha_{m(ne)}^{p1}, \alpha_{m(ne)}^{p2}].$$

We can prove $[\alpha_{m(ne)}^{p1}, \alpha_{m(ne)}^{p2}] > \max[\alpha_{m(hne)}^{pi}, \alpha_{m(hne)}^{di}, \alpha_{m(hne)}^m]$ when $(1+\theta)(\alpha_{t2} - \alpha_{t1}) + 2(c_2 - c_1) = 0$ by comparing each component in both sides one by one. Due to extreme complexity, we show only one example as follows.

$$\alpha_{m(ne)}^{p1} - \alpha_{m(hne)}^{p1} = D_0 \frac{(1+\theta)(\alpha_{t2} - \alpha_{t1}) + 2(c_2 - c_1)}{\alpha_{t2} - c_2} + F_0(\frac{\alpha_{t1} - c_1}{\alpha_{t2} - c_2} + E_0),$$

where D_0 , F_0 , and E_0 are lengthy and thus omitted. Given $\max[\alpha_{m(hne)}^{pi}, \alpha_{m(hne)}^{di}, \alpha_{m(hne)}^m] = \hat{\alpha}_{m(h)}^H$, we have $\hat{\alpha}_m^H > \hat{\alpha}_{m(h)}^H$ if $(1+\theta)(\alpha_{t2} - \alpha_{t1}) + 2(c_2 - c_1) = 0$. Because $\hat{\alpha}_{m(h)}^H$ strictly increases with α_h , $\hat{\alpha}_{m(h)}^H > \hat{\alpha}_m^H$ as long as α_h is sufficiently high. Therefore, there exists a threshold value $\tilde{\alpha}_{h1}$, such that if $\alpha_h \geq \tilde{\alpha}_{h1}$, then $\hat{\alpha}_{m(h)}^H \geq \hat{\alpha}_m^H$. Similarly, there exists a threshold value $\tilde{\alpha}_{h2}$ such that $\min[\hat{\alpha}_{m(h)}^H, \hat{\alpha}_{m(h)}^{L1}, \hat{\alpha}_{m(h)}^{L2}] \geq \min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}]$. Define $\tilde{\alpha}_h = \max\{\tilde{\alpha}_{h1}, \tilde{\alpha}_{h2}\}$. Therefore, if $\alpha_h \geq \tilde{\alpha}_h$, then $\hat{\alpha}_{m(h)}^H \geq \hat{\alpha}_m^H$ and $\min[\hat{\alpha}_{m(h)}^H, \hat{\alpha}_{m(h)}^{L1}, \hat{\alpha}_{m(h)}^{L2}] \geq \min[\hat{\alpha}_m^H, \hat{\alpha}_m^{L1}, \hat{\alpha}_m^{L2}]$. Q.E.D.

Proof of Proposition 7: Compare two different exclusive referral scenarios where either Retailer 1 or Retailer 2 is selected in the referral. Similarly, our following discussion assumes nonnegative demand in both traditional and referral markets, and an exclusive referral will not occur if either the manufacturer or the referred retailer cannot benefit from such a referral.

Without loss of generality, we mainly consider the case of referring to Retailer 1 and the results can be easily extended to referring to Retailer 2. The profit functions are given by

$$\Pi_1^{r1} = (p_1 - w - c_1) \left(\frac{\alpha_{t1} - \theta\alpha_{t2} - p_1 + \theta p_2}{1 - \theta^2} + \rho_1 \alpha_m - p_1 \right),$$

$$\begin{aligned}\Pi_2^{r1} &= (p_2 - w - c_2) \left(\frac{\alpha_{t2} - \theta\alpha_{t1} - p_2 + \theta p_1}{1 - \theta^2} \right), \\ \Pi_m^{r1} &= w \left(\frac{\alpha_{t1} - \theta\alpha_{t2} - p_1 + \theta p_2}{1 - \theta^2} + \rho_1 \alpha_m - p_1 + \frac{\alpha_{t2} - \theta\alpha_{t1} - p_2 + \theta p_1}{1 - \theta^2} \right).\end{aligned}$$

The second order conditions are given by

$$\frac{\partial^2 \Pi_1^{r1}}{\partial p_1^2} = -\frac{2(2 - \theta^2)}{1 - \theta^2} < 0 \quad \text{and} \quad \frac{\partial^2 \Pi_2^{r1}}{\partial p_2^2} = -\frac{2}{1 - \theta^2} < 0.$$

Solving the FOCs, we have

$$\begin{aligned}p_1^{r1}(w^{r1}) &= \frac{(4 + \theta - 2\theta^2)w^{r1} + (2 - \theta^2)\alpha_{t1} - \theta\alpha_{t2} + 2(1 - \theta^2)\rho_1\alpha_m + 2(2 - \theta^2)c_1 + \theta c_2}{8 - 5\theta^2}, \\ p_2^{r1}(w^{r1}) &= \frac{(4 + 2\theta - 2\theta^2 - \theta^3)w^{r1} - (3\theta - 2\theta^3)\alpha_{t1} + (4 - 3\theta^2)\alpha_{t2} + (\theta - \theta^3)\rho_1\alpha_m + \theta(2 - \theta^2)c_1 + 2(2 - \theta^2)c_2}{8 - 5\theta^2}.\end{aligned}$$

Substituting the best response retail prices in terms of the wholesale price into the manufacturer's profit function, we obtain the optimal wholesale price that optimizes the manufacturer's profit:

$$w^{r1} = \frac{(4 + \theta(5 - \theta - 2\theta^2))\rho_1\alpha_m + (4 + \theta(1 - \theta(3 + \theta)))\alpha_{t1} + (4 + (2 - \theta)\theta)\alpha_{t2} - (4 + 3\theta)(2 - \theta^2)c_1 - (4 + 2\theta - \theta^2)c_2}{2(2 + \theta)(6 + \theta(1 - 3\theta))}.$$

Substituting w^{r1} into $p_1^{r1}(w^{r1})$ and $p_2^{r1}(w^{r1})$, we obtain the corresponding optimal retail prices. Similarly, we can obtain the results for the case where the manufacturer exclusively refers to Retailer 2. Denote Π_m^{ri} as the manufacturer's profit when the manufacturer exclusively refers to Retailer i . Due to the structure symmetry, we obtain the optimal profits as follows:

$$\Pi_m^{ri} = \frac{((4 + \theta(5 - \theta - 2\theta^2))\rho_i\alpha_m + (4 + \theta(1 - \theta(3 + \theta)))\alpha_{ti} + (4 + (2 - \theta)\theta)\alpha_{tj} - (4 + 3\theta)(2 - \theta^2)c_i - (4 + 2\theta - \theta^2)c_j)^2}{4(1 + \theta)(2 + \theta)(8 - 5\theta^2)(6 + \theta - 3\theta^2)}, j = 3 - i, i = 1, 2.$$

For exclusive referral to Retailer 1 to be the equilibrium choice, both the manufacturer and Retailer 1 must be better off than in no referral. The discussion is also conditional on nonnegative demand. We enumerate on them sequentially.

(i) That the manufacturer is better off in the exclusive referral to Retailer 1 than in no referral is equivalent

to

$$\begin{aligned} \sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{nr}} &= \frac{2(4 + \theta(5 - \theta - 2\theta^2))\rho_1\alpha_m + 2((4 + \theta(1 - \theta(3 + \theta)))\alpha_{t1} \\ &\quad + (4 + (2 - \theta)\theta)\alpha_{t2} - (4 + 3\theta)(2 - \theta^2)c_1 - (4 + (2 - \theta)\theta)c_2)}{4\sqrt{(1 + \theta)(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))}} - \frac{\alpha_{t1} + \alpha_{t2} - c_1 - c_2}{2\sqrt{2}\sqrt{2 + \theta - \theta^2}} \\ &\geq 0. \end{aligned}$$

Hence, $\sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{nr}}$ is a linear function of α_m . Meanwhile,

$$\frac{\partial \left(\sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{nr}} \right)}{\partial \alpha_m} = \frac{(4 + \theta(5 - \theta - 2\theta^2))\rho_1}{2\sqrt{(1 + \theta)(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))}} > 0.$$

Thus, there exists a single crossing point, defined as $\alpha_{m(e)}^{m1}$, such that $\sqrt{\Pi_m^{r1}} = \sqrt{\Pi_m^{nr}}$ when $\alpha_m = \alpha_{m(e)}^{m1}$.

$\alpha_{m(e)}^{m1}$ is decreasing with ρ_1 because $\frac{\partial(\sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{nr}})}{\partial \alpha_m}$ is increasing with ρ_1 .

(ii) That Retailer 1 is better off in the exclusive referral to Retailer 1 than in no referral is equivalent to

$$\begin{aligned} \sqrt{\Pi_1^{r1}} - \sqrt{\Pi_1^{nr}} &= \frac{\sqrt{2 - \theta^2}((1 - \theta)(1 + \theta)(32 + \theta(16 - 3\theta(5 + 2\theta)))\rho_1\alpha_m + (32 + \theta(32 - \theta(19 + 3\theta(8 - \theta^2))))\alpha_{t1} \\ &\quad - (16 + \theta(28 - \theta(2 + 3\theta(5 + \theta))))\alpha_{t2} - (64 + \theta(48 - \theta(66 + \theta(46 - 3\theta(5 + 3\theta))))c_1 \\ &\quad + (16 + \theta(28 - \theta(2 + 3\theta(5 + \theta))))c_2)}{2\sqrt{1 - \theta^2}(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))} \\ &\quad - \frac{(6 + \theta - 3\theta^2)\alpha_{t1} - (2 + (3 - \theta)\theta)\alpha_{t2} - (6 + \theta(1 - 3\theta))c_1 + (2 + (3 - \theta)\theta)c_2}{4\sqrt{1 - \theta^2}(4 - \theta^2)} \geq 0. \end{aligned}$$

Similarly, $\sqrt{\Pi_1^{r1}} - \sqrt{\Pi_1^{nr}}$ is a linear function of α_m , and

$$\frac{\partial \left(\sqrt{\Pi_1^{r1}} - \sqrt{\Pi_1^{nr}} \right)}{\partial \alpha_m} = \frac{\sqrt{(1 - \theta^2)(2 - \theta^2)}(32 + \theta(16 - 3\theta(5 + 2\theta)))\rho_1}{2(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))} > 0.$$

Thus, there exists a single crossing point, defined as $\alpha_{m(e)}^{p1}$, such that $\sqrt{\Pi_1^{r1}} = \sqrt{\Pi_1^{nr}}$ when $\alpha_m = \alpha_{m(e)}^{p1}$.

$\alpha_{m(e)}^{p1}$ is decreasing with ρ_1 because $\frac{\partial(\sqrt{\Pi_1^{r1}} - \sqrt{\Pi_1^{nr}})}{\partial \alpha_m}$ is increasing with ρ_1 .

(iii) We also ensure that the realized demand from the referral market is nonnegative. The demand is a linear function of α_m as below:

$$\begin{aligned} &((128 + \theta(72 - \theta(125 + \theta(65 - 2\theta(15 + 7\theta))))\rho_1\alpha_m - (64 + \theta(40 - \theta(63 + \theta(37 - \theta(15 + 8\theta))))\alpha_{t1} \\ &\quad - (16 - \theta(12 + \theta(26 - \theta(5 + 8\theta))))\alpha_{t2} - (2 - \theta^2)(32 + \theta(16 - 3\theta(5 + 2\theta)))c_1 \\ &\quad + (16 - \theta(12 + \theta(26 - \theta(5 + 8\theta))))c_2) \\ &\quad \frac{}{2(2 + \theta)(8 - 5\theta^2)(6 + \theta(1 - 3\theta))} \geq 0. \end{aligned}$$

The corresponding slope with respect to α_m is

$$\frac{1}{240} \left(112 + \frac{45}{2+\theta} + \frac{144}{8-5\theta^2} + \frac{45-25\theta}{6+\theta-3\theta^2} \right) \rho_1 > 0.$$

Define $\alpha_{m(e)}^{d1}$ such that the referral market demand is nonnegative as long as $\alpha_m \geq \alpha_{m(e)}^{d1}$. If $\alpha_m < \alpha_{m(e)}^{d1}$, in our analysis, Retailer 1 receives zero demand rather than a negative demand. $\alpha_{m(e)}^{d1}$ is decreasing with ρ_1 because $\frac{1}{240} \left(112 + \frac{45}{2+\theta} + \frac{144}{8-5\theta^2} + \frac{45-25\theta}{6+\theta-3\theta^2} \right) \rho_1$ is increasing with ρ_1 .

Let $\tilde{\alpha}_m^{L1} \equiv \max[\alpha_{m(e)}^{m1}, \alpha_{m(e)}^{p1}, \alpha_{m(e)}^{d1}]$. Therefore, if $\alpha_m \geq \tilde{\alpha}_m^{L1}$, exclusive referral to Retailer 1 is the equilibrium choice as compared to no referral. Otherwise, no referral is the equilibrium choice. Similar analysis applies to Retailer 2, and $\tilde{\alpha}_m^{L2} \equiv \max[\alpha_{m(e)}^{m2}, \alpha_{m(e)}^{p2}, \alpha_{m(e)}^{d2}]$. According to the definition, $\tilde{\alpha}_m^{Li}$ is decreasing with $\rho_i, i = 1, 2$.

Moreover,

$$\sqrt{\Pi_m^{r1}} - \sqrt{\Pi_m^{r2}} = \frac{(\theta+1) \left((\theta(1-2\theta)+4)(\rho_1-\rho_2)\alpha_m - ((\alpha_{t1}-\alpha_{t2})\theta(\theta+1) + (c_1-c_2)(4-3\theta^2)) \right)}{2\sqrt{(\theta+1)(\theta+2)(8-5\theta^2)(\theta(1-3\theta)+6)}}.$$

Given $\sqrt{\Pi_m^{r1}} + \sqrt{\Pi_m^{r2}}$ and $\frac{(\theta+1)}{2\sqrt{(\theta+1)(\theta+2)(8-5\theta^2)(\theta(1-3\theta)+6)}}$ are positive, the result in Proposition 7 is concluded. Q.E.D.

Proof of Proposition 8: In nonexclusive referral, the equilibrium is given by

$$\begin{aligned} p_i^{ne} &= \frac{(10+\theta(1-4\theta))\alpha_{ti} + 2(6-\theta)c_i + (2-3\theta)(\alpha_{tj}-2c_j) + ((10+\theta(1-4\theta))\rho_i + (2-3\theta)\rho_j)\alpha_m}{8(4-\theta^2)}, \\ w^{ne} &= \frac{1}{8}(\alpha_{t1} + \alpha_{t2} + (\rho_1 + \rho_2)\alpha_m - 2c_1 - 2c_2), \\ \Pi_m^{ne} &= \frac{((\rho_1 + \rho_2)\alpha_m + \alpha_{t1} + \alpha_{t2} - 2c_1 - 2c_2)^2}{16(2-\theta)(1+\theta)}. \end{aligned}$$

For nonexclusive referral to be the equilibrium referral type, it must be a mutual choice of the manufacturer and both retailers. For the manufacturer, it is equivalent to prove that

$$\sqrt{\Pi_m^{ne}} - \max[\sqrt{\Pi_m^j}] \geq 0, j = nr, r1, r2.$$

We find that $\sqrt{\Pi_m^{ne}} - \max[\sqrt{\Pi_m^j}]$ is a linear function of α_m (here we skip the lengthy equations for parsimony). We obtain

$$\frac{\partial (\sqrt{\Pi_m^{ne}} - \sqrt{\Pi_m^{nr}})}{\partial \alpha_m} = \frac{(\rho_1 + \rho_2)}{4\sqrt{(2-\theta)(1+\theta)}} > 0$$

and

$$\frac{\partial \left(\sqrt{\Pi_m^{ne}} - \sqrt{\Pi_m^{ri}} \right)}{\partial \alpha_m} = \frac{(\rho_1 + \rho_2)}{4\sqrt{(2-\theta)(1+\theta)}} - \frac{((1+\theta)(4+\theta(1-2\theta)))\rho_i}{2\sqrt{(1+\theta)(2+\theta)(8-5\theta^2)(6+\theta(1-3\theta))}}, i = 1, 2.$$

Denote $\widehat{\rho}_m(\theta) \equiv \frac{\sqrt{(2+\theta)(8-5\theta^2)(6+\theta-3\theta^2)}}{2\sqrt{2-\theta}(1+\theta)(4+\theta(1-2\theta))-\sqrt{(2+\theta)(8-5\theta^2)(6+\theta-3\theta^2)}}$ which solves $\frac{\partial(\sqrt{\Pi_m^{ne}}-\sqrt{\Pi_m^{ri}})}{\partial \alpha_m} = 0$. It means that $\frac{\partial(\sqrt{\Pi_m^{ne}}-\sqrt{\Pi_m^{ri}})}{\partial \alpha_m} > 0$ when $\frac{\rho_i}{\rho_j} < \widehat{\rho}_m(\theta)$.

Therefore, comparing nonexclusive referral (ne) to no referral (nr), there exists a single crossing point, $\alpha_{m(ne)}^{mr}$, such that $\sqrt{\Pi_m^{ne}} \geq \sqrt{\Pi_m^{nr}}$ as long as $\alpha_m \geq \alpha_{m(ne)}^{mr}$. Meanwhile, comparing nonexclusive referral (ne) to exclusive referral (ri), when $\frac{\rho_i}{\rho_j} < \widehat{\rho}_m(\theta)$, there exists a single crossing point, $\alpha_{m(ne)}^{mi}$, such that $\sqrt{\Pi_m^{ne}} > \sqrt{\Pi_m^{ri}}$ as long as $\alpha_m > \alpha_{m(ne)}^{mi}$, $i = 1, 2$.

For both retailers, $\sqrt{\Pi_i^{ne}} - \sqrt{\Pi_i^{rj}}$ is a linear function of α_m . We obtain

$$\frac{\partial \left(\sqrt{\Pi_i^{ne}} - \sqrt{\Pi_i^{rj}} \right)}{\partial \alpha_m} = \frac{(6+\theta-3\theta^2)\rho_i - (2+(3-\theta)\theta)\rho_j}{4\sqrt{2(1-\theta^2)}(4-\theta^2)} + \frac{\sqrt{1-\theta^2}(16+\theta(-12+\theta(-26+\theta(5+8\theta))))\rho_j}{2(2+\theta)(8-5\theta^2)(6+\theta(1-3\theta))}.$$

Denote $\widehat{\rho}_p(\theta) \equiv \frac{(2+\theta)(8-5\theta^2)(6+\theta-3\theta^2)^2}{(2+\theta)(8-5\theta^2)(6+\theta-3\theta^2)(2+(3-\theta)\theta)-2\sqrt{2}(4-\theta^2)(1-\theta)(1+\theta)(16+\theta(-12+\theta(-26+\theta(5+8\theta))))}$, which solves $\frac{\partial(\sqrt{\Pi_i^{ne}}-\sqrt{\Pi_i^{rj}})}{\partial \alpha_m} = 0$. It means that $\frac{\partial(\sqrt{\Pi_i^{ne}}-\sqrt{\Pi_i^{rj}})}{\partial \alpha_m} > 0$ when $\frac{\rho_i}{\rho_j} < \widehat{\rho}_p(\theta)$. Therefore, when $\frac{\rho_i}{\rho_j} < \widehat{\rho}_p(\theta)$, there exists a single crossing point, $\alpha_{m(ne)}^{pi}$, such that $\sqrt{\Pi_i^{ne}} > \sqrt{\Pi_i^{rj}}$ as long as $\alpha_m > \alpha_{m(ne)}^{pi}$, $i = 1, 2$.

We require that the realized referral demand in nonexclusive referral must be nonnegative. Given that the demand D_{ri}^{ne} is a linear function of α_m , and the slope of α_m is

$$\frac{(22+\theta-7\theta^2)\rho_i - (2+\theta(19-\theta-4\theta^2))\rho_j}{8(4-5\theta^2+\theta^4)} > 0.$$

Denote $\widehat{\rho}_d(\theta) \equiv \frac{22+\theta-7\theta^2}{2+\theta(19-\theta-4\theta^2)}$, which solves $\frac{\partial D_{ri}^{ne}}{\partial \alpha_m} = 0$. It means that $\frac{\partial D_{ri}^{ne}}{\partial \alpha_m} > 0$ when $\frac{\rho_i}{\rho_j} < \widehat{\rho}_d(\theta)$. Therefore, when $\frac{\rho_i}{\rho_j} < \widehat{\rho}_d(\theta)$, there exists a single crossing point, $\alpha_{m(ne)}^{di}$, such that $D_{ri}^{ne} > 0$ as long as $\alpha_m > \alpha_{m(ne)}^{di}$, $i = 1, 2$.

We now define $\tilde{\alpha}_m^{H1} \equiv \min[\alpha_{m(ne)}^{Ki}, \infty]$, $i = 1, 2$, where $K \in \{m, p, d\}$, which satisfies the condition that $\frac{\rho_i}{\rho_j} > \widehat{\rho}_K(\theta)$; $\tilde{\alpha}_m^{H2} \equiv \max[\alpha_{m(ne)}^{Ki}, \alpha_{m(ne)}^{mr}]$, $i = 1, 2$, where $K \in \{m, p, d\}$, which satisfies the condition that $\frac{\rho_i}{\rho_j} < \widehat{\rho}_K(\theta)$. According to the definition, nonexclusive referral will be the equilibrium only when $\tilde{\alpha}_m^{H1} \leq \alpha_m < \tilde{\alpha}_m^{H2}$. If $\tilde{\alpha}_m^{H2} < \tilde{\alpha}_m^{H1}$, nonexclusive referral will never be the equilibrium. When $\alpha_m \geq \max[\tilde{\alpha}_m^{H1}, \tilde{\alpha}_m^{H2}]$, nonexclusive referral will not be the equilibrium, and the manufacturer will provide exclusive referral to Retailer 1 for more profit.

When $\alpha_m < \tilde{\alpha}_m^{H2}$, the exclusive referral emerges as the equilibrium referral type. Based on Proposition 7, if $\min[\tilde{\alpha}_m^{H2}, \tilde{\alpha}_m^{L1}] \leq \alpha_m < \tilde{\alpha}_m^{H2}$, the manufacturer will exclusively refer to Retailer 1, since it is more profitable than to exclusively refer to Retailer 2.

If $\min[\tilde{\alpha}_m^{H2}, \tilde{\alpha}_m^{L1}, \tilde{\alpha}_m^{L2}] \leq \alpha_m < \min[\tilde{\alpha}_m^{H2}, \tilde{\alpha}_m^{L1}]$, the manufacturer will exclusively refer to Retailer 2, because exclusively refer to Retailer 2 would be the only viable referral choice and manufacturer's profit is more than that in no referral. When $\alpha_m < \min[\tilde{\alpha}_m^{H2}, \tilde{\alpha}_m^{L1}, \tilde{\alpha}_m^{L2}]$, the referral segment market size is too small such that no referral is the equilibrium choice. Q.E.D.