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Quality Inspectability Investment on Imperfect Production Processes under Limited Capital

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Abstract — Imperfect production processes lead to imperfect products and decrease the profit of a business. Improvement in the production process by increasing the investment cost will decrease the defective percentage of items. This study develops an EPQ model with investments on imperfect production processes under limited capital. An algorithm is developed to derive the replenishment and investment policies such that the expected unit time profit is maximized. An alternative approach for the solution procedure is provided.

Keywords — Inventory, Economic production quantity; Imperfect quality; Investment.

1. INTRODUCTION

There exists a vast literature on imperfect items, among which most assume a random defective percentage to discuss related issues. However, very few studies considered how to improve the defective percentage to increase profits. Relative costs are necessary to improve the defective percentage. This study discusses the trade-off between increased production costs and increased revenue.

Imperfect production processes lead to imperfect products and decrease the profit of a business. Most studies during the past decades assumed that perfect items were produced. Rosenblatt and Lee (1986) were early researchers who considered that defective items and imperfect quality existed in production processes. Salameh and Jaber (2000) developed an inventory model considering imperfect items using the EPQ/EOQ formulae. Wee et al. (2007) extended the approach by Salameh and Jaber (2000) and developed a generalized production lot size model with backordering. Maddah and Jaber (2008) rectified a flaw in the work of Salameh and Jaber (2000) on the EOQ model and proposed a new model that rectified this flaw using renewal theory. Lo et al. (2007) developed an integrated production and inventory model. The model assumed a varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. Castro (2008) considered a combined maintenance strategy to find the optimal interval in which the repair of system failures is performed only in an interval of time of the working period.

Recently, Sarkar *et al.* (2010) considered the joint determination of optimal production lot size, safety stock and reliability parameter under the realistic assumptions that the production facility was subject to random machinery system breakdown and changes in the variable reliability parameter. Sarkar *et al.* (2011) dealt with an economic manufacturing quantity model for a time-dependent (quadratic) demand pattern. By using the Euler–Lagrange theory to build up the necessary and sufficient conditions for the optimality of dynamic variables they determined the optimal product reliability and production rate that achieves the biggest total integrated profit for an imperfect manufacturing process. However, large percentages of defective items are a perplexing problem in a supply chain. Improving the production processes by increasing the investment cost will decrease the defective percentage of the items. This study focuses on determining the investment cost on production processes for optimal profit.

Improving the firm's business by increasing the investment cost is needed to be considered by managers. Affisco et al. (2002) investigated the potential impact of investments in quality improvement and setup cost reduction. Gurnani *et al.* (2007) studied the impact of product pricing and timing of investment decisions on supply chain co-operation. Hsu *et al.* (2010) developed a deteriorating inventory policy when retailers invest on preservation technology to reduce the rate of product deterioration. Kulkarni (2008) considered a multi-product environment where production lot-sizing and investing on quality improvements in several production processes were desired. Klingelhöfer (2009) offered a general approach to valuating investments in end-of-pipe-technologies (EOP-technologies) with special regard to an emissions trading scheme. Other researchers such as Liu and Çetinkaya (2007), Mathur and Shah (2008), and Lin (2009) considered different investment issues. However, little attention has been paid to research in the investment on imperfect production processes.

In this study, we develop an EPQ model with the investment on imperfect production processes under limited investment cost. The renewal theory is considered in the model. An algorithm is developed to derive replenishment and investment polices under limited capital such that the expected unit time profit is maximized.

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2. ASSUMPTIONS AND NOTATION

The mathematical models presented in this study have the following assumptions:

(1) The customer's demand is $D(t) = a$, constant.
(2) The production rate, M, is known and constant with $M > a$
(3) The lead-time is known and constant.
(4) The screening process and demand proceeds simultaneously.
(5) The defective items exist in each production. The defective percentage, p, has a uniform distribution over $[0, \beta]$, where $0 \le \beta < 1$.
(6) No shortages are allowed.
(7) A single product is considered.

The following notations are used:

T	production cycle length
t_1	production run time per cycle, decision variable
M	production rate
x	screening rate, $x > D(t)$
c	production cost per unit
K	setup cost per production
p	defective percentage in per production, which is a random variable with uniformly distribution over $[0, \beta(r)]$ which the defective percentage function, $\beta(r)$, is a decreasing function of the investment cost of r.
s	selling price of good quality items per unit $s > c$
v	selling price of defective items per unit, $c > v$
r	investment cost on production processes, decision variable
d	screening cost per unit
h	inventory holding cost per item
TR	total revenue per cycle; which is the sum of total sales of good quality and imperfect quality items
TC	total cost per cycle
TPU	net profit per unit time
$ETPU$	expected value of TPU with investing on production processes
$ETPUw$	expected value of TPU without investing on production processes

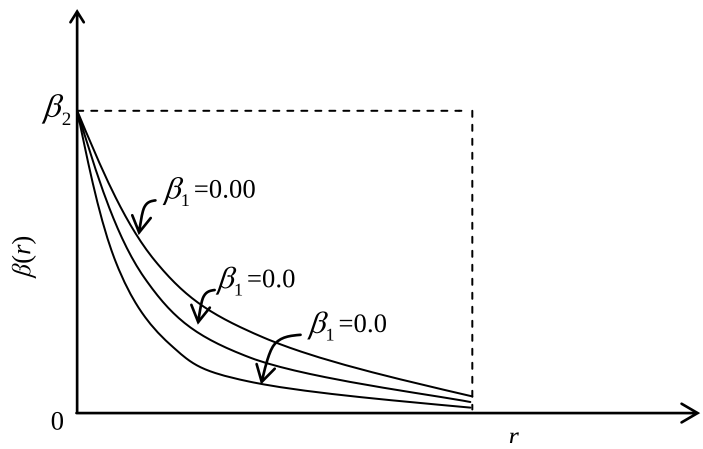

Figure 1. The defective percentage function, $\beta(r)$, against the investment cost for different value of β_1 .

3. MATHEMATICAL MODEL

In this study, the customer's demand of $D(t) = a$ is considered. We assume an imperfect production process with a constant production rate of M, the production cost of c per unit and a setup cost of K per production. Each lot produced contains some percentage of defectives, p, with uniformly distribution over $[0, \beta(r)]$ which $\beta(r)$ is a decreasing function of the investment cost of r (Please refer to Figure 1). The selling price of good quality items is s per unit. The items with imperfect quality assumed a 100% screening of the production process at a constant rate of x per unit time. Items of poor quality are kept in stock and sold prior to the next production as a single batch at a discounted price of v . No shortages are allowed. Two cases may occur: first, when the screening rate is higher than the production rate (i.e., $x \ge M$), the screening time of t_x is shorter than the production run time of t_1, the behavior of the inventory level is illustrated in Figure 2. Second, when the screening rate is lower than the production rate (i.e., $x < M$), the screening time of t_x is longer than the production run time of t_1, the behavior of the inventory level is illustrated in Figure 5. In relation to the real world, the investment cost is limited; we need to consider the limited capital in the study. The optimum operating inventory strategy is obtained by trading off the total revenues per unit time, the production cost per unit time, the inventory holding cost per unit time and the item screening cost per unit time so that the sum is the maximum.

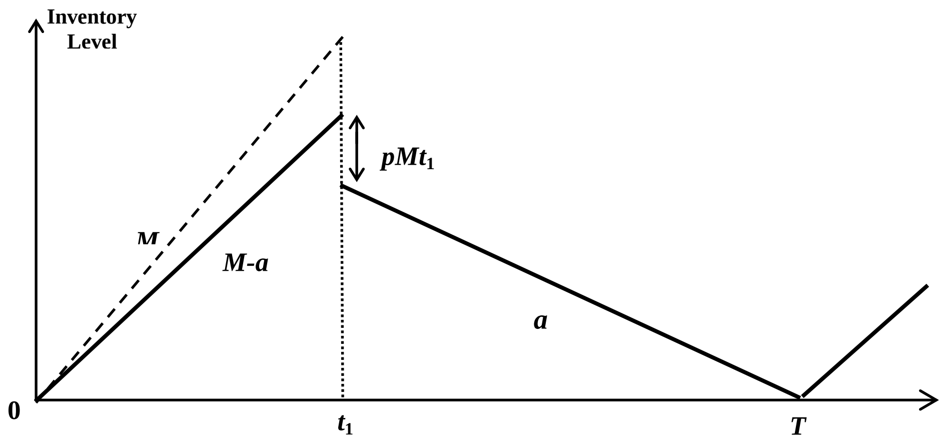

Figure 2. Inventory system when $x \geq M$.

3.1. Case I when $x \geq M$

To avoid shortages, it is assumed that the number of good items at t_1 is at least equal to the demand during production time, that is $(1-p)Mt_1 - at_1 \geq 0,$ and can be rewritten as

$$(1-p)M - a \geq 0. \ (1-p)M - a \geq 0. \tag{1}$$

The random variable p is uniformly distributed over $\left[0, \beta(r)\right]$, where $0 < \beta(r) < 1$, $\beta(r)$ is assumed to be a decreasing function of the investment cost of r. We define $TR(t_1, p)$ as the total revenue that is the sum of total sales of good quality and the imperfect quality items. One has

$$TR(t_1, p) = aT(t_1, p)s + pMt_1\nu \text{ . (Please refer to (5))} \tag{2}$$

$TC(r, t_1, p)$ is the sum of setup cost per cycle, investment cost per cycle, production cost per cycle, screening cost per cycle, and holding cost per cycle. One has

$$TC(r, t_1, p) = K + r + cMt_1 + dMt_1 + h\left\{\frac{(M-a)t_1^2}{2} + \frac{[(1-p)M - a]t_1(T(t_1, p) - t_1)}{2}\right\}. \tag{3}$$

The total profit per unit time of $TPU(r, t_1, p)$ given by dividing the total profit per cycle by the cycle length of T is

$$TPU(r, t_1, p) = \frac{TR(t_1, p) - TC(r, t_1, p)}{T(t_1, p)}. \tag{4}$$

here $\left(M - a\right)t_1 - pM\ t_1 = a(T - t_1)$ (please refer to Figure 2), and T can be rewritten as

$$T(t_1, p) = \frac{(M - pM)t_1}{a}. \tag{5}$$

The expected value of $TPU(r, t_1, p)$ is

$$ETPU(r, t_1) = E[\frac{TR(t_1, p) - TC(r, t_1, p)}{T(t_1, p)}]. \tag{6}$$

Since the process of generating the profit is renewal (with renewal points at production epochs), the expected profit per unit time is given by the renewal-reward theorem (Maddah and Jaber, 2008) as

$$ETPU(r,t_1)=\frac{E[TR(t_1,p)-TC(r,t_1,p)]}{E[T(t_1,p)]}, \tag{7}$$

Where

$$E(TR(t_1,p))=Mt_1[s-E(p)(s-v)] \tag{8}$$

$$E(TC(r,t_1,p))=K+r+cMt_1+dMt_1+h\{\frac{(M-a)t_1^2}{2}$$
$$+\frac{[1-2E(p)+E(p^2)]M^2-2(1-E(p)Ma+a^2}{2a}t_1^2\}. \tag{9}$$

$$E(T(t_1,p))=\frac{[1-E(p)]Mt_1}{a}. \tag{10}$$

And

$$E(p)=\int_0^{\beta(r)}p\frac{1}{\beta(r)}dp=\frac{\beta(r)}{2}. \tag{11}$$

$$E(p^2)=\int_0^{\beta(r)}p^2\frac{1}{\beta(r)}dp=\frac{\beta(r)^2}{3}. \tag{12}$$

Due to the constraint of (1), and since the defective percentage p is a random variable with uniformly distribution over $[0,\beta(r)]$, and if the investment cost, r, is limited by R, then our problem can be formulated as:

Max: $ETPU(r, t_1)$ (13)
Subject to: $(1-\beta(r))M-a\geq 0,\ R\geq r\geq 0, t_1\geq 0.$ (14)

3.2. Optimal solution

In order to confirm the optimal solution in $ETPU(r, t_1)$, the following sufficient conditions must be satisfied:

$$\left(\frac{\partial^2 ETPU}{\partial r\partial t_1}\right)^2-\left(\frac{\partial^2 ETPU}{\partial r^2}\right)\left(\frac{\partial^2 ETPU}{\partial t_1^{\,2}}\right)\leq 0, \tag{15}$$

and one or both

$$\frac{\partial^2 ETPU}{\partial r^2}\leq 0,\quad \frac{\partial^2 ETPU}{\partial t_1^{\,2}}\leq 0.$$

However, (15) is hard to prove. As a result, a solution procedure is developed. We first prove that the optimal production run time per cycle, t_1, is unique for any given invested capital r. Next, we provide a simple algorithm to find the optimal investment cost and production schedule for the proposed model. Since

$$\frac{\partial}{\partial t_1}ETPU(r,t_1)=\frac{hM^2t_1^{\,2}[E^2(p)-3E(p)+3]-3hMt_1^{\,2}a[1-E(p)]-6a(K+r)}{3Mt_1^{\,2}[-2+E(p)]}. \tag{16}$$

$$\frac{\partial^2}{\partial t_1^{\,2}}ETPU(r,t_1)=\frac{-2a(K+r)}{M(1-E(p))t_1^{\,3}}<0. \tag{17}$$

Where $0\leq E(p)<1$. Therefore, $ETPU\left(r,t_1\right)$ is concave in t_1, and gets an optimal solution of t_1 for any given invested capital, r. The optimal solution, $t_{1r}\,*$, by setting $\frac{\partial}{\partial t_1}ETPU(r,t_1)=0$ can be formulated as:

$$t_{1r}{}^{*} = \sqrt{\frac{6a(K+r)}{hM^2[E^2(p)-3E(p)+3]-3hMa[1-E(p)]}}\,. \tag{18}$$

Therefore, the solution procedure of optimal (r*, t_1*) is described as follows:

Solution Procedure

Step 1 Start with j=1

Step 2 Set $r=j$, If r satisfies the domain constraint (i.e., $(1-\beta(r))M-a\geq 0$), then find the optimal production run time per cycle, $t_{1r}{}^{*}$ by (18). Otherwise go to Step 4.

Step 3 Use the result in Step 2 to calculate the $ETPU\left(r,t_{1r}{}^{*}\right)$ by (7).

Step 4 $j=j+1$, if $j<R$, then go to Step 2, otherwise go to Step 5.

Step 5 Determine the maximal value Ω of $ETPU\left(r,t_{1r}{}^{*}\right)$ derived from Step 2, Ω is the optimal solution. Stop.

Example 1. In this example, a=50000, M=80000, x=90000, K=500, h=5, d=0.5, c=35, s=50, v =5, and R=2000. The percentage defective random variable, p, can take any value in the range [0, $\beta(r)$] with $\beta\left(r\right)=..=\dfrac{0.1}{1+0.01r}$, where β_1 and β_2 are constants (β_2 denotes the original probability of defective items). With the given data, the optimal decision is obtained by using the software MATHCAD, the solution is r*=\$1090 and t_1*=0.115 year. The EPQ is Q*=Mt_1*=9248 units, and the maximum profit per year $ETPU$(r*, t_1*)=\$701299. When considering no investment on production processes, the optimal solution is t_1*=0.066 year, $ETPU$(0, t_1*)=\$635017. This result shows that the effects of the investment are significant; the percentage of profit increase is (701299/635017)-1=10.4%.

Alternative approach for solution procedure

The solution procedure of optimal (r*, t_1*) described above is calculated step by step. In this section, an alternative approach for the solution procedure referred to Teng et al. (2012) is developed. As in (18), for a given invested capital, r, the optimal solution of $ETPU$ is t_{1r} *. The optimal expected value function $ETPU$(r, t_1r*(r)) is well-defined to illustrate the optimal value behavior of $ETPU$ when r varies. Figure 3 depicts the optimal expected value behavior in [0,2000] with the parameters used in Example 1. It shows the approximate position of optimal expected value. Figure 4 depicts the more accurate position of optimal expected value behavior in [1080,1100]. The details are described in Example 2.

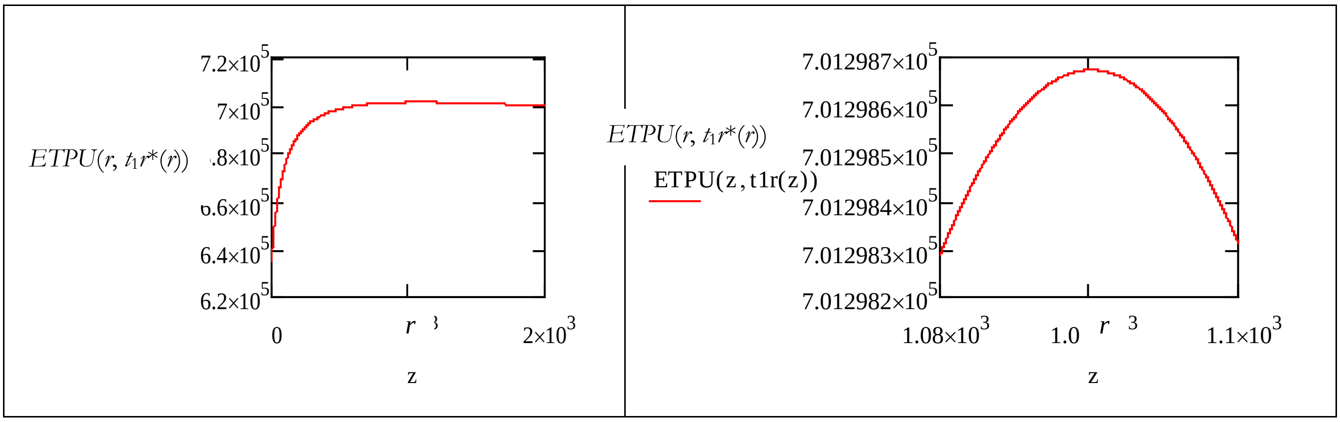

Figure 3. Optimal expected value behavior in [0,2000] when x>M.

Figure 4. Optimal expected value behavior in [1080,1100] when x>M.

Example 2. In order to compare with Example 1, the parameters in Example 1 are also used in this example. With the parameters, $ETPU(r,t_1)$ can be calculated by software Maple as follows:

$$ETPU(r,t_1) = -0.00625\ (1.0907\times 10^{11} t_1 - 9.8\times 10^{11} t_1 - 2.14\times 10^{10} t_{1r} + 2.28\times 10^{9} t_1^{\,2} r - 1.16\times 10^{8} t_1 r^2$$

$$+1.2\times 10^{7} t_1^{\,2} r^2 + 5\times 10^{8} + 1.1\times 10^{7} r + 70000 r^2 + 100 r^3) / \left[(95+r)\ (100+r) t_1\right]$$

From (18), $t_{1r}*(r)$ can be calculated by software Maple as follows:

$$t_{1r}*(r) = 10\sqrt{(2.28\cdot 10^9 r + 1.0907\cdot 10^{11} + 1.2\cdot 10^7 r^2)(r+500)}(100+r) / (2.28\cdot 10^9 r + 1.0907\cdot 10^{11} + 1.2\cdot 10^7 r^2)\,.$$

From (17), the optimal r^* =\$1090 is derived by setting $\frac{\partial}{\partial t_1} ETPU(r, t_1^{\,*}(r)) = 0$, and $t_{1r}^{\ *}\left(r^*\right)$ = 0.115 year. These results are the same as that of Example 1. These results are the same as that of Example 1 and the approach is easier.

3.3. Case II when $x < M$

To avoid shortages, it is assumed that the number of good items, (1-p)Mt_1, is equal or greater than the demand during [0, t_x], that is (please refer to Figure 5).

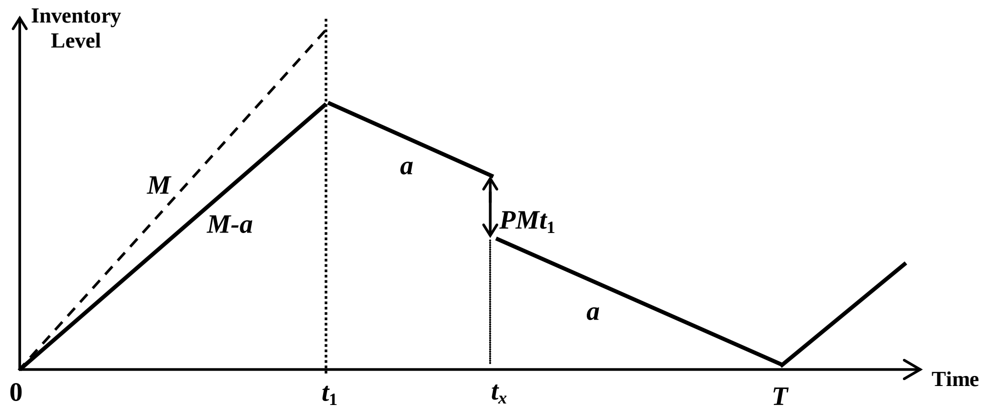

Figure 5. Inventory system when x $<$ M

$$(1-\beta(r))Mt_1 - at_x \geq 0\,, \tag{19}$$

where $t_x = \frac{Mt_1}{x}$. (20)

$$TR(t_1,p) = a(T(t_1,p)s + pMt_1\nu\,. \tag{21}$$

$$TC(r,t_1,p) = K + r + cMt_1 + dMt_1 + h\left\{[(M-a)t_1]\frac{t_1}{2} + \left[2(M-a)t_1 - a(t_x - t_1)\right]\frac{(t_x - t_1)}{2}\right.$$

$$\left. +[(1-p)Mt_1 - at_x]\frac{(T(t_1,p) - t_x)}{2}\right\}. \tag{22}$$

$$T(t_1,p) = \frac{(M-pM)t_1}{a}\,. \tag{23}$$

One has

$$ETPU(r,t_1) = \frac{E[TR(t_1,p) - TC(r,t_1,p)]}{E[T(t_1,p)]}\,. \tag{24}$$

Due to the constraint of (19), and since the defective percentage p is a random variable with uniformly distribution over $[0, \beta(r)]$, and if the investment cost, r, is limited by R, then our problem can be formulated as:

Max: $ETPU(r,t_1)$

Subject to: $(1-\beta(r))Mt_1 - at_x \geq 0$, $R \geq r \geq 0, t_1 \geq 0$. (25)

Since

$$\frac{\partial}{\partial t_1}ETPU(r,t_1) = \frac{hM^2t_1^2[2xE(p)-2aE(p)-x-hxE(p^2)]+hMt_1^2xa+2xa(K+r)}{2Mxt_1^2[1-E(p)]}. \quad (26)$$

$$\frac{\partial^2}{\partial t_1^2}ETPU(r,t_1) = \frac{-2a(K+r)}{M(1-E(p))t_1^3} < 0. \quad (27)$$

Therefore, it means that $ETPU(r,t_1)$ is concave in t_1, and gets an optimal solution of t_1 for any given invested capital, r. The optimal solution, t_{1r}^*, can be formulated as:

$$t_{1r}* = \sqrt{\frac{-2xa(K+r)}{hMxa+hM^2[2xE(p)-2aE(p)-x-hxE(p^2)]}}. \quad (28)$$

Example 3.

In this example, a=50000, M=80000, x=60000, K=500, h=5, d=0.5, c=35, s=50, v =5, and R=1200. The percentage defective random variable, p, can take any value in the range $[0, \beta(r)]$ with $\beta(r)\frac{\beta_2}{1+\beta_1 r} = \frac{0.1}{1+0.01r}$. With the given data and the solution procedure, the optimal decision is obtained by using the software MATHCAD, the solution is r*=$1093 and t_1*=0.115 year. The EPQ is Q*=Mt_1*=9234 units, and the maximum profit per year $ETPU(r^*, t_1^*)$=$701258.

4. SENSITIVITY ANALYSIS

Sensitivity analysis is carried out when the parameters β_1, β_2 and x are changed. Table 1 (Figure 6), Table 2 (Figure 7), and Table 3 (Figure 8) show the changes in the investment cost, r, the production run time, t_1, the expected net profit, *ETPU* and the expected net profit without investing, *ETPUw*. The % profit increase is [(*ETPU*- *ETPUw*)/*ETPUw*)]*100%.

Table 1 and Figure 6 show (β_1) at 0.003, 0.004,…,0.017 with other variables unchanged. It is shown that as β_1 increases, the investment cost decreases, while the expected net profit increases. Table 2 and Figure 7 show (β_2) at 0.03, 0.04,…,0.17 with other variables unchanged. It is shown that as β_2 increases, the investment cost increases, while the expected net profit decreases. Table 3 and Figure 8 show the screening rate, x, at 53000, 54000,…, 67000 with other variables unchanged. It is shown that as x increases, the investment cost decreases, while the expected net profit increases. Furthermore, from Table 1 to Table 3, we can see the significant % profit increase.

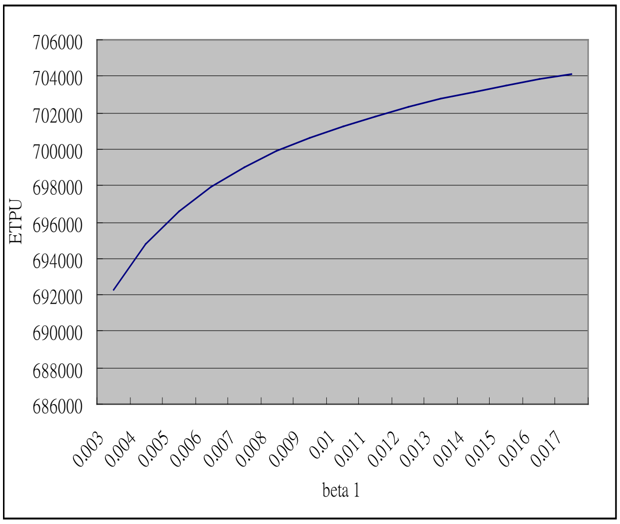

Figure 6. The effect of β_1 on the expected value of *TPU*.

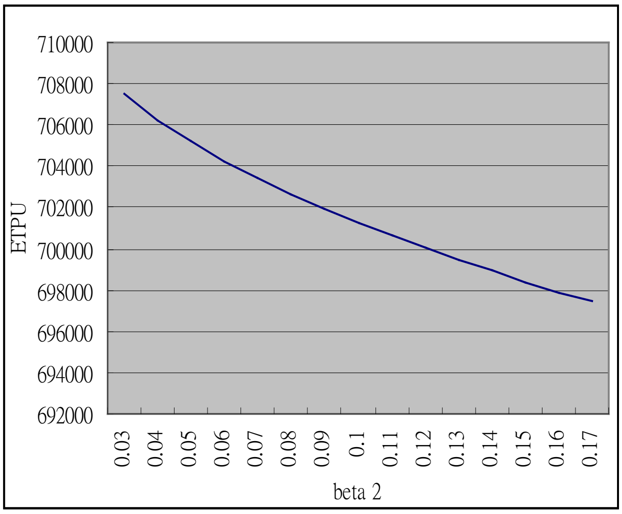

Figure 7. The effect of β_2 on the expected value of *TPU*.

Table 1. Sensitivity analysis of β_1.

a=50000, M=80000, x=60000, k=500, h=5, d=0.5, c=35, s=50, v=5, β_2=0.1.					
β_1	r	t_1	*ETPU*	*ETPUw*	% profit increase
0.003	2147	0.149	692320	634728	9.1%
0.004	1828	0.14	694807	634728	9.5%
0.005	1613	0.133	696575	634728	9.7%
0.006	1456	0.128	697921	634728	10%
0.007	1336	0.124	698994	634728	10.1%
0.008	1239	0.121	699876	634728	10.3%
0.009	1160	0.118	700620	634728	10.4%
0.01	1093	0.115	701258	634728	10.5%
0.011	1036	0.113	701815	634728	10.6%
0.012	987	0.112	702301	634728	10.6%
0.013	943	0.11	702744	634728	10.7%
0.014	905	0.108	703137	634728	10.8%
0.015	871	0.107	703494	634728	10.8%
0.016	840	0.106	703819	634728	10.9%
0.017	812	0.105	704118	634728	10.9%

Table 2. Sensitivity analysis of β_2.

a=50000, b=40000, M=80000, x=60000, k=500, h=5, d=0.5, c=35, s=50, v=5, β_1=0.01.					
β_2	r	t_1	*ETPU*	*ETPUw*	% profit increase
0.03	478	0.09	707471	692009	2.2%
0.04	584	0.095	706248	684079	3.2 %
0.05	682	0.099	705195	676067	4.3 %
0.06	773	0.103	704262	667971	5.4%
0.07	859	0.107	703418	659791	6.6%
0.08	940	0.11	702645	651524	7.8%
0.09	1018	0.113	701928	643170	9.1%
0.1	1093	0.115	701258	634728	10.5%
0.11	1166	0.118	700628	626195	11.9%
0.12	1236	0.12	700033	617570	13.4%
0.13	1304	0.123	699467	608852	14.9%
0.14	1370	0.125	698928	600040	16.5%
0.15	1435	0.127	698412	591131	18.1%
0.16	1499	0.129	697917	582125	19.9%
0.17	1561	0.131	697441	573019	21.7%

Table 3. Sensitivity analysis of screening rate, x.

a=50000, b=40000, M=80000, k=500, h=5, d=0.5, c=35, s=50, V=5, β_1=0.01, β_2=0.1.					
x	r	t_1	*ETPU*	*ETPUw*	% profit increase
53000	1095	0.115	701237	634576	10.5%
54000	1094	0.115	701240	634600	10.5%
55000	1094	0.115	701243	634623	10.5%
56000	1094	0.115	701247	634646	10.5%
57000	1094	0.115	701250	634667	10.5%
58000	1093	0.115	701253	634688	10.5%
59000	1093	0.115	701255	634708	10.5%
60000	1093	0.115	701258	634728	10.5%
61000	1093	0.115	701261	634747	10.5%
62000	1093	0.115	701263	634765	10.5%
63000	1092	0.115	701266	634783	10.5%
64000	1092	0.115	701268	634800	10.5%
65000	1092	0.115	701271	634817	10.5%
66000	1092	0.115	701273	634833	10.5%
67000	1092	0.116	701275	634849	10.5%

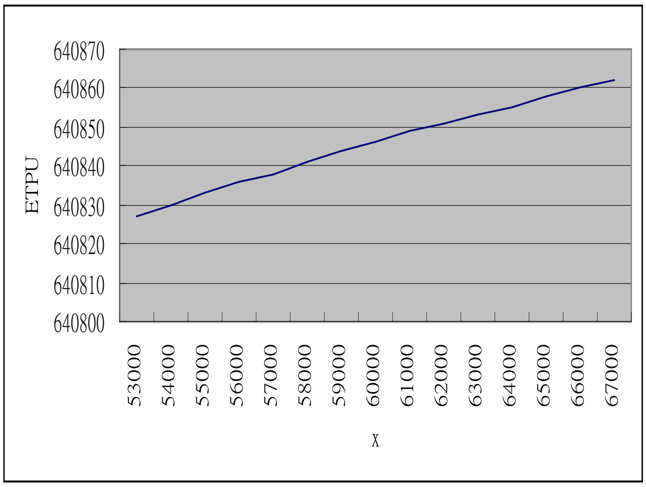

Figure 8. The effect of screening rate, x, on the expected value of *TPU*.

5. CONCLUSION

Under limited capital conditions, managers are faced with the key problem of how to improve the firm's business by increasing the investment cost. This study develops an EPQ model with an investment on improving the production process under the limited capital. An algorithm is developed to derive replenishment and investment policies such that the expected profit per unit time is maximized. Numerical examples and sensitivity analysis are provided to illustrate the theory. Sensitivity analysis shows that as x increases, the investment cost decreases, while the expected net profit increases. In addition, a significant % profit increase is observed when the investment cost is considered. The results will provide managerial insights to managers for improved decision making. Future research can be done for stochastic demand.

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