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Multi-Objective Decision Making With A Large Number of Objectives. An Application For Europe 2020

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Abstract — The European Union attempts to reach a growing and sustainable economy by 2020, a much more moderate target than the 2010 target of becoming the most competitive and dynamic knowledge-based economy in the world. This intention has to be supported by an adequate Optimization and Decision Support System, such as MULTIMOORA. MULTIMOORA is a quantitative method, comparing multiple objectives, expressed in different units. In opposition to similar methods, MULTIMOORA does not need normalization, being based on dimensionless measures. MULTIMOORA is composed of three approaches: Ratio System, Reference Point and Multiplicative Form Methods, all of the same importance and each controlling each other. Twenty two objectives characterize the EU-Countries economies as a preparation for 2020. Unless important structural Decision Making will change their international position, "which of these countries are the best prepared for 2020" can be concluded from this study on basis of Ordinal Dominance.

Keywords — Introduction, Multi-Objective Decision Making, MULTIMOORA, Ordinal Dominance Theory, Discordance, European Union.

1. INTRODUCTION

Twenty two objectives, 10 originating from statistics and 12 from statistics and forecasts important for the future, will characterize the 27 EU-Countries economies as a preparation for 2020. Nevertheless these data concern only the economic guidelines of Strategy Europe 2020 and neither purely social nor climate or energy targets.

As all these data are expressed in different units the exercise needs an adequate Decision Support System for optimization.

Twenty two Objectives from 27 countries, grouped in Objectives and Super Objectives, result in 594 data, which means that methods of partial aggregation cannot be used:

The Super Objective of Economic Importance represented by 10 objectives:

1. Current account deficit of the Balance of Payments (obj. 3, a MIN.)
2. GDP per capita in Purchasing Power Parity (obj. 4, a MAX.)
3. GDP growth rate (percentage to a base year, obj. 5, a MAX.)
4. Inflation (obj. 6, a MIN.)
5. Employment rate (obj. 8, a MAX.)
6. Unemployment rate (obj. 9, a MIN.)
7. GDP per capita with basis EU-15=100, prospective compared to a base year (obj.15, a MAX.)
8. GDP growth rate, prospective, percentage compared to last year, , obj. 16, a MAX.)
9. GDP per capita index number with basis EU-15=100, prospective, compared to last year (obj. 19, a MAX.)
10. GDP growth rate, prospective compared to last year (obj. 20, a MAX.)

The Super Objective of Public Finance represented by 7 objectives:

1. Government Budget Deficit as a % of GDP (obj. 1, a MIN.)
2. Government Debt as a % of GDP (obj. 2, a MIN.)
3. Government bond yields as a % (obj. 7, a MIN.)
4. Government Budget Deficit as a % of GDP, prospective compared to a base year (obj. 17 a MIN.)
5. Government Debt as a % of GDP prospective compared to a base year (obj. 18 a MIN.)
6. Government Budget Deficit as a % of GDP compared to last year (obj.21 a MIN.)
7. Government Debt as a % of GDP compared to last year (obj. 22 a MIN.)

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The Super Objective of the Population Pyramid represented by 5 objectives:

1. Tertiary education as a % of age group 30-34 (obj. 10 a MAX.)
2. Median Age of total population (obj. 11 a MIN)
3. Proportion to population aged 0-14 of total population (obj. 12 a MAX.)
4. Proportion to population aged 15-64 of total population (obj. 13 a MAX.)
5. Proportion to population aged 65 or more of total population (obj. 14 a MIN.)

The data come from:

- IMF and World Bank, 2011.
- EUROSTAT, 2011.
- European Commission, 2011.
- European Central Bank, 2011.

Following table 1 summarizes research by Brauers and Baležentis (2012) on basis of these data.

2. CHOICE OF A METHOD FOR MULTI-OBJECTIVE OPTIMIZATION

Methods of partial aggregation and methods based on weights are excluded for large decision matrices. Why weights have to be excluded? It would be extremely difficult and time consuming for stakeholders to reach convergence in the choice of weights for the 594 data of the European example.

How is it possible to find substitutes for Weights? Therefore it is necessary to read the decision matrix, composed of 594 elements, not in a horizontal but in a vertical way with objectives headings ranged horizontally and alternative solutions headings vertically. Dimensionless ratios are obtained per objective by making averages per column of the decision matrix.

MOORA (Multiple Objectives Optimization by Ratio Analysis) reads the decision matrix in that way. The Reference Point Method uses the obtained ratios as inputs in order to compare them with the coordinates of a Reference Point.

To make the series of dimensionless measures methods complete MULTIMOORA adds a Full Multiplicative Form, whereby its factors loose their identity and ipso facto become dimensionless.

Table 1. Data for the 22 objectives for the European Union Member States (2010-2012)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
	MIN	MIN	MIN	MAX	MAX	MIN	MIN	MAX	MIN	MAX	MIN	MAX	MAX	MIN	MAX	MAX	MIN	MIN	MAX	MAX	MIN	MIN
Belgium	4.1	96.8	-1.04	32,600	2.2	2.3	3.9	62.0	8.3	44.4	40.9	16.9	65.9	17.2	106	2.4	3.7	97	106	2.2	4.2	97.5
Bulgaria	3.2	16.2	0.99	4,800	0.2	3.0	5.5	59.7	10.2	27.7	41.4	13.6	68.9	17.5	40.5	2.8	2.7	18	41.6	3.7	1.6	18.6
Czech R.	4.7	38.5	3.84	14,200	2.3	1.2	3.8	65.0	7.3	20.4	39.4	14.2	70.6	15.2	75	2	4.4	41.3	76.1	2.9	4.1	42.9
Denmark	2.7	43.6	-5.48	42,200	1.7	2.2	2.9	73.4	7.4	47.0	40.5	18.1	65.6	16.3	110	1.7	4.1	45.3	109	1.5	3.2	47.1
Germany	3.3	83.2	-5.71	30,300	3.7	1.2	2.8	71.1	7.1	29.8	44.2	13.5	65.8	20.7	109	2.6	2	82.4	110	1.9	1.2	81.1
Estonia	-0.1	6.6	-3.76	10,700	2.3	2.7	5.4	61.0	16.9	40.0	39.5	15.1	67.8	17.1	60.6	4.9	0.6	6.1	62	4	2.4	6.9
Ireland	32.4	96.2	0.71	34,900	-0.4	-1.6	9.2	60.0	13.7	49.9	34.3	21.3	67.4	11.3	111	0.6	10.5	112	111	1.9	8.8	118
Greece	10.5	142.8	10.5	20,100	-4.5	4.7	13.2	59.6	12.6	28.4	41.7	14.4	66.7	18.9	75.9	-3.5	9.5	158	75.5	1.1	9.3	166
Spain	9.2	60.1	4.56	23,100	-0.1	2.0	5.1	58.6	20.1	40.6	39.9	14.9	68.3	16.8	91	0.8	6.3	68.1	90.7	1.5	5.3	71
France	7	81.7	1.74	29,800	1.5	1.7	3.3	64.0	9.8	43.5	39.8	18.5	64.9	16.6	97.2	1.8	5.8	84.7	97.2	2	5.3	86.8
Italy	4.6	119	3.29	25,600	1.3	1.6	4.7	56.9	8.4	19.8	43.1	14.1	65.7	20.2	92.8	1	4	120	92.3	1.3	3.2	120
Cyprus	5.3	60.8	7.74	21,600	1.0	2.6	5.0	69.7	6.3	45.1	36.2	16.9	70.0	13.1	88	1.5	5.1	62.3	88.1	2.4	4.9	64.3
Latvia	7.7	44.7	-3.58	8,000	-0.3	-1.2	7.0	59.3	18.7	32.3	40.0	13.8	68.8	17.4	47.6	3.3	4.5	48.2	49.2	4	3.8	49.4
Lithuania	7.1	38.2	-1.85	8,400	1.3	1.2	5.1	57.8	17.8	43.8	39.2	15.0	68.9	16.1	52.7	5	5.5	40.7	54.8	4.7	4.8	43.6
Luxemburg	1.7	18.4	-7.82	79,500	3.5	2.8	3.1	65.2	4.5	46.1	38.9	17.7	68.3	14.0	248	3.4	1	17.2	251	3.8	1.1	19
Hungary	4.2	80.2	-2.06	9,700	1.2	4.7	7.3	55.4	11.2	25.7	39.8	14.7	68.7	16.6	59.7	2.7	-1.6	75.2	60.6	2.6	3.3	72.7
Malta	3.6	68.0	4.17	14,800	2.7	2.0	4.4	56.0	6.9	18.6	39.2	15.6	69.6	14.8	75	2	3	68	75.2	2.2	3	67.9
Netherlands	5.4	62.7	-7.16	35,400	1.8	0.9	3.1	74.7	4.5	41.4	40.6	17.6	67.1	15.3	119	1.9	3.7	63.9	118	1.7	2.3	64
Austria	4.6	72.3	-2.71	34,100	2.3	1.7	3.3	71.7	4.4	23.5	41.7	14.9	67.5	17.6	114	2.4	3.7	73.8	114	2	3.3	75.4
Poland	7.9	55.0	4.47	9,300	3.8	2.7	5.9	59.3	9.6	35.3	37.7	15.2	71.3	13.5	57.6	4	5.8	55.4	58.8	3.7	3.6	55.1
Portugal	9.1	93.0	9.88	16,200	1.3	1.4	8.4	65.6	12.0	23.5	40.7	15.2	66.9	17.9	70	-2.2	5.9	102	67.8	-1.8	4.5	107
Romania	6.4	30.8	4.06	5,700	-1.3	6.1	7.1	58.8	7.3	18.1	38.3	15.2	69.9	14.9	40.9	1.5	4.7	33.7	41.9	3.7	3.6	34.8
Slovenia	5.6	38.0	0.84	17,300	1.4	2.1	4.3	66.2	7.3	34.8	41.4	14.0	69.5	16.5	79.6	1.9	5.8	42.8	80.2	2.5	5	46
Slovakia	7.9	41.0	3.44	12,100	4.0	0.7	4.2	58.8	14.4	22.1	36.9	15.3	72.4	12.3	69.3	3.5	5.1	44.8	71.2	4.4	4.6	46.8
Finland	2.5	48.4	-3.09	33,600	3.6	1.7	3.1	68.1	8.4	45.7	42.0	16.6	66.4	17.0	105	3.7	1	50.6	106	2.6	0.7	52.2
Sweden	0	39.8	-6.26	37,000	5.7	1.9	3.0	72.7	8.4	45.8	40.7	16.6	65.3	18.1	114	4.2	-0.9	36.5	115	2.5	-2	33.4
U. K.	10.4	80.0	3.18	27,400	1.4	3.3	3.3	69.5	7.8	43.0	39.6	17.5	66.0	16.5	101	1.7	8.6	84.2	101	2.1	7	87.9

3. MULTIPLE OBJECTIVES OPTIMIZATION BY RATIO ANALYSIS (MOORA)

3.1 The two parts of MOORA

The method starts with the decision matrix of different alternatives on different objectives:

$$(x_{ij})$$

with: x_{ij} as the response of alternative *j* on objective *i*
i=1,2,…,n as the objectives
j=1,2,…,m as the alternatives

MOORA goes for a ratio system in which the response of an alternative on an objective is compared to a denominator, which is representative for all alternatives concerning that objective.

If for this denominator the sum of each alternative per objective is chosen, the traditional formula of averages is obtained:

$$x_{ij}^{\circ} = \frac{x_{ij}}{\sum_{j=1}^{m} x_{ij}} \tag{1}$$

This formula may lead to unexpected results. Indeed, for instance in the case of productivity growth some sectors, regions or countries may show a decrease instead of an increase in productivity i.e. a negative number. In this way the sum in the denominator could become negative and ipso facto all ratios become negative. In addition, a negative numerator with a negative denominator will result in a positive ratio, in contradiction with the original negative numerator. Even worse, the denominator can become zero and division by zero means a senseless operation.

Brauers and Zavadskas (2006) tested eight other formulas. They all had main disadvantages with exception of the formula where for the denominator the square root of the sum of squares of each alternative per objective was chosen:

$$x_{ij}^{*} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}^{2}}} \tag{2}$$

with: x_{ij} = response of alternative *j* on objective *i.*
j = 1,2,...,m; m the number of alternatives.
i = 1,2,…n; n the number of objectives.
x_{ij}^* = a dimensionless number representing the response of alternative j on objective i.

Dimensionless numbers, having no specific unit of measurement, are obtained for instance by deduction, multiplication or division.

In formula (2) the responses of the alternatives on the objectives belong to the interval [0; 1]. However, sometimes the interval could be [-1; 1]. Indeed, for instance in the case of productivity growth some sectors, regions or countries may show a decrease instead of an increase in productivity leading to a negative dimensionless number, up to -1.

For optimization these responses are added in case of maximization and subtracted in case of minimization:

$$y_{j}^{*} = \sum_{j=1}^{j=g} x_{ij}^{*} - \sum_{j=g+1}^{j=n} x_{ij}^{*} \tag{3}$$

with: i = 1,2,…,g as the objectives to be maximized.
i = g+1, g+2,…, n as the objectives to be minimized.
y_j^* = the normalized assessment of alternative j with respect to all objectives.

An ordinal ranking in a descending order of the y_j^* shows the final preference.

For the second part of MOORA the Reference Point Theory is chosen with the Metric of Tchebycheff as given by the following formula (Karlin and Studden, 1966, 280):

$$\underset{(j)}{Min}\left\{\underset{(i)}{\max}\left|r_i - x_{ij}^{*}\right|\right\} \tag{4}$$

with $\left|r_i - x_{ij}^{*}\right|$ the absolute value if x_{ij}^* is larger than r_i for instance in minimization.

This Reference Point Theory starts from the ratios defined in the MOORA method, namely formula (2). Contrary to other points of view the reference point is preferred here which possesses as co-ordinates the dominating co-ordinates per attribute of the candidate alternatives and which is designated as the *Maximal Objective Reference Point*. This approach is called realistic and non-subjective as the co-ordinates, which are selected for the reference point, are realized in one of the candidate alternatives. The alternatives A (10;100), B (100;20) and C (50;50) will result in the Maximal Objective Reference Point R_m (100;100).

Given equation (4) the results are ranked in an ascending order.

The Importance given to an Objective by the Attribution Method in MOORA

It may look that one objective cannot be much more important than another one as all their ratios are smaller than one (see formula 2) Nevertheless it may turn out to be necessary to stress that some objectives are more important than others. In order to give more importance to an objective its ratios could be multiplied with a *Significance Coefficient*.

In the Ratio System, in order to give more importance to an objective, its relation with an alternative under the form of a dimensionless number could be multiplied with a Significance Coefficient:

$$\ddot{y}_j^{\,*} = \sum_{i=1}^{i=g} s_i x_{ij}^{\,*} - \sum_{i=g+1}^{i=n} s_i x_{ij}^{\,*} \tag{5}$$

with: i = 1,2,…,g as the objectives to be maximized.

i = g+1, g+2,…, n as the objectives to be minimized.

s_i = the significance coefficient of objective i.

$\ddot{y}_j^{*}$ = the total assessment with significance coefficients of alternative j with respect to all objectives.

For the Reference Point Approach importance is a consequence of formula 5 of the ratio system.

The *Attribution of Sub-Objectives* represents another solution to give importance to an objective. Take the example of the purchase of fighter planes (Brauers, 2002). For economics, the objectives concerning the fighter planes are threefold: price, employment and balance of payments, but there is also military effectiveness. In order to give more importance to military defense, effectiveness is broken down in, for instance, the maximum speed, the power of the engines and the maximum range of the plane. Anyway, the *Attribution Method* is more refined than that a significance coefficient method could be as the attribution method succeeds in characterizing an objective better. For instance, for employment two sub-objectives replace a significance coefficient of two and in this way characterize the direct and indirect side of employment separately.

After the given military example the importance given to an objective in the application for the Economies of the EU-countries was not done by the introduction of significance coefficients but by sub-objectives.

4. MULTIMOORA

As MOORA is based on dimensionless measures why not going further by adding the remaining form which uses dimensionless measures namely the Multiplicative Form? We even prefer to speak of the "Full-Multiplicative Form". Otherwise it could refer to a combination with linearity as has been done by Keeney and Raiffa (1993, 234).

The following *n*-power form for multi-objectives is called from now on **Full-Multiplicative Form**:

$$U_j = \prod_{j=1}^{n} x_{ij} \tag{6}$$

with: j = *1,2,...,m; m* the number of alternatives.

i = *1,2,…,n; n* being the number of objectives.

x_{ij} = response of alternative *j* on objective *i.*

U_j = overall utility of alternative *j.*

The overall utilities (U_j), obtained by multiplication of different units of measurement, become dimensionless. The outcome of this presentation is nonlinear, which presents an advantage, as the utility function of human behavior toward several objectives has to be considered as nonlinear.

Rule

In the full-multiplicative form the relation between the utilities U_j does not change if more importance is given to an objective by multiplying it by a factor. Indeed, at that moment all alternatives are multiplied with that factor.

Consequence 1

In the full-multiplicative form the introduction of weights is meaningless. Indeed weights are here in fact multiplying coefficients.

Consequence 2

In the full-multiplicative form an attribute of the size 10, 10^2, 10^3, 10^6, 10^9 etc. can be replaced by the unit size without changing the relationship between the utilities of the alternatives.

This consequence is extremely important for attributes expressed in monetary units. Instead of expressing an attribute in tens, hundreds, thousands, millions, billions for instance of dollars, the use of one digit in the integer part is sufficient.

How is it then possible to give more importance to an objective? Allocating an exponent to an objective signifies stressing the importance of this objective (Miller and Star, 1969).

How to combine a minimization problem with the maximization of the other objectives? Therefore, the objectives to be minimized are denominators in the formula:

$$U_j^{'} = \frac{A_j}{B_j} \tag{7}$$

with: $A_j = \prod_{i=1}^{g} x_{ij}$

j = 1,2,...,m; m the number of alternatives
i = 1,2,…,n; n the number of objectives
g = the number of objectives to be maximized

with: $B_j = \prod_{i=g+1}^{n} x_{ij}$

$n - g$ = the number of objectives to be minimized

$U_j^{'}$: the utility of alternative j with objectives to be maximized and objectives to be minimized.

In the Full Multiplicative Form a problem may arise for zero and negative values making the results senseless. Therefore the index number 100 replaces the zero number. At that moment for instance 96.6 substitutes the negative value of minus 3.4. Consequently, 103.4 represent the positive value of 3.4. This operation has to be done for the whole column involved.

Summarizing one can conclude that the MULTIMOORA method is composed of three parts: the Ratio System, the Reference Point Method and the Full Multiplicative Form. The three methods were already separately discussed by Brauers (2004a), whereas the name of MULTIMOORA was given by Brauers and Zavadskas in 2010.

5. MULTIMOORA APPLIED FOR MULTI-OBJECTIVE OPTIMIZATION OF THE ECONOMIES OF THE EU MEMBER COUNTRIES

The initial data were translated in dimensionless ratios according to Eqs. 2 and 3, i.e. the Ratio System of MOORA. Subsequently Eq. 4 used the ratios obtained in Eq. 2 to calculate the distances to the Reference Point of MOORA. Finally, the Full Multiplicative Form used the initial data to rank the Member States according to Eq. 6 and 7. Following Table 2 presents the results of multi-objective optimization.

How is it possible to obtain a final ranking for 594 elements approached after three methods? Therefore another three possibilities are open: 1) Correlation of Ranks 2) the Median Method and 3) Ordinal Dominance.

Table 2. The three Approaches of MULTIMOORA as applied for the Economy of the EU Member States (a)

	Member States	Ratio S.	Ref. Point	Full Multipl. F. (b)	**Rank RS**	**Rank RP**	**Rank MF**
1	Belgium	-0.07166	0.3196606	1.741	**12**	**6**	**15**
2	Bulgaria	-0.17729	0.5091396	12.667	**13**	**22**	**9**
3	Czech Republic	-0.11831	0.4534052	2.857	**11**	**13**	**11**
4	Denmark	0.865113	0.2549106	30.994	**6**	**3**	**7**
5	Germany	0.373762	0.335337	14.656	**7**	**8**	**8**
6	Estonia	0.172228	0.4689265	8,283.459	**4**	**16**	**3**
7	Ireland	-0.50645	0.7270675	0.028	**25**	**26**	**25**
8	Greece	-1.1709	0.710736	0.0000004	**27**	**27**	**26**
9	Spain	-0.38842	0.4814092	0.0714	**24**	**19**	**23**
10	France	0.11367	0.3719685	0.792	**18**	**9**	**18**
11	Italy	-0.33378	0.4322718	0.0366	**22**	**12**	**24**
12	Cyprus	-0.0528	0.6053159	2.151	**19**	**24**	**14**
13	Latvia	-0.3024	0.4873291	0.261	**15**	**21**	**22**
14	Lithuania	-0.16221	0.4846028	1.383	**10**	**20**	**16**
15	Luxemburg	1.609567	0.5091396	490,600.533	**1**	**7**	**2**
16	Hungary	-0.26461	0.4757423	447.281	**20**	**17**	**5**
17	Malta	-0.21317	0.466408	0.772	**17**	**15**	**19**
18	Netherlands	0.75226	0.3005764	31.987	**5**	**2**	**6**
19	Austria	0.378763	0.3094369	6.722	**8**	**4**	**10**
20	Poland	-0.33806	0.4784686	0.932	**16**	**18**	**17**
21	Portugal	-0.83554	0.6884153	0.00000001	**26**	**25**	**27**
22	Romania	-0.38694	0.5030054	0.288	**23**	**23**	**21**
23	Slovenia	0.054059	0.4239422	2.400	**14**	**10**	**12**
24	Slovakia	-0.29411	0.4593844	2.228	**9**	**14**	**13**
25	Finland	0.567699	0.3128448	713.446	**3**	**5**	**4**
26	Sweden	1.716535	0.2896712	3,371,715,911.324	**2**	**1**	**1**
27	United Kingdom	-0.00365	0.427989	0.499	**21**	**11**	**20**

(a) Details of the calculations can eventually be asked from the authors. Table 1 forms the basis of these calculations.
(b) The differences between the numbers in the Full Multiplicative Form are very large. They are not comparable in a cardinal way but only in the ordinal way under the form of a ranking as given in the last column. For more information see the text after table 5 further on.

6. THE RANK CORRELATION METHOD

The method of correlation of ranks consists of totalizing ranks. Rank correlation was introduced first by psychologists such as Spearman (1904, 1906 and 1910) and later taken over by the statistician Kendall in 1948. He argues (Kendall 1948: 1): "we shall often operate with these numbers as if they were the cardinals of ordinary arithmetic, adding them, subtracting them and even multiplying them," but he never gives a proof of this statement. In his later work this statement is dropped (Kendall and Gibbons 1990).

In ordinal ranking 3 is farther away from 1 than 2 from 1, but Kendal (1948: 1) goes too far. For Kendal B is very far away from A as it has 7 ranks before and A only 4, whereas it is not true cardinally (see table 3).

Table 3. Ordinal versus cardinal: comparing the price of one commodity

	Ordinal	Cardinal
	1	
	2	
	3	
	4	
A	5	6.03$
	6	6.02$
	7	6.01$
B	8	6$

In addition a supplemental notion, the statistical term of Correlation, is introduced. Suppose the statistical universe is just represented by two experts, for us it could be two methods. If they both rank in a same order different items to reach a certain goal, it is said that the correlation is perfect. However, perfect correlation is a rather exceptional situation. The problem is then posited: how in other situations correlation is measured. Therefore, the following Spearman's coefficient is used (Kendall 1948: 8):

$$\rho = 1 - \frac{6\sum D^2}{N(N^2-1)} \tag{8}$$

where D stands for the difference between paired ranks, and N for the number of items ranked.

According to this formula, perfect correlation yields the coefficient of one. An acceptable correlation reaches the coefficient of one as much as possible. No correlation at all yields a coefficient of zero. If the series are exactly in reverse order, there will be a negative correlation of minus one, as shown in the following example (Table 4).

Table 4. Negative rank order correlations

Items	Expert 1	Expert 2	D	D
1	1	7	-6	36
2	2	6	-4	16
3	3	5	-2	4
4	4	4	0	0
5	5	3	2	4
6	6	2	4	16
7	7	1	6	36
Σ				112

This table shows that the sum of ranks in the case of an ordinal scale has no sense. Correlation leads to:

$$\rho = 1 - \frac{6 \times 112}{7(49-1)} = -1$$

However, as addition of ranks is not allowed also a subtraction, the difference D, is not permitted.

Most people will better understand the ordinal problem by the way of a qualitative scale, e. g.:

1st very good;
2nd moderate;
3rd very bad.

But equally one could say:

1st very good;
2nd good;
3rd more or less good;
4th moderate;
5th more or less low;
6th low;
7th very low.

How is the first 2nd comparable with the second 2nd? Etc.

Arbitrary Methods to go from an Ordinal Scale to a Cardinal Scale

1. Arithmetical Progression: 1, 2, 3, 4, 5……
2. Geometric Progression: 1, 2, 4, 8, 16……..
3. Fundamental Scale of Saaty (1987): 1, 3, 5, 7, 9
4. Normal Scale of Lootsma (1987)
   $e^{\circ} = 1$
   $e^1 = 2.7$
   $e^2 = 7.4$
   $e^3 = 20.1$………..
5. Stretched Scale of Lootsma (1987)
   $e^{\circ} = 1$
   $e^2 = 7.4$
   $e^4 = 54.6$
   $e^6 = 403.4$………..
6. Point of View of the Psychologists (Miller, 1965)
   Ordinal Scales: 1, 2, 3, 4, 5, 6, 7. After 7 an individual would no more know the cardinal significance compared to the previous seven numbers.

In fact infinite variations are possible. All stress an acceleration or a dis-acceleration process but are not aware of possible trend breaks. The full multiplicative method with its huge numbers illustrates the best this trend break as shown in Table 5.

Table 5. Ranking of Scenarios for the Belgian Regions by the Full-Multiplicative Method in the Year 1996.

1	Scenario IX	Optimal Economic Policy in Wallonia and Brussels	203,267
2	Scenario X	Optimal Economic Policy in Wallonia and Brussels even agreeing on the Partition of the National Public Debt	196,306
3	Scenario VII	Flanders asks for the Partition of the National Public Debt	164,515
4	Scenario VIII	No Solidarity at all	158,881
5	Scenario II	Unfavorable Growth Rate for Flanders	90
6	Scenario IV	an Unfavorable Growth Rate for Flanders and at that moment asks also for the Partition of the National Public Debt	87
7	Scenario III	Partition of the National Public Debt	54
8	Scenario I	the Average Belgian	51
9	Scenario V	Average Belgian but as compensation Flanders asks for the Partition of the National Public Debt	49
10	Scenario O	Status Quo	43
11	Scenario VI	Flanders asks for the Partition of the National Public Debt	42

Source: Brauers and Ginevičius 2010.

We may conclude from table 5:

1. There is no correlation between the ordinal and the cardinal scales.
2. Note the trend break after rank 4.

Axioms on Ordinal and Cardinal Scales

"Obviously, a cardinal utility implies an ordinal preference but not *vice versa"* (Arrow 1974).

1. A deduction of an Ordinal Scale, a ranking, from cardinal data is always possible
2. An Ordinal Scale can never produce a series of cardinal numbers.
3. An Ordinal Scale of a certain kind, a ranking, can be translated in an ordinal scale of another kind.

7. THE METHOD OF THE MEDIAN

Given a series of numbers the *Median* is then defined as the middle measurement after the measurements have been arranged in order of magnitude.

It is clear that for three methods, like in the MULTIMOORA example, the median value alone is not representative for the composition of the whole series. In order to characterize the series of numbers better the notion of *Skewness* is introduced.

To measure Skewness one could find two other values beside the median. The *First Quartile* is the middle measurement between the origin and the median. The *Third Quartile* is the middle measurement between the median and the end of the scale. In fact the median itself is the Second Quartile (the theory on quartiles is already mentioned by Mills, previously Professor of Columbia University, 1924, 114). In this way one may speak of skewness to the left and skewness to the right. A task could be to find ways and means to decrease skewness by trying to bring the quartiles nearer to each other. A series of numbers is the most equilibrated if the First and the Third Quartile move as much as possible to the Median. For instance convergence of various opinions of a group of discussants is reached by approaching the quartiles as much as possible to the median. This approach forms the basis of the method which is called Delphi where this convergence is reached after several rounds (for Delphi see: Brauers, 2004, 39-46).

In order to speak of a First Quartile one has to have at least 3 numbers and another three for the Third Quartile. With the Median itself the Median Method needs to have at least 6 numbers to be operational. At that moment the median is a point midway the third and the fourth number (Mueller et al. 1970, 122-123). Consequently the Median Method is not valuable for application in the study of MULTIMOORA, counting only three numbers every time.

If assuming that there would be no overlapping in methods additionally TOPSIS (Hwang and Yoon, 1981) and VIKOR (Opricovic and Tzeng, 2004), as other methods with dimensionless measures, could be added. Even then 5 methods would not be sufficient.

8. THE THEORY OF ORDINAL DOMINANCE

How to make a synthesis between the results of the three approaches: Ratio System, Reference Point Method, which uses the ratios obtained in the ratio system as coordinates, and the Full Multiplicative Form? Brauers and Zavadskas developed a Theory of Dominance (2011).

In application of axiom 3 we shall translate the ordinal scales of the three methods of MULTIMOORA in another one based on Dominance, being Dominated, Transitivity and Equability.

Stakeholders or their representatives may give a different importance to objectives in a multi-objective problem but this is not the case with the three methods of MULTIMOORA. These three methods represent up till now the only methods, not overlapping each other, with dimensionless measures in multi-objective optimization and one can not argue that one method is better than or is of more important than the other ones.

Dominance

Absolute Dominance means that an alternative, solution or project is dominating in ranking all other alternatives, solutions or projects which are all being dominated. This absolute dominance shows as rankings for MULTIMOORA: (1-1-1).

General Dominance in two of the three methods with a P b P c Pd (P preferred to) is for instance of the form:

(d-a-a) is generally dominating (c-b-b).
(a-d-a) is generally dominating (b-c-b).
(a-a-d) is generally dominating (b-b-c)
and further on transitiveness plays fully.

Transitiveness

If a dominates b and b dominates c than also a will dominate c.

Overall Dominance of one alternative on another

For instance (a-a-a) is overall dominating (b-b-b) which is overall being dominated by (a-a-a).

Equability

Absolute Equability has the form: for instance (e-e-e) for 2 alternatives.
Partial Equability of 2 on 3 exists e. g. (5-e-7) and (6-e-3).

Circular Reasoning

Despite all distinctions in classification some contradictions remain possible in a kind of Circular Reasoning.
We can cite the case of:
Object A (11-20-14) dominates generally object B. (14-16-15).
Object B. (14-16-15) dominates generally Object C (15-19-12)
but Object C (15-19-12) dominates generally Object A (11-20-14).
In such a case the same ranking is given to the three objects.

The following TABLE 6 presents the final classification of MULTIMOORA.

Table 6. The final classification of MULTIMOORA for the Economies of the EU Countries (a)

MULTIMOORA	Countries	Rank RS	Rank RP	Rank MF
1	**Sweden**	2	**1**	**1**
2	**Luxemburg**	**1**	7	**2**
3	**Finland**	**3**	5	**4**
4	**Estonia**	**4**	16	**3**
5	**Netherlands**	**5**	**2**	**6**
6	**Denmark**	**6**	**3**	7
7	**Germany**	**7**	8	**8**
8	**Austria**	**8**	**4**	**10**
9	**Slovakia**	**9**	14	**13**
10	**Belgium**	**12**	**6**	15
11	**Slovenia**	**14**	**10**	12
12	**Czech Republic**	11	**13**	**11**
13	**Lithuania**	**10**	20	**16**
14	**France**	**18**	**9**	18
15	**Hungary**	20	**17**	**5**
16	**Bulgaria**	**13**	22	**9**
17	**Poland**	**16**	18	**17**
18	**Malta**	17	**15**	**19**
19	**Latvia**	**15**	**21**	22
20	**Cyprus**	**19**	24	**14**
21	**United Kingdom**	**21**	**11**	**20**
22	**Italy**	**22**	**12**	24
23	**Romania**	**23**	23	**21**
24	**Spain**	**24**	**19**	**23**
25	**Ireland**	**25**	26	**25**
26	**Portugal**	**26**	**25**	27
27	**Greece**	27	27	26

(a) If there is General Dominance in two of the three methods compared to the following country the domination is indicated with a large bold figure.

The group of ten which only joined the EU in 2004 is doing quite well and especially the countries Estonia (who even joined the EURO Group), Slovakia, Slovenia, Lithuania and the Czech Republic.

The so called PIIGS are as expected last classified together with Romania but also the United Kingdom is not very well classified.

9. MEDIAN, CORE AND DISCORDANCE

9.1 The Median Method is not applicable

As said before the Median Method needs at least 6 observations and MULTIMOORA offers only 3 observations. Nevertheless we can see that some exaequo's have to be noted such as: Germany and Austria (8), Hungary, Poland and Malta (17), Romania and Spain (23).

9.2 The CORE States

All 27 Member States of the European Union were assigned either one of three roles in the European world-system. Best performing states with ranks from 1 to 9 were considered as **CORE** States, those possessing ranks 10-18 as **Semi-Peripheral** States and those with ranks 19-27 as **Periphery** States. It should be noted that European CORE States are not necessarily CORE States at the world level. Nevertheless, the European States will maintain their ranking between them even at the world level (for the global world-system we mention for instance Clark 2010).

Consequently, Sweden, Luxemburg, Finland, Estonia, The Netherlands, Denmark, Germany, Austria and Slovakia form the group of the European CORE States.

9.3 Discordance

Discordance, a term much used in Multi-Objective Theory, means disagreement, inconsistency or lack of harmony (Webster's new universal unabridged dictionary).

For Estonia Discordance as the lack of harmony or inconsistency is very large, namely: 3-16. One could argue that for that reason Estonia can not belong to the European CORE-group. Estonia could be replaced by Belgium, ranked 10 with a Discordance of (6-15). For instance Belgium could be ranked 9 and Estonia would take the place of Belgium (rank 10) with a regrouping of the other CORE-members at that moment.

10. CONCLUSION

We prefer to estimate the economic worth of the European Union Member States towards 2020 by Multi-Objective Optimization. Therefore 22 objectives with an actual and future outlook are selected to characterize each EU Member State.

Next problem is the choice of an effective method of Multi-Objective Optimization. This method has to use complete and not partial aggregation, as an overall view of all countries is needed, and to avoid the use of weights. Therefore, methods based on dimensionless measures are preferred. MULTIMOORA responding to all these conditions is finally chosen. In addition MULTIMOORA is composed of three approaches each controlling each other. In this way all methods based on dimensionless measures are included.

Having the results of the three approaches, Ratio Analysis System, Reference Point Approach and Full Multiplicative Form, the problem remains how to come to a final and unique solution. For that purpose the correlation of ranks is senseless, whereas the method of the median is not possible. A Theory of Dominance is preferred.

The final results classify Sweden first followed by Luxemburg and then by Finland, Estonia, the Netherlands, Denmark, Germany, Austria and Slovakia. They can be considered as the CORE States of the European Union. However some objections could be made for Estonia as it shows a huge Discordance of (3-16) between the three observations. Eventually, Estonia could be replaced by Belgium, ranked tenth, but with a Discordance of (6-15).

Some of the ten countries, which joined the EU in 2004, are doing quite well, leaded by Estonia which even joined the EMU and Slovakia. As expected the PIIGS countries are classified at the bottom, but joined by an unforeseen United Kingdom.

In this way we have an idea how the European countries on economic terms are advancing to the European Strategy for 2020 and which of these countries are the best prepared for 2020, unless important political decisions of a structural kind would change their international position.

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