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Min-Max Goal Programming Approach For Solving Multi-Objective De Novo Programming Problems

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Abstract **—** Despite being an important approach for establishing an optimal system, De Novo Programming, does not have its unique general solution algorithm. Especially when multi-objective problems are discussed in the light of De Nova hypothesis, the solving method directs decision maker to different solutions. The De Novo Programming model suggested in our study includes De Novo Programming and Min-max Goal Programming approaches and uses positive and negative ideals. Problem-solving phases of the model are explained through illustrative examples.

Keywords**—**Optimal System Design, De Novo programming, Min-max Goal Programming

1. INTRODUCTİON

Multi-objective Decision Making (MODM) techniques have an important role in solving decision problems, which include more than one objective function. These techniques use priority or weight factor according to information obtained from the decision maker and provide the decision maker with solutions. However, due to presence of multiple objectives, it is very hard to obtain optimal solutions with such techniques. One, therefore, should seek for satisfactory or compromising solutions.

In mathematical techniques, both solving method and constraints affect the solution. If constraint resources are not used at full capacity in a mathematical model, unused resources reduce the realization level of objectives. Therefore, it is very important to ensure that all objectives are realized at optimal levels, and constraint resources are used at full capacity. De Novo Programming proposed by Zeleny (1986) ensures creation of an optimal-level model by reorganizing the constraint resources within the frame of a given budget. The major characteristic of De Novo hypothesis is to realize optimal system design instead of optimizing a given system (Zeleny, 1990). Besides the classic solutions for De Nova Programming problems, Li and Lee (1990) analyzed problems in a fuzzy environment. Babic and Pavic (1996), Shi (1999), Chen and Hsieh (2006), Huang, *et al.* (2006), Zhang *et al.* (2009), and Chen and Tzeng (2009) have contributed De Novo Programming literature with their studies.

Li and Lee (1990) utilized a two-phase approaching their fuzzy solution. That approach an extension of "min-max" version consists of two phases. First phase of the approach is identical to max-min fuzzy model introduced by Zimmermann (1978), and the possible solution obtained from the first phase is tested with averaging operator to determine whether it is the only solution or not. However, in our study we employed min-max operator in the proposed solving process to examine closeness to the positive ideal solution with regards to Goal Programming.

In addition to the approaches proposed for the solution of Multi-objective De Novo Problems, one can also utilize Goal Programming approaches for solving Multi-objective De Novo Problems. In our study, we used positive-ideal solutions and turned each objective of the illustrative problem turned into goals. Moreover, normalization procedure was initiated by using positive and negative ideal solutions in order to denominate goal deviations in a single unit. Also we used min-max Goal Programming, which is a special and effective approach in Goal Programming. It was found that this approach provided satisfaction for all goals, and therefore, the decision point got the highest closeness to positive ideal solutions.

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2. MULTI-OBJECTIVE DE NOVO PROGRAMMING FORMULATION

In this section, we will give the basic formulation of Multi-objective De Novo programming proposed by Zeleny (1990). In the formulation process, we will discuss maximization and minimization type objectives. Mathematical expression of Multi-objective De Novo programming is as follows:

$$\max Z_k = C^1 x$$

$$\min W_s = C^2 x$$

Subject to (1)

$$Ax - b \leq 0$$

$$pb \leq B$$

$$x \geq 0,$$

Where $Z_k = C^1 x = \sum_{j=1}^{n} c_{kj} x_j$, *k=1,2,…,l*, are *l* objective functions Z_k to be maximized simultaneously. $W_s = C^2 x = \sum_{j=1}^{n} c_{sj} x_j$, s=*1,2,…r*, are *r* objective functions W_s to be minimized simultaneously. $C^1 \in \mathbb{R}^{l \times n}$, $C^2 \in \mathbb{R}^{r \times n}$ and $A \in \mathbb{R}^{m \times n}$ are matrices of dimensions $l \times n$, $r \times n$ *and* $m \times n$ respectively. $b \in \mathbb{R}^m$ is the *m*-dimensional unknown resource vector, $p \in \mathbb{R}^m$, is the vector of unit prices of *m* resources, and *B* is the given total budget. Problem (1)'s solution result and best performances for each objective function are determined according to resource amount reformulated depending on the budget. Problem (1) is formulated as continuous "knapsack" problem by using unit price of constraint resource. The formulation is as follows:

$$\max Z_k = C^1 x,$$

$$\min W_s = C^2 x,$$

Subject to (2)

$$Vx \leq B,$$

$$x \geq 0,$$

Where $Z_k = \left(Z_1, \ldots, Z_l\right) \in \mathbb{R}^l$, $Z_k = \left(Z_1, \ldots, Z_l\right) \in \mathbb{R}^l$, $W_s = \left(W_1, \ldots, W_r\right) \in \mathbb{R}^r$ and $V = \left(V_1, ,, .V_n\right) = pA \in \mathbb{R}^n$.

Using the methodology of De Novo single-criterion optimal, Problem (2) can be solved, for *x* and *b*, with respect to each to objective functions Z_k and W_s, respectively. Let vector $Z_k^* = \left(Z_1^*, \ldots, Z_l^*\right)$ and vector $W_s^* = (W_1^*, \ldots, W_r^*)$ denote the multi-criteria performance of the idea design relative to given *B*. Obviously, Z_k^* and W_s^* must be attainable for a given budget level. *B*. Z^* and W^* represent the meta-optimum performance. In the light of this information, corresponding x^* and b^* can be found for meta-optimum by solving the following problem:

$$Min \;\; Vx$$

Subject to (3)

$$C^1 x \leq Z_k^*,$$

$$C^2 x \leq W_s^*,$$

$$x \geq 0,$$

Solving (3) identifies the minimum budget B^* at which the meta-optimum performance Z_k^* and W_s^* can be realized through x^* and b^*. Solving problem (3) B^* must exceed any given budget *B*. At a specific B budget, the optimum-path ratio *r* can be used:

$$r = B \big/ B^* . \tag{4}$$

Final solution with (4) is determined by using these formulations: $x = rx^*$, $b = rb^*$, $Z = rZ_k^*$,ve $W = rW_s^*$.

Besides the meta-optimum solution for multi-objective De Novo Programming, different multi-objective decision making techniques can be used for reaching the solution. Zimmermann (1978) suggested the Utility Approach, Goal Programming, Interactive Approaches and Fuzzy Approach for solving multi-objective decision problems. Li and Lee (1990), and Lee and Li (1993) suggested solutions for De Nova Programming problems by applying fuzzy methodology to the problem (2). In this study, a satisfactory solution is specified for multi-objective De Novo Programming by using min-max approach a specific type of goal programming.

In Goal Programming approaches, different types of goals must be denominated in the same unit. Goals can be denominated in a single unit by using normalization techniques. There are a few normalization techniques in the literature, such as Percentage normalization, Euclidean normalization, Summation normalization, and Zero-one normalization. Tamiz *et al.* (1998) explained these techniques in detail in their study. Zeleny's (1976) technique developed basing on the difference between positive-ideal solution and negative-ideal solution can be an alternative to these techniques. When practicing Zeleny's above mentioned technique in Goal Programming, it should be applied to only negative or positive deviations, because goal deviations need to be expressed in the same unit. That process is indicated with (10) equation.

According to (Li and Lee 1990), positive and negative ideal solutions can be summarized as in the following:

Positive ideal solutions for Z_k and $W_s Z_k^* = \max Z_k$ and $W_s^* = \min W_s$

$$I^* = \left\{ Z_1^*, \ldots, Z_l^*; W_1^*, \ldots, W_r^* \right\} \tag{5}$$

Negative ideal solutions for Z_k and W_s, $Z_k^* = \min Z_k$ and $W_s^* = \max W_s$

$$I^- = \left\{ Z_1^-, \ldots, Z_l^-; W_1^-, \ldots, W_r^- \right\} . \tag{6}$$

The purpose is to identify the best and the worst performances for each objective function, and thus, to determine a satisfactory solution depending on these performances.

2.1. Min-max Goal Programming

Goal Programming is one of the most important techniques in the MOD Mprocess. This technique is an extension of classical linear programs and includes achievement of target values for each objective, instead of maximization or minimization of the objective function. The term Goal Programming was first used by Charnes and Cooper (1961).With the studies of Lee (1972), Ijiri (1972), and Ignizio (1982); Goal Programming has become a strong and well-accepted technique in literature.

The overall purpose of Goal Programming is to minimize the deviation between the achievement of the goals and their aspirational levels. The minimization process can be accomplished with different methods (Romero, 1991).In the first type the unwanted deviations are assigned weights according to their relative importance to the DM and minimized as an Archimedian sum. This is known as weighted GP (WGP) (Tamiz *et. al.*,1998). The second Goal Programming variant is also sometimes termed pre-emtive Goal Programming in literature. The distinguishing feature of Lexicographic Goal Programming (LGP) is the existence of number of priority levels (Jones and Tamiz, 2010). The third Goal Programming variant was introduced by Flavell (1976). In this variant, the maximum deviation from amongst the weighted set of deviations is minimized rather than the sum of the deviations themselves (Jones and Tamiz 2003). Mathematical expression of Min-max Goal Programming is as follows:

$$Min \;\; d$$

Subject to (7)

$$f_i(x) + n_i - p_i = b_i$$

$$\alpha_i n_i + \beta_i p_i \leq d$$

$$x \in F$$

$$x \geq 0, n_i, p_i \geq 0, i = 1, 2, \ldots, k$$

Where, d is maximum deviation, b_i is the precise aspirations level for the ith goal, n_i, p_i are negative and positive deviations from aspiration value of ith goal. For obtaining satisfactory solution for multi-objective de novo programming based on Min-max Goal Programming, problem (7) is reformulated as in the following. According to new formulation $b_i = Z_k^*$ for maximization objectives while $b_i = W_s^*$ for minimization objectives.

If positive ideal solutions are used for maximization-type objectives, positive deviation should be zero. Because objectives cannot pass over the ideal solution,

$$Z_k + n_k - p_k = Z_k^* \text{ and } n_k \leq d . \tag{8}$$

Also as this type of objectives cannot drop below positive ideal solution, if positive ideal solutions are used for minimization objectives, negative deviation should be zero.

$$W_s + n_s - p_s = W_s^* \text{ and } p_s \leq d . \tag{9}$$

Furthermore, positive ideal solutions and negative ideal solutions have been used for the normalization of objectives. That process is as follows:

$$t_k = Z_k^* - Z_k^- \text{ and } t_s = W_s^- - W_s^* \tag{10}$$

Here the normalization is invariant for, t_k and t_s maximization and minimization objectives. By taking problem (7) into consideration and using (8), (9) and (10), De Novo Goal Programming model can be written as in the following:

$$Min \ d$$

Subject to (11)

$$Z_k + n_k - p_k = Z_k^*$$

$$\alpha_k \frac{n_k}{t_k} \leq d$$

$$W_s + n_s - p_s = W_s^*$$

$$\beta_s \frac{p_s}{t_s} \leq d$$

$$Vx \leq B$$

$$k = 1,2,\ldots,l, s = 1,2,\ldots r$$

Where d is maximum deviation, $Z_k = \left(Z_1,\ldots,Z_l\right) \in \mathbb{R}^l$, $W_s = \left(W_1,\ldots,W_r\right) \in \mathbb{R}^r$, $V = \left(V_1,,,,.V_n\right) = pA \in \mathbb{R}^n$ and B is the given total budget, α_k and β_s are the respective positive weights, t normalization is invariant. Solution of problem (11) shows how much each objective function deviates from the ideal solution, and the deviation rate is specified with parameter *d*.

In min-max solution: $0 \leq d \leq 1$ .When the *d* value is zero, it means that the positive ideal value for objective function is achieved. If the *d* value is equal to one, it means that objective functions are equal to negative ideal values. By taking these two conditions into consideration, the normalized rate of deviation shows the achievement percentage of objective functions according to ideal values (Ballestero and Romero, 1998).

3. ILLUSTRATIVE EXAMPLE

Consider the numerical problem of Zeleny (1986).Relative weights were considered equal in De Novo Goal Programming solutions for (P1). Problem was solved in the light of both min-max Goal Programming approach and the approaches proposed by Zeleny (1990), and Li and Lee (1990). Table 3 and Table 4 show comparison of all results obtained in the study.

Max $Z_1 = 50x_1 + 100x_2 + 17.5x_3$ (Profits)

Max $Z_2 = 92x_1 + 75x_2 + 50x_3$ (Quality)

Max $Z_3 = 25x_1 + 100x_2 + 75x_3$ (Workers Satisfaction)

Subject to: (P1)

$12x_1 + 17x_2 \leq 1400$ (Milling Machine)

$3x_1 + 9x_2 + 8x_3 \leq 1000$ (Lathe)

$10x_1 + 13x_2 + 15x_3 \leq 1750$ (Grinder)

$6x_1 + 16x_3 \leq 1325$ (Jig Saw)

$12x_2 + 7x_3 \leq 900$ (Drill Press)

$x_1, x_2, x_3 \geq 0$

Where $x_1, x_2, x_3 \geq 0$, with the price of resources $p_1 = \$0.75$, $p_2 = \$0.6$, $p_3 = \$0.6$, $p_3 = \$0.35$, $p_4 = \$0.50$, $p_5 = \$1.15$, and $p_6 = \$0.65$, and the budget level $B = \$4658.75$. Firstly, (P1) is formulated in multi-objective De Novo programming model by using problem (2) as in the following.

$Z_1 = 50x_1 + 100x_2 + 17.5x_3$

$Z_2 = 92x_1 + 75x_2 + 50x_3$

$Z_3 = 25x_1 + 100x_2 + 75x_3$

Subject to: (P2)

$23.475x_1 + 42.675x_2 + 28.7x_3 = 4658.75$

$x_1, x_2, x_3 \geq 0$

In (P2) solution, each objective function has taken different decision-variable values. Table 1 shows obtained results.

Table 1. Decision Variables and Objective Functions

Decision Variables	Z_1	Z_2	Z_3
x_1	0	198.455	0
x_2	109.168	0	0
x_3	0	0	162.325
	10916.81	18257.93	12174.43

Due to the fact that variable value is different for each objective function in Table 1, it is not possible to achieve the optimal solution. Therefore, ideal solutions obtained by using (5) and (6) for determining satisfactory solution are indicated below:

$I^* = \{10916.81, 18257.93, 12174.43\}$ and $I^- = \{2840.701, 8116.289, 4961.395\}$.

Depending on positive and negative ideal solutions, De Novo Goal Programming model is written for problem (1.1)as the following:

$Min \;\; d$

Subject to (P3)

$50x_1 + 100x_2 + 17.5x_3 + n_1 - p_1 = 10916.81$

$$92x_1 + 75x_2 + 50x_3 + n_2 - p_2 = 18257.93$$

$$25x_1 + 100x_2 + 75x_3 + n_3 - p_3 = 12174.43$$

$$\frac{n_1}{10916.81 - 2840.701} \leq d$$

$$\frac{n_2}{18257.93 - 8116.289} \leq d$$

$$\frac{n_3}{12174.43 - 4961.395} \leq d$$

$$23.475x_1 + 42.675x_2 + 28.7x_3 = 4658.75$$

$$x_1, x_2, x_3 \geq 0$$

Suggested solution for (P3) in terms of De Novo Goal Programming is indicated in Table 2. The values in the table occur basing on that *d = 0.505127*. It can be inferred from this result that the satisfactory solution is between negative and positive ideal solution values.

Table 2. Satisfactory Solution Values

Decision Variables	Z_1	Z_2	Z_3
x_1	98.124	98.124	98.124
x_2	6.69	6.69	6.69
x_3	72.116	72.116	72.116
	6837,348	13135.11	8530.93

Considering the decision-variable values in Table 2, satisfactory amounts of (P1) constraints' right-side invariables are indicated in terms of De Novo Programming in Table 3. Last column of Table 3 shows the resource utilization in the case where relative importance of goals is accepted as $\alpha_1 = 0.45$, $\alpha_3 = 0.35$ and $\alpha_2 = 0.20$.

Table 3. Satisfactory Resource Amounts

Machine Type	Original Availability	Zeleny (1990)	Li and Lee (1990)	Minmax Solution 1	Minmax Solution 2
Milling Mach.	1400	1465.06	1306.651	1291.23926	1788.4147
Lathe	1000	910.42	929.118	931.52301	807.858
Grinder	1750	2030.65	2140.908	2149.97534	1989.61
Jig Saw	1325	1444.64	1719.618	1742.61401	1299.155
Drill Press	900	640.07	587.553	585.104004	453.61
Band Saw	1075	1299.55	1286.886	1284.2135	1484.822

When (P1) is reformulated according to the amount of resource use suggested in Table 3, the most important outcome is that right-side value of each constraint is being used at full capacity. It means that all constraints are active, and feasible area actualizes only on a single point. Suggested resource amounts have been created with the given budget. As you see in Table 3, first phase of Li and Lee (1990)'s two-phase approach is similar to Zimmermann(1978)'s approach. If the problem is solved in the first phase; decision variables, resource utilization, and objective function values are exactly the same min-max Goal programming. Table 4 shows decision variable and objective function values of all obtained solutions.

Table 4. Overall Assessment of Solutions

Objective Functions	Zeleny (1990)	Zimmerman (1978)	Li and Lee (1990)	Min-max Solution 1	Min-max Solution 2
Z_1	7686.87	6837,348	6906.35	6837,348	8570.375
Z_2	12855.89	13135.11	1327.44	13135.11	14470.04
Z_3	8572.40	8530.93	8524.3	8530.93	7459.75
Closeness to the ideal	-	$\alpha = 0.494873$	$\alpha = 0.4966$	$d = 0.505127$	$d = 0.1307$
Weights	-	-	-	$\alpha_1 = \alpha_2 = \alpha_3$	$\alpha_1 = 0.45, \alpha_3 = 0.35,$ $\alpha_2 = 0.20$
Variables	$x_1 = 92.48$ $x_2 = 20.90$ $x_3 = 55.61$	$x_1 = 98.124$ $x_2 = 6.690$ $x_3 = 72.116$	$x_1 = 97.97$ $x_2 = 7.70$ $x_3 = 70.738$	$x_1 = 98.124$ $x_2 = 6.690$ $x_3 = 72.116$	$x_1 = 104.56$ $x_2 = 24.410$ $x_3 = 40.503$

4. CONCLUSION

De Novo hypothesis actualizing Optimal System Design provide meta-optimum solutions at optimal level for both single-objective and multi-objective mathematical models. Also Multi-objective Decision problem solutions obtained by using Goal Programming/Compromise Programming in accordance with De Novo hypothesis produce satisfactory/compromising system design. In this study, positive and negative ideal solutions have been used for Min-max approach in the scope of Goal Programming utilized during the study. With such a reformulation, we have tried to achieve a satisfactory result between best and worst performances of the problem.

In Table 4, first phase of Li and Lee (1990)'s solution is exactly the same as Zimmermann (1978)'s solution. Result of this solution is identical to the result of min-max solution-1 where relative weights are equally important. This result and result of min-max solution-1 are the same, because while min-max examined the closeness to positive-ideal solutions, other approach examined the closeness to negative-ideal solutions. If $\alpha = 0.494873$ figure shows the closeness to the negative-ideal solution, closeness of this solution to the positive-ideal solutions is 0.505127, and it shows that solutions of Zimmermann (1978) and min-max Goal Programming are different only in terms of philosophy. Li and Lee (1990) calculated almost the same result in the second phase of the solution. If the problem is solved with min-max Goal Programming according to the relative weights of $\alpha_1 = 0.45$, $\alpha_3 = 0.35$ and $\alpha_2 = 0.20$, important results are obtained in Z_1 and Z_2 values. In the view of that fact we can say that use of relative weights in decision problems affect the result according to the relative weights. When proposed solution method is used, the closeness to positive-ideal values is higher.

Besides identifying compromising solutions of de novo programming problems with the use of min-max approach, compromise programming yield important results in preliminary examination conducted by our working group. Especially when relative weights are equally important for both Goal Programming and Compromise Programming, obtained solution yields the same results as Zimmermann's fuzzy approach in terms of *distance function model*. On the other hand, if relative importance is different, more efficient results can be achieved.

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