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Optimal Production and Inspection Strategy with Inspection Time and Reworking for a Deteriorating Process

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Abstract— The paper considers an integrated production, inspection, preventive maintenance, and inventory problem and determines the optimal inspection interval, inspection frequency, and production quantity yielding the maximum unit expected profit in an imperfect production process with inspection time and reworking. When the process in an out-of-control state produces a certain percentage of non-conforming items, this study assumes that a certain proportion of the non-conforming items can be reworked into conforming items. Furthermore, in a system with process deterioration, this study examines the effectiveness of imperfect preventive maintenance, and conducts numerical analysis to explore the influence of reworking and inspection time on profit.

Keywords— inventory, preventive maintenance, imperfect rework, imperfect process, inspection time

1. INTRODUCTION

Traditional economic production quantity (EPQ) models assume that all production system outputs consist of conforming items (Silver and Peterson, 1985), failure-free production equipment, and that all products conform to quality requirements. However, numerous studies have incorporated restrictive assumptions into traditional EPQ models to make their models more consistent with actual production conditions. Salameh and Jaber (2000) proposed a model incorporating a cost model that considers the production of non-conforming items in production processes. Freimer *et al.* (2006) examined how setup costs and process improvements impact EPQ models. Moreover, Jaber (2006) explored the impact of quality improvements and setup costs on a production quantity model by incorporating an imperfect production process. Darwish (2008) also investigated how setup costs influence an EPQ model and found that setup costs can significantly affect the production cycle.

Preventive maintenance (PM) can enhance system reliability and reduce the incidence of failure. Many PM models assume that a system returns to perfect conditions after each PM implementation, but in practical situations, the failure rate of a system may be altered after PM is performed. Pham & Wang (1996) observed that maintenance is frequently imperfect and reviewed various optimal strategies under imperfect maintenance. Tseng *et al.* (1998) proposed an imperfect PM strategy model for deteriorating production systems. Moreover, Ben-Daya and Makhdoum (1998) investigated how different PM strategies can influence an integrated production and quality model; they examined the effect of different PM strategies relative to EPQ and economic control diagram design. Ben-Daya (1999) developed an integrated economic design and optimal maintenance standard model, incorporating EPQ and $\overline{X}$ control diagrams. Ben-Daya (2002) later proposed an integrated model encompassing EPQ and the level of PM. Moreover, this model considered the optimal inspection interval, inspection frequency, and production quantity in generalized situations involving a distribution that degenerated with increasing failure rate. Darwish and Ben-Daya (2007) examined how inspection errors and PM affect a production inventory system. Charlot *et al.* (2007) considered issues involving the manufacturing system, preventive maintenance error and restoration rate, and proposed an optimal manufacturing and maintenance strategy. Lee (2008) investigated the effect of PM activities on cost in multiple productions.

In some production systems, a portion of non-conforming items can be reworked. For instance, in the case of copper-plated circuit boards with an incorrect the copper thickness, the copper can be washed off and the board reused, thereby avoiding the cost of substrate scrapping. Hayek and Salameh (2001) proposed a production lot strategy that enabled the reworking of products. Chiu (2003) also proposed an optimal production lot strategy when reworking can be performed. In addition, Chiu *et al.* (2007) proposed an optimal production quantity strategy for situations in which a certain percentage of reworked and processed products remain non-conforming and must be scrapped. Biswas and Sarker (2008) proposed optimal EPQ models for a lean production system with in-cycle reworking and scrapping. They developed several inventory models for

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a single-stage production process in which defective items are reworked and scraps are detected and discarded during the entire process. Sarker *et al.* (2008) expanded on Biswas and Sarker's theory to consider models for optimum batch quantity in a multi-stage system with a rework process. Chen *et al.* (2010) also derive the optimal production quantity and inspection policy for an imperfect production system with a rework and scrap rate, and investigate the effect of reworking on profit.

This study extends the work of Chen *et al.* (2010) to consider inspection time. We attempt to optimize the inspection interval, inspection frequency, and production quantity for maximizing expected profit in an imperfect production process that involves reworking and inspection time. We assume that a percentage of non-conforming items can be reworked, while the rest of the items are regarded as scrap. In particular, we perform numerical simulations to explore the effects of a rework option on the profit and the important aspects of the developed model.

The rest of this paper is organized as follows. Section 2 presents a mathematical model. Section 3 then introduces the proposed optimal solution. Next, Section 4 summarizes numerical analysis results. Conclusions are drawn in Section 5.

2. MODEL DEVELOPMENT

2.1 Notation

This study uses the following symbols:

D	: Demand rate in units per unit time
P	: Production rate in units per unit time ($P > D$)
P_r	: Non-conforming items reworked rate in units per unit time
Q	: The expected production quantity
T	: Production time per cycle
T_r	: Reworking time of non-conforming items
CT	: Inventory time per cycle
s	: Inspection time
S	: Setup costs per production cycle
C_h	: Storage cost per product per unit time
C_I	: Cost of each inspection
C_r	: Unit cost of reworking non-conforming items
C_d	: Production cost of each scrapped non-conforming item
P_u	: Retail price of each product
d	: Ratio of non-conforming items produced when the process is in the out-of-control state
d_1	: Ratio of non-conforming items that cannot be reworked and will be scrapped ($0 \le d_1 \le 1$)
d_2	: Ratio of non-conforming items able to be reworked that will be scrapped during the reworking process ($0 \le d_2 \le 1$)
I_j	: Inventory level after the jth inspection
$R(t)$	: Restoration cost
k	: Inspection frequency during each production cycle
h_j	: Interval reached for the jth inspection
t_j	: jth inspection time point, where $t_j = \sum_{i=1}^{j} h_i + (j-1)s$
N_i	: Number of non-conforming items produced due to out-of-control process between time points t_{i-1} and t_i ($i = 1, 2, \ldots, k$)
b_i	: Actual system age before the ith preventive maintenance
a_i	: Actual system age after the ith preventive maintenance
$f(t)$	: Probability density function of the time that the process will be from the in-control state to the out-of-control state.
$F(t)$	: Cumulative distribution function
$\overline{F}(t)$	: Survival function, $\overline{F}(t) = 1 - F(t)$

$r(t)$: Hazard function, $r(t) = f(t) / \bar{F}(t)$

$I(t)$: Inventory level at time t

C_{apm} : Cost of implemented preventive maintenance

C_{mpm} : Cost of implementing largest-scale preventive maintenance

p_j : Conditional probability that the process shifts to the out-of-control state during the time interval ($t_{j-1} + s$, t_j) given that the process is in in-control state at time t_{j-1}

$ET(\pi)$: Total expected profit from each cycle

$EU(\pi)$: Expected profit per unit time

λ : The scale parameter of Weibull distribution

ν : The shape parameter of Weibull distribution

2.2 Assumptions

The assumptions used for this integrated model are as follows:

(1) Consider a production process producing a single product.

(2) The process is in either the in-control or out-of-control state. At the beginning of a production cycle, the system is assumed to be in the in-control state, producing items of good quality.

(3) The process will shift to the out-of-control state. Moreover, inspection can be used to determine the process state. The inspection time is a fixed value s.

(4) If the process is judged to be in the in-control state, PM is implemented. PM occurs at time $t_j + s$, $j = 1,2,\ldots,k-1$. To simplify the formula, it is assumed that the PM time is not considered. The process failure rate will decrease with PM. The reduction in the effective age of the process depends on the level of PM performed.

(5) When the process is in an out-of-control state, a percentage of non-conforming items are scrap items. The other portion of the non-conforming items can be reworked and reworking starts immediately after the end of regular production. The reworking is assumed to be imperfect, implying that a percentage of those non-conforming items fail reworking and also become scrap.

(6) The elapsed time for a process to shift is a random variable that follows a general distribution with an increasing hazard rate.

(7) A production cycle ends either when the process is in the out-of-control state or after the *k*th inspection.

2.3 Mathematical model

The total expected cost during each cycle includes setup cost S, holding cost $E(HC)$, non-conforming item reworking cost $E(RW)$, preventive maintenance cost $E(PM)$, inspection cost $E(IC)$, cost of manufacturing scrapped non-conforming items $E(DC)$, and restoration cost $E(RC)$. To derive these costs, we must first derive the expected production time, non-conforming item reworking time, and inventory time. Figure 1 describes the inventory cycle and the jth inspection.

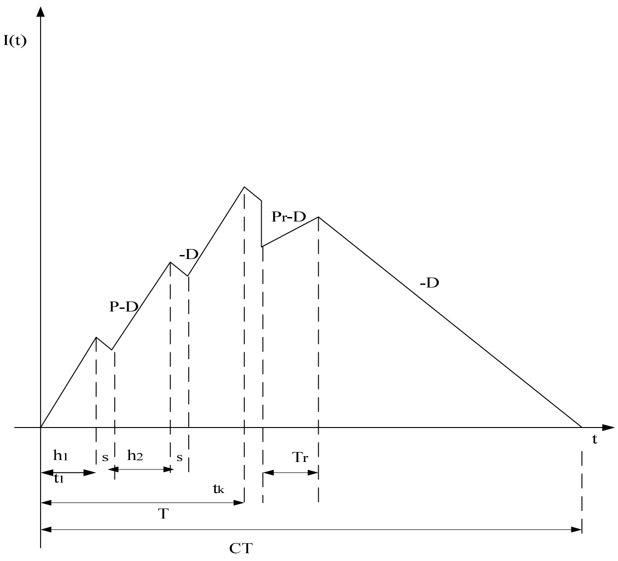

Figure 1. Inventory cycle

Proposition 1.
The expected production time of one cycle is

$$E(T) = \sum_{j=1}^{k-1} (h_j + s) \prod_{i=1}^{j-1} (1 - p_i) + h_k \prod_{i=1}^{k-1} (1 - p_i) \tag{1}$$

Proof.
Let $E(t_j)$ be the expected residual time during the cycle after time t_j, given that the process is in an in-control state at time t_{j-1}, $E(t_0) = E(T)$. Let $p_j = [F(b_j) - F(a_{j-1})] / \overline{F}(a_{j-1})$, consider the process when the first inspection is completed (at time $t_1 = h_1 + s$). For each possible state, the following table describes the expected residual time left in the cycle and the associated probabilities.

State	Probability	Expected residual time
In-control state	$1 - p_1$	$E(t_1)$
Out-of-control state	p_1	0

Consequently,

$E(T) = E(t_0)$

$\quad = h_1 + s + (1 - p_1) E(t_1),$

$E(t_1) = h_2 + s + (1 - p_2) E(t_2),$

$E(t_2) = h_3 + s + (1 - p_3) E(t_3),$

and by the same reasoning

$E(t_j) = h_{j+1} + s + (1 - p_{j+1}) E(t_{j+1}), \;\; j = 1, 2, \ldots k - 2.$

Notably,

$E(t_{k-1}) = h_k, \;\; E(t_k) = 0.$

Therefore,

$$E(T) = \sum_{j=1}^{k-1} (h_j + s) \prod_{i=1}^{j-1} (1 - p_i) + h_k \prod_{i=1}^{k-1} (1 - p_i).$$

The expected production quantity in one cycle, being the product of the production rate and the expected production time, is

$$Q = P \cdot [E(T) - (k-1)s] \tag{2}$$

Next, to derive the non-conforming item reworking time, we need to ascertain the total expected number of non-conforming items per production cycle. The expected number of non-conforming items during the *j*th interval is

$$E(N_j) = \int_{a_{j-1}}^{b_j - s} d \cdot P \cdot (b_j - s - t) \frac{f(t)}{\bar{F}(a_{j-1})} dt \tag{3}$$

Proposition 2.
The expected number of non-conforming items during one cycle is

$$E(N) = \sum_{j=1}^{k} p_j E(N_j) \prod_{i=1}^{j-1} (1 - p_i) \tag{4}$$

Proof.
Let $E(B_j)$ be the expected quantity of non-conforming product remaining at time t_j when the process is in an in-control state and maintenance has been performed correctly, so that $E(B_0) = E(N)$. Consider the process when the first inspection is completed (at time $t_1 = h_1 + s$). For each possible state, the following table describes the expected residual quantity for the cycle and the associated probabilities.

Process State	Probability	Expected quantity of non-conforming product remaining
In-control state	$1 - p_1$	$E(B_1)$
Out-of-control state	p_1	$E(N_1)$

Consequently,

$$E(N) = E(B_0)$$
$$= p_1 E(N_1) + (1 - p_1) E(B_1),$$
$$E(B_1) = p_2 E(N_2) + (1 - p_2) E(B_2),$$
$$E(B_2) = p_3 E(N_3) + (1 - p_3) E(B_3),$$

and by the same reasoning

$$E(B_j) = p_{j+1} E(N_{j+1}) + (1 - p_{j+1}) E(B_{j+1}),\ j = 1, 2, \ldots., k-2.$$

Notably,

$$E(B_{k-1}) = p_k E(N_k),\ E(B_k) = 0.$$

Therefore,

$$E(N) = \sum_{j=1}^{k} p_j E(N_j) \prod_{i=1}^{j-1} (1 - p_i).$$

Then, we can obtain the expected non-conforming item reworking time for one cycle:

$$E(T_r) = (1 - d_1) E(N) \big/ P_r \tag{5}$$

The expected inventory time equals the expected number of conforming items divided by the demand rate per unit time, and is thus

$$E(CT) = \frac{1}{D} \{P[E(T) - (k-1)s] - E(N) + (1 - d_1)(1 - d_2) E(N)\} \tag{6}$$

The production cost for each product is fixed and can be ignored. Among the various costs making up of the expected total cost per cycle, only the setup cost S is fixed. The formulas for other costs are derived as follows. We assume the storage method to be the same for all items, both conforming and non-conforming. Therefore, the same storage cost per product per unit time (C_h) is applied to both conforming and non-conforming items in calculating the expected holding cost during each production cycle. The expected holding cost is thus

$$\begin{aligned} E(HC) &= C_h \int_0^{CT} I(t)dt \\ &= C_h E(H) \end{aligned} \tag{7}$$

Where $E(H)$ is the expected inventory in the production cycle under the function $I(t)$, which itself is given by

$$\begin{aligned} E(H) = \frac{1}{2}\{&\sum_{j=1}^{k}[h_j(I_{j-1} + I_j + Ds)[\prod_{i=1}^{j-1}(1-p_i)] + s(2I_j + Ds)[\prod_{i=1}^{j-1}(1-p_i)] \\ &+\sum_{j=1}^{k-1}[\prod_{i=1}^{j-1}(1-p_i)]p_j\{[2I_j - 2(1-d_1)E(N) + (P_r - D)E(T_r)]E(T_r) \\ &+\frac{[I_j - (1-d_1)E(N) + (P_r - D)E(T_r)]^2}{D}\} \\ &+\prod_{j=1}^{k-1}(1-p_j)\{[2I_k - 2(1-d_1)E(N) + (P_r - D)E(T_r)]E(T_r) \\ &+\frac{[I_k - (1-d_1)E(N) + (P_r - D)E(T_r)]^2}{D}\}\} \end{aligned} \tag{8}$$

Where I_j is the inventory level at time $t_j + s$, $I_j = \sum_{i=1}^{j}[(P-D)h_i - Ds]$, for $j = 1, 2, ..., k$ and $I_0 = 0$. If $I_j < 0$, we set it to be zero. After PM, the system will become younger, despite its age not returning to a new state. The change in system age is correlated with the degree of maintenance. Let

$$\delta_k = \eta^{k-1}\frac{C_{apm}}{C_{mpm}} \tag{9}$$

The parameter η ($0 \le \eta \le 1$) is an imperfect factor expressing the deterioration of system age due to the performance of preventative maintenance. Ben-Daya (1999) suggests that the correlation between reduction in age and PM is either linear or nonlinear. We assume the correlation is linear, yielding the following:

$$a_k = (1-\delta_k)b_k \tag{10}$$

At time t_j, the effective system age is

$$\begin{aligned} &b_1 = h_1, \\ &b_j = a_{j-1} + h_j + s,\ j = 2, 3, ..., k \end{aligned} \tag{11}$$

Because the change in system age due to PM influences the quantity of non-conforming items, repair cost, and manufacturing cycle length, an integrated model can be developed.

Proposition 3.
The expected preventive maintenance cost during one cycle will be

$$E(PM) = C_{apm}\sum_{j=1}^{k-1}\prod_{i=1}^{j}(1-p_i) \tag{12}$$

Proof.

Let $E(pm_j)$ represent the expected remaining PM cost after time t_j in the production cycle. Assuming that at point t_j the production process is judged to be in the in-control state, then $E(pm_0) = E(PM)$. At the end of the inspection cycle, the probability of the process being judged to be in a particular state and the corresponding expected remaining PM cost is as shown below:

State	Probability	Expected remaining PM cost
In-control state	$1 - p_1$	$E(pm_1)$
Out-of-control state	p_1	0

Consequently,

$E(PM) = E(pm_0)$

$= (1 - p_1)E(pm_1)$,

$E(pm_1) = C_{apm} + (1 - p_2)E(pm_2)$,

$E(pm_2) = C_{apm} + (1 - p_3)E(pm_3)$.

Similarly,

$E(pm_j) = C_{apm} + (1 - p_{j+1})E(pm_{j+1})$, $j = 1, 2, ..., k - 2$.

Notably,

$E(pm_{k-1}) = C_{apm}$, $E(pm_k) = 0$.

Therefore,

$E(PM) = C_{apm} \sum_{j=1}^{k-1} \prod_{i=1}^{j} (1 - p_i)$.

Because PM is performed after each inspection, but not at the end of a cycle, the inspection cost is

$$E(IC) = [\sum_{j=1}^{k-1} \prod_{i=1}^{j} (1 - p_i) + 1]C_I \tag{13}$$

The ratio of non-conforming items that cannot be reworked and must be scrapped to all non-conforming items is d_1 ($0 \le d_1 \le 1$). Accordingly, the non-conforming item reworking cost is

$$E(RW) = C_r \cdot (1 - d_1)E(N) \tag{14}$$

The ratio of non-conforming items able to be reworked that must be scrapped during the reworking process to all non-conforming items able to be reworked is d_2 ($0 \le d_2 \le 1$). Therefore, the cost of manufacturing scrapped non-conforming items is

$$E(DC) = C_d \cdot [d_1 + (1 - d_1)d_2]E(N) \tag{15}$$

When the process is in the out-of-control state, it must be terminated for maintenance. Therefore the restoration cost $E(RC)$ should be included in the total cost. According to the concepts of Ben-Daya (2002), when the restoration cost is assumed to have a linear relationship with delay time, we have

$$R(b_j - t) = r_0 + r_1(b_j - t) \tag{16}$$

Where r_0 and r_1 remain constant, and the restoration cost for the *j*th interval is

$$E(RC_j) = \int_{a_{j-1}}^{b_j} R(b_j - t)\frac{f(t)}{\bar{F}(a_{j-1})}dt$$
$$= (r_0 + r_1 b_j)[1 - \frac{\bar{F}(b_j)}{\bar{F}(a_{j-1})}] - r_1\int_{a_{j-1}}^{b_j}\frac{tf(t)}{\bar{F}(a_{j-1})}dt \tag{17}$$

Consequently, the expected restoration cost for a single cycle is

$$E(RC) = \sum_{j=1}^{k} p_j\{(r_0 + r_1 b_j)[1 - \frac{\bar{F}(b_j)}{\bar{F}(a_{j-1})}] - r_1\int_{a_{j-1}}^{b_j}\frac{tf(t)}{\bar{F}(a_{j-1})}dt\}\prod_{i=1}^{j-1}(1 - p_i) \tag{18}$$

The proof is similar to that for **Proposition 3.** The total expected costs during each cycle are thus

$$E(TC) = S + E(HC) + E(PM) + E(IC) + E(RW) + E(DC) + E(RC) \tag{19}$$

The total expected income from each cycle equals the expected number of conforming items multiplied by the retail price per product and is

$$E(TR) = P_u \cdot \{P[E(T) - (k-1)s] - [d_1 + (1-d_1)d_2]E(N)\} \tag{20}$$

Subtracting the total expected costs from the total expected income, we obtain the total expected profit from each cycle as

$$ET(\pi) = E(TR) - E(TC) \tag{21}$$

Finally, expected profit per unit time is

$$EU(\pi) = \frac{E(TR) - E(TC)}{E(CT)} \tag{22}$$

3. OPTIMAL SOLUTION

The optimal solution maximizes total expected profit per unit time. This study first specifies the effect of C_{apm} on profit, and then determines its optimal value. Next, this study identifies how h_1 and k affect profit using numerical analysis, and calculates the maximum expected profit, which yields the optimum solution. The optimal Q value is then calculated after determining h_1 and k. Applying the ideas of Banerjee and Rahim (1988), each inspection arrival interval has an identical cumulative risk rate, and the correlation between each inspection arrival interval is

$$\int_{t_{j-1}+s}^{t_j+s} r(t)dt = \int_0^{t_1+s} r(t)dt,\ j = 2,3,....,k \tag{23}$$

Owing to the effect of PM, the failure rate decreases progressively at the end of each arrival interval, and thus

$$\int_{a_{j-1}}^{b_j} r(t)dt = \int_0^{h_1+s} r(t)dt,\ j = 2,3,....,k \tag{24}$$

Ben-Daya (2002) suggested that the length of time for which a process can be maintained in a controlled state follows a Weibull distribution, i.e., the probability density function is $f(t) = \lambda v t^{v-1} e^{-\lambda t^{v}}$, $t > 0$, $v \geq 1$, $\lambda > 0$. The inspection arrival interval is thus

$$h_j = [(a_{j-1})^{v} + (h_1 + s)^{v}]^{1/v} - a_{j-1} - s,\ j = 2, 3, \ldots, k \tag{25}$$

This model employs the following derivation steps:

(1) When $k = 1$, calculate the expected profit per unit time for different h_1 values, enabling the determination of the maximum expected profit per unit time $EU_1(\pi)$ under these conditions.

(2) When $k = 2, 3, \ldots, k_{\max}$ ($k_{\max}$ denotes the upper limit of the inspection frequency), $EU_2(\pi), EU_3(\pi), \ldots, EU_{k_{\max}}(\pi)$ can be obtained.

(3) $EU(\pi) = Max\{EU_j(\pi), j = 1, 2, \ldots, k_{\max}\}$, enabling the optimal values h_1^* and k^* to be obtained.

4. NUMERICAL ANALYSIS

In this section, we use the production of the copper plating of a printed circuit board as an example to illustrate important aspects of the developed model. We assume the setup costs for each production cycle to be \$150, the production rate in units per unit time to be 1000, the demand rate in units per unit time to be 500, and non-conforming items reworked rate in units per unit time to be 750. The cost of each inspection is \$10, the cost of the maximum preventive maintenance is \$30; the storage cost per product per unit time is \$0.5; the production cost of each scrapped non-conforming item is \$20; the unit cost of reworking non-conforming items is \$5. The retail price of each product is \$10; the ratio of non-conforming items able to be reworked but which will be scrapped during the reworking process is assumed to be 0.1. The probability that the system shifts from an in-control state to an out-of-control state is assumed to follow a Weibull distribution. The Weibull scale and shape parameters are $\lambda = 5$ and $v = 2.5$ respectively.

The assumed values of the parameters and variables are summarized as follows:

$\lambda = 5$, $v = 2.5$, $D = 500$, $P = 1000$, $D = 500$, $P_r = 750$, $C_h = \$0.5$, $S = \$150$, $C_{mpm} = \$30$, $C_r = \$5$, $P_u = \$10$, $C_I = \$10$, $r_0 = \$10$, $r_1 = 0.5$, $\eta = 0.99$, $d_2 = 0.1$.

Table 1. Effect of preventive maintenance level on expected profit per unit time[a]

C_{apm}/C_{mpm}	0.0	0.25	0.5	0.75	1.0
$EU(\pi)$	4625	4655	4678	4695	4706

[a] $k = 4, h_1 = 0.2550, d = 0.6, d_1 = 0.5, s = 0.05.$

The effect of different PM levels is shown in Table 1. Clearly, raising PM level increases the expected profit per unit time, which implies that maximizing the level of PM yields the best results (i.e., under the same conditions, maximizing PM maximizes profit). Hence, our numerical simulations confirm the positive role played by PM activities in the production process. Indeed, the installation of PM equipment has been increasingly recognized as an essential part of overall planning in many manufacturing industries.

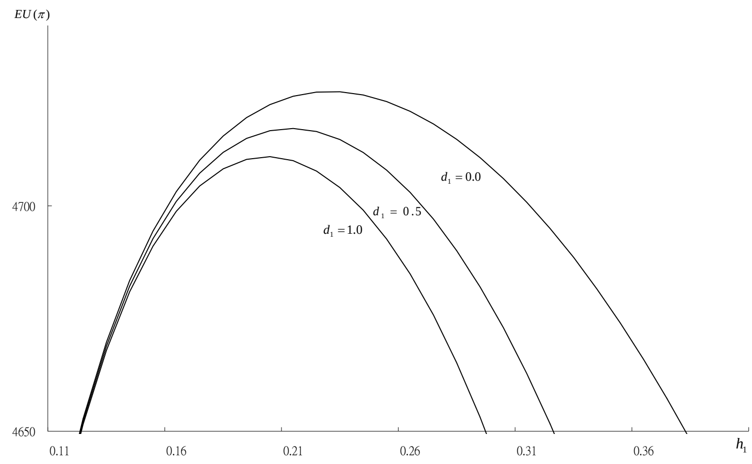

Figure 2. Effect of the rate of scrapping non-conforming items that cannot be reworked on unit expected profit ($k = 4, d = 0.6, s = 0.05$)

Figure 2 indicates that the scrapping rate of non-conforming items which cannot be reworked significantly affects expected profit per unit time. Furthermore, profits increase with decreasing scrapping rate of non-conforming items. This implies that the expected profit from each cycle increases with the reworking ratio. Therefore, it is imperative, in practice, to raise the ratio of non-conforming items able to be reworked, in order to save costs and reduce waste of materials.

Table 2. Optimal solutions under different conditions

		$d = 0.2$				$d = 0.4$				$d = 0.8$			
		k^*	h_1^*	Q^*	$EU(\pi)$	k^*	h_1^*	Q^*	$EU(\pi)$	k^*	h_1^*	Q^*	$EU(\pi)$
$d_1 = 0.0$	$s = 0.00$	2	0.3291	570	4742	3	0.2669	672	4732	3	0.2441	635	4722
	$s = 0.01$	2	0.3301	571	4743	3	0.2663	666	4733	3	0.2436	630	4723
	$s = 0.05$	2	0.3339	576	4748	3	0.2633	641	4737	3	0.2409	607	4727
	$s = 0.10$	2	0.3376	580	4753	2	0.3048	541	4741	3	0.2357	576	4729
$d_1 = 0.5$	$s = 0.00$	3	0.2677	673	4732	3	0.2448	636	4722	3	0.2228	595	4710
	$s = 0.01$	3	0.2671	667	4733	3	0.2443	631	4724	3	0.2224	591	4711
	$s = 0.05$	3	0.2640	642	4737	3	0.2416	609	4728	3	0.2195	571	4715
	$s = 0.10$	2	0.3059	542	4742	3	0.2365	577	4730	3	0.2138	540	4715
$d_1 = 1.0$	$s = 0.00$	3	0.2544	652	4727	3	0.2320	613	4716	4	0.1982	694	4702
	$s = 0.01$	3	0.2539	647	4728	3	0.2316	608	4717	4	0.1971	683	4703
	$s = 0.05$	3	0.2511	623	4732	3	0.2288	587	4721	4	0.1914	635	4706
	$s = 0.10$	3	0.2461	591	4735	3	0.2234	556	4722	3	0.2009	517	4704

Table 2 summarizes the optimal inspection frequency, first inspection interval, EPQ, and expected profit per unit time for different conditions. Clearly, with increase in the ratio of non-conforming items produced when the process is in the out-of-control state and of non-conforming items that cannot be reworked and must be scrapped to all non-conforming items, the optimal inspection frequency also increases. Furthermore, the optimal inspection frequency reduces with increased inspection time. The first inspection interval decreases with increasing ratios of non-conforming items produced when in an out-of-control state to total production, and of non-conforming items that cannot be reworked to total production. The expected profit per unit time decreases when there is an increase in the ratio of non-conforming items produced and the ratio of non-conforming items that cannot be reworked and will be scrapped. For example, the expected profit per unit time is NT$4,753 when $d = 0.2, d_1 = 0.0, s = 0.1$, and NT$4,702 when $d = 0.8, d_1 = 1.0, s = 0.0$, representing a drop in expected profit per unit time of NT$51. This shows that the these three factors of inspection time, the ratio of non-conforming items produced, and the ratio of non-conforming items that cannot be reworked and will be scrapped significantly affect the expected profit per unit time.

5. CONCLUSIONS

By integrating issues relating to production, inspection, preventive maintenance, and inventory, this study proposes a production and inspection strategy to maximize the expected profit per unit time for an imperfect production process with inspection time and reworking. Consumers increasingly demand high-quality products. During the current era of rising consumer consciousness, providing defective products to customers not only increases service and replacement costs, but also severely damages company credibility. Therefore, it is extremely important that companies plan and adjust production procedures and establish inspection, service, and maintenance systems to comply with customer quality requirements. Deteriorating production systems are a reality in manufacturing. Clarifying how production, inspection, preventive maintenance, and inventory are related in these systems can help managers perform operation control and quality assurance more effectively. After investigating the effect of reworking and inspection time on unit expected profit, this study has found that inspection time and the reworking ratio significantly influence unit expected profit. Future work must address the issue of the preventive maintenance time.

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