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Transient Analysis of Single Machine Production Line Dynamics

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Abstract **—** The importance of studying the system's transient behaviors is evident through its practical and theoretical applications. In this paper, a discrete material flow production process which consists of a single station with station break down is considered. Efficient approximations are proposed to determine the first and second order transient performance measures of a production line which can be in one of the two states: up (working) or down (failed). The method is based on modeling the production line by an alternating renewal process which enables generalization from exponential time to failures and time to repairs to a general case when uptimes and downtimes have both arbitrary distributions. A due date performance measure is derived and discussed. This study also discusses availability analysis in the production line. Numerical examples are provided to illustrate the procedure.

Keywords **—** production line, alternating renewal process, transient analysis, availability function, due date performance.

1. INTRODUCTION

In the modern days of mass production systems, production lines play a key role to achieve customers' satisfaction. Mathematical and stochastic models which address to the manufacturing system design and performance measures for the production line control have been the subject of intense investigation of researchers. The major focus of such investigations has been to apply queuing networks models. These studies however have been restricted to steady-state analysis. These analyses provide in a compact form of certain measures like the production rate, throughput, buffer etc. For a good review of production models one can refer to Papadopoulos and Heavy (1996) and Dallery and Gershwin (1992). Moreover, Li and his team of researchers (2009) in one of their papers summarized studies in the field of throughput analysis of production systems and discussed approximation methods for more complicated systems. They continue to imply that transient analysis is still an open area with many potential research topics.

One cannot ignore the importance of the closed form solutions necessary for the computer implementation provided by the steady-state analysis. Nowadays, customer orders are to be met with minimum lead time. Further, production systems are governed by JIT deliveries. With such changing environments, the planning periods are decidedly reduced. As a result, in many production lines with a limited planning horizon the steady-state measures may not be in synchronization with the actual situation and what really needed may be the transient measures. Transient analysis of production line has been addressed by Mitra (1988) and Mocanu (2005). Narahari and Viswanadham (1994) summarized different situations where transient analysis plays a key role. Zhang *et al.* (2013) and Meerkov *et al.* (2012) discussed transient analysis in Bernoulli and Geometric production lines, respectively. As an example of practical application of transient analysis one can refer to Wang and Hu (2010). Food industry in general and dairy industry in particular, can be considered as one of the most important industries with high impact on our daily life and economy. Wang and Hu (2010) applied transient analysis in their research to determine the capacity of buffer in dairy filling and packing production lines. Production rate of the system and work-in process during transient have been investigated in their study. Transient analysis of throughput for a system of machines in series was done by Chen and Yuan (2004). The proposed approaches relied on the assumption that each machine of the system should be modeled in an alternating renewal process when up and down times following exponential distribution.

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The objective of the manager of production systems has generally been to optimize for the first order performance measures such as the expected throughput and expected buffer. Such an approach has been practiced because the first order measures are amenable for a compact and closed form. While it is important that the production system delivers on the average a pre-specified number of items, it is equally important that the variation in the output remains under control. For instance, between two production lines whose average output in a given time period is the same, production managers will prefer the production line which exhibits lesser variation. Gershwin (1993) observes that the production line output has high variability often lying in the interval mean $\pm 10\%$ of standard deviation. Miltenburg (1987) was perhaps the first to present a method which determines the asymptotic variance of the output per unit time. He used the Markov chain theory to determine the asymptotic mean and variance of the time spent in each of states. Hendricks (1992) developed an analytical approach which unfortunately was computer intensive and thus was not useful for large number of machines. Tan (1997a) modeled production lines with finite buffer using Markov reward systems and computed the asymptotic variance rate of production. He has also considered production lines with no inter station buffers (Tan, 1997b). It is also interesting to note that Tan (1999) has dealt with discrete flow production line with cycle dependent failures. A crucial result which enables one to extend the basic results for a single machine to *N* station production lines was given by Gershwin (1993). His approach was based on the exact calculation of the production variance for a single machine and then items of decomposition technique were developed for larger production lines. He *et al.* (2007) addressed production variability of a production line with *M* machines and *M-1* buffers of finite size. They proposed approximation approach based on Markovian arrival process to determine production variability.

All the literatures cited so far have the Markov property built into the model. More specifically, the residence times are assumed to be exponentially distributed whose lack of memory property gives the modeler lots of flexibility leading to explicit analytical results. However, in real life situations one is confronted with arbitrary distributions which render the analysis intractable. Thus, it is not surprising that researchers resort to Markov modeling. There are several obvious extensions to these studies. The case of arbitrary distributed up times and down times is the worthwhile extension of the exponential case which allows to model many realistic situations. Although there has been an increasing amount of literature dealing with random failures in production systems over the past decade, non-exponential failure and repair times distributions deserve more in-depth investigation.

Kenne and Gharbi (2000) studied a manufacturing system subject to random failures and repairs. The aim of their study was to extend the hedging point policy to non-exponential failure and repair times distributions and random demand rates models. The cost related to the inventory and back order penalties chose to be the performance measure in their study. Pham-Gia and Turkkan (1999) evaluated effectiveness of a repairable system. In their research, the system was modeled by the gamma alternating renewal process and the effectiveness was measured by the proportion of time the system is on. Unknown distribution functions for the repair and failure times were studied by Gamiz and Roman (2008). The instantaneous availability considered as the most significant measure for evaluating the effectiveness of a repairable system in this research. Bierbooms *et al.* (2013) have recently developed a method to approximate the throughput and the mean buffer content of the production line. They analyzed the production line consisting of a number of machines in series with a finite buffer between each pair of machines. The up and down times assumed to be generally distributed. Colledani and Tolio (2011) proposed a decomposition method for evaluating the performance of transfer lines. They stated that times to repair are very rarely observed to be exponentially distributed in actual systems. Hence, the repair times assumed to be non-exponential in their study.

This paper attempts to obtain the first and second order characteristic of the throughput of a single station production line with arbitrary up and down times. We do hope Gershwin`s decomposition technique will help us to obtain the corresponding characteristics of a N station production line. Our modeling could also be viewed as a generalization of the Markov reward model for a discrete material flow production line of Tan (1999). The single machine that we consider could be operational (up state) or failed (down state), so that we model the system using an alternating renewal process.

In a dynamic and fast changing environment, we are of the considered view that the steady-state as well as the first order measures alone is not sufficient to give a correct picture of the production dynamics. We believe that a transient analysis incorporating the first and second order performance measures will provide a powerful decision support tool which alone can bring out the nuances in the production dynamics. The characteristics of interest require computation of performance measures such as the mean and variance of the number of visits to the up state as well as the availability function. However, no analytical solutions are available for these measures excepting when the up and down times are exponentially distributed which corresponds to the Markov model. A notable contribution of this study lies in developing useful approximations to determine (i) the expected number of visits to the up state known as the renewal function and (ii) the availability function which gives the probability that the system is found in the up state at any arbitrary time.

2. MATHEMATICAL MODEL

For our model, a discrete material flow production process which consists of a single station with station break downs is considered. We assume that the station is neither starved nor blocked. The station works for a random amount of time (up time) before it fails. The station is sent for repair which takes a random amount of time (down time) when it becomes operational again. The sum of the up and down time will be referred to as a cycle. Let the duration of the two states in the nth cycle be specified by the sequence of independently and identically distributed random variables $X_n$with distribution function $F_U(.)$ and Y_n with distribution $F_D(.)$. The sequence of random variables $\{X_n; n \geq 1\}$ and $\{Y_n; n \geq 1\}$ are assumed to be independent. Denote by $N_i(t)$, $i = U, D$, the number of renewals of state i in $[0, t]$. Then $\{N_D(t), t \geq 0\}$ is an ordinary renewal process generated by the sequence of random variable $\{X_i + Y_i\}$ having distribution $H = F_U * F_D$ (i.e. H is given by convolution of F_U and F_D) and $\{N_U(t), t \geq 0\}$ is a modified renewal process with initial distribution F_U (i.e. initial inter arrival time X_1) and subsequent distribution $H = F_U * F_D$ (i.e. subsequent inter arrival times $X_{i+1} + Y_i;\ \ i = 1,2, \ldots$). The single station then can be described by an alternating renewal process. In a cycle, when the production line operates, it is assumed that an item is produced. Therefore, the number of items produced in an arbitrary time interval $[0, \mathrm{t})$ equals the number of times the up and down states have been visited by the process in the said interval. We wish to observe that the production of one unit in an up state during a cycle is only a convenient assumption for the sake of clarity. However, one can assume that a fixed number of items are produced during an up time in a cycle or assign a production rate during the up time with very minor changes to the model. One can even assume a reward for each of the visits to the up state as done in Tan (1999). We also wish to observe that Tan constructed a Markov reward model where the single station material flow production line has been modeled as a discrete time Markov chain. The present model is clearly a generalization of his model to continuous time processes. Also the restrictive assumptions of exponential up and down times are relaxed to accommodate general distributions.

With the above model assumptions, the single station can be represented by an alternating renewal process with two states {U, D}. We assume that the station has become just operational so that it is in state U initially. This is not a restrictive assumption as the model could be easily be worked out starting with the down state as well. The expectation of the random variable $N(t)$ denoted by the function $M(t) = E[N(t)]$ plays a crucial role in the theory of renewal processes. This function is referred to as the renewal function of the process. Using elementary probability arguments one can show that $M(t)$ satisfy the following integral equation.

$$M(t) = F(t) + \int_0^t M(t-x) dF(x) \tag{1}$$

Therefore, $M_U(t)$ and $M_D(t)$, respectively, denote the expected number of times the up and down states have been visited in $[0, t)$ which are given by the following equations.

$$M_U(t) = F_U(t) + \int_0^t M_U(t-x) dF_{U+D}(x) \tag{2}$$

$$M_D(t) = F_{U+D}(t) + \int_0^t M_D(t-x) dF_{U+D}(x) \tag{3}$$

With the assumption of the production of one unit in each of the up states, $M_U(t)$ also gives the number of units produced in the same interval. The Laplace transform of $M_U(t)$ and $M_D(t)$ are given by

$$M_U^*(s) = \frac{f_U^*(s)}{s[1 - f_U^*(s).f_D^*(s)]} \tag{4}$$

$$M_D^*(s) = \frac{f_U^*(s).f_D^*(s)}{s[1-f_U^*(s).f_D^*(s)]} \tag{5}$$

There is no explicit solution for these renewal equations excepting in the case of alternating renewal processes driven by exponential up and down times. While there have been approximations available for the renewal function $M_D(t)$, we are not aware of any approximation to the function $M_U(t)$ perhaps because of its structure. In a *GI/M/1* queue system, Whitt (1984) showed that there is a considerable reduction in the range of possible values of σ (the steady-state probability that a customer will have to wait to begin his service) and *L* (the expected equilibrium queue length) when the third moment is also used, compared with using just two moments of *F*. Thereupon, we present below a theorem which gives an efficient approximation procedure to compute the renewal function $M_U(t)$ based on the first three moments of the distribution function F_U and F_{U+D} in the absence of any knowledge of the form of F_U and F_{U+D}.

Theorem 1. *Assume that the first three raw moments (about the origin) of the random variables* U ($\mu'_{1(U)}, \mu'_{2(U)}$, and $\mu'_{3(U)}$) *and* U+D ($\mu'_{1(U+D)}, \mu'_{2(U+D)}$, and $\mu'_{3(U+D)}$) *exist and are known. Then the following results hold for renewal function* $M_U(t)$.

$$M_U(t) = A.t - \frac{B.\left(1-e^{s'_0.t}\right)}{s'_0} \tag{6}$$

where

$$A = \frac{1}{\mu'_{1(U+D)}} \tag{7}$$

$$B = -S_0{}' \, \frac{\mu'_{2(U+D)} - 2\mu'_{1(U)}\mu'_{1(U+D)}}{2\mu'^{2}_{1(U+D)}} \tag{8}$$

and

$$S_0{}' = -\frac{6\left(\mu'_{2(U+D)} - 2\mu'_{1(U)}\mu'_{1(U+D)}\right)\mu'_{1(U+D)}}{2\mu'_{3(U+D)}\mu'_{1(U+D)} - 3\mu'^{2}_{2(U+D)} + 6\mu'_{2(U+D)}\mu'_{1(U+D)}\mu'_{1(U)} - 6\mu'_{2(U)}\mu'_{1(U+D)}} \tag{9}$$

Proof. We know that the Laplace transform of the renewal density $m_U(t)$ is given by

$$m_U^*(s) = \frac{f_U^*(s)}{1-f_U^*(s).f_D^*(s)} \tag{10}$$

We note that there is a singularity at the origin for the function $m_U^*(s)$. Thus, the function $m_U^*(s)$ is approximated with the help of ration function as below.

$$m_U^*(s) = \frac{A}{s} + \frac{B}{s-s'_0} \tag{11}$$

Inverting the above equation results in (6). Now the constants A, B, and s'_0 are obtained as follows:

We express $f^*(s)$, the Laplace transform of the *pdf* as a power series as below.

$$f^*(s) = \sum_{n=0}^{\infty} \frac{(-1)^n . s^n}{n!} \mu'_n = 1 - \frac{s^1 \mu'_1}{1!} + \frac{s^2 \mu'_2}{2!} - \frac{s^3 \mu'_3}{3!} + \cdots$$

Using (10) and (11) we obtain

$$\frac{A}{s} + \frac{B}{s - s_0{}'} = \frac{1 - \frac{S^1 . \mu'_{1(U)}}{1!} + \frac{S^2 . \mu'_{2(U)}}{2!} - \frac{S^3 . \mu'_{3(U)}}{3!} + \cdots}{1 - \left[1 - \frac{S^1 . \mu'_{1(U+D)}}{1!} + \frac{S^2 . \mu'_{2(U+D)}}{2!} - \frac{S^3 . \mu'_{3(U+D)}}{3!} + \cdots\right]} \tag{12}$$

Comparing the coefficients of *S*, S^2, S^3 on both the sides of (12) and after some algebra we obtain the constants A, B, and S'_0 as given in (7), (8), and (9), respectively. Hence, finally we obtain

$$M_U(t) = \frac{t}{\mu'_{1(U+D)}} + \frac{\left(\mu'_{2(U+D)} - 2\mu'_{1(U)} \mu'_{1(U+D)}\right) . \left(1 - e^{S_0{}' . t}\right)}{2\mu'^2_{1(U+D)}} \tag{13}$$

Using a similar analysis as in the previous theorem, we can obtain the renewal function $M_D(t)$ as

$$M_D(t) = \frac{t}{\mu'_{1(U+D)}} + \frac{\left(\mu'_{2(U+D)} - 2\mu'^2_{1(U+D)}\right) . \left(1 - e^{s_0 . t}\right)}{2\mu'^2_{1(U+D)}} \tag{14}$$

where

$$S_0 = -\frac{6(\mu'_{2(U+D)} - 2\mu'^2_{1(U+D)}) \mu'_{1(U+D)}}{2\mu'_{3(U+D)} \mu'_{1(U+D)} - 3\mu'^2_{2(U+D)}} \qquad \blacksquare$$

It should be noted that for the approximations (13) and (14) to be valid, the constants S_0 and $S_0{}'$ must be negative. The restriction that S_0 and $S_0{}'$ are less than zero is not very restrictive since we have seen that these conditions are satisfied by many well-known distribution functions like gamma, mixture of exponential, lognormal, Weibull, and phase type distributions which are commonly used in production and reliability analysis. The condition is also met for distributions like Truncated Normal and Inverse Gaussian but under certain conditions.

The availability function $A(t)$ which gives the probability that the system is in up state at an arbitrary time *t* is given by

$$A(t) = M_D(t) - M_U(t) + 1 \tag{15}$$

Using (13) and (14) in the above equation and after some algebra we obtain

$$A(t) = -\frac{\mu'_{2(U+D)} (e^{s_0 . t} - e^{S_0{}' . t}) - 2\mu'^2_{1(U+D)} e^{s_0 . t} - 2\mu'_{1(U)} \mu'_{1(U+D)} \left(1 - e^{S_0{}' . t}\right)}{2\mu'^2_{1(U+D)}} \tag{16}$$

Finally, our interests lie not only on the first order characteristics of the number distributions but on the second order as well. It is well-known that

$$var[N_D(\mathrm{t})] = M_D(t) + 2\int_0^t M(t-x) dM(x) - [M(t)]^2 \tag{17}$$

Use of the above equation with $M_D(t)$ as specified in (14) yields

$$var[N_D(\mathrm{t})] = \frac{\sigma^2}{\left[\mu'_{1(U+D)}\right]^3} t + \left\{ \frac{2\sigma^2}{\left[\mu'_{1(U+D)}\right]^2} + \frac{3}{4} + \frac{5\sigma^4}{4\left[\mu'_{1(U+D)}\right]^4} - \frac{2\left[\mu'_{3(U+D)}\right]}{3\left[\mu'_{1(U+D)}\right]^3} \right\}$$
$$- \left\{ \frac{5\sigma^2}{2\left[\mu'_{1(U+D)}\right]^2} + \frac{1}{2} + \frac{\sigma^4}{\left[\mu'_{1(U+D)}\right]^4} - \frac{2\mu'_{3(U+D)}}{3\left[\mu'_{1(U+D)}\right]^3} \right\} e^{s_0 t} + 2tv\left(\frac{1}{\mu} + vs\right) e^{s_0 t} - v^2 e^{2s_0 t} \tag{18}$$

where $v = \frac{\mu'_{2(U+D)} - 2\mu'^2_{1(U+D)}}{2\mu'^2_{1(U+D)}}$ and $\sigma^2 = \mu'_{2(U+D)} - \mu'^2_{1(U+D)}$

3. SPECIAL CASES

Having obtained the approximations for the first and second order of characteristics of the production process, in the following we proceed to obtain these characteristics for two certain special cases.

Case 1. We assume in this case, the length of operating period (up times) and period under repair (down times) are independent random variables having negative exponential distributions with mean $1/\lambda$ and $1/\mu$, respectively, so that the single station production line can be described by a two state Markov process. We choose

$$f_U(x) = \lambda e^{-\lambda x}$$

$$f_D(x) = \mu e^{-\mu x}$$

The proposed approximations yields

$$M_D(t) = \frac{\mu\lambda[t\mu + t\lambda - 1 + e^{S_0 t}]}{(\mu+\lambda)^2}$$

$$M_U(t) = \frac{\mu[t\mu\lambda + t\lambda^2 + \mu - \mu e^{S'_0 t}]}{(\mu+\lambda)^2}$$

$$var[N_D(\mathrm{t})] = \frac{(\mu^2+\lambda^2)\lambda\mu t}{(\mu+\lambda)^3} - \frac{2(\mu^2+\lambda^2)}{(\mu+\lambda)^2} + \frac{3}{4} + \frac{5}{4}\frac{(\mu^2+\lambda^2)^2}{(\mu^2+\lambda^2)^4} + \frac{3}{2}\frac{(\mu^2+\lambda^2)e^{S_0 t}}{(\mu+\lambda)^2} - \frac{(\mu^2+\lambda^2)^2 e^{S_0 t}}{(\mu+\lambda)^4} - \frac{4t\mu^2\lambda^2 e^{S_0 t}}{(\mu+\lambda)^3} - \frac{\mu^2\lambda^2 e^{2S_0 t}}{(\mu+\lambda)^4} - \frac{e^{S_0 t}}{2}$$

$$A(t) = \frac{\lambda}{\lambda+\mu} + \frac{\mu}{\lambda+\mu} e^{S_0 t}$$

where $S_0 = S'_0 = -(\mu+\lambda)$

The availability function $A(t)$ for a two state Markov process is well known in the literature. (See page 242 of Ross, 1996). Our approximation gives the same expression for the availability function.

Case 2. In the second case, we assume the up and down times to be distributed according to gamma distributions. Such a case arises when the system failure can be identified with a sequence of stages with each stage being exponentially distributed. Further the repairs are carried out in stages with each stage being exponentially distributed. Specifically, we assume

$$f_U(x) = \frac{1}{\Gamma(k)} \lambda^k x^{k-1} e^{-\lambda x}$$

$$f_D(x) = \frac{1}{\Gamma(k)}\lambda^k x^{k-1} e^{-\lambda x}$$

we obtain

$$\mu'_{1(U)} = \mu'_{1(D)} = \frac{k}{\lambda}, \mu'_{2(U)} = \mu'_{2(D)} = \frac{k(k+1)}{\lambda^2}, \mu'_{3(U)} = \mu'_{3(D)} = \frac{k(k+1)(k+2)}{\lambda^3}$$

$$\mu'_{1(U+D)} = \frac{2k}{\lambda}, \mu'_{2(U+D)} = \frac{2k(2k+1)}{\lambda^2}, \mu'_{3(U+D)} = \frac{4k(k+1)(2k+1)}{\lambda^3}$$

Using the approximations method, characteristics of the production process are given by

$$M_D(t) = \frac{t\lambda}{2k} - \frac{(2k-1)(1-e^{S_0 t})}{4k}$$

where $s_0 = \frac{-6\lambda}{2k+1}$,

$$M_U(t) = \frac{t\lambda}{2k} + \frac{(1-e^{s'_0 t})}{4k}$$

where $s'_0 = \frac{-6\lambda}{2k^2+1}$ and $A(t) = \frac{2k+e^{S_0 t}(2k-1)+e^{s'_0 t}}{4k}$

Lukas (2008) has given explicit formula for the computation of $M_U(t)$ and $M_D(t)$ when the up and down time are gamma distributed by expressing an infinite series in terms of finite sum that involves complex numbers. It is interesting to note that our approximation provides the exact results for the formula given by Lukas.

4. APPLICATION: Due Date Performance Measure

In this part, a general problem arises in a single machine production line has evaluated using the purposed approximations. One of the main jobs of an operations manager in a production line is in fulfilling the orders on time without recourse to back log or lost sales. Therefore, a good due date performance measure to know whether the output matches the demand can be defined to be the probability that the customer`s demands are fulfilled on time. Let $Q$ be the ordered quantity and $T_Q$ the due date of the same order. If the quantity produced in $(0, T_Q)$ exceeds $Q$, then the production line is able to meet the customer`s order on time. Thus, a due date performance measure can be defined as

$$D_d = \Pr[N(T_Q) \geq Q]$$

To compute this measure one should be know the probability distribution of $N(t)$. However, if T_Q is sufficiently large, central limit theorem can be invoked to establish that the random variable $N(t)$ is asymptotically normal. This gives us

$$D_d = \Pr[N(T_Q) \geq Q] = 1 - \frac{1}{\sqrt{2\pi}.\sigma_{N(T_Q)}}\int_{-\infty}^{Q} e^{-\frac{\left[x-E[N(T_Q)]\right]^2}{2\sigma^2_{N(T_Q)}}} dx = 1 - \Phi(Q)$$

where Φ is cumulative normal probability.

5. NUMERICAL ILLUSTRATION

The numerical results are intended to show the variations in the optimal decision variables owing to the selected distributions for up and down times as well as the variations in the parametric value of the same distribution. Here, we continue with some numerical result for the first case which was discussed in section 3.

Figure 1 plots the renewal function for various failure rates using equations 13. Firstly, we note that the renewal function is a monotonically non-decreasing function. Also for a specified μ (0.2), as the failure rate λ increases (from 0.01 to 0.08) so that the mean working time decreases, the number of units produces increases. Although this may look counterintuitive, with the assumption of one unit produced in each working interval, this is to be expected. However, if we make the assumption of a repair rate so that the number of units produced varies with the length of the working times such a result cannot be expected.

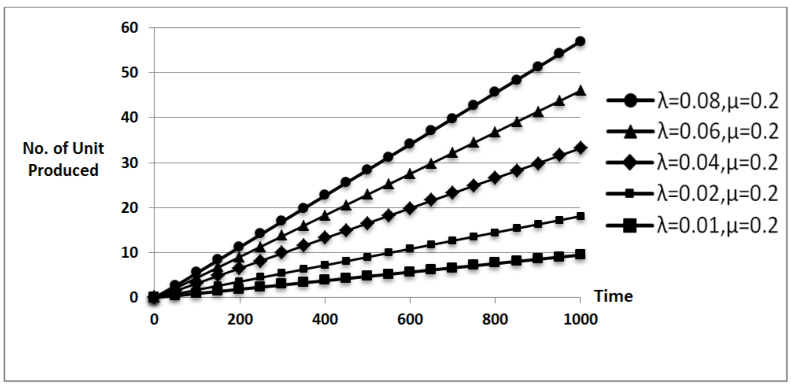

Figure 1: The Expected Number of Units Produced in a Time Interval T.

In figure 2, we plot the availability function for various values of repair rates λ (from 0.01 to 0.08) and a specified μ (0.2) using equation (16). We note that as the mean working time increases the probability of the system being found in a working state increases. We also observe that the availability function reaches the steady-state availability A given in (16).

In figure 3, we present the due date performance measure which is specified by the probability that a given order size Q is fulfilled within the due date given for certain values of Q. The values of λ and μ were fixed to be 0.04 and 2, respectively. It is seen immediately that as the due date t increases for a given order size Q, the probability of fulfilling that order is an increasing function of t and tends to unity as t tends to infinity. Also for a given t such a probability is a decreasing function of Q. The due date curve exhibits more shoulder for smaller values of Q and is steeper for larger values of Q.

In Figure 4, we depict variance of the output per unit time as a function of time. As we can see, by increasing the value of time, the ratio $[N(t)]/t$ increases and reaches to its asymptotic amount. It is worth to mention that the amount of this ration is sensitive to the value of failure rate. By increasing the failure rate λ, the amount of this ration increases as well.

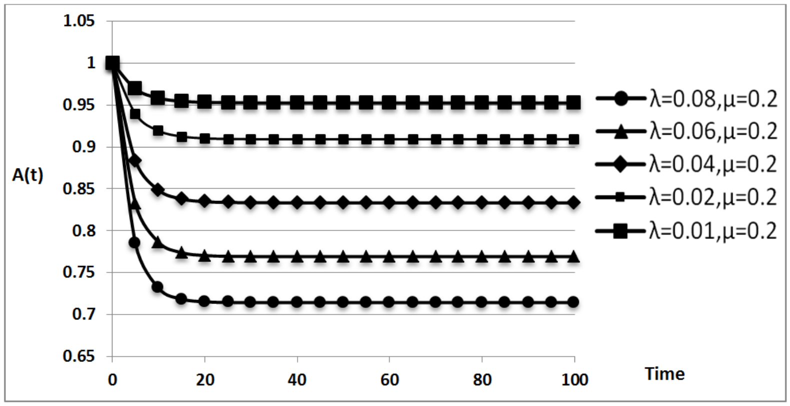

Figure 2: Availability Function A(t).

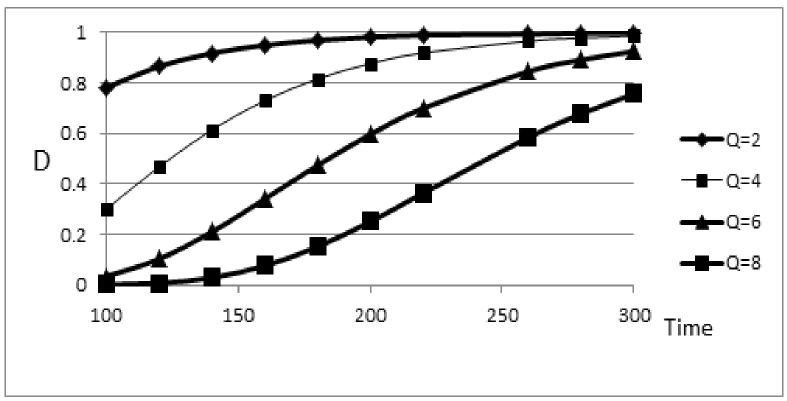

Figure **3**: The Due Date Performance Measure Pr [$N(T_Q) \geq Q$].

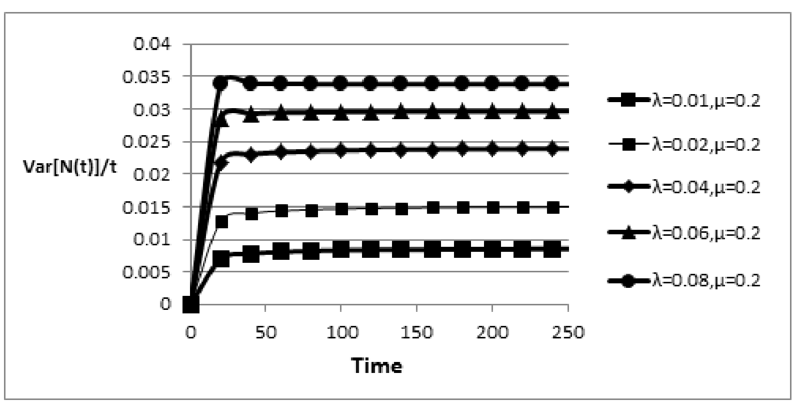

Figure 4: Variance [N(t)]/t as a Function of Time.

6. CONCLUDING REMARKS

This paper presents a transient analysis of a single machine production line, modeling the system using an alternating renewal process. The existing models have the Markov property of the up and down time built in. Also these models make use of the steady-state analysis of the system. Moreover, the study of short time production variability has all along been considered to be a difficult problem in the literature for quite some time (Tan, 1999). Hence, we have moved forward using arbitrary probability distributions as well as using transient analysis in our study. The major contribution of our work is in deriving some useful approximations for the renewal functions as well as the availability function. It is not possible to obtain explicit solutions for these functions for arbitrary up and downtime distributions. These approximations are easy to implement and depend only on the first three moments of the underlying distributions. We also analyze availability in the production line. Using the average and variability of the throughput, we have suggested a due date performance measure for relatively large values of due date. This is because we invoke central limit theorem for $N(t)$, the number of units produced in an arbitrary time *t*.

We conclude the study with some direction for future work. The generalization form the exponential to arbitrary distributions necessitated the use of approximations for the performance measures. However, one could approximate the arbitrary distribution functions by phase type distributions. There are extensive literatures available for such approximations (see Kambo *et al.,* 2012, for example). The advantage of such approximations is that the performance measures for phase type distributions are available in explicit form. Secondly, we have assumed the up times and down times to be mutually independent. It may be interesting to make a study if these variables are correlated. Such a case arises when there are different types of failures and the repair times depend on the type of the failure. Finally, it is natural that this study is carried forward for N-station production lines and later on with buffer in between the stations.

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