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# A Robust Method for Designing the Parameters of Genetic Algorithms

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***Abstract*** — Genetic algorithms are commonly used in mathematical programming to deal with problems related to optimal values; however, the success of the search process depends largely on the values selected for the parameters. This paper proposes a robust method for determining the parameters of genetic algorithms, in which the Taguchi method is applied over continuous or discrete areas using a simple optimal-point-search strategy. Thus, the proposed method is well-suited to both continuous and discrete parameters. Numerical results for two well-known test cases demonstrate the effectiveness of the proposed method. We also applied this method to a complex, practical binary-integer problem dealing with scheduling the Taiwan High Speed Rail system and compared the results with two existing methods, the Taguchi method and the response surface method. The solution obtained from the genetic algorithm with parameter settings provided by the proposed method proved superior to the original scheduling solution.

***Keywords*** — Robust decision, Parameter design, Taguchi methods, Genetic algorithms, Evolutionary algorithms

## 1. INTRODUCTION

Genetic algorithms, which search for optimal solutions, are important tools in mathematical programming. Problems to which they can be effectively applied include examination timetables (Pillay and Banzhaf, 2010), flexible job-shop scheduling (Zhang et al., 2011) and multi-depot and periodic vehicle routing (Vidal et al., 2012). The values selected for the parameters of a genetic algorithm largely determine the performance of the search process and assigning appropriate values can be very difficult, particularly when dealing with large-scale programming problems that often require excessive computational time.

De Jong (1975) and Grefenstette (1986) proposed methods for the selection of parameters based on the relationship between parameter values and the performance of algorithms using a full factorial experimental design. These studies remain an important reference for the developers of genetic algorithms. Other researchers developed methods to control the value of parameters in each generation of a run (Julstrom, 1995; Hinterding et al., 1996; Lobo and Goldberg, 2004). However, optimal parameter settings vary from problem to problem (Schaffer et al., 1989). This has led to the application of the Taguchi method, which uses orthogonal arrays for the design of parameters to optimize search performance (Pongcharoen et al., 2002; Anagun and Qzcelik, 2005; Hippolyte et al., 2008).

The results obtained using the Taguchi method can often overcome the effects of interaction among parameters; however, this approach is applicable only to the selection of optimal parameter values from several discrete levels. Thus, the response surface method was developed for cases in which the range of parameters are continuous intervals (Najafi et al., 2009; Niaki and Ershadi, 2012; Shahsavar et al., 2010). This method uses experimental data to formulate a mathematical model to represent the relationship between the performance of the algorithm and parameter values, whereupon the optimal parameter settings are selected from extreme points in the model. However, the response surface method often produces a mathematical model that inaccurately describes the relationship. In addition, this approach often breaks down when it reaches the parameter boundaries, where the value is fixed but does not necessarily provide an optimal solution.

The paper proposes a robust new method, which is free from mathematical modeling. The proposed method features two main advantages. First, it combines the Taguchi method with a simple optimal-point-search strategy in either continuous or discrete areas, and is not susceptible to the boundary problems associated with the response surface method. Second, it can provide hints regarding the establishment of factors for use in the Taguchi experiment. The optimal parameter settings often vary according to the different runtimes (i.e., different phases of a run), which can undermine their overall reliability. This may explain why so few empirical studies addressing the issue of parameter design in genetic algorithms have performed experiments to confirm their results. The effects of parameter settings must be considered with regard to the performance of an algorithm in a fixed runtime. Genetic algorithms generate a greater number of solutions as

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the population size (PS) or the maximum number of generations (MG) is increased; therefore, we combined these two runtime-related parameters into a single factor (PSMG) to fix the runtime of each experiment in order to obtain more reliable parameter settings. The use of PSMG can also derive parameter settings applicable to a group of problems with similar mathematical formulations. This is helpful as designing parameter settings for a single problem is inefficient; the time used for the design of parameters could be used to prolong the runtime of the algorithm in order to obtain better solutions. In Section 3.1, an example is provided to illustrate the advantages of PSMG in dealing with these types of problems.

This paper is organized as follows. Section 2 presents a brief introduction to the processes of genetic algorithms and the Taguchi method. We also outline the Taiwan High Speed Rail Timetabling Problem (THSRTP) and explain why this problem in particular was selected to evaluate the efficacy of the proposed method. In Section 3, we use the THSRTP to illustrate some of the issues encountered when searching for optimal solutions without the introduction of the factor PSMG. We then introduce the proposed parameter setting method with an embedded feature for fixing the runtime in each experiment. In Section 4, the proposed method is demonstrated using the THSRTP as well as two problems commonly used for the evaluation of genetic algorithms. We then compare our results with those obtained using the conventional Taguchi method and response surface method as well as the research that first introduced the THSRTP. The paper is concluded in Section 5.

## 2. PRELIMINARY

In the following, we provide a brief introduction to genetic algorithms and the Taguchi method. We then introduce the THSRTP and our reasons for selecting it.

### 2.1 Genetic algorithm

Genetic algorithms (Whitley, 1994; Man et al., 1996; Melanie, 1998) mimic genetic processes by evolving a set of candidate solutions toward an optimal solution. A fitness function is defined to measure the fitness of each solution to the problem, such that solutions with a better fitness value have a higher probability of being selected for the next generation. The fitness of candidate solutions is therefore improved with each new generation. Genetic algorithms operate according to the following five steps.

**Step1. Initializing the algorithm:** Parameter values are provided for the PS, the selection rate (SR), the crossover rate (CR), the mutation rate (MR) and the MG. According to the value assigned to PS, the initial population is randomly and uniformly generated from the feasible solution space using an appropriate encoding method, such as binary encoding or value encoding.

**Step2. Parent Selection:** Parent solutions are selected from the current generation to generate child solutions in the following crossover step. Several selection methods are commonly used, including the tournament method, the roulette wheel method, and the rank selection method.

**Step3. Crossover:** Crossover is the process of generating the next generation, using methods such as the one-point method, the two-point method, and the arithmetic method.

**Step4. Mutation:** To avoid the convergence of solutions to local optimums, mutations can be introduced using methods such as flip bit, uniform, and Gaussian.

**Step5. Termination:** The algorithm is terminated if the number of generations reaches the given MR or if the current highest fitness value is not improved over several consecutive generations.

### 2.2 Taguchi method

The aim of the Taguchi method (Antony and Antony, 2001; Ross, 1988) is to design parameter settings such that the average outputs are improved and variance is minimized. The results obtained using the Taguchi method tend to be robust. The Taguchi method is used to arrange experiments in the most efficient manner through the use of orthogonal arrays. Experiment data is evaluated according to its signal-to-noise (S/N) ratio in order to simplify the process of analysis and obtain optimal parameter settings. Unfortunately, the Taguchi method selects optimal parameter settings from finite discrete levels, and therefore provides suboptimal parameter settings when parameters are continuous intervals.

### 2.3 Taiwan High Speed Rail Timetabling Problem

The goal of the THSRTP is to produce a train schedule capable of maximizing revenue. The rail system operates between Taipei and Kaohsiung from 6:00 to 24:00 every day. A northbound subsystem operates from Kaohsiung to Taipei and a southbound subsystem operates from Taipei to Kaohsiung. A total of 11 stations are included in the system and no more than 88 trains operate along either of the subsystems. To deal with differences between stations, each subsystem has the option to run five types of trains with a maximum capacity of 870 people. Trains stop for 2 minutes in all stations except Taichung, in which the stop is 3 minutes. We refer readers to Yu and Li (2009) for further details of the THSRTP.

This study selected the THSRTP because it is a complex binary integer problem for which it is nearly impossible to obtain a satisfying solution in an acceptable period of time, particularly when using algorithms without well-designed parameter settings. The rail system has two subsystems, which produce two similar scheduling subproblems. Parameter settings should be applicable to other problems with a similar mathematical formulation, rather than being limited to individual problems. The two subproblems are used to illustrate how the conventional Taguchi method obtains different parameter settings for each, while the Taguchi method with PSMG obtains the same settings. This makes it possible to apply one set of parameters to multiple scenarios, such as different customer rates or different combinations of stations.

## 3. PROPOSED METHOD

In the following, we use the THSRTP to illustrate the necessity of PSMG and thereafter provide a detailed description of the proposed method.

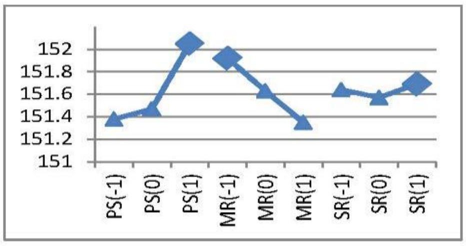
### 3.1 Necessity of combining factor PSMG

In this section, numerical results related to the THSRTP are used to illustrate the necessity of combining PS and MG into the combined factor PSMG. In order to designate runtimes, we required that the products of PS and MG values be the same in all experiments. The Taguchi method was used to search for optimal settings for both the northbound and southbound problems. PS and MG were used directly as factors in the first run and then combined as PSMG in the second run. The first run included three factors (PS, SR, and MR), with MG fixed at 500 and CR at 100%. The experiment levels of the three factors in the first run are presented in Table 1.

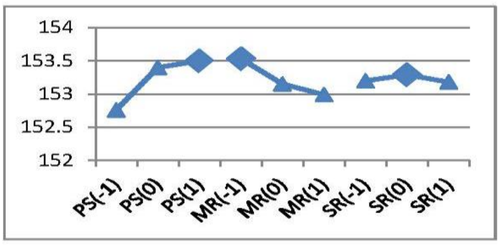
Table 1. Levels of factors in the first run

| Factors | Level -1 | Level 0 | Level 1 |
|---|---|---|---|
| PS | 50 | 100 | 200 |
| MR | 0.2 | 0.5 | 0.8 |
| SR | 20% | 50% | 80% |

We used an L9 orthogonal array (see Table A1 in the Appendix) to arrange the experiments, each of which was repeated three times. This provided observations related to each treatment, in the form of the values of objective functions with solutions obtained using the genetic algorithm using the parameter settings in Tables A2 and A3 in the Appendix. As shown in Figure 1(a), the parameter values obtained for the northbound problem were 200 for PS, 0.2 for MR, and 80% for SR. As shown in Figure 1(b), the parameter values obtained for the southbound problem were 200 for PS, 0.2 for MR, and 50% for SR. This shows that the parameter settings obtained using the conventional Taguchi method (without PSMG) produces distinctly different solutions for two similar subproblems.




a) The southbound problem




b) The northbound problem

Figure 1. Graphs showing the effect of each level of the three factors in the first run: a) southbound problem; b) northbound problem. The Taguchi method produced different parameter settings for very similar problems. The values were obtained using Eq. (1) in Section 3.2.

The levels for each factor in the second run are presented in Table 2. We fixed the runtime in each Taguchi experiment by fixing the products of the values in all three levels of PS and MG at 100,000.

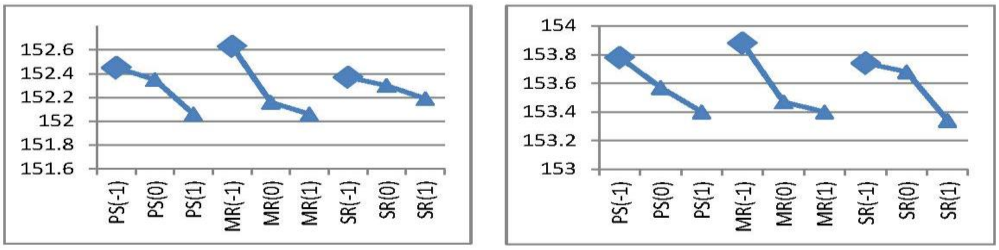
We used an L9 orthogonal array and repeated each experiment three times. The resulting observations are shown in Tables A4 and A5 in the Appendix. As shown in Figure 2, the parameter settings obtained using PSMG are the same for the two subproblems. PSMG is 50*2000, MR is 0.2, and SR is 20%. These parameter settings could be reliably applied to other problems with similar mathematical formulations, which could help to save time. It should be noted that the runtimes were approximately 5 hours for the northbound problem and approximately 8 hours for the southbound problem. Therefore it is possible to fix the runtimes through fixing the products of the values of PS and MG.

Table 2. Levels of factors in the second run

| Factors | Level -1 | Level 0 | Level 1 |
|---|---|---|---|
| PSMG(=PS*MG) | 50∗2000 | 100∗1000 | 200∗500 |
| MR | 0.2 | 0.5 | 0.8 |
| SR | 20% | 50% | 80% |

### 3.2 Proposed method

This section presents a detailed description of the proposed method, which is based on the Taguchi method. Employing a simple optimal-point-search strategy, the Taguchi method is run iteratively in order to obtain optimal parameter settings. The proposed method includes the following six steps:




a) The southbound problem b) The northbound problem

Figure 2. Graphs showing the effect of each level of the three factors in the second run: a) southbound problem; b) northbound problem. The optimal parameter settings obtained using the Taguchi with PSMG factor are the same for the very similar problems in a) and b). The values were obtained using Eq. (1) in Section 3.2.

**Step1.** Select factors for experiments;
**Step2.** Establish initial experiment levels for each factor;
**Step3.** Conduct experiments with orthogonal array;
**Step4.** Analyze the observations obtained in the experiment;
**Step5.** Establish new levels for each factor;
**Step6.** Conduct confirmation experiment.

Steps 1 and 2 involve the selection of factors and the establishment of initial levels for the factors selected in the Taguchi experiment. Steps 3 and 4 are the processes belonging to the original Taguchi method and a stop criterion. Step 5 establishes the levels of each factor for the following Taguchi experiment. Step 6 evaluates the reliability of the parameter setting. Figure 3 presents a flow chart of the proposed method.

The steps are detailed in the following:

**Step 1. Select factors for experiments:** With the exception of PS and MG, which are combined as factor PSMG, each of the other parameters can be considered as a factor in the Taguchi experiments.

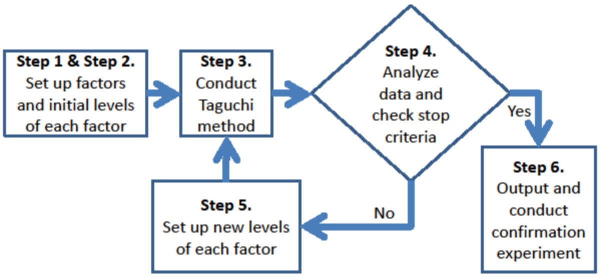



Figure 3. Flow chart of proposed method

**Step 2. Establish initial levels for each factor:** Three levels were selected for each factor, denoted as -1, 0, and 1 (Figure 4). First, we selected appropriate values for factors at level 0. We then selected a suitable distance between two consecutive levels for each of the factors, referred to as the *permissible distance*. Finally, the permissible distance is added to level 0 to obtain level 1, and subtracted from level 0 to obtain level -1. We denoted the point where all factors are level 0 as *the initial point*.

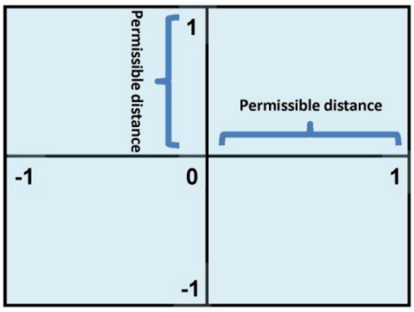



Figure 4. Illustration of the three levels of each factor in the proposed method

**Step 3. Experiments using orthogonal array:** Having established the number of factors and number of levels, we were then able to select an appropriate orthogonal array from the Taguchi orthogonal array selector matrix. We had four factors (SR, CR, MR, and PSMG), each of which had 3 levels; therefore, we selected array L9 (see Table A1 in the Appendix) for our experiment.

**Step4. Analysis of observations:** Assume that each treatment in the experiment repeats $n$ times. Let $y_{i1}, y_{i2}, \ldots, y_{in}$ denote $n$ observations with treatment $i$. The S/N ratio for treatment $i$ is

$$SN_i = -10\log_{10}\left(\frac{1}{n}\sum_{j=1}^{n}\frac{1}{y_{ij}^{2}}\right).$$

Let $s(k,h)$ denote the set of treatments in which the level of factor $k$ is $h$. We then calculate the effect of the factor $k$ at level $h$ as follows:

$$E_{k,h} = \frac{\sum_{i\in s(k,h)} SN_i}{\text{the size of } s(k,h)}, \text{ for } h = -1, 0, 1. \tag{1}$$

In this experiment, the best level for factor $k$ is the one that maximizes its effect. Let $p_k^*$ denote the best level in the Taguchi experiment. Thus, we have

$$E_{k,p_k^*} = \max_{h\in\{-1,0,1\}} E_{k,h} \tag{2}$$

Eqs. (1) and (2) are used to obtain the ideal parameter setting for the Taguchi experiment. Let $E_{opt}$ be *the predicted* S/N *ratio* of the obtained parameter settings, calculated as follows:

$$E_{opt} = \bar{E} + \sum_{i=1}^{m}(E_{k,p_k^*} - \bar{E}) \tag{3}$$

where $\bar{E}$ represents the average of the S/N ratio of all treatments and $m$ is the number of factors. If the ideal level for each factor is 0 and we are satisfied with the current permissible distance, then the algorithm is stopped. The current best level is referred to as *the NT optimal parameter setting*. Proceed to Step 6 to confirm the results with an experiment using the NT optimal parameter settings. Otherwise, proceed to Step 5.

**Step5. Establish new levels for each factor:** If the best level for each factor is 0, but we are not satisfied with the current permissible distance, the current permissible distance is reduced by half. We then take the best level obtained for each factor in Step 4 as Level 0 and add/subtract the new permissible distance to/from Level 0 to obtain Levels 1/-1. We then proceed to Step 3 for the next iteration. Figure 5 illustrates how the new levels of each factor are arranged for next iteration of the Taguchi process in the case where the best level for each factor is 0 and optimal parameter settings of greater accuracy are still desired.

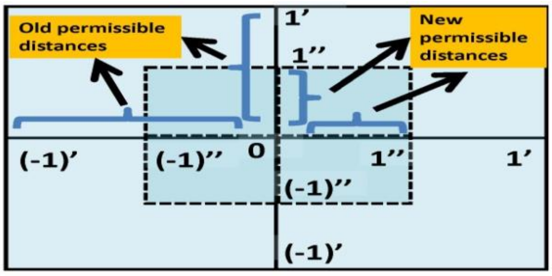



Figure 5. Illustration of arrangement of new levels for the next iteration of the Taguchi process when the best level for each factor is 0 but greater accuracy is desired (from 1'/(-1)' to 1"/(-1)")

If one of the ideal levels is not 0, we take the best level obtained for each factor in Step 4 as the new Level 0 and add/subtract the permissible distance to obtain Level 1/-1. We then proceed to Step 3 to initiate the next iteration. Figure 6 illustrates how the new levels of each factor are arranged for the next iteration of the Taguchi experiment as well as the means by which the center point (the point where the level of each factor is 0) evolves.

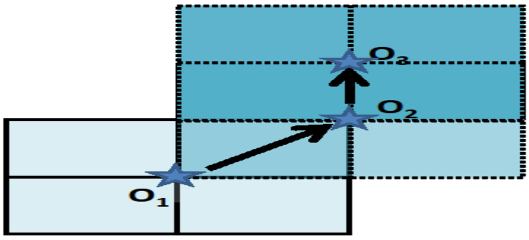



Figure 6. Arrangement of new levels for each factor in the next iteration of the Taguchi process as well as the evolution of the center point ($O_1 \rightarrow O_2 \rightarrow O_3$)

In this step, we set up Levels 0 and 1/-1 in the case where Level -1/1 is beyond the range of the factor in order to avoid the boundary problem of the response surface method. Figure 7 illustrates how only two levels are used when Level -1/ 1 is beyond the lower/upper boundary. The figure also shows the evolution of the center point, in which the trajectory is not trapped when it encounters a boundary.

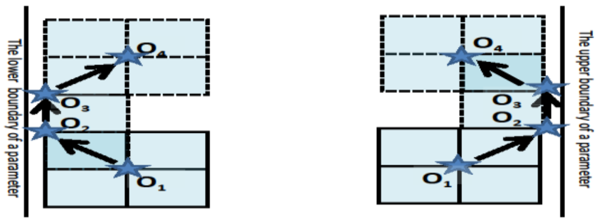



Figure 7. Illustration of the setting of factor levels when the trajectory of center points meets a boundary

**Step6. Confirmation experiment:** The predicted S/N ratio of the NT optimal parameter settings is used to calculate a 95% confidence interval, c.f. (Tseng et al., 2013). We then conduct a confirmatory experiment to obtain the real S/N ratio for the NT optimal parameter settings. If the real S/N ratio falls within the predicted confidence interval, then the NT optimal parameter setting can be regarded as reliable in improving search performance of the genetic algorithm with a given fixed runtime.

## 4. NUMERICAL RESULTS

Numerical results were used to illustrate the advantages of the proposed method in two well-known test cases and

THSRTP. Example 1 is the second test problem presented by De Jong (1975). Example 2 is a problem proposed by Michalewicz (1992). To solve these problems, we used the value encoding method for generation of the initial population, the roulette wheel method for selection, the two-point method for crossover, and the uniform method for mutation. To solve the THSRTP, we used the binary method, the roulette wheel method, the two-point method, and the flip bit method. Sections 4.1 and 4.2 present the results for examples 1 and 2, respectively. Section 4.3 outlines the advantages of the proposed method in solving THSRTP. We also provide a comparison of results from the proposed method, the conventional Taguchi method, the response surface method, and the research that originally proposed the THSRTP.

### 4.1 Example 1

The mathematical formulation of Example 1 is as follows:

$$\max.\ f(x_1, x_2) = 100(x_1^2 - x_2)^2 + (1 - x_1)^2;$$
$$\text{subject to} \quad -2.048 \le x_i \le 2.048,\ i = 1, 2.$$

To obtain the optimal solution of example 1, we used a genetic algorithm with a flat unimodal objective function. Most algorithms take a long time to obtain optimal results; therefore, the parameter settings must be well selected to maximize search efficiency. The proposed method was implemented with the following initial points: PS (250), MG (200), SR (0.5), and MR (0.5). The permissible distances were as follows: PS (50), SR (0.1), and MR (0.1). CR was fixed at 100% and the products of PS and MG were fixed at approximately 50,000. After 9 iterations, the proposed method reached the NT optimal parameter settings, as follows: PS (550), MG (91), SR (0.3), and MR (0.6). We then ran the algorithm fifteen times using the NT optimal parameter settings as well as other parameter settings. The data in Table 3 shows that the NT optimal parameter settings enhanced the efficiency and robustness of the genetic algorithm. In addition, the results obtained using the NT optimal parameter settings are equal to or better than those using the larger products of PS (500) and MG (500), which resulted in a longer runtime.

Table 3. Statistics for each of the fifteen observations after running the genetic algorithm with NT optimal parameter setting as well as other parameter settings

| Parameter settings | | | | Statistics of 15 observations | | | | |
|---|---|---|---|---|---|---|---|---|
| **PS** | **MG** | **SR** | **MR** | **Average** | **Standard Deviation** | **Max** | **Min** | **Note** |
| 550 | 91 | 0.3 | 0.6 | 3895.179 | 4.831087411 | 3905.286 | 3889.111 | The NT optimal |
| 200 | 250 | 0.9 | 0.1 | 3856.475533 | 34.32418192 | 3894.977 | 3803.319 | |
| 200 | 250 | 0.5 | 0.5 | 3887.2032 | 12.21736083 | 3904.498 | 3857.002 | |
| 200 | 250 | 0.1 | 0.9 | 3886.714733 | 16.46737656 | 3903.132 | 3850.682 | |
| 750 | 67 | 0.9 | 0.1 | 3876.5374 | 17.95609041 | 3900.914 | 3844.551 | |
| 750 | 67 | 0.5 | 0.5 | 3891.9514 | 7.617956211 | 3905.435 | 3877.7 | |
| 750 | 67 | 0.1 | 0.9 | 3896.353733 | 6.730309656 | 3904.092 | 3882.585 | |
| 500 | 100 | 0.9 | 0.1 | 3872.686267 | 20.75982521 | 3900.658 | 3830.021 | |
| 500 | 100 | 0.5 | 0.5 | 3885.017333 | 15.57543425 | 3897.938 | 3832.857 | |
| 500 | 100 | 0.1 | 0.9 | 3894.754067 | 8.44823424 | 3904.713 | 3873.308 | |
| 500 | 500 | 0.9 | 0.1 | 3883.968267 | 12.67353769 | 3897.134 | 3857.755 | |
| 500 | 500 | 0.5 | 0.5 | 3896.472467 | 6.775929149 | 3905.309 | 3876.291 | |
| 500 | 500 | 0.1 | 0.9 | 3900.676067 | 4.222486733 | 3905.305 | 3891.085 | |

### 4.2 Example 2

The mathematical formulation of Example 2 is as follows:

$$\max.\ f(x_1, x_2) = 21.5 + x_1 \sin(4\pi x_1) + x_2 \sin(20\pi x_2);$$
$$\text{subject to} \quad \text{i) } -3 \le x_1 \le 12.1;$$
$$\text{ii) } 4.1 \le x_2 \le 5.8.$$

A genetic algorithm was used to determine the optimal solution of Example 2. We started with the following initial points: PS (75), MG (67), SR (0.5), and MR (0.5). The permissible distances were as follows: PS (25), SR (0.1), and MR (0.1). CR was fixed at 100% and the products of PS and MG were fixed at approximately 5,000. The proposed method obtained the following NT optimal parameter settings after 9 iterations: PS (225), MG (22), SR (0.2), and MR (0.6). We then ran the algorithm fifteen times using the NT optimal parameter settings as well as other parameter settings. The data in Table 4 show that the NT optimal parameters enhanced the efficiency and robustness of the algorithm. In addition, the results obtained using the NT optimal parameter settings were superior to those obtained using the larger product of the values of PS (100) and MG (100), which resulted in a longer runtime.

Table 4. Statistics for each of the fifteen observations after running the genetic algorithm with NT optimal parameter settings as well as other parameter settings

| **Parameter settings** | | | | **Statistics of 15 observations** | | | | |
|---|---|---|---|---|---|---|---|---|
| **PS** | **MG** | **SR** | **MR** | **Average** | **Standard deviation** | **Max** | **Min** | **Note** |
| 225 | 22 | 0.2 | 0.6 | 38.74336933 | 0.111274339 | 38.84788 | 38.45883 | The NT optimal |
| 100 | 50 | 0.1 | 0.9 | 38.66122133 | 0.171744523 | 38.84418 | 38.19388 | |
| 100 | 50 | 0.5 | 0.5 | 38.47978333 | 0.422183827 | 38.8338 | 37.28982 | |
| 100 | 50 | 0.9 | 0.1 | 37.73982333 | 0.805082343 | 38.7396 | 36.3143 | |
| 200 | 25 | 0.1 | 0.9 | 38.66834733 | 0.170036202 | 38.82777 | 35.93082311 | |
| 200 | 25 | 0.5 | 0.5 | 38.53962933 | 0.258780486 | 38.81623 | 35.81610927 | |
| 200 | 25 | 0.9 | 0.1 | 37.884068 | 0.462452011 | 38.62415 | 35.20625214 | |
| 300 | 17 | 0.1 | 0.9 | 38.728376 | 0.080760479 | 38.8467 | 35.98797618 | |
| 300 | 17 | 0.5 | 0.5 | 38.50215933 | 0.323958726 | 38.8309 | 35.85158415 | |
| 300 | 17 | 0.9 | 0.1 | 38.24645333 | 0.300650301 | 38.77262 | 35.55911312 | |
| 100 | 100 | 0.1 | 0.9 | 38.69361133 | 0.094649554 | 38.84861 | 38.54477 | |
| 100 | 100 | 0.5 | 0.5 | 38.58577867 | 0.223413075 | 38.84736 | 37.99703 | |
| 100 | 100 | 0.9 | 0.1 | 37.78635067 | 0.53904741 | 38.59559 | 36.45794 | |

### 4.3 Numerical results of THSRTP

In this section, the proposed method was applied to the THSRTP and the results were compared with those obtained using the conventional Taguchi method, the response surface method, and the exhaustive method proposed by Yu and Li (2009). We then conducted an experiment to confirm our results and check for consistency.

PSMG, MR, and SR were the three factors in Step 1 of the proposed method with CR fixed at 100%. The three initial levels are presented in Table 5.

Table 5. Selected factors and their initial levels

| Factors | Level -1 | Level 0 | Level 1 |
|---|---|---|---|
| PSMG(=PS*MG) | 25＊4000 | 50＊2000 | 75＊1334 |
| MR | 0.4 | 0.5 | 0.6 |
| SR | 40% | 50% | 60% |

The product of the values of PS and MG in all three levels was 100,000. The permissible distance was 25 for PSMG between two population sizes, 0.1 for MR, and 10% for SR. As shown in Figure 8, the northbound/southbound problem was halted after 9/10 iterations, resulting in the following settings: PS (20), MG (5000), MR (0.005), and SR (10%). This demonstrates that the proposed method with PSMG resulted in the same NT optimal parameter settings for each of the two similar problems. Figure 9 shows the improvement in the S/N ratios with each iteration.

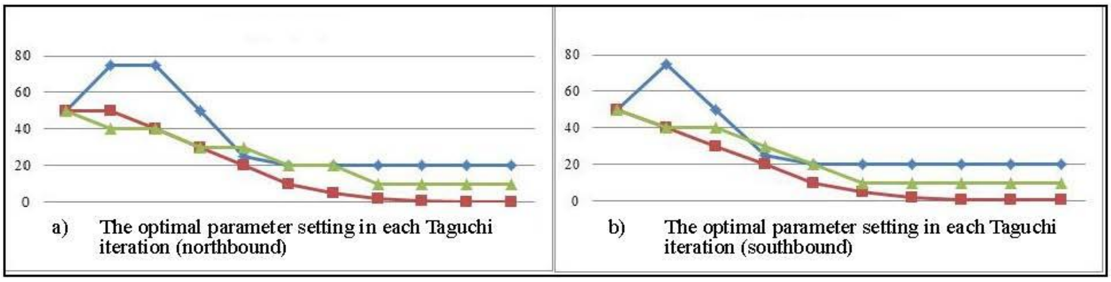



Figure 8. Optimal levels identified by the proposed method for the three parameters PS (diamonds), MR (squares), and SR (triangles) to solve a) the northbound problem and b) the southbound problem

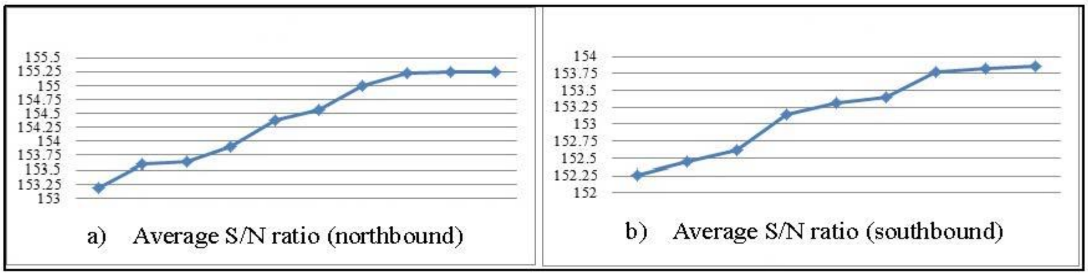



Figure 9. S/N ratios in each iteration: a) northbound problem; b) southbound problem

The effects of the NT optimal parameter settings were compared with those obtained using the conventional Taguchi method with factor PSMG (using results in Section 3.1) and the response surface method (to design the parameter settings for the THRTP). Revenue values in Tables 6 and 7 were averaged from the results obtained after running the genetic algorithm ten times using the selected parameter settings. For both subproblems, the average revenue obtained using the NT optimal parameter settings is higher than that obtained using the other methods. As shown in Tables 6 and 7, the runtimes are the same as long as the products of the PS and MG values are equal.

Table 6. Effects of parameter settings obtained from the three methods for northbound problem

| Methods | PS | MG | MR | SR | Average revenue | Average Runtime |
|---|---|---|---|---|---|---|
| Taguchi method | 50 | 2000 | 0.2 | 20% | 52,524,165 | 5hours |
| Response surface method | 20 | 5000 | 0.04 | 10% | 59,401,789 | 5hours |
| Proposed method | 20 | 5000 | 0.005 | 10% | 59,813,924 | 5hours |

We then compared our results with those of Yu and Li (2009), in which a variety of parameter combinations was evaluated. The results are presented in Table 8, and show that the NT optimal parameter settings significantly improved upon the results obtained in the original study for the northbound problem. The results for the southbound problem are nearly equal.

The 95% confidence interval for the NT optimal parameter settings is [155.23, 155.83] for the northbound and [154.14, 154.36] for the southbound problem. Experiments were conducted three times to confirm the results obtained for both of

Table 7. Effects of parameter settings obtained from the three methods for southbound problem

| Methods | PS | MG | MR | SR | Average revenue | Average Runtime |
|---|---|---|---|---|---|---|
| Taguchi method | 50 | 2000 | 0.2 | 20% | 44,623,679 | 8hours |
| Response surface method | 20 | 5000 | 0.1 | 10% | 50,052,425 | 8hours |
| Proposed method | 20 | 5000 | 0.005 | 10% | 51,547,170 | 8hours |

Table 8. Effects of NT optimal parameter settings and parameter settings of Yu and Li (2009)

| Methods | Northbound problem | | Southbound problem | |
|---|---|---|---|---|
| | Best revenue | Average revenue | Best revenue | Average revenue |
| Exhaustive method (Yu and Li, 2009) | 58,626,301 | 58,144,641 | 52,161,178 | 51,977,269 |
| Proposed method | 60,927,919 | 59,813,924 | 52,268,905 | 51,547,170 |

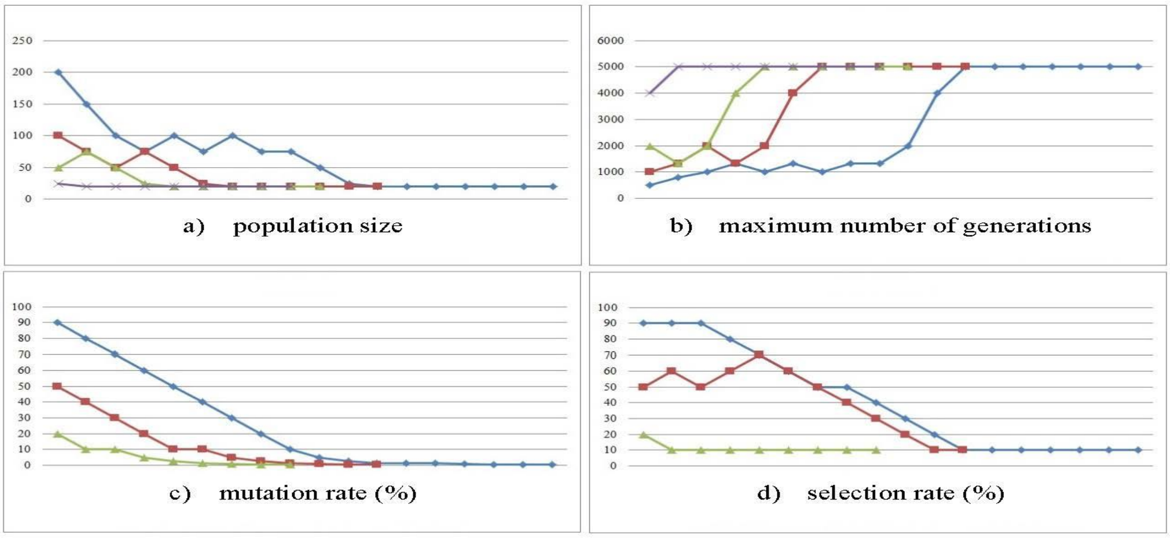



Figure 10. Convergence conditions of the proposed method using different initial points

the subproblems. The resulting average S/N ratio of the observations in the three experiments was 155.58 for the northbound problem and 154.22 for the southbound problem, both of which fall within their corresponding confidence interval, indicating the robustness and reliability of the NT optimal parameter settings.

We then tested the consistency of the results obtained using the proposed method by starting iterations from different initial points. We randomly selected four different sets of initial points for PS, MG, MR, SR: (200, 500, 0.9, 90%), (100, 1000, 50%, 50%), (50, 2000, 50%, 50%) and (25, 4000, 20%, 20%). As shown in Figure 10, all four sets converged to the same values: (20, 5000, 0.005, 10%).

## 5. CONCLUSION

This paper presents a robust method for the design of parameters used in genetic algorithms, in which the Taguchi method is augmented with a simple optimal-point-search strategy in either continuous or discrete areas. We also propose an innovative combination of PS and MG into a single factor to enhance the search performance of the Taguchi method. The numerical results from two well-known test examples demonstrate the effectiveness of the proposed method. The complex THSRTP also showed that the NT optimal parameter settings obtained using the proposed method are more reliable and more effective than those obtained using the two popular methods, the Taguchi method and the response surface method. The NT optimal parameter settings enabled the genetic algorithm to optimize revenue with superior results to those featured in the paper that originally proposed the THSRTP.

It is worth noting that the proposed method is applicable not only to genetic algorithms but also other evolutionary algorithms that deal with continuous and/or discrete parameters. This paper makes a significant contribution to the development of evolutionary algorithms and the field of parameter design.

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## APPENDIX

Table A1. The L9 experimental array. There are four factors, A, B, C and D, in this experiment and each factor has three experimental levels, -1,0 and 1.

| No. of experiments | Factors | | | |
|---|---|---|---|---|
| | A | B | C | D |
| E1 | -1 | -1 | -1 | -1 |
| E2 | -1 | 0 | 0 | 0 |
| E3 | -1 | 1 | 1 | 1 |
| E4 | 0 | -1 | 0 | 1 |
| E5 | 0 | 0 | 1 | -1 |
| E6 | 0 | 1 | -1 | 0 |
| E7 | 1 | -1 | 1 | 0 |
| E8 | 1 | 0 | -1 | 1 |
| E9 | 1 | 1 | 0 | -1 |

Table A2. The Taguchi experimental result for the southbound problem in the first run (without PSMG)

| The southbound problem | | | | | | |
|---|---|---|---|---|---|---|
| No of Experiments | Observations | | | Average of observations | Variance of observations | S/N ratio |
| | ob1 | ob2 | ob3 | | | |
| E1 | 39680499 | 37892701 | 39657721 | 39076974 | 1025673 | 151.83 |
| E2 | 37699541 | 36627222 | 35854895 | 36727219 | 926380 | 151.29 |
| E3 | 35840264 | 36182632 | 34520266 | 35514387 | 877788 | 151.00 |
| E4 | 36741612 | 39720469 | 36996292 | 37819458 | 1651242 | 151.54 |
| E5 | 39541124 | 39151902 | 36836122 | 38509716 | 1462382 | 151.70 |
| E6 | 35441799 | 34970256 | 38645215 | 36352423 | 1999565 | 151.19 |
| E7 | 42670416 | 40384531 | 41792925 | 41615957 | 1153172 | 152.38 |
| E8 | 39459469 | 41139219 | 37657107 | 39418598 | 1741416 | 151.90 |
| E9 | 38016366 | 40173939 | 39624427 | 39271577 | 1121230 | 151.87 |

Table A3. The Taguchi experimental result for the northbound problem in the first run (without PSMG)

| The northbound problem | | | | | | |
|---|---|---|---|---|---|---|
| No of Experiments | Observations | | | Average of observations | Variance of observations | S/N ratio |
| | ob1 | ob2 | ob3 | | | |
| E1 | 45354974 | 44895572 | 44957682 | 45069409 | 249248 | 153.08 |
| E2 | 43144989 | 42477304 | 43352349 | 42991547 | 457257 | 152.67 |
| E3 | 43881788 | 40986334 | 42523764 | 42463962 | 1448653 | 152.55 |
| E4 | 49052047 | 49319883 | 49361342 | 49244424 | 167888 | 153.85 |
| E5 | 46222786 | 46610139 | 46096534 | 46309820 | 267635 | 153.31 |
| E6 | 46422554 | 43768872 | 44673542 | 44954989 | 1349043 | 153.05 |
| E7 | 47643498 | 48496125 | 48655641 | 48265088 | 544189 | 153.67 |
| E8 | 47071185 | 46879091 | 47417324 | 47122533 | 272766 | 153.46 |
| E9 | 45968447 | 47199862 | 46583693 | 46584001 | 615708 | 153.36 |

Table A4. The Taguchi experimental result for the southbound problem in the second run (with PSMG)

| The southbound problem | | | | | | |
|---|---|---|---|---|---|---|
| No of Experiments | Observations | | | Average of observations | Variance of observations | S/N ratio |
| | ob1 | ob2 | ob3 | | | |
| E1 | 44616251 | 43402308 | 44623679 | 44214079 | 703024.4 | 152.91 |
| E2 | 42695183 | 41230962 | 39939153 | 41288433 | 1378914 | 152.31 |
| E3 | 41161375 | 39385630 | 40640033 | 40395679 | 912742.6 | 152.12 |
| E4 | 43554659 | 43221043 | 42669807 | 43148503 | 446863.9 | 152.70 |
| E5 | 40209740 | 40910720 | 40719394 | 40613285 | 362336.4 | 152.17 |
| E6 | 40599841 | 40334194 | 40891915 | 40608650 | 278964.8 | 152.17 |
| E7 | 41436280 | 39419583 | 42702714 | 41186192 | 1655791 | 152.28 |
| E8 | 40085743 | 38222693 | 41542886 | 39950441 | 1664227 | 152.02 |
| E9 | 39450605 | 38960498 | 39585521 | 39332208 | 328902.5 | 151.89 |

Table A5. The Taguchi experimental result for the northbound problem in the second run (with PSMG)

| The northbound problem | | | | | | |
|---|---|---|---|---|---|---|
| No of Experiments | Observations | | | Average of observations | Variance of observations | S/N ratio |
| | ob1 | ob2 | ob3 | | | |
| E1 | 52519176 | 52524165 | 52127744 | 52390362 | 227447.3 | 154.38 |
| E2 | 46092171 | 48855954 | 50412142 | 48453422 | 2187935 | 153.69 |
| E3 | 46401261 | 45624929 | 46269989 | 46098726 | 415537 | 153.27 |
| E4 | 50065495 | 49055525 | 49341413 | 49487478 | 520587.2 | 153.89 |
| E5 | 48178480 | 45822550 | 45753996 | 46585009 | 1380412 | 153.36 |
| E6 | 47973345 | 46167851 | 47198357 | 47113184 | 905755.4 | 153.46 |
| E7 | 48339735 | 47037324 | 44802773 | 46726611 | 1788835 | 153.38 |
| E8 | 46233312 | 46102286 | 47518894 | 46618164 | 782801.3 | 153.37 |
| E9 | 46845563 | 47112543 | 47282428 | 47080178 | 220223.5 | 153.46 |