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Two-Heterogeneous Server Markovian Queueing Model with Discouraged Arrivals, Reneging and Retention of Reneged Customers

Rakesh Kumar[1]* and Sumeet Kumar Sharma[2]

[1]Assistant Professor, School of Mathematics, Shri Mata Vaishno Devi University, Katra, Sub-Post Office, Katra, J&K, India 182320

[2] Research Scholar, School of Mathematics, Shri Mata Vaishno Devi University, Sub-Post Office, Katra, J&K, India 182320

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Abstract — Customer impatience (reneging) has negative impact on the performance of queueing systems. If we talk from business point of view, firms lose their potential customers due to customer impatience which affects their business as a whole. It is envisaged that if the firms employ certain customer retention strategies then there are chances that a certain fraction of impatient customers can be retained in the queueing system for their further service. Recently, Kumar and Sharma (2012c) study a finite capacity multi-server Markovian queueing system with discouraged arrivals, reneging and retention of reneged customers. They consider homogeneous servers. In real life, the servers that are not mechanically controlled may not possess same service rates. Thus, it is more appropriate to consider heterogeneous servers. In this paper, we extend the work of Kumar and Sharma (2012c) by considering heterogeneous servers. We consider a two-heterogeneous servers' finite capacity Markovian queueing system with discouraged arrivals, reneging and retention of reneged customers. The steady-state probabilities of system size are derived explicitly by using iterative method. Some useful measures of effectiveness are also derived and discussed. Finally, some important queueing models are derived as the special cases of this model.

Keywords — Queueing system, Reneging, Discouraged arrivals, Steady-state solution, Retention of reneged customers

1. INTRODUCTION

Queueing theory plays an important role in modelling real life problems involving congestions in wide areas of science, technology and management. Applications of queueing with customer impatience can be seen in traffic modelling, business and industries, computer-communication, health sector and medical science etc. Service stations that are not mechanically controlled like checkout counters, grocery stores, banks etc. have heterogeneous service rates because one cannot expect human servers to work at constant rate. Queueing modellers consider this aspect and study queueing systems with heterogeneous servers following pioneering work of Morse (1958) who considers the situation of some hyper-exponential distributions for service times with parallel service channels. Saaty (1960) expresses the concept of heterogeneous servers in finding the time dependent solution of multi-server Markovian queue. Krishanamoorthy (1963) considers a Poisson queue with two-heterogeneous servers with modified queue disciplines. The steady-state solution, transient solution and busy period distribution for the first discipline and the steady-state solution for the second discipline are obtained. Singh (1970) extends the work of Krishanamoorthy (1963) on heterogeneous servers by incorporating balking to compares results with homogeneous server queue to show the conditions under which the heterogeneous system is better than the corresponding homogeneous system. Sharma and Dass (1989) analyze M/M/2/N queueing system with heterogeneous servers to derive the probability density function of the busy period. The mean and variance of the Queueing system is also found. Moreover, the time-dependent solution of a limited space double channel Markovian Queueing model with heterogeneous servers using matrix method is found by Sharma and Dass (1990).

Queues with discouraged arrivals have applications in computers with batch job processing where job submissions are discouraged when the system is used frequently and arrivals are modelled as a Poisson process with state dependent arrival rate. The discouragement affects the arrival rate of the queueing system. Queueing models where potential customers are discouraged by queue length are studied by many researchers in their research work. Natvig (1975) studies the single server birth-death queueing process with state dependent parameters $\lambda_n = \frac{\lambda}{n+1}, n \geq 0 \; and \; \mu_n = \mu, n \geq 1.$ Raynolds (1968) studies multi-server queueing model with discouragement. He obtains equilibrium distribution of queue length and derives

* Corresponding author's email: rakesh_stat_kuk@yahoo.co.in

other performance measures. Courtois and Georges (1971) study finite capacity M/G/1 queueing model where the arrival and the service rates are arbitrary functions of the number of customers in the system. They obtain results for expected value of time needed to complete a service including waiting time distribution. Von Doorn (1981) obtains exact expressions for transient state probabilities of the birth death process with parameters $\lambda_n = \frac{1}{n+1}\lambda, n \geq 0$ and $\mu_n = \mu, n \geq 1$. Ammar et al. (2012) study single server, finite capacity Markovian queue with discouraged arrivals and reneging and obtain the transient solution of the model by using matrix method.

The notion of customer impatience appears in queueing theory in last few decades. The pioneer work on reneging and balking in single-server Markovian queueing systems is carried out by Haight (1957, 1959) and Ancker and Gafarian (1963a, 1963b). El-Paoumy (2008) considers a queueing system with batch arrival, balking, reneging and two heterogeneous servers. El-Paoumy and Nabwey (2011) study a Poisson queue with balking function, reneging and two heterogeneous servers. El-Sherbiny (2012) incorporates balking and general balking function in a Markovian queueing system with reneging and two heterogeneous servers and derives steady-state solution using iterative method. Customer impatience leads to the loss of potential customers which affects the business as a whole. Kumar and Sharma (2012a, 2012b) envisage that a reneged customer may be retained for his further service by employing certain convincing mechanisms and study the retention of reneged customers. Kumar and Sharma (2013) study an M/M/c/N queueing model with reneging and retention of reneged customers. Kumar (2013) obtains the transient solution of an M/M/c/N queueing model with balking, reneging and retention of reneged customers. He also performs the economic analysis of the model.

Recently, Kumar and Sharma (2012c) study a finite capacity multi-server Markovian Queueing system with discouraged arrivals, reneging and retention of reneged customers. They consider homogeneous servers. In real life, the servers that are not mechanically controlled may not possess same service rates. Thus, it is more appropriate to consider heterogeneous servers. In this paper, we extend the work of Kumar and Sharma (2012c) by considering heterogeneous servers. We consider a two-heterogeneous servers' finite capacity Markovian Queueing system with discouraged arrivals, reneging and retention of reneged customers. We derive the steady-state probabilities of system size explicitly by using iterative method.

Rest of the paper is structured as follows: In section 2, we describe the queueing model. The differential-difference equations are derived and solved iteratively in section 3. Measures of effectiveness are derived in section 4. The special cases of the model are derived in section 5. The conclusions and future work are provided in section 6.

2. QUEUEING MODEL DESCRIPTION

The customers arrive to the Queueing system according to a Poisson process with parameter λ. There are two servers and service times are exponentially distributed with parameters μ_1 and μ_2 at server 1 and server 2 respectively ($\mu_1 > \mu_2$). A arriving unit finding both the servers busy waits in the queue in order of arrival and the unit in front of the queue occupies the server that falls vacant first. When only one of the servers is free, the arriving unit occupies the free server. When both the channels are free, an arriving unit chooses fast server with probability π_1 and slow server with probability π_2, such that $\pi_1 + \pi_2 = 1$. A unit finding every server busy arrives with arrival rate that depends on the number of customers present in the system at that time i.e. if there are $n,\ n > 2$ customers in the system, the customer enters the system with rate $\frac{\lambda}{(n-2)+1}$. The capacity of the system is finite (say, N). That is, the system can accommodate at most N customers. A queue gets developed when the number of customers exceeds the number of servers, that is, when $n > 2$. Each customer upon joining the queue will wait a certain length of time for his service to begin. If it has not begun by then, he will get impatient (reneged) and may leave the queue without getting service with probability p and may remain in the queue for his service with probability $q\ (= 1 - p)$. The reneging times follow exponential distribution with parameter ξ.

Define,

- $P_n(t), n = 0,1,23\ldots N$ be the probability that there are n unit in the system at time t.
- $P_{10}(\mathrm{t})$ be the probability that first server is engaged and second server is free with no waiting line at time t.
- $P_{01}(\mathrm{t})$ be the probability that first server is free and second server is engaged, there is no waiting line at time t.
- $P_{00}(\mathrm{t})$ be the probability that there are no units in the system at time t.
- Also, $P_2(t) = P_{11}\ (t)$ and $P_1(t) = P_{10}(t) + P_{01}(t)$.

3. DIFFERENTIAL-DIFFERENCE EQUATIONS AND SOLUTION OF THE QUEUEING MODEL

In this section, the mathematical framework of the queueing model is presented. Let $P_n(t)$ be the probability that there are n customers in the system at time t. The differential-difference equations are derived by using the general birth-death arguments. These equations are solved iteratively in steady-state in order to obtain the steady-state solution.

The differential-difference equations of the model are:

$$\frac{dP_{00}(t)}{dt} = -\lambda P_{00}(t) + \mu_1 P_{10}(t) + \mu_2 P_{01}(t) \tag{1}$$

$$\frac{dP_{10}(t)}{dt} = -(\lambda + \mu_1)P_{10}(t) + \mu_2 P_{11}(t) + \lambda\pi_1 P_{00}(t) \tag{2}$$

$$\frac{dP_{01}(t)}{dt} = -(\lambda + \mu_2)P_{01}(t) + \mu_1 P_{11}(t) + \lambda\pi_2 P_{00}(t) \tag{3}$$

$$\frac{dP_2(t)}{dt} = -(\lambda + \mu_1 + \mu_2)P_2(t) + (\mu_1 + \mu_2 + \xi p)P_3(t) + \lambda P_1(t) \tag{4}$$

$$\frac{dP_n(t)}{dt} = -\left[\left(\frac{\lambda}{(n-2)+1}\right) + \mu_1 + \mu_2 + (n-2)\xi p\right] P_n(t) + [\mu_1 + \mu_2 + \{(n+1) - 2\}\xi p]P_{n+1}(t) + \left(\frac{\lambda}{(n-3)+1}\right) P_{n-1}(t);\ 3 \le n \le N-1 \tag{5}$$

$$\frac{dP_N(t)}{dt} = \left(\frac{\lambda}{(N-2)+1}\right) P_{N-1}(t) - \left\{\mu_1 + \mu_2 + (N-2)\xi p\right\} P_N(t) \tag{6}$$

In steady-state, $\text{Lim}_{n\to\infty} P_n(t) = P_n$ .Therefore, the steady-state equations corresponding to equations (1) - (6) are as follows:

$$0 = -\lambda P_{00} + \mu_1 P_{10} + \mu_2 P_{01} \tag{7}$$

$$0 = -(\lambda + \mu_1)P_{10} + \mu_2 P_{11} + \lambda\pi_1 P_{00} \tag{8}$$

$$0 = -(\lambda + \mu_2)P_{01} + \mu_1 P_{11} + \lambda\pi_2 P_{00} \tag{9}$$

$$0 = -(\lambda + \mu_1 + \mu_2)P_2 + (\mu_1 + \mu_2 + \xi p)P_3 + \lambda P_1 \tag{10}$$

$$0 = -\left[\left(\frac{\lambda}{(n-2)+1}\right) + \mu_1 + \mu_2 + (n-2)\xi p\right] P_n + [\mu_1 + \mu_2 + \{(n+1) - 2\}\xi p]P_{n+1} + \left(\frac{\lambda}{(n-3)+1}\right) P_{n-1};\ 3 \le n \le N-1 \tag{11}$$

$$0 = \left(\frac{\lambda}{(N-2)+1}\right) P_{N-1} - \left\{\mu_1 + \mu_2 + (N-2)\xi p\right\} P_N \tag{12}$$

On solving equations (7)-(9), we have

$$P_{10} = \frac{\lambda + (\mu_1+\mu_2)\pi_1}{2\lambda + \mu_1 + \mu_2} \frac{\lambda}{\mu_1} P_{00} \tag{13}$$

$$P_{01} = \frac{\lambda + (\mu_1+\mu_2)\pi_2}{2\lambda + \mu_1 + \mu_2} \frac{\lambda}{\mu_2} P_{00} \tag{14}$$

Adding (13) and (14), we get

$$P_1 = \frac{\lambda + \mu_1\pi_2 + \mu_2\pi_1}{2\lambda + \mu_1 + \mu_2} \frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2} P_{00} \tag{15}$$

Solving recursively equations (10)-(12), we get

$$P_n = \frac{1}{((n-2)+1)!} \prod_{k=2}^{n} \frac{\lambda}{[\mu_1 + \mu_2 + (k-2)\xi p]} \frac{\lambda + \mu_1\pi_2 + \mu_2\pi_1}{2\lambda + \mu_1 + \mu_2} \frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2} P_{00}\ ; 3 \le n \le N-1 \tag{16}$$

Using the normalization condition, $\sum_{n=0}^{N} P_n = 1$,we get

$$P_{00} = \left\{1 + \frac{\lambda + \mu_1\pi_2 + \mu_2\pi_1}{2\lambda + \mu_1 + \mu_2} \frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2} + \sum_{n=2}^{N} \frac{1}{((n-2)+1)!} \prod_{k=2}^{n} \frac{\lambda}{[\mu_1 + \mu_2 + (k-2)\xi p]}\right\}^{-1} \tag{17}$$

Hence, the steady-state probabilities of the system size are derived explicitly.

4. MEASURES OF EFFECTIVENESS

In this section, some important measures of effectiveness are derived. These can be used to study the performance of the queueing system under consideration.

The Expected System Size (Ls):

The expected number of customers in the system is given as:

$$L_s = \frac{\lambda + \mu_1\pi_2 + \mu_2\pi_1}{2\lambda + \mu_1 + \mu_2} \frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2} P_{00} + \sum_{n=2}^{N} \frac{1}{((n-2)+1)!} \prod_{k=2}^{n} \frac{\lambda}{[\mu_1 + \mu_2 + (k-2)\xi p]} \frac{\lambda + \mu_1\pi_2 + \mu_2\pi_1}{2\lambda + \mu_1 + \mu_2} \frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2} P_{00}$$

The Expected Number of Customers Served, $\mathbf{E(C.S.)}$:

The expected number of customers served is given by:

$$E(C.S.) = \mu_1 \left\{\frac{\lambda + (\mu_1+\mu_2)\pi_1}{2\lambda + \mu_1 + \mu_2} \frac{\lambda}{\mu_1}\right\} P_{00} + \mu_2 \left\{\frac{\lambda + (\mu_1+\mu_2)\pi_2}{2\lambda + \mu_1 + \mu_2} \frac{\lambda}{\mu_2}\right\} P_{00} + (\mu_1 + \mu_2) \sum_{n=2}^{N} \frac{1}{((n-2)+1)!} \prod_{k=2}^{n} \frac{\lambda}{[\mu_1 + \mu_2 + (k-2)\xi p]} \frac{\lambda + \mu_1\pi_2 + \mu_2\pi_1}{2\lambda + \mu_1 + \mu_2} \frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2} P_{00}$$

Rate of Abandonment, $\mathbf{R_{aband}}$:

The average rate at which the customers abandon the system is given by:

$$R_{aband} = \lambda - \mu_1 \left\{\frac{\lambda + (\mu_1+\mu_2)\pi_1}{2\lambda + \mu_1 + \mu_2} \frac{\lambda}{\mu_1}\right\} P_{00} - \mu_2 \left\{\frac{\lambda + (\mu_1+\mu_2)\pi_2}{2\lambda + \mu_1 + \mu_2} \frac{\lambda}{\mu_2}\right\} P_{00} - (\mu_1 + \mu_2) \sum_{n=2}^{N} \frac{1}{((n-2)+1)!} \prod_{k=2}^{n} \frac{\lambda}{[\mu_1 + \mu_2 + (k-2)\xi p]} \frac{\lambda + \mu_1\pi_2 + \mu_2\pi_1}{2\lambda + \mu_1 + \mu_2} \frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2} P_{00}$$

Expected Number of Customers who Actually Wait, $E(\text{Actual Cust. Waiting})$:
The Expected number of waiting customers, who actually wait is given by:

$$E(\text{Actual Cust. Waiting}) = \frac{\sum_{n=3}^{N}(n-2)P_n}{\sum_{n=3}^{N}P_n} = \frac{\sum_{n=3}^{N}(n-2)\left[\frac{1}{(n-2+1)!}\prod_{k=2}^{n}\frac{\lambda}{[\mu_1+\mu_2+(k-2)\xi p]}\frac{\lambda+\mu_1\pi_2+\mu_2\pi_1}{2\lambda+\mu_1+\mu_2}\frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2}\right]P_{00}}{\sum_{n=3}^{N}\left[\frac{1}{(n-2+1)!}\prod_{k=2}^{n}\frac{\lambda}{[\mu_1+\mu_2+(k-2)\xi p]}\frac{\lambda+\mu_1\pi_2+\mu_2\pi_1}{2\lambda+\mu_1+\mu_2}\frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2}\right]P_{00}}$$

Average Reneging Rate (Rr)

$$R_r = \sum_{n=2}^{N}\xi p\frac{1}{((n-2)+1)!}\prod_{k=2}^{n}\frac{\lambda}{[\mu_1+\mu_2+(k-2)\xi p]}\frac{\lambda+\mu_1\pi_2+\mu_2\pi_1}{2\lambda+\mu_1+\mu_2}\frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2}P_{00}$$

Average Retention Rate (R_R)

$$R_R = \sum_{n=2}^{N}\xi q\frac{1}{((n-2)+1)!}\prod_{k=2}^{n}\frac{\lambda}{[\mu_1+\mu_2+(k-2)\xi p]}\frac{\lambda+\mu_1\pi_2+\mu_2\pi_1}{2\lambda+\mu_1+\mu_2}\frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2}P_{00}$$

where P_n and P_{00} are given by equations (16) and (17) respectively.

5. SPECIAL CASES

In this section, we derive and discuss some important special cases of the model.

5.1. When there is no customer discouragement

The model reduces to an M/M/2/N queueing system with two-heterogeneous server, reneging and retention of reneged customers with

$$P_1 = \frac{\lambda+\mu_1\pi_2+\mu_2\pi_1}{2\lambda+\mu_1+\mu_2}\frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2}P_{00}$$

$$P_n = \prod_{k=2}^{n}\frac{\lambda}{[\mu_1+\mu_2+(k-2)\xi p]}\frac{\lambda+\mu_1\pi_2+\mu_2\pi_1}{2\lambda+\mu_1+\mu_2}\frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2}P_{00}\,,\ 2\le n\le N$$

and

$$P_{00} = \left\{1+\frac{\lambda+\mu_1\pi_2+\mu_2\pi_1}{2\lambda+\mu_1+\mu_2}\frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2}+\sum_{n=2}^{N}\prod_{k=2}^{n}\frac{\lambda}{[\mu_1+\mu_2+(k-2)\xi p]}\right\}^{-1}.$$

5.2. When there is no customer retention (i.e. q=0).

In this case, the queueing model gets reduced to an M/M/2/N queueing model with two-heterogeneous server, discouraged arrivals and reneging with

$$P_1 = \frac{\lambda+\mu_1\pi_2+\mu_2\pi_1}{2\lambda+\mu_1+\mu_2}\frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2}P_{00}$$

$$P_n = \frac{1}{((n-2)+1)!}\prod_{k=2}^{n}\frac{\lambda}{[\mu_1+\mu_2+(k-2)\xi]}\frac{\lambda+\mu_1\pi_2+\mu_2\pi_1}{2\lambda+\mu_1+\mu_2}\frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2}P_{00}\,;\ 3\le n\le N-1$$

and

$$P_{00} = \left\{1+\frac{\lambda+\mu_1\pi_2+\mu_2\pi_1}{2\lambda+\mu_1+\mu_2}\frac{\lambda(\mu_1+\mu_2)}{\mu_1\mu_2}+\sum_{n=2}^{N}\frac{1}{((n-2)+1)!}\prod_{k=2}^{n}\frac{\lambda}{[\mu_1+\mu_2+(k-2)\xi]}\right\}^{-1}.$$

5.3. When $\mu_1=\mu_2$ and $\pi_1=\pi_2=\frac{1}{2}$:

The resulting model is an M/M/2/N queueing model with discouraged arrivals, reneging and retention of reneged customers which is a special case of model studied by Kumar and Sharma (2012c).

6. CONCLUSIONS AND FUTURE WORK

This paper studies a two-heterogeneous servers' Markovian queueing model with discouraged arrivals, reneging and retention of reneged customers. The stationary probabilities of the system size are derived explicitly. Finally, some significant queueing models are derived as special cases of this model. The queueing model studied in this paper finds its applications in banking industry, manufacturing, and in hotel management.

The model analysis is limited to finite capacity. The infinite capacity case of the model can also be studied. Further, the model can be solved in transient state to get time-dependent results. The same idea can be extended to some non-Markovian queueing models.

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