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Multi-Choice Goal Programming with Trapezoidal Fuzzy Numbers

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Abstract **—**This manuscript aims at studying a multi objective multi-choice goal programming with symmetric trapezoidal fuzzy numbers. This situation arises in many practical situations where the decision maker likes to set multi aspiration levels for objective functions and the decision variables are fuzzy in nature. The problem is solved by converting it into the standard MODM problem by Mehar's method and MCGP method. Two numerical examples are presented for illustrating the proposed problem.

Keywords **—** Goal programming, Multi-choice aspiration levels, Ranking function, Trapezoidal fuzzy numbers.

1. INTRODUCTION

The goal programming approaches, which assume that the DM can specify the goals of the objective functions, first appeared in a 1961 text by Charnes and Cooper (1961) to deal with multi objective linear programming (MOLP) problems. Subsequent works on goal programming approaches have been numerous including Lee (1972), Charnes and Cooper (1977) and Ignizio (1976, 1982).

A new area of research was originated by Healy (1964) known as a multiple choice programming problem. In these problems there is a requirement to choose, among several possible combinations as an alternative to optimize an objective function subject to a set of constraints. It is a mixed binary programming in which all binary variables constitute a number of mutually exclusive choices where only one variable is to be selected. Multi-choice linear programming problems has been discussed by several authors such as Ravindran et al. (1987), Hiller and Liebermen (1990) etc. Chang et al. (2007, 2008) introduces multi-choice goal programming in which DM has multiple choices of aspiration levels of objective functions and recently Biswal and Acharya (2009, 2011) consider the problem where the right hand side parameter has multiple choices.

In many practical situations values of most parameters of an optimization problem are assigned by DM but in reality DM itself do not precisely know the exact values of those parameters. In such situations the problem arises where the decision parameters are not well defined or imprecise or uncertain which results in uncertain decision variables. Using fuzzy data/fuzzy numbers of uncertain parameters are proving to be helpful to deal with such situations. Fuzzy theory was initiated by Lotfi A. Zadeh in 1965 with his seminal paper "Fuzzy Sets" (Zadeh [1965]). He has done a lot of work in this field. After that much research has been taking place. Zimmermann (1978) was the first who applies fuzzy concept in linear programming. In many fields fuzzy theories have been applied such as transportation, bi-level programming, assignment problems etc. Ganesan and Veeramani (2006) solve a fuzzy linear program with trapezoidal numbers, Kumar and Kaur (2011) gave a new method called Mehar's method for the same problem and there are many more authors who work on fuzzy problems some of them are Allahviranloo et al. (2008), Kumar et al. (2011), Ezzati et al. (2013), Gupta et al. (2012, 2013a, 2013b, 2014) etc.

In the present manuscript, we consider multi objective multi-choice goal programming in which coefficients of objective functions and decision variables are assumed to be trapezoidal fuzzy numbers. Using Mehar's method and MCGP method the problem has been solved and demonstrated with the help of two numerical examples. The whole manuscript is organized as follows: section 1 gives the brief introduction. Section 2 shows the formulation and solution procedure of the problem and section 3 has been devoted to numerical examples for demonstrating the problem. Finally, in section 4 the work has been concluded.

2. FUZZY MULTI-CHOICE GOAL PROGRAMMING

2.1 Formulation of the problem

Chang proposed MCGP approach in the literature of goal programming and allows DMs to set multi-choice aspiration

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levels for each goal. Then, Tabrizi et al. (2012) shows that these aspiration levels can be imprecise or fuzzy and solve the typical FMCGP problem. However, in real world situations most of the parameters as well as decision variables can be imprecise or fuzzy due to some uncontrollable and unavoidable circumstances. Therefore, a general formulation of MCGP with fuzzy numbers is as follows:

$$\left.\begin{array}{l} Minimize\ \tilde{f}_i(X) \approx w_i \left| \sum_{i=1}^{n} \tilde{c}_i \otimes \tilde{x}_i - g_{i1}\ or\ g_{i2}\ or \ldots g_{im} \right| \\ subject\ to \sum_{i=1}^{n} a_{ij} \tilde{x}_i \preceq, \approx, \succeq b_j,\ j = 1,2,\ldots,m \\ \qquad \tilde{x}_i \succeq \tilde{0} \end{array}\right\} \quad (1)$$

where $w_i, i = 1,2,\ldots,n$ is the relative importance of objective function and $\tilde{x}_i = (x_i, y_i \alpha_i, \alpha_i), \tilde{c}_i = (p_i, q_i, \beta_i, \beta_i)$ are symmetric trapezoidal fuzzy numbers†.

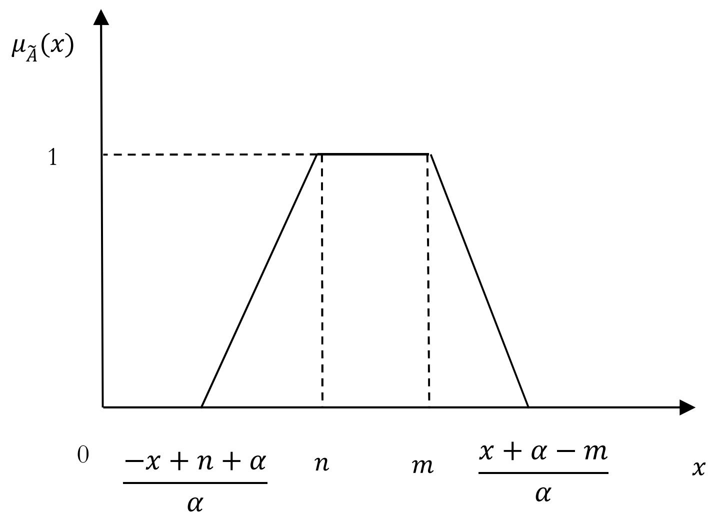

Figure 1: Trapezoidal fuzzy membership function‡

2.1.1 Fuzzy optimal solution of the problem

A set of symmetric trapezoidal fuzzy numbers $\{\tilde{x}_i\}$ is said to be fuzzy optimal solution of (1) if the following properties are satisfied: (defined by Kumar *et al.*, 2011)

- $\sum_{i=1}^{n} a_{ij} \tilde{x}_i \preceq, \approx, \succeq b_j,\ j = 1,2,\ldots,m$
- $\tilde{x}_i \succeq \tilde{0} \quad \forall i$

† A fuzzy set $\tilde{A} = (m, n, \alpha, \alpha)$ on real numbers $\mathbb{R}$ is said to be a symmetric trapezoidal fuzzy number if there exist real numbers $m, n, m \le n$ and $\alpha > 0$ such that (also depicted in figure 1)

$$\mu_{\tilde{A}(x)} = \begin{cases} \frac{x+\alpha-m}{\alpha}, & x \in [m-\alpha, m] \\ 1, & x \in [m, n] \\ \frac{-x+n+\alpha}{\alpha}, & x \in [n, n+\alpha] \\ 0, & \text{otherwise} \end{cases}$$

‡ This is a footnote image.

If there exist any set of symmetric trapezoidal fuzzy numbers $\{\tilde{y}_i\}$ such that $\sum_{i=1}^{n} a_{ij}\tilde{x}_i \preceq, \approx, \succeq b_j,\ j=1,2,\ldots,m$ and $\tilde{x}_i \succeq \tilde{0}\ \ \forall i$, then $\sum_{i=1}^{n} \tilde{c}_i \otimes \tilde{x}_i \succeq \sum_{i=1}^{n} \tilde{c}_i \otimes \tilde{y}_i$ (in case of maximization problem) and $\sum_{i=1}^{n} \tilde{c}_i \otimes \tilde{x}_i \preceq \sum_{i=1}^{n} \tilde{c}_i \otimes \tilde{y}_i$ (in case of minimization problem).

2.2 Solution procedure of the problem

Now to solve the problem (1), first the symmetric trapezoidal fuzzy numbers converted into crisp numbers using the ranking function§ discussed by Kumar and Gaur (2011) as follows:
By putting the values of $\tilde{x}_i = (x_i, y_i, \alpha_i, \alpha_i)$ and $\tilde{c}_i = (p_i, q_i, \beta_i, \beta_i)$ in problem (1), we get:

$$\left.\begin{aligned}
&\textit{Minimize}\ \tilde{f}_i(X) \approx w_i\left|\sum_{i=1}^{n}(p_i,q_i,\beta_i,\beta_i)\otimes(x_i,y_i,\alpha_i,\alpha_i) - g_{i1}\ or\ g_{i2}\ or \ldots g_{im}\right| \\
&\textit{subject to} \sum_{i=1}^{n} a_{ij}(x_i,y_i,\alpha_i,\alpha_i) \preceq,\approx,\succeq b_j,\ j=1,2,\ldots,m \\
&\qquad (x_i,y_i,\alpha_i,\alpha_i) \succeq \tilde{0}
\end{aligned}\right\} \tag{2}$$

Now using the ranking function problem (2) can be written as:

$$\left.\begin{aligned}
&\textit{Minimize}\ \Re(\tilde{f}_i(X)) = w_i\left|\sum_{i=1}^{n}\Re\{(p_i,q_i,\beta_i,\beta_i)\}\Re\{(x_i,y_i,\alpha_i,\alpha_i)\} - g_{i1}\ or\ g_{i2}\ or \ldots g_{im}\right| \\
&\textit{subject to} \sum_{i=1}^{n} a_{ij}\Re\{(x_i,y_i,\alpha_i,\alpha_i)\} \le,=,\ge b_j,\ j=1,2,\ldots,m \\
&\qquad \Re\{(x_i,y_i,\alpha_i,\alpha_i)\} \ge 0\ \ \forall i \\
&\qquad y_i - x_i \ge 0,\ \ \alpha_i \ge 0
\end{aligned}\right\} \tag{3}$$

The above problem (3) is a linear multi-choice goal programming problem and to demonstrate the MCGP figure 2 has been given below:

	One aspiration level for each goal	Two aspiration level for each goal	Multi aspiration level for each goal
A	g_1	$g_1\ g_5$	$g_1\ \ g_5$ g_7
B	g_2	$g_2\ g_4$	$g_2\ \ g_4$ g_8
C	g_3	$g_3\ g_6$	$g_3\ \ g_6$ g_9

Figure 2: Example of MCGP

§ A ranking function is a function $\Re : F(\mathbb{R}) \to \mathbb{R}$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (m, n, \alpha, \alpha)$ be a trapezoidal fuzzy number then $\Re(\tilde{A}) = \frac{m_1 + n_1}{2}$. Let $\tilde{A} = (m_1, n_1, \alpha, \alpha)$ *and* $\tilde{B} = (m_2, n_2, \beta, \beta)$ be two symmetric trapezoidal fuzzy numbers. Then

a) $\tilde{A} \succeq \tilde{B}$ *iff* $\Re(\tilde{A}) \ge \Re(\tilde{B})$
b) $\tilde{A} \succ \tilde{B}$ *iff* $\Re(\tilde{A}) > \Re(\tilde{B})$
c) $\tilde{A} \approx \tilde{B}$ *iff* $\Re(\tilde{A}) = \Re(\tilde{B})$

Now according to the above description, three cases can be considered as:

(i) Suppose one aspiration level is there for each goal i.e. from the figure there are three aspiration levels g_1, g_2 and g_3 corresponding to goals A, B and C, then this is a simple MODM problem and can be formulated using GP as follows:

$$\left.\begin{aligned}
&\textit{Minimize} \quad \sum_{i=1}^{3}(d_i^+ + d_i^-)\\
&\textit{subject to} \ \ \Re(\tilde{f}_i(X)) - d_i^+ + d_i^- = g_i,\ i = 1,2,3\\
&\qquad \sum_{i=1}^{n} a_{ij}\Re\{(x_i, y_i, \alpha_i, \alpha_i)\} \le,=,\ge b_j,\ j = 1,2,\ldots,m\\
&\qquad \Re\{(x_i, y_i, \alpha_i, \alpha_i)\} \ge 0 \ \ \forall i\\
&\qquad y_i - x_i \ge 0,\ \ \alpha_i \ge 0\\
&\qquad d_i^+, d_i^- \ge 0,\ \ i = 1,2,3
\end{aligned}\right\} \tag{4}$$

where d_i^+ and d_i^- are the over and under deviational variables respectively.

(ii) Two aspiration levels for each goal. This is a case of MODM problem with either or choice. For instance goal A targets to choose an appropriate aspiration level from either g_1 *or* g_5, goal B is to choose from either g_2 *or* g_4, while goal C targets to choose an appropriate aspiration level from either g_3 *or* g_6. This problem is formulated by using Chang's (2007) approach and three extra binary variables should be added as follows:

$$\left.\begin{aligned}
&\textit{Minimize} \quad \sum_{i=1}^{3}(d_i^+ + d_i^-)\\
&\textit{subject to} \ \ \Re(\tilde{f}_1(X)) - d_1^+ + d_1^- = z_1 g_1 + (1 - z_1) g_5\\
&\qquad \Re(\tilde{f}_2(X)) - d_2^+ + d_2^- = z_2 g_2 + (1 - z_2) g_4\\
&\qquad \Re(\tilde{f}_3(X)) - d_3^+ + d_3^- = z_3 g_3 + (1 - z_3) g_6\\
&\qquad \sum_{i=1}^{n} a_{ij}\Re\{(x_i, y_i, \alpha_i, \alpha_i)\} \le,=,\ge b_j,\ j = 1,2,\ldots,m\\
&\qquad \Re\{(x_i, y_i, \alpha_i, \alpha_i)\} \ge 0 \ \ \forall i\\
&\qquad y_i - x_i \ge 0,\ \ \alpha_i \ge 0\\
&\qquad z_i = 0/1;\ d_i^+, d_i^- \ge 0,\ \ i = 1,2,3
\end{aligned}\right\} \tag{5}$$

where $z_i, i = 1,2,3$ are the binary variables and d_i^+ and d_i^- are the over and under deviational variables respectively.

(iii) Multi aspiration levels for each goal. This is a case of MODM problem with multiple choice selection. For instance goal A targets to choose an appropriate aspiration level between g_1, g_5 and g_7, goal B is to choose between g_2, g_4 and g_8, while goal C targets to choose an appropriate aspiration level between g_3, g_6 and g_9. This problem is formulated by using Chang's (2007) approach and six extra binary variables should be added as follows:

$$\left.\begin{array}{ll} \textit{Minimize} & \sum_{i=1}^{3}(d_i^+ + d_i^-) \\ \textit{subject to} & \Re(\tilde{f}_1(X)) - d_1^+ + d_1^- = z_1z_2g_1 + z_1(1-z_2)g_5 + (1-z_1)z_2g_7 \\ & \Re(\tilde{f}_2(X)) - d_2^+ + d_2^- = z_3z_4g_2 + z_3(1-z_4)g_4 + (1-z_3)z_4g_8 \\ & \Re(\tilde{f}_3(X)) - d_3^+ + d_3^- = z_5z_6g_3 + z_5(1-z_6)g_6 + (1-z_5)z_6g_9 \\ & \sum_{i=1}^{n} a_{ij}\Re\{(x_i, y_i, \alpha_i, \alpha_i)\} \le, =, \ge b_j,\ j = 1,2,\ldots,m \\ & \Re\{(x_i, y_i, \alpha_i, \alpha_i)\} \ge 0\ \ \forall i \\ & y_i - x_i \ge 0,\ \ \alpha_i \ge 0 \\ & z_i = 0/1;\ d_i^+, d_i^- \ge 0,\ \ i = 1,2,3 \end{array}\right\} \tag{6}$$

where $z_i, i = 1,2,\ldots,6$ are the binary variables and d_i^+ and d_i^- are the over and under deviational variables respectively.

By the reference of Chang (2000), the quadratic binary terms z_1z_2, z_3z_4 and z_5z_6 can be converted into the equivalent linear form as follows:

Let $x = z_iz_j$, where x satisfy the following inequalities:

$$(z_i + z_j - 2) + 1 \le x \le (2 - z_i - z_j) + 1 \tag{7}$$

$$x \le z_i; x \le z_j; x \ge 0 \tag{8}$$

The above inequalities can be checked as follows:

- if $z_i = z_j = 1$ then $x = 1$ (from 7)
- if $z_i = z_j = 0$ then $x = 0$ (from 8)

Now the achievement function of crisp model of MCGP can be expressed based on the above mentioned discussion as follows:

$$\left.\begin{array}{ll} \textit{Minimize} & \sum_{i=1}^{n}(d_i^+ + d_i^-) \\ \textit{subject to} & \Re(\tilde{f}_i(X)) - d_i^+ + d_i^- = \sum_{j=1}^{m} g_{ij}S_{ij}(B),\ i = 1,2,\ldots,n \\ & \sum_{i=1}^{n} a_{ij}\Re\{(x_i, y_i, \alpha_i, \alpha_i)\} \le, =, \ge b_j,\ j = 1,2,\ldots,m \\ & \Re\{(x_i, y_i, \alpha_i, \alpha_i)\} \ge 0\ \ \forall i \\ & y_i - x_i \ge 0,\ \ \alpha_i \ge 0 \\ & d_i^+, d_i^- \ge 0,\ \ i = 1,2,\ldots,n \end{array}\right\} \tag{9}$$

where $S_{ij}(B)$ represents a function of binary serial number which guarantees only one aspiration level is chosen from each goal. d_i^+ and d_i^- are the over and under deviational variables respectively.

3. NUMERICAL EXAMPLES

3.1 Example 1

To illustrate the FMCGP, assume that a manager of the manufacturing company establishes the following goals, goal 1, goal 2 and goal 3 with multiple choices for aspiration levels as:

goal 1: To achieve at either 26 or 32 rupees of total profit.

goal 2: To keep the pollution level below either 14 or 18 units.

goal 3: To produce at least 5 or 8 or 9 tons of the three products.

for the production planning problem in which three products P1, P2 and P3 are produced utilizing three different materials M1, M2 and M3. The material required producing one ton of each product and the limitations of the materials are given below:

Table 1: Production conditions

Materials	Material per ton			Materials limitation
	P1	P2	P3	
M1	2	6	3	27
M2	3	2	4	16
M3	4	1	2	18

Note that the material availability can vary from day to day due to wastage, defective items etc. Finally the profit for each product can also vary due to variations in price. At the same time the company wants to keep the profit somewhat close to 3 million yen/ton for P1, 8 million yen/ton for P2 and 5 million yen/ton for P3. In the production process, it is pointed out that product P1 yields somewhat close to 5 units of pollution per ton, product P2 yields 4 units of pollution per ton and product P3 yields 3 units of pollution per ton. The company wants to determine the range of each product to be produced per day to maximize its profit. It is assumed that all the amounts produced are consumed in the market.

Since the profit from each product and the material availability are uncertain, the number of units to be produced on each product will also be uncertain. So we will model the problem as a fuzzy linear programming problem. We use symmetric trapezoidal fuzzy numbers for each uncertain value.

Coefficients c_1, c_2, c_3 for goal 1, which is close to 3, 8, 5 are modeled as [2, 4, 2, 2], [7, 9, 3, 3], [4, 6, 2, 2]. Similarly, the other coefficients for goal 2 and goal 3 are also modeled as symmetric trapezoidal fuzzy numbers taking into account the nature of the problem and other requirements.

Fuzzy model: The MCGP problem with trapezoidal fuzzy numbers can be written as:

$$\left.\begin{array}{l}(goal\,1)\;\; [2,4,2,2]\otimes\tilde{x}_1\oplus[7,9,3,3]\otimes\tilde{x}_2\oplus[4,6,2,2]\otimes\tilde{x}_3=26\,or\,32\\ (goal\,2)\;\; [4,6,1,1]\otimes\tilde{x}_1\oplus[3,5,2,2]\otimes\tilde{x}_2\oplus[2,4,2,2]\otimes\tilde{x}_3=14\,or\,18\\ (goal\,3)\;\; \tilde{x}_1\oplus\tilde{x}_2\oplus\tilde{x}_3=8\,or\,10\,or\,11\\ subject\,to\;\; 2\tilde{x}_1\oplus6\tilde{x}_2\oplus3\tilde{x}_3\preceq27\\ \qquad\qquad 3\tilde{x}_1\oplus2\tilde{x}_2\oplus4\tilde{x}_3\preceq16\\ \qquad\qquad 4\tilde{x}_1\oplus\tilde{x}_2\oplus2\tilde{x}_3\preceq18\\ \qquad\qquad \tilde{x}_1\succeq0,\tilde{x}_2\succeq0,\tilde{x}_3\succeq0\end{array}\right\} \quad (10)^{**}$$

** Let $\widetilde{A}=(m_1,n_1,\alpha,\alpha)$ *and* $\widetilde{B}=(m_2,n_2,\beta,\beta)$ be two symmetric trapezoidal fuzzy numbers. Then the arithmetic operations on $\widetilde{A}$ *and* $\widetilde{B}$ are given by:

a) Multiplication-

$$\widetilde{A}\otimes\widetilde{B}=(m_1,n_1,\alpha,\alpha)\otimes(m_2,n_2,\beta,\beta)=\left(\left(\frac{m_1+n_1}{2}\right)\left(\frac{m_2+n_2}{2}\right)-w,\left(\frac{m_1+n_1}{2}\right)\left(\frac{m_2+n_2}{2}\right)+w,\left|n_1\beta+n_2\alpha\right|,\left|n_1\beta+n_2\alpha\right|\right)$$

$$where\; w=\left(\frac{k-h}{2}\right) and\; h=\min\{m_1m_2,m_1n_2,m_2n_1,n_1n_2\}, k=\max\{m_1m_2,m_1n_2,m_2n_1,n_1n_2\}$$

b) Addition- $\widetilde{A}\oplus\widetilde{B}=(m_1,n_1,\alpha,\alpha)\oplus(m_2,n_2,\beta,\beta)=(m_1+m_2,n_1+n_2,\alpha+\beta,\alpha+\beta)$

c) Scalar multiplication- $\lambda\widetilde{A}=\begin{cases}(\lambda m_1,\lambda n_1,\lambda\alpha,\lambda\alpha), & \lambda\geq0\\ (\lambda m_1,\lambda n_1,-\lambda\alpha,-\lambda\alpha), & \lambda\leq0\end{cases}$

Above fuzzy model of the problem has been solved according to the procedure discussed in section 3.2. Using LINGO (2013) software the optimal solutions obtained as:

$$(x_1, y_1, \alpha_1) = (0.1765, 0.1765, 0)$$
$$(x_2, y_2, \alpha_2) = (2.2059, 2.2059, 0)$$
$$(x_3, y_3, \alpha_3) = (2.7647, 2.7647, 0)$$

Substituting these values in $\tilde{x}_i; i = 1,2,3$, we get the fuzzy optimal solution as:

$$\tilde{x}_1 = (0.1765, 0.1765, 0, 0),\ \tilde{x}_2 = (2.2059, 2.2059, 0, 0),\ \tilde{x}_3 = (2.7647, 2.7647, 0, 0)$$

and fuzzy optimal values as:

$$goal\ 1 = (26.8531, 37.1473, 12.50, 12.50),\ goal\ 2 = (12.8531, 23.1473, 10.12, 10.12),\ goal\ 3 = (5.1471, 5.171, 0, 0)$$

The values of goals are in the form of symmetric trapezoidal fuzzy numbers, by converting them into crisp numbers we get our required goal values as:

$$goal\ 1 = 32,\ goal\ 2 = 18,\ goal\ 3 = 5.1471$$

Crisp model: The MCGP problem can be written as:

$$\left.\begin{array}{l}
(goal\,1)\ \ 3x_1 + 8x_2 + 5x_3 = 32 \\
(goal\,2)\ \ 5x_1 + 4x_2 + 3x_3 = 14 \\
(goal\,3)\ \ x_1 + x_2 + x_3 = 8 \\
subject\,to\ \ 2x_1 + 6x_2 + 3x_3 \le 27 \\
\qquad\qquad 3x_1 + 2x_2 + 4x_3 \le 14 \\
\qquad\qquad 4x_1 + x_2 + 2x_3 \le 18 \\
\qquad\qquad x_i \ge 0;\ i = 1,2,3
\end{array}\right\} \tag{11}$$

Solving above problem using LINGO (2013) software the optimal solutions obtained as:

$$(x_1, x_2, x_3) = (0, 4, 0)$$

and optimal values as

$$goal\,1 = 32,\ goal\,2 = 2,\ goal\,3 = 4$$

It has been seen that goal 1 & 2 has reached the aspiration levels 32 & 18 exactly and goal 3 has 5.1471 achieved to reach the aspiration level 8 for fuzzy model. However, the results of crisp model show that goal 1 has reached the aspiration level 32 exactly and goal 2 has a positive value (+2) over aspiration level 14 and goal 3 has 4 achieved to reach aspiration level 8. This clarifies that fuzzy model gives better result than crisp model for decision making because fuzzy model balanced on the three goals better than crisp model.

3.2 Example 2

To illustrate FMCGP problem in this example we assume an artificial data and formulate the fuzzy and crisp model given below. Both the model is solved in accordance of the example 1.

Coefficients c_1, c_2, c_3 for goal 1 which is close to 14, 13, 16 are modeled as [13, 15, 2, 2], [12, 14, 3, 3], [15, 17, 2, 2]. Similarly the other coefficients for goal 2 and goal 3 are also modeled as symmetric trapezoidal fuzzy numbers taking into account the nature of the problem and other requirements.

Fuzzy model: The MCGP problem with trapezoidal fuzzy numbers can be written as:

$$\left.\begin{aligned}
&(goal\,1)\ \ [13,15,2,2]\otimes\tilde{x}_1\oplus[12,14,3,3]\otimes\tilde{x}_2\oplus[15,17,2,2]\otimes\tilde{x}_3 = 100\,or\,120\\
&(goal\,2)\ \ [11,13,2,2]\otimes\tilde{x}_1\oplus[10,12,5,5]\otimes\tilde{x}_2\oplus[16,18,3,3]\otimes\tilde{x}_3 = 80\,or\,100\\
&(goal\,3)\ \ [14,16,1,1]\otimes\tilde{x}_1\oplus[16,18,3,3]\otimes\tilde{x}_2\oplus[10,12,3,3]\otimes\tilde{x}_3 = 70\,or\,90\,or\,110\\
&subject\ to\ \ \tilde{x}_1\oplus\tilde{x}_3\succeq 10\\
&\qquad\qquad\ \tilde{x}_2\succeq 4\\
&\qquad\qquad\ \tilde{x}_1\oplus\tilde{x}_2\oplus\tilde{x}_3\succeq 15\\
&\qquad\qquad\ \tilde{x}_1\succeq 0,\tilde{x}_2\succeq 0,\tilde{x}_3\succeq 0
\end{aligned}\right\} \tag{12}$$

Above fuzzy model of the problem has been solved according to the procedure discussed in section 3.2. Using LINGO (2013) software the optimal solutions obtained as:

$$(x_1, y_1, \alpha_1) = (0,0,0)$$
$$(x_2, y_2, \alpha_2) = (15,15,0)$$
$$(x_3, y_3, \alpha_3) = (0,0,0)$$

Substituting these values in $\tilde{x}_i; i = 1,2,3$, we get the fuzzy optimal solution as:

$$\tilde{x}_1 = (0,0,0,0),\ \tilde{x}_2 = (15,15,0,0),\ \tilde{x}_3 = (0,0,0,0)$$

and fuzzy optimal values as:

$$goal\,1 = (180,210,45,45),\ goal\,2 = (150,180,75,75),\ goal\,3 = (240,270,45,45)$$

The values of goals are in the form of symmetric trapezoidal fuzzy numbers, by converting them into crisp numbers we get our required goal values as:

$$goal\,1 = 195,\ goal\,2 = 82.5,\ goal\,3 = 255$$

Crisp model: The MCGP problem can be written as:

$$\left.\begin{aligned}
&(goal\,1)\ \ 14x_1 + 13x_2 + 16x_3 = 100\\
&(goal\,2)\ \ 12x_1 + 11x_2 + 17x_3 = 100\\
&(goal\,3)\ \ 15x_1 + 17x_2 + 11x_3 = 90\\
&subject\ to\ \ x_2 + x_3 \ge 10\\
&\qquad\qquad x_2 \ge 4\\
&\qquad\qquad x_1 + x_2 + x_3 \ge 15\\
&\qquad\qquad x_i \ge 0;\ i = 1,2,3
\end{aligned}\right\} \tag{13}$$

Solving above problem using LINGO (2013) software the optimal solutions obtained as:

$$(x_1, x_2, x_3) = (5,10,0)$$

and optimal values as

$$goal\,1 = 200,\ goal\,2 = 170,\ goal\,3 = 245$$

It has been seen that goal 1 has 195 achieved to reach the aspiration level 120, goal 2 has 82.5 achieved to reach the aspiration level 100 & goal 3 has a positive value (+145) over aspiration level 110 for fuzzy model. However, the results of crisp model shows that goal 1 has a positive value (+100) over aspiration level 100 and goals 2 & 3 also has a positive values (+70) and (+155) over aspiration levels 100 & 90. This clarifies that fuzzy model gives better result than crisp model for decision making because fuzzy model balanced on the three goals better than crisp model.

4. CONCLUSION

This manuscript considers a multi-choice goal programming problem in which parameters and decision variables are symmetric trapezoidal fuzzy numbers. The problem is converted into a traditional MODM problem using Mehar's method and then it is solved by MCGP method. Finally, two numerical problems have been solved using optimization software LINGO 13 (LINGO-User's Guide). For more information one can visit the site: http://www.lindo.com. On the basis of the results obtained from the two numerical problems it can be said that the fuzzy form of the problem gives more efficient result than the crisp form.

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