International Journal of
Operations Research

International Journal of Operations Research Vol. 11, No. 3, 091−099 (2014)

EPQ Model for Time-Declining Demand with Imperfect Production Process under Inflationary Conditions and Reliability

Nita H. Shah*[1] and Bhavin J. Shah[2]

[1]Department of Mathematics, Gujarat University, Ahmedabad – 380 009, Gujarat, India

[2]Indian Institute of Management, Indore–453556, Madhya Pradesh, India

Received June 2014; Revised July 2014; Accepted September 2014

Abstract — An inventory model with imperfect production process is developed for time-declining demand pattern. Imperfect production process results into a mix of good quality and defective items. Some of the defective items can be corrected to an extent with additional reworking cost. Reliability of the production process is considered as the decision variable. Optimum value of the reliability parameter helps in long-term strategic decisions regarding investments in production technology; process know-how and product design complexity. Model is developed with an objective to maximize total profit using Euler-Lagrange theory in a single item setting. Effects of inflation and time value of money on total profit are also considered. Results obtained are validated by numerical example. Sensitivity analysis of various parameters is carried out to derive managerial insights.

Keywords — Imperfect production process, Time-declining demand, Process reliability, Inflation

1. INTRODUCTION AND PROBLEM DEFINITION

In the literature of inventory modeling, most of the models consider demand for the product to be constant. However, this may not be true in general in today's fast changing world.

Demand for many products like cellphones, lifestyle products like shoes, watches, apparels decline after sometime. They follow time declining demand pattern. In this paper, this demand pattern is captured in the proposed model. Also classical EOQ models assume that quality of the item manufactured is good (conforming to specifications) and it remains good throughout the production cycle. This assumption is not always true. It is inherent for certain production processes that they deviate from their set values over a period of time and start producing items which are not of accepted quality, i.e. of imperfect quality. As a result, it gives a mixed basket of items; items that are of accepted quality that can go to market and items with imperfect quality which need to be reworked at an additional cost before correcting them to make market ready. This phenomenon is captured by defining reliability parameter of the production process in the proposed model. Higher value of the reliability parameter requires more long term investments to commit for production technology; process know-how and product design complexity. This will ensure higher ratio of good quality items but in turn increases per unit production cost dramatically due to higher initial investment in technology. On the other hand it reduces reworking cost to an extent. However, when demand for the item is declining as per time it is difficult to justify higher long term investment. There exists a trade-off between increasing cost due to investment in technology resource vis-à-vis reworking cost of imperfect quality items based on the reliability parameter of the production process. This paper attempts to address this issue and seeks to find an optimum level of reliability parameter in order to help decision making in such situations. Proposed model seeks to maximize profit using Euler-Lagrange theory after considering effects of inflation and time value of money.

Rest of the paper is organized as follows: Section 2 describes literature review; section 3 gives notations and assumptions; section 4 contains model formulation followed by numerical analysis and sensitivity analysis in section 5. At the end of paper managerial insights are discussed in section 6 before concluding the paper in section 7.

2. LITERATURE REVIEW

Schrady (1967) was amongst the first ones who studied rework or remanufacturing in inventory decisions. Porteus (1986) studied optimal lot sizing and setup cost reduction in an imperfect production process environment. Khouja (2000) proposed economic lot size and delivery scheduling problem for reworking of defective items. Chung and Hou (2003) discussed allowing shortage for the imperfect production process for deteriorating items. Chiu et al. (2004) suggested

* Corresponding author's email: nitahshah@gmail.com

framework for imperfect production process resulting into repairable and scrapped items. Rework policies for imperfect quality items in a particular cycle was studied by Jamal et al. (2004). Cardenas-Barron (2009) extended Jamal et al. (2004)'s work for N cycles. Sana et al. (2007) suggested EMQ model with reduction in selling price in an imperfect production process. Yoo et al. (2009) proposed EPQ model with imperfection with respect to quality, inspection and rework. Widyadana and Wee (2010) suggested algebraic approach to rework lot sizing models.

Giri and Dohi (2005) proposed generalized EMQ model with stochastic machine breakdown and repair with corrective and preventive repair times. Chiu et al. (2010) discussed optimization of the finite production rate model with scrap, rework and stochastic machine breakdown. Sarkar et al. (2010) extended Giri and Dohi (2005)'s work and introduced unit production cost as a function of reliability parameter in imperfect production process.

Silver and Meal (1969) studied demand rate as a function of time. Other significant contribution in the field of time dependent demand are that of Donaldson (1977), Buchanan (1980), Silver and Peterson (1985), Hariga (1996). These models were further extended to capture shortages, backlogging, and linearly increasing or decreasing demand by various researchers like Deb and Chaudhuri (1987), Teng and Thompson (1996), and Giri et al. (2000). Shah et al. (2011) proposed inventory model for time-sensitive as well as price dependent demand.

Inflation results into erosion of purchasing power of money and one cannot ignore it in inventory planning. Buzacott (1975) and Misra (1975) studied effects of inflation on inventory policies. Other significant research papers in the same area are that of Bierman and Thomas (1977), Misra (1979), Aggarwal (1981), Chandra and Bahner (1985), Ray and Chaudhuri (1997), Chung and Lin (2001), and Yang (2004). Dey et al. (2008) discussed an inventory problem with dynamic demand under inflation and time-value of money.

3. NOTATIONS AND ASSUMPTIONS

3.1 Notations

$R(t) = ae^{-bt}$ Demand at time t $(a > 0, 0 < b < 1)$
$Q(t) =$ On hand inventory at time $t \geq 0$
$\dfrac{\partial Q(t)}{\partial t} =$ Rate of change in inventory level with respect to t
$C_m =$ Material cost per unit item
$C_1(\eta) =$ Development cost of production system based on reliability parameter η
$C_p(\eta, t) =$ Production cost per unit item
$h =$ Holding cost per unit time
$C_{rw} =$ Reworking cost per defective item in the production system
$P =$ Selling price per unit item
$\alpha =$ Variation constant for tool/die cost
$P(t) =$ Production rate at time t
$\eta =$ Reliability parameter of production process
$\beta =$ Rate of defective items produced during $[0, T]$
$\eta_{\min} =$ Minimum value of reliability parameter η
$\eta_{\max} =$ Maximum value of reliability parameter η
$\upsilon =$ Difficulty factor in increasing reliability of production process
$C_{LE} =$ Labor and energy costs (fixed – independent of reliability parameter η)
$C_T =$ Cost of technology, resource and design complexity for production when $\eta = \eta_{\max}$
$T =$ Production-inventory cycle time (fixed)
$k = r - i$;
$r =$ Interest rate per unit of currency
$i =$ Inflation per unit of currency

3.2 Assumptions

1. η - Reliability parameter of the production process is given by the ratio of number of failures (defective items) and total number of operating hours. It can also be described as number of defective items per operating hour of production process.
2. Production process is assumed to be an imperfect production process which produces two types of items: perfect and imperfect; in terms of quality. Perfect quality items are ready for sell. Items with imperfect quality require some amount of reworking before being classified as perfect quality items. These items attract reworking costs to be incurred.
3. Production cost is s a function of reliability parameter and production rate while development cost is assumed to be a function of reliability parameter.
4. Time horizon is infinite.
5. Model is developed for time-declining demand for a single item.
$$R(t) = ae^{-bt} \text{ where } a > 0, 0 < b < 1$$
6. Effects of inflation and time value of money (TVM) are considered.

4. NOTATIONS AND ASSUMPTIONS

Production process is considered to be an imperfect production process that produces two types of items. Items confirming to quality (perfect quality) and not confirming to quality (imperfect quality). Items with perfect quality are considered to be ready to go to market. Items with imperfect quality are reworked at an additional cost to correct them to perfect quality.

Reliability parameter of the production system η is considered to be a decision variable. It is defined as the number of items with imperfect quality per operating hour of the system. It is obvious that smaller value of η will require more investments in technology and knowhow which will result in higher production cost per unit. Higher value of η indicates relatively smaller investments in technology and knowhow but will increase the burden on reworking cost. Keeping this trade-off in mind η is considered as a decision variable which aids to other long-term strategic decisions like investment in production technology, process know-how and product design complexity.

Development cost for production system which is a function of reliability parameter (see Sarkar et al. (2010, 2011)) is given by:

$$C_1(\eta) = C_{LE} + C_T e^{\vartheta(\eta_{\max} - \eta)/(\eta - \eta_{\min})} \tag{1}$$

Clearly we observe that $\eta \propto \dfrac{1}{C_1(\eta)}$. Also $C_1(\eta)$ is minimum when $\eta = \eta_{\max}$ and $C_1(\eta)$ is maximum when $\eta = \eta_{\min}$

Unit production cost is given by

$$\mathrm{C}_p(\eta, t) = C_m + \frac{C_1(\eta)}{P(t)} + \alpha P(t) \tag{2}$$

Governing differential equation considering time-declining demand for this production system is given by

$$\frac{dQ(t)}{dt} = P(t) - R(t) \text{ with } Q(0) = 0 \text{ and } Q(T) = 0$$

Production rate at time *t* is

$$\mathrm{P}(t) = \frac{dQ(t)}{dt} + R(t) = \mathrm{Q}'(t) + R(t) \tag{3}$$

Revenue from selling items during [0, T] is given by

$$SR = \int_0^T \left[P - C_p(\eta, P(t)\right] P(t) dt \tag{4}$$

Total holding cost of items produced [0, T] is given by

$$HC = \int_0^T hQ(t) dt \tag{5}$$

Defective items produced at item *t* is represented by $\beta\, \mathrm{e}^{\eta t}\, P(t)$ where $\beta\, \mathrm{e}^{\eta t} < 1$. In this kind of system production of defective items increases with time and on the other side demand assumed to decline with time. Reworking cost of defective items during [0, T] is given by

$$C_{rw} \int_0^T \beta\, \mathrm{e}^{\eta t}\, P(t)\, dt \tag{6}$$

Total profit function after considering inflation and time value of money (TVM) during $[0,T]$ is

$$\pi = \int_0^T e^{-kT} \left(P - C_p(\eta, P(t))P(t) - hI(t) - C_{rw}\beta e^{\eta t} P(t)\right) dt \tag{7}$$

It represented by $\pi = \int_0^T \lambda(Q', Q, t)dt$ where

$$\lambda(Q', Q, t) = e^{-kt} \left\{ (P - C_m - C_{rw}\beta e^{\eta t})(Q'(t) + R(t)) - \alpha(Q(t) + R(t))^2 - hQ(t) - C_1(\eta) \right\} \tag{8}$$

For finding an optimal path we consider a path (or curve) C_o for which the profit function π will be maximum. Consider any curve C_w. C_o is given by $Q = Q_\xi(t)$, $0 \le t \le T$. C_w is given by

$$Q(t) = Q_w(t) = Q_\xi(t) + w\,f(t) \tag{9}$$

where *f(t)* is an arbitrary differentiable function of *t* and *w* is sufficiently small quantity. $\therefore$ $\pi(w) = \int_0^T \lambda_w dt$ where

$$\lambda_w = \lambda\left(Q_\xi(t) + w\,f(t), Q_\xi'(t) + w\,f'(t), t\right) \tag{10}$$

For optimum value of $\pi(w)$, we must have $\dfrac{d\pi(w)}{dw} = 0$

$$\begin{aligned} \frac{d\pi_w}{dw} &= \int_0^T \left\{ f(t)\frac{\partial \lambda_w}{\partial Q} + f'(t)\frac{\partial \lambda_w}{\partial Q} \right\} dt \\ &= \int_0^T f(t)\frac{\partial \lambda_w}{\partial Q} dt + \left[f(t)\frac{\partial w}{\partial Q} \right]_0^T - \int_0^T f(t)\frac{d}{dt}\left(\frac{\partial \lambda_w}{\partial Q}\right) dt \\ &= \left[f(t)\frac{\partial \lambda_w}{\partial Q} \right]_0^T + \int_0^T f(t)\left\{ \frac{\partial \lambda_w}{\partial Q} - \frac{d}{dt}\left(\frac{\partial \lambda_w}{\partial Q}\right) \right\} dt \end{aligned} \tag{11}$$

Arbitrary function *f (t)* vanishes at end points as *Q* is zero at end points. Therefore, $f(0) = f(T) = 0$. Considering $\left.\dfrac{d\pi(w)}{dw}\right|_{w=0} = 0$; we get

$$\frac{\partial \lambda_w}{\partial Q} - \frac{d}{dt}\left(\frac{\partial \lambda_w}{\partial Q}\right) = 0 \tag{12}$$

which is necessary condition for optimum values of π. Differentiating π again with respect to w;

$$\frac{d^2\pi(w)}{dw^2} = \int_0^T \left\{ f^2 \frac{\partial^2 \lambda_w}{\partial Q^2} + 2f \cdot f' \frac{\partial^2 \lambda_w}{\partial Q \partial Q'} + \frac{\partial^2 \lambda_w}{\partial Q'^2} \right\} dt$$

At $w = 0$

$$\left.\frac{d^2\pi(w)}{dw^2}\right|_{w=0} = \int_0^T \left\{ f^2 \frac{\partial^2 \lambda}{\partial Q^2} + 2f \cdot f' \frac{\partial^2 \lambda}{\partial Q \partial Q'} + \frac{\partial^2 \lambda}{\partial Q'^2} \right\} dt \tag{13}$$

where $\dfrac{\partial \lambda}{\partial Q} = -he^{-kt}$, $\dfrac{\partial^2 \lambda}{\partial Q^2} = 0$, $\dfrac{\partial \lambda}{\partial Q'} = e^{-kt}\left(P - C_m - C_{rw}\beta e^{\eta t}\right) - 2\alpha(Q'(t) + R(t))$, $\dfrac{\partial^2 \lambda}{\partial Q'^2} = -2\alpha e^{-kt}$,

$\dfrac{\partial^2 \lambda}{\partial Q \partial Q'} = 0$. Hence

$$\frac{d^2\pi}{dw^2} = -\int_0^T 2\alpha\, f'^2 e^{-kt} dt < 0 \tag{14}$$

Equation (14) is a sufficient condition establishing the fact that π has a maximum value in [0, T].
For finding optimal path, following Euler-Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial \lambda}{\partial Q'}\right) - \frac{\partial \lambda}{\partial Q} = 0$$

$$\Rightarrow \frac{d}{dt} \; e^{-kt} \;\; (P - C_m - C_{rw}\beta e^{\eta t}) - 2\alpha(Q'(\mathrm{t}) + \mathrm{R}(\mathrm{t})) \quad + he^{-kt} = 0$$

Simplifying this equation, we get

$$Q'' - kQ' = L_1 + L_2 e^{-bt} + L_3 e^{\eta t} \tag{15}$$

where $L_1 = \dfrac{h - k(P - C_m)}{2\alpha}$, $L_2 = a(k+b)$, $L_3 = \dfrac{C_{rw}\beta(k-\eta)e^{\eta t}}{2\alpha}$. Simplifying equation (15), we get

$$\begin{aligned} Q(t) = {} & \frac{-1}{e^{-kT}-1}\left[\frac{L_2}{b(k+b)}(1-e^{-bT}) - \frac{L_3}{\eta(\eta-k)}(e^{\eta T}-1) + \frac{L_1 T}{k}\right] - \frac{L_2}{b(k+b)} - \frac{L_3}{\eta(\eta-k)} \\ & + \frac{e^{kt}}{e^{kT}-1}\left[\frac{L_2}{b(k+b)}(1-e^{-bT}) - \frac{L_3}{\eta(\eta-k)}(e^{\eta T}-1) + \frac{L_1 T}{k}\right] - \frac{L_1 t}{k} + \frac{L_2 e^{-bt}}{b(k+b)} + \frac{L_3 e^{\eta t}}{\eta(\eta-k)} \end{aligned} \tag{16}$$

$$P(t) = Q'(t) + R(t) = Q'(t) + ae^{-bt} \tag{17}$$

$$\pi = \int_0^T e^{-kt} \;\; (P - C_m - C_{rw}\beta e^{\eta t})(Q'(\mathrm{t}) + \mathrm{R}(\mathrm{t})) - \alpha(Q'(\mathrm{t}) + \mathrm{R}(\mathrm{t}))^2 - hQ(t) - C_1(\eta) \;\; dt \tag{18}$$

5. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

A numerical example and sensitivity analysis is worked out to illustrate developed model using Maple 14 software. Consider following parametric value in their appropriate units.

a=500 units, b=20%, C_m =\$4, h=1, C_{rw} =4, P=40, α =5 %, β =15%, υ =0.50, C_{LE} =200,

C_T =500, T =1 year, $k = r - i$ =0.03, $\eta_{\min}$ = 10 % , $\eta_{\max}$ = 90 %.

We get $\eta^* = 0.52$ and $\pi^* = 4702$

For the developed model, it is not possible to derive analytical formula for optimality. However, Figure 1 clearly shows concavity of the profit function. Behavior of optimal development cost versus optimal reliability is shown in the curve given in Figure 2.

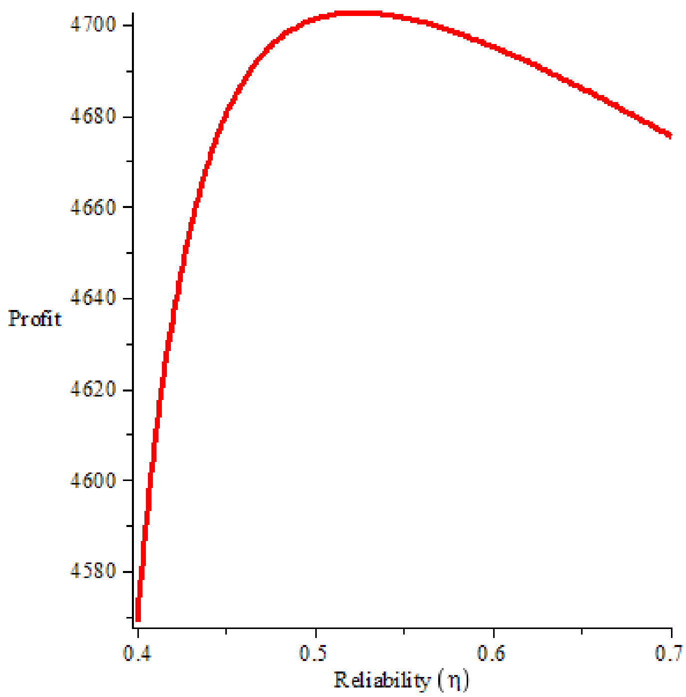

Figure 1. Profit function versus Reliability

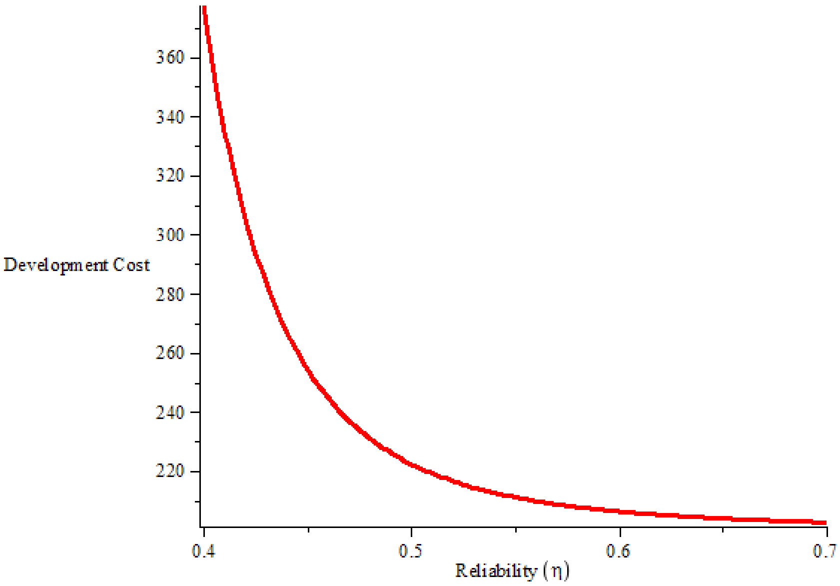

Figure 2. Development cost function versus Reliability

Sensitivity analysis of optimal result is carried out by changing various inventory parameters. Figure 3 gives effects on optimum profit value and Figure 4 gives effects on reliability parameter when various inventory parametric values are changed within range $\pm 20\%$ of the base value.

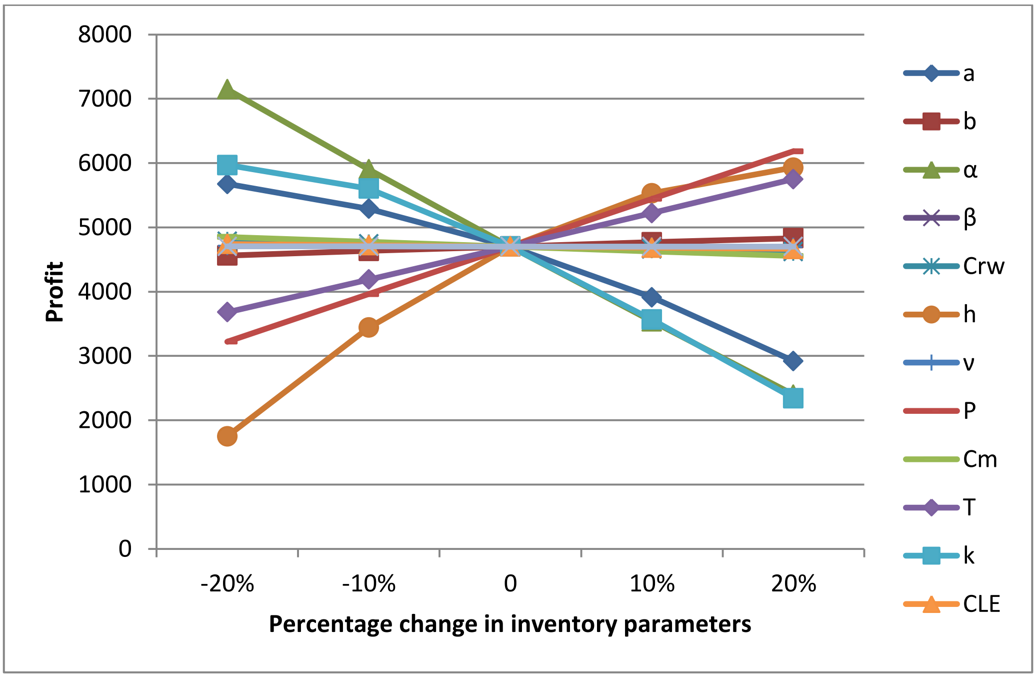

Figure 3. Changes in optimum profit versus changes in inventory parameters

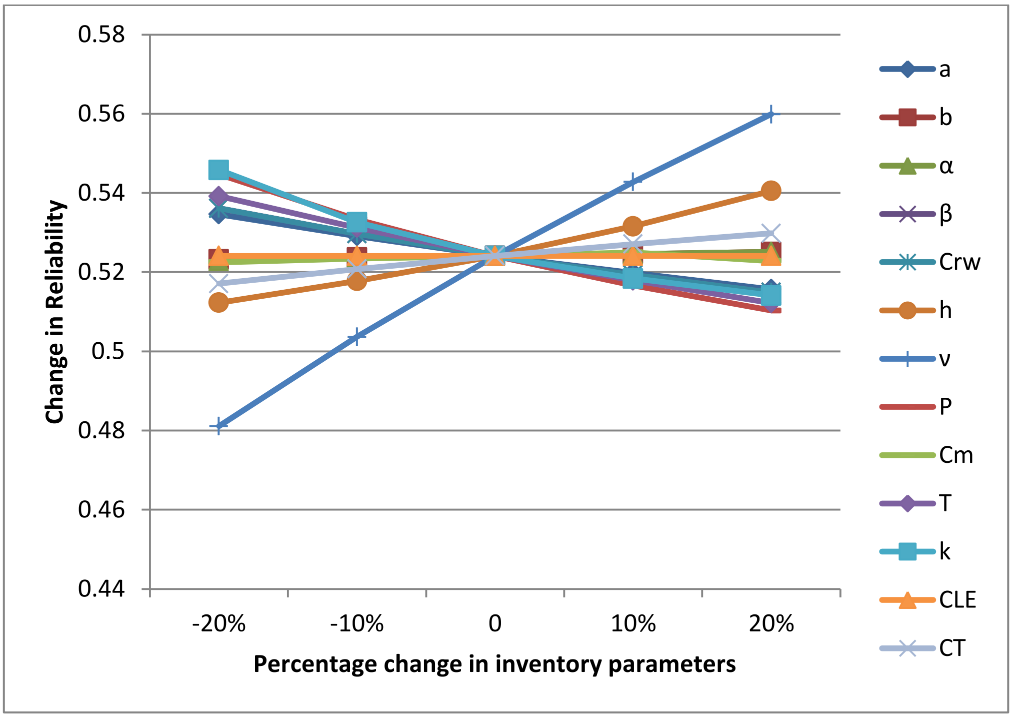

Figure 4. Changes in Reliability Parameter versus changes in inventory parameters

It is clear from Figure 3 that optimum profit is much more sensitive to changes in parametric values of a, k, C_m, h, P and T. It reaffirms the fact that profit is more impacted due to initial demand (a) as demand starts declining as per time. Profit is also be much sensitive to gross operating margin (Selling Price ($P - C_m$) – Cost of material) given the fact that cost of labor and energy are relatively fixed in nature. As time declining demand pattern is considered, profit is also more sensitive to holding cost and effects of inflation and the time value of money (k).

It is evident from Figure 3 that optimum profit is more sensitive to base initial demand a (constant in demand function). Effects on profit because of inflation and due to time value of money (k) and material cost of item are also significant. Cost of reworking of an item, time-declining demand constant (b), and defective production rate also impact profit in a limited way.

Figure 4 gives sensitivity analysis carried out on optimum reliability values due to changes in various inventory parameters. It is very obvious that optimum reliability is very sensitive to v, as it is associated with difficulty factor in increasing reliability. It is also sensitive to changes in production rate of defectives (β), cost of reworking (C_{rw}). Optimum reliability is relatively less sensitive to effects of α, C_{LE} .

6. MANAGERIAL INSIGHTS

1. Most manufacturing systems faces malfunction or items produced deviate from specification after sometime. The fraction of defective items produced during this time goes up and it adds a burden on system in the form of reworking cost. This model captures this very inherent imperfect nature of production system. It helps in committing funds beforehand for long term strategic investments in production technology, process know-how and product design complexity based upon reliability parameter.
2. Cost of reworking for items with imperfect quality and production rate of such items during a given production runtime determines optimum level of reliability parameter for the production process.
3. Due to time declining demand pattern, gross operating margin (selling price minus cost of material) plays an important role to decide optimum profit and optimum reliability parameter.

7. CONCLUSION

In this paper, inventory model with imperfect production process is developed for the time declining demand. Major contribution of this research paper is in deciding optimal reliability parameter. Developed model helps to take long term strategic decisions regarding degree of involvement and investments in production technology, process know-how and product design complexity which will have a tradeoff with incurring reworking cost for correcting items with imperfect quality. Associated profit function is maximized by Euler-Lagrange function method.

This research work can further be extended by considering multi-level inventory model with credit financing options, stochastic demand, etc.

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