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The global Malmquist productivity index under the optimistic-pessimistic approach of DEA

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Abstract —In this paper, an optimistic-pessimistic approach of data envelopment analysis (DEA) is used to provide a new global Malmquist productivity index. To this end, we estimate the score of each DMU in two periods under the assumption that the sum of scores of DMUs at two periods equals to unity. Then, the scores of each DMU in two periods are compared to determine its productivity change.

Keywords —Data envelopment analysis, Optimistic-pessimistic approach, Productivity, Malmquist productivity index.

1. INTRODUCTION

Data envelopment analysis (DEA) is a linear programming (LP) technique for measuring the relative efficiency of peer decision making units (DMUs) when multiple inputs and outputs are present. This objective method was originated by Charnes et al. (1978) (CCR). Since this pioneering model, other DEA models have been introduced. For instance, the variable return to scale version of the CCR model, namely BCC model, was presented by Banker et al. (1984). As another example, we can point to the additive model proposed by Charnes et al. The slacks-based measure of efficiency (SBM) was proposed by Tone (2001). The super efficiency models were proposed by Andersen and Petersen (1993) for ranking only efficient units in the DEA. Khodabakshi (2007, 2009b, 2010a, 2011) and Khodabakshi et al. (2010b) extend some DEA models from the deterministic environment to the stochastic one. Also, Khodabakshi (2009a) and, Khodabakshi and Asgharian (2009), and Asgharian et. al (2010) proposed a DEA model based on relaxed combinations of inputs.

The Malmquist Productivity Index (MPI) is a bilateral index that can be used to compare the production technology of two economies. The MPI was first suggested by Malmquist (1953) as a quantity index for use in the analysis of consumption of inputs. The MPI is based on the concept of the production function which gives the maximum possible production, with respect to a set of inputs. So, if S_a is the set of inputs to the production function of economy A, and Q is the production function of economy A, we could write $Q = f_a(S_a)$. To calculate the Malmquist index of economy A with respect to economy B, we must substitute the labor and capital inputs of economy A into the production function of B, and vice versa. The formula for MPI is given below:

$$\mathbf{MPI} = \sqrt{(\mathbf{Q_1 Q_2})(\mathbf{Q_3 Q_4})} \tag{1}$$

where, $Q_1 = f_a(S_a)$, $Q_2 = f_a(S_b)$, $Q_3 = f_b(S_a)$, $Q_1 = f_b(S_b)$.

In addition to comparing the relative performance of a set of DMUs at a specific period, DEA can also calculate the productivity change of a DMU over time. While the efficiency measures are calculated by DEA, the productivity is measured by the MPI and defined as the ratio between efficiency, as calculated by the DEA, for the same DMU in two different time periods. This index was developed by Caves et al. (1982a, b) as a ratio of two distance functions for the measurement of productivity change. The most popular method is the one proposed by Färe et al. (1994) which takes the geometric mean of the efficiency values calculated from two base periods. This approach uses four different frontier facets to calculate MPI. Also, the Färe et al.'s Malmquist index only uses radial DEA efficiency scores. Chen (2003) developed a non-radial Malmquist productivity index that incorporates the preference over the performance improvement and integrates the inefficiency represented by slacks. Pastor and Lovell (2005, 2007) proposed a global MPI, which uses the data of all units at all periods to construct the frontier for calculating the MPI. Different facets of the production frontiers also result in different bases for calculating MPI, which makes units under different facets incomparable. To solve this problem, Kao (2010) proposed a common-weight MPI. This common-weight method changes the DEA based MPI from a nonparametric

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measure to a parametric one. Also, Jahanshahloo et al. (2006) proposed a method for measuring MPI when interval or fuzzy data are present. Emrouznejad et al. (2011) proposed another approach to estimate the overall profit MPI of units with fuzzy and interval data. The concept of this index has further been studied and developed in the non-parametric framework by several authors. See for example, among others, Chung et al (1997); Emrouznejad and Thanassoulis (2010); Grifell-Tatje and Lovell (1998); Simar and Wilson (1999); Thrall (2000). Portela and Thanassoulis (2006, 2010) applied the Malmquist indexes to estimate the productivity change of bank branches. To see some applications of this index, we refer the readers to the papers Changa et al. (2009); Chen et al. (2004); Odeck (2000, 2006). In this paper, another method is presented to estimate the global productivity change. This method is based on a single model which has recently been introduced by Khodabakhshi and Aryavash (2012, 2014a,b) (KA model). The KA model is based on both optimistic and pessimistic approaches of DEA.

The rest of the paper is organized as follows. In the next section, the KA model is reviewed. In Section 3, we put forward the methodological basis of the proposed method. In Section 4, the empirical results are presented and discussed. The final section contains brief concluding remarks and future extensions.

2. The KA model

Assume that there are n decision making units DMU_j $(j = 1, \ldots, n)$ which convert m inputs x_{ij} $(i = 1, \ldots, m)$ into s outputs y_{rj} $(r = 1, \ldots, s)$ and DMU_o is an under evaluation DMU. Also, suppose that all inputs and outputs are non-negative deterministic numbers. In the DEA, the nonnegative weights v_i and u_r are assigned to inputs and outputs, respectively, and the efficiency score of DMU_o (θ_o) is obtained as follows:

$$\theta_o = \frac{\sum_{r=1}^{s} y_{ro} u_r}{\sum_{i=1}^{m} x_{io} v_i} \tag{2}$$

In the DEA models, the weights are obtained in an objective way. In the pioneering DEA models, the following set of normalizing constraints reflects the condition that the efficiency score of every DMU must be less than or equal to unity.

$$\theta_j = \frac{\sum_{r=1}^{s} y_{rj} u_r}{\sum_{i=1}^{m} x_{ij} v_i} \le 1, j = 1, \ldots, n \tag{3}$$

Hence, in DEA the following fractional model is used to evaluate the performance of DMU_o:

$$\begin{aligned} & Max\ \theta_o \\ & s.t.\ \theta_j = \frac{\sum_{r=1}^{s} y_{rj} u_r}{\sum_{i=1}^{m} x_{ij} v_i} \le 1, j = 1, \ldots, n \\ & \quad u_r, v_i, \theta_j \ge 0. \end{aligned} \tag{4}$$

Under conditions (3), the scores of all efficient DMUs be equal to unity. So, model (4) cannot discriminate among efficient DMUs. Also, if the number of DMUs is less than the combined number of inputs and outputs, a large portion of the DMUs will be identified as efficient by model (4), and the efficiency discrimination among DMUs is questionable. Recently, Khodabakhshi and Aryavash (2012, 2014a, b) removed the normalizing constraints (3) from the model (4) and replaced them by following condition:

$$\sum_{j=1}^{n} \theta_j = 1 \tag{5}$$

This replacement removes the discrimination trouble of DEA. Furthermore, under the condition (5) both minimum and maximum possible efficiency values of the objective function (θ_o) can be achieved. So, using single model the efficiency scores of each DMU can be evaluated in both pessimistic and optimistic attitudes. Using this replacement, the model (4) is transformed to the following model:

$$\begin{aligned} & Min\ and\ Max\ \theta_o \\ & s.t.\ \theta_j = \frac{\sum_{r=1}^{s} y_{rj} u_r}{\sum_{i=1}^{m} x_{ij} v_i}, j = 1, \ldots, n \\ & \quad \sum_{j=1}^{n} \theta_j = 1 \\ & \quad u_r, v_i, \theta_j \ge 0. \end{aligned} \tag{6}$$

This model must be run two times. First, θ_o must be minimized to determine its minimum value(θ_o^L), and then θ_o must be maximized to determine its maximum value (θ_o^U). In fact, the value θ_o^L and value θ_o^U are respectively obtained using the pessimistic and optimistic approaches.

Theorem 1. *Using the transformations* $w_{ij} = v_i\theta_j$, *model* (6) *can be rewritten as following LP*:

$$\begin{aligned} & Min\ and\ Max\ \theta_o = \sum_{r=1}^{s} y_{ro}u_r \\ & s.t. \sum_{i=1}^{m} x_{io}v_i = 1 \\ & \sum_{i=1}^{m} x_{ij}w_{ij} - \sum_{r=1}^{s} y_{rj}u_r = 0, \qquad j = 1, \dots, n \\ & \sum_{j=1}^{m} w_{ij} - v_i = 0, \qquad i = 1, \dots, m \\ & u_r, v_i, w_{ij} \geq 0 \end{aligned} \tag{7}$$

Proof. To see the proof of this theorem, the readers are referred to Khodabakhshi and Aryavash (2012).

3. THE PROPOSED METHOD

The motivation of this section is to estimate the productivity change of n DMUs at two time periods t and t + 1. To this end, we estimate the efficiency score of DMUs in two periods using a global production function. To this end, the DMUs in the second time period are considered as new DMUs, and they are merged with DMUs of the first time period. So, we determine the efficiency values of 2n DMUs. The first n DMUs are the DMUs at time period t, and the second n DMUs are these same DMUs at time period t + 1. The DMUs at time periods t are depicted by $\mathrm{DMU}_j(\mathrm{j} = 1, \dots, \mathrm{n})$. Also, these same DMUs at time periods t + 1 are depicted by $\mathrm{DMU}_{n+j}(\mathrm{j} = 1, \dots, \mathrm{n})$. For example, the inputs of the ninth DMU in the first and second time periods are respectively depicted by x_{i9} and $\mathrm{x}_{i(n+9)}$. As another example, the score of ninth DMU in the first and second time periods are respectively shown by θ_9 and θ_{n+9}.

To use the KA model, it is sufficient the condition (5) be replaced with following condition:

$$\sum_{j=1}^{2n} \theta_j = \sum_{j=1}^{n} \theta_j + \sum_{j=1}^{n} \theta_{n+j} = 1. \tag{8}$$

The minimum and maximum possible scores of θ_o are obtained by using KA model (7). In fact, $\theta_o^L$and θ_o^U are the efficiency values of DMU_o from the pessimistic and optimistic points of view, respectively. Hence, the efficiency score of DMU_o can be any value of interval $[\theta_o^L, \theta_o^U]$. We now aggregate θ_o^L and θ_o^U into an integrated score θ_o to reflect the performance of DMU_o as a deterministic number. We have the following interval for each θ_j:

$$\theta_j^L \leq \theta_j \leq \theta_j^U, j = 1, \dots, 2n. \tag{9}$$

Using the parameters $\lambda_j(\mathrm{j} = 1, \dots, 2\mathrm{n})$, intervals (9) can be written as following convex combinations:

$$\theta_j = \theta_j^L\lambda_j + \theta_j^U(1 - \lambda_j), 0 \leq \lambda_j \leq 1, j = 1, \dots, 2n \tag{10}$$

To aggregate θ_j^L and θ_j^U into a single number, a value of interval [0, 1] must be assigned to parameter λ_j. To determine the efficiency of DMUs in an *equitable* way, the values of all parameters must be *equally* selected. So, we must have $\lambda = \lambda = \lambda_1 = \cdots = \lambda_n$ in (10). On the other hand, based on assumption (8), we have$\sum_{j=1}^{2n} \theta_j = 1$. Therefore, the values of all θ_j can be determined by solving the following linear equations system:

$$\begin{cases} \theta_j = \theta_j^L\lambda + \theta_j^U(1 - \lambda), j = 1, \dots, 2n \\ \sum_{j=1}^{2n} \theta_j = 1. \end{cases} \tag{11}$$

Now, the results of system (11) are used to determine the productivity change of n DMUs between two time periods t and t + 1. The values $\theta_1, \dots, \theta_{n+1}, \dots, \theta_{2n}$are obtained as the efficiency scores of these n DMUs at two time periods. In fact values $\theta_1, \dots, \theta_n$ are the scores of DMUs at time period t, and $\theta_{n+1}, \dots, \theta_{2n}$are the scores of these same DMUs at time period t + 1. Therefore, θ_j and $\theta_{n+j}$are the scores of DMU_jat time periods t and t + 1, respectively, and their difference is defined as a productivity change of DMU_j:

$$\Delta\theta_j = \theta_{n+j} - \theta_j, j = 1, \dots, n. \tag{12}$$

Productivity change: DMU_ohas productivity gain from period t to period t + 1 if $\Delta\theta_o > 0$. DMU_ohas productivity loss from period t to period t + 1 if $\Delta\theta_o < 0$. DMU_ohas no productivity change from period t to period t + 1 if $\Delta\theta_o = 0$. In addition, the ratio $\frac{\Delta\theta_j}{\theta_j}$ can be expressed as a percentage of progress or regress of DMU_jat the time period t + 1 in relation to the time period t. Also, the total productivity change of the set of DMUs from period t to period t + 1 can be estimated by:

$$\Delta\theta = \sum_{j=1}^{n} \theta_{n+j} - \sum_{j=1}^{n} \theta_j. \tag{13}$$

Total productivity change: We define that $\Delta\theta > 0$ indicates total productivity gain; $\Delta\theta < 0$ indicates total productivity loss; and $\Delta_\theta = 0$ means no change in total productivity from period t to period t + 1. In fact, the value of $\Delta\theta$ shows the regress or progress situation of the set of all DMUs between two time periods. Moreover, the ratio $\frac{\Delta\theta}{\sum_{j=1}^{n}\theta_j}$ can be expressed as a percentage of total productivity change of the set of all DMUs at period t + 1 in relation to period t. Finally, our method can be used, not only for comparing the performance of a unit in two periods, but also for comparing the performance of two different units at the same or different time periods.

4. Application

In 1989 Taiwan Forestry Bureau (TFB) reorganized the forest districts in order to implement a comprehensive management plan (See Kao (2000)). We determine the productivity change of eight forest districts (n = 8) after reorganization. Since the data may fluctuate from year to year, ten-year averages from 1979 to 1989 are presented in Table 1 as the data of the eight forest districts before reorganization (period t). Three years after the reorganization, the inputs and outputs were collected for the eight districts, and the averages of the years 1989-1992 were calculated as presented in the Table 1 (period t + 1). The inputs include land: area in hectares (unit=10^3ha), Labor: number of employees, expenditures: expenses per year in US dollars (unit=\$$10^6$), and initial stock: volume of forest stock before the evaluation in cubic meters (unit=10^6m^3). The output include timber production: timber harvested each year in cubic meters (unit=10^3m^3), soil conservation: forest stock for conserving soil in cubic meter (unit=10^6m^3), and recreation: visitors served by forests every year in number of visits (unit=10^3visits).

Table 1. Data of the forest districts before and after the reorganization.

period	DMU		Inputs				Outputs		
	j	Districts	Land	Labor	Expend	Initial Stock	Harvest	Stock	Visitors
t	1	Lo Tung	189.36	2803.5	15.75	18.82	19.73	20.12	84.00
	2	Hsin Chu	151.07	787.1	12.18	17.50	58.28	22.79	280.85
	3	Tung Shi	138.71	1281.0	18.63	19.22	29.98	22.18	43.36
	4	Nan Tou	211.78	863.0	14.78	23.29	96.15	26.07	0.00
	5	Chia Yi	121.20	1018.0	13.76	10.04	47.76	13.24	399.83
	6	Pin Tung	187.10	981.7	12.58	17.44	89.49	15.41	1238.98
	7	Tai Tung	227.20	216.1	5.87	24.04	44.08	27.28	0.00
	8	Hua Lien	319.91	979.9	16.67	38.78	46.93	42.83	41.88
t+1	9	Lo Tung	175.73	442.5	11.67	16.04	3.09	16.04	119.46
	10	Hsin Chu	162.81	417.9	12.93	26.10	12.45	26.10	287.26
	11	Tung Shi	138.41	561.3	20.87	23.48	4.51	23.48	247.53
	12	Nan Tou	211.82	462.4	17.30	23.53	11.16	23.53	0.00
	13	Chia Yi	139.52	587.1	8.30	13.16	3.52	13.21	845.38
	14	Pin Tung	196.05	345.8	12.17	15.88	11.61	15.88	964.04
	15	Tai Tung	226.55	202.3	5.91	26.80	15.11	26.80	159.31
	16	Hua Lien	320.85	525.9	12.02	44.13	3.72	44.11	61.70

The third and fifth columns of Table 2 exhibit the efficiency intervals of DMUs in time periods t and t + 1, respectively. The system (11) is used to integrate these intervals into unique numbers. The fourth and sixth columns of Table 2 show the unique efficiency scores of DMUs in the periods t and t + 1, respectively. For example, the pessimistic and optimistic scores of the first DMU at period t are 0.004 and 0.067, respectively. Hence, its scores at the first time period is located in the interval [0.004, 0.067]. The system (11) determines the unique number $\theta_1 = 0.024$ as its efficiency scores at time period t. In the same way, $\theta_{8+1} = \theta_9 = 0.029$ is obtained as its scores at time period t + 1. Comparing θ_1 with θ_9, we find that Lo Tung district has better efficiency at the second time period. Hence, it has experienced productivity growth after the reorganization. Its performance improves by 20.83% after the reorganization, as is seen from the eighth column of Table 2. Similarly, we can find that, Hsin Chu district has experienced 37.98% productivity decrease, and Tung Shi district has no change in its productivity from time period t to time period t + 1. In the last column of Table 2, the forest districts are ranked according to their productivity change.

In the last row of Table 2, the situation of the set of all DMUs between two time periods is generally considered. The total scores of all DMUs in the first and second time periods are 0.512 and 0.488, respectively. Hence, the performance of the set of forest districts reduces by 4.69% after the reorganization. This means that, the reorganization at least in the first three years has bad effect on the performance of Taiwan national forests set.

Our method can also be used for comparing the performance of two different units at the same or different time periods. For instance, DMU_4 and DMU_5 have a same performance at the first period, whereas the performance of DMU_5 is better than the performance of DMU_4at the second period.

Table 2. The productivity change of the forest districts.

DMU		Efficiency at Period t		Efficiency at Period t+1		MPI		
j	District	$[\theta_j^L, \theta_j^U]$	θ_j	$[\theta_{n+j}^L, \theta_{n+j}^U]$	θ_{n+j}	$\Delta\theta_j$	$\Delta\theta_j\%$	Rank
1	Lo Tung	[0:004, 0:067]	0.024	[0:005, 0:079]	0.029	+0.005	+20.83%	4
2	Hsin Chu	[0:036; 0:168]	0.079	[0:017, 0:118]	0.049	-0.030	-37.98%	7
3	Tung Shi	[0:004; 0:107]	0.037	[0:005, 0:102]	0.037	0.000	0.00%	5
4	Nan Tou	[0:000; 0:221]	0.072	[0:000, 0:092]	0.030	-0.042	-58.33%	8
5	Chia Yi	[0:017; 0:186]	0.072	[0:006, 0:269]	0.092	+0.020	+27.78%	2
6	Pin Tung	[0:020; 0:269]	0.101	[0:019, 0:354]	0.128	+0.027	+26.73%	3
7	Tai Tung	[0:000; 0:279]	0.091	[0:020; 0:182]	0.073	-0.018	-19.78%	6
8	Hua Lien	[0:003; 0:105]	0.036	[0:003; 0:147]	0.050	+0.014	+38.89%	1
Total		-	0.512	-	0.488	-0.024	-4.69%	-

Kao (2000) used two methods for estimating the MPI scores of DMUs according to the common weight and global common weight methods. The results of these methods can be seen in the Table 3. As can be seen, there are many differences between these results and ours. We expect these differences, because our models are based on an optimistic-pessimistic attitude whereas the Kao's models are based on optimistic approach. Also, the Kao's method does not deal with the productivity situation of the set of DMUs.

Table 3. The MPI in the Kao's approach.

DMU		Common-weights method		Global Common-weights method	
j	District	MPI Score	Rank	MPI Score	Rank
1	Lo Tung	0.9496	2	1.4342	1
2	Hsin Chu	0.7777	8	1.0060	6
3	Tung Shi	0.8640	6	1.1630	3
4	Nan Tou	0.7969	7	0.9691	7
5	Chia Yi	0.8807	4	1.1074	4
6	Pin Tung	0.9805	1	1.1772	2
7	Tai Tung	0.8670	5	0.9488	8
8	Hua Lien	0.9330	3	1.0356	5

5. CONCLUSION

In this study, only single model is applied to calculate productivity index instead of using four different frontier facets, so all DMUs have a common basis for comparison. Our approach can be used, not only for comparing the performance of a unit in two time periods, but also for comparing the performance of two different units at the same or different time periods. Also, our method measures the efficiency score by using an optimistic-pessimistic attitude. Finally, the proposed method can be used for determining the total productivity change of the set of all DMUs between two time periods. In the future researches, the proposed model can be developed to solve the similar problem with imprecise data.

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