International Journal of Operations Research

International Journal of Operations Research Vol. 12, No. 1, 001–006(2015)

The Optimistic-Pessimistic Ranking in the Chance Constrained DEA

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Received August 2014; Revised February 2015; Accepted February 2015

Abstract **—** Recently, Khodabakhshi and Aryavash have introduced a ranking method [Applied Mathematics Letters, 25 (2012) 2066-2070.], which is based on an optimistic-pessimistic approach of data envelopment analysis (DEA). This method ranks all decision making units according to a combination of their minimum and maximum possible efficiency scores which are determined by solving two linear programming models. This method can only rank decision making units (DMUs) with deterministic input and output indexes, whereas in the real application the data are often imprecise. In this paper, the chance constrained version of this ranking method is presented.

Keywords **—** Data envelopment analysis, Stochastic ranking model, Chance constrained programming, Optimistic-pessimistic approach

1. INTRODUCTION

Data envelopment analysis (DEA) is a decisional technique for performance analysis of decision making units (DMUs). This mathematical instrument was introduced by Charnes et al. (1978) and then developed by other researchers. Nowadays, DEA is widely used in management science, operational research, system engineers, decision analysis and so on. One of the applications of DEA is to rank DMUs according to their efficiencies. The traditional DEA models divide DMUs into two efficient and inefficient groups, whereas in practice, there is often a need to fully rank them. This is one of the shortcomings of the traditional DEA models to rank DMUs. Another drawback of the traditional DEA models is that they cannot discriminate among DMUs when the number of inputs and outputs is too high relative to the number of DMUs. In such situations, the real position of some DMUs cannot be reflected. Furthermore, the traditional DEA models evaluate the DMUs only from the optimistic viewpoint. In the other words, they do not pay attention to the weak points of DMUs. This lack of consideration from the pessimistic attitude misses some important information about the performance of DMUs.

So far, several ranking method have been introduced in the DEA context. A thorough review on ranking methods up 2002 can be found in Adler et al. (2002). Sexton et al. (1986) proposed the ranking method of DMUs based on a cross-efficiency ratio matrix. Cook et al. (1992) developed prioritization models to rank only the efficient units in DEA. Andersen and Petersen (1993) proposed super-efficiency models for ranking only efficient units in the DEA (see also Li et al. (2007)). The benchmarking ranking of efficient DMUs initially developed by Torgersen et al. (1996). Cooper and Tone (1997) ranked the DMUs according to scalar measures of inefficiency in DEA, based on the slack variables. Jahanshahloo et al. (2007) proposed a DEA ranking system based on changing the reference set. Liu and Peng (2008) proposed the common weights analysis (CWA) to determine a set of indices for common weights to rank efficient DMUs of DEA. Khodabakhshi and Aryavash (2012) introduced another ranking method which is based on both pessimistic and optimistic attitudes of DEA.

Although the DEA models have many advantages, they don't allow variations in input and output data. Recently, some models have been presented which incorporate data variation in the DEA models. Jahanshahloo et al. (2009) proposed a method which Ranks DMUs by l_1-norm with fuzzy data in DEA. Kao and Liu (2003) designed a mathematical programming approach to fuzzy efficiency ranking. Ma and Li (2008) introduced a fuzzy ranking method with range reduction techniques. Also, Zerafat Angiz et al. (2010) proposed another ranking method in the fuzzy environment.

Chance constraints are incorporated in DEA in Olesen and Petersen (1999) leading to a stochastic (chance constrained) programming model. The traditional DEA model envelopes observed input-output vectors for each DMU and the chance constrained DEA model envelopes confidence regions for each DMU. This paper analysis the relationship between the probability levels specified in the chance constraints and the corresponding probability levels defining the confidence regions. The efficiency by stochastic data has been analyzed by Sengupta (2000). In addition Morita and Seiford (1999) have discussed DEA efficiency reliability and probability. Also, Cooper et al. (1996) and Cooper et al. (1998) introduced stochastic formulation of the original models which incorporate possible uncertainty in the inputs and/or outputs. Jess et al. (2001) introduced a semi-infinite programming model in DEA to study an interesting chemical engineering problem. Khodabakhshi

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(2007) presented a super-efficiency model based on improved outputs in DEA. Also, Khodabakhshi (2010a, 2011) and Khodabakhshi et al. (2010b) proposed some super efficiency models in the stochastic environment. Khodabakhshi and Asgharian (2009) presented an input relaxation measure of efficiency in stochastic DEA. In addition, Khodabakhshi (2009) estimated most productive scale size with stochastic data in DEA.

Recently, Khodabakhshi and Aryavash (2012, 2014a,b) have introduced an optimistic-pessimistic approach in the DEA literature which can be applied for ranking DMUs. In this approach, DMUs are ranked according to a combination of the minimum and maximum possible efficiency scores of DMUs which are computed by solving two linear programming models. The main idea of this method is to estimate the efficiency scores of DMUs under the assumption that the sum of scores of all DMUs equals to one. This method does not have the mentioned limitations of the pervious DEA based ranking methods. In this paper, this ranking method is extended with stochastic inputs and outputs. Then a deterministic equivalent for this model is obtained and converted to a quadratic program.

The rest of the paper has been structured as follows. In Section 2, the optimistic-pessimistic ranking method is briefly demonstrated. In Section 3, this ranking method is extended by according its chance constrained programming formulations. In Section 4, our method is illustrated using an example. In Section 5, we conclude the paper with a summary and a sketch of further research opportunities.

2. THE OPTIMISTIC-PESSIMISTIC APPROACH

Assume that there are n decision making units DMU_j $(j = 1,\cdots,n)$ which convert m inputs x_{ij} (i = 1,…, m) into s outputs y_{rj} (r = 1,…, s) and DMU_0 is an under evaluation DMU. Also, suppose that all inputs and outputs are non-negative deterministic numbers. Khodabakhshi and Aryavash (2012, 2014a, b) introduced a new DEA models which determines the efficiency scores of DMU_0 under the assumption that the sum of scores of all DMUs equals to unity, this means that:

$$\sum_{j=1}^{m} \theta_j = 1 \tag{1}$$

Based on this assumption, they have derived the following linear programming model:

$$\begin{aligned} & Min \;\; and \;\; Max \;\; \theta_o = \textstyle\sum_{r=1}^{s} y_{ro} u_r \\ & s.t. \textstyle\sum_{i=1}^{m} x_{ij} w_{ij} - \sum_{r=1}^{s} y_{rj} u_r = 0, \;\; j = 1,\ldots,n \\ & \quad \textstyle\sum_{i=1}^{m} x_{io} \;\; v_i = 1 \\ & \quad \textstyle\sum_{j=1}^{n} w_{ij} = v_i, \; i = 1,\ldots,m \\ & \quad w_{ij} \;, u_r, \, v_i \geq 0 \end{aligned} \tag{2}$$

This model must be run two times. First, θ_o is minimized to determine its minimum value (θ_o^L) and then θ_o is maximized to determine its maximum value (θ_o^U). In fact, these scores are obtained using the pessimistic and optimistic viewpoints, respectively.

Now, we turn to the dual (envelopment) model associated with model (2). Considering the objective function of this model, two dual models must be obtained, one for minimizing problem and the other for maximizing problem. The dual models of (2) can be written as follows:

$$\begin{aligned} & \theta_o^U = \text{Min } \theta \\ & s.t. \textstyle\sum_{i=1}^{m} \;\; y_{rj}\lambda_j \;\; \geq \;\; y_{ro} \quad r = 1,\ldots,s \\ & \quad x_{ij} \;\; \lambda_j \leq x_{io}\theta_o, \; i = 1,\ldots,m \;, \; j = 1,\ldots,n \\ & \quad \lambda_j \geq 0, \; j = 1,\ldots,n, \end{aligned} \tag{3}$$

and,

$$\begin{aligned} & \theta_o^L = \text{Max } \theta \\ & s.t. \textstyle\sum_{i=1}^{m} \;\; y_{rj}\lambda_j \;\; \leq \;\; y_{ro} \quad r = 1,\ldots,s \\ & \quad x_{ij} \;\; \lambda_j \geq x_{io}\theta_o, \; i = 1,\ldots,m \;, \; j = 1,\ldots,n \\ & \quad \lambda_j \geq 0, \; j = 1,\ldots,n, \end{aligned} \tag{4}$$

The minimum (θ_o^L) and maximum (θ_o^U) possible efficiency scores of DMU_0 are respectively estimated using the models (3,4). The models (3,4) are respectively called optimistic and pessimistic models. In fact, the (θ_o^L) and (θ_o^U) are obtained using the pessimistic and optimistic viewpoints, respectively. Hence, we have the following interval for the efficiency score of DMU_j:

$$\theta_j^L \le \theta_j \le \theta_j^U, \; j = 1,\ldots,n. \tag{5}$$

The intervals (5) can be rewritten as following convex combinations:

$$\theta_j = \theta_j^L \lambda_j + \theta_j^U (1-\lambda_j), \; 0 \le \lambda_j \le 1, \; j = 1,\ldots,n. \tag{6}$$

To aggregate θ_j^L and θ_j^U into a single score, a value of interval [0, 1] must be assigned to parameter λ_j. To determine the efficiency of DMUs in an *equitable* way, the values of all λ_j must be *equally* selected. So, we must have $\lambda_1 = \ldots = \lambda_n$ in (6). Hence, the system of n equations in the 2n variables equations (6) is transformed to the following system of n equations with $n+1$ variables:

$$\theta_j = \theta_j^L \lambda + \theta_j^U (1-\lambda), \; 0 \le \lambda \le 1, \; j = 1,\ldots,n. \tag{7}$$

On the other hand, based on assumption (1), we have $\sum_{j=1}^n \theta_j = 1$. Therefore, the values of all θ_j can be determined by solving the following system of n + 1 linear equations with n + 1 variables:

$$\begin{cases} \theta_j = \theta_j^L \lambda + \theta_j^U (1-\lambda), \; j = 1,\ldots,n. \\ \sum_{j=1}^n \theta_j = 1 \end{cases} \tag{8}$$

We rank DMUs according the answer of this system which are depicted by $\theta_j^M \; (j = 1,\cdots,n)$.

3. THE CHANCE CONSTRAINED RANKING

In what follows, we introduce stochastic version of the models (3) and (4). Let $\tilde{x}_{ij}$ $i = 1,\ldots,m$ and $\tilde{y}_{rj}$ $r = 1,\ldots,s$ be the random inputs and outputs related to $DMU_j \; (j = 1,\cdots,n)$. These components have been deemed to be normally distributed that $\tilde{x}_{ij} \sim N(x_{ij},(\sigma_{ij}^I)^2)$ and $\tilde{y}_{rj} \sim N(y_{rj},(\sigma_{rj}^o)^2)$. In accordance with the notation which have been proposed in Cooper et al. (2004), the chance constrained version of model (3) is as follows:

$$\begin{aligned} &\theta_o^U = \text{Min}\ \theta \\ &s.t.\ P\{\textstyle\sum_{i=1}^m \tilde{y}_{rj}\lambda_j \ge \tilde{y}_{ro}\} \ge 1\text{-}\alpha \quad r = 1,\ldots,s \\ &\quad P\{\tilde{x}_{ij}\lambda_j \le \tilde{x}_{io}\theta_o\} \ge 1-\alpha, \; i = 1,\ldots,m, \; j = 1,\ldots,n \\ &\quad \lambda_j \ge 0, \; j = 1,\ldots,n, \end{aligned} \tag{9}$$

In this model, P means probability, α is a predetermined value between 0 and 1 which specifies the significance level. Now, we exploit the normality assumption to introduce a deterministic equivalent to this model. For each $r \in \{1,\ldots,s\}$ we have

$$\begin{aligned} 1\text{-}\alpha \le P\{\textstyle\sum_{i=1}^m \tilde{y}_{rj}\lambda_j \ge \tilde{y}_{ro}\} &= P\{\dfrac{\sum_{i=1}^m \tilde{y}_{rj}\lambda_j - \tilde{y}_{ro} - \sum_{i=1}^m y_{rj}\lambda_j + y_{ro}}{\sigma_r^o} \ge \dfrac{-\sum_{i=1}^m y_{rj}\lambda_j + y_{ro}}{\sigma_r^o}\} \\ &= P\{Z \ge \dfrac{-\sum_{i=1}^m y_{rj}\lambda_j + y_{ro}}{\sigma_r^o}\} = 1 - \varphi(\dfrac{-\sum_{i=1}^m y_{rj}\lambda_j + y_{ro}}{\sigma_r^o}) \end{aligned} \tag{10}$$

which,

$$\begin{aligned} (\sigma_r^o)^2 &= Var\,(\sum_{j=1}^n \tilde{y}_{rj}\lambda_j - \tilde{y}_{ro}) \\ &= \sum_{j=1, j\ne o}^n \lambda_j^2 (\sigma_{rj}^o)^2 + (\lambda_o - 1)^2 (\sigma_{ro}^o)^2 + 2\sum_{i=1, i\ne o}^n \sum_{j=1, j\ne o, i<j}^n \lambda_i\lambda_j Cov\,(\tilde{y}_{ri},\tilde{y}_{rj}) + 2\sum_{j=1, j\ne o}^n (\lambda_o - 1)\lambda_j Cov\,(\tilde{y}_{rj},\tilde{y}_{ro}) \end{aligned} \tag{11}$$

The inequality (10) can be rewritten as follows:

$$\varphi(\frac{-\sum_{i=1}^m y_{rj}\lambda_j + y_{ro}}{\sigma_r^o}) \le \alpha \Rightarrow -\textstyle\sum_{i=1}^m y_{rj}\lambda_j + y_{ro} \le \varphi^{-1}(\alpha)\sigma_r^o \Rightarrow \sum_{i=1}^m y_{rj}\lambda_j + \varphi^{-1}(\alpha)\sigma_r^o \ge y_{ro} \tag{12}$$

In inequality (12), φ is the cumulative distribution function (cdf) of a standard Normal random variable and φ^{-1} is its inverse. Similarly, for each $i \in \{1,\ldots,m\}$ and for each $j \in \{1,\ldots,n\}$, we have:

$$\begin{aligned}1\text{-}\alpha \le P\{\tilde{x}_{ij}\lambda_j \le \tilde{x}_{io}\theta\} &= P\{\frac{\tilde{x}_{ij}\lambda_j - \tilde{x}_{io}\theta - x_{ij}\lambda_j + \mathrm{x}_{io}\theta}{\sigma^I_{ij}} \le \frac{-x_{ij}\lambda_j + \mathrm{x}_{io}\theta}{\sigma^I_{ij}}\} \\ &= P\{Z \le \frac{-x_{ij}\lambda_j + \mathrm{x}_{io}\theta}{\sigma^I_{ij}}\} = \varphi(\frac{-x_{ij}\lambda_j + \mathrm{x}_{io}\theta}{\sigma^I_{ij}})\end{aligned} \tag{13}$$

which,

$$(\sigma^I_{ij})^2 = Var\,(\tilde{x}_{ij}\lambda_j - \tilde{x}_{io}\theta) = \lambda_j^{\,2}(\sigma^I_{ij})^2 + \theta^2(\sigma^I_{io})^2 + 2\lambda_j\theta\ Cov\ (\tilde{x}_{ij}, \tilde{x}_{io}) \tag{14}$$

The inequality (13) can be rewritten as follows:

$$\varphi(\frac{-x_{ij}\lambda_j + \mathrm{x}_{io}\theta}{\sigma^I_{ij}}) \ge 1\text{-}\alpha \Rightarrow -x_{ij}\lambda_j + \mathrm{x}_{io}\theta \ge -\sigma^I_{ij}\varphi^{-1}(\alpha) \Rightarrow x_{ij}\lambda_j - \varphi^{-1}(\alpha)\sigma^I_{ij} \le \mathrm{x}_{io}\theta \tag{15}$$

Therefore, the deterministic equivalent of model (9) can be represented by:

$$\begin{aligned}&\theta^U_o = \text{Min}\ \theta \\ &s.t.\ \textstyle\sum_{i=1}^m y_{rj}\lambda_j + \varphi^{-1}(\alpha)\sigma^o_r \ge y_{ro},\ r = 1,\dots,s \\ &\quad x_{ij}\lambda_j - \varphi^{-1}(\alpha)\sigma^I_{ij} \le \mathrm{x}_{io}\theta,\ i = 1,\dots,m,\ j = 1,\dots,n \\ &\quad \lambda_j \ge 0,\ j = 1,\dots,n,\end{aligned} \tag{16}$$

Also, it can be shown that the corresponding stochastic version of the model (4) is as follows:

$$\begin{aligned}&\theta^L_o = \text{Max}\ \theta \\ &s.t.\ P\{\textstyle\sum_{i=1}^m \tilde{y}_{rj}\lambda_j \le \tilde{y}_{ro}\} \ge 1\text{-}\alpha\ \ r = 1,\dots,s \\ &\quad P\{\tilde{x}_{ij}\lambda_j \ge \tilde{x}_{io}\theta_o\} \ge 1-\alpha,\ i = 1,\dots,m,\ j = 1,\dots,n \\ &\quad \lambda_j \ge 0,\ j = 1,\dots,n,\end{aligned} \tag{17}$$

In a similar way, it can be shown that the deterministic equivalent of model (17) can be represented by:

$$\begin{aligned}&\theta^L_o = \text{Max}\ \theta \\ &s.t.\ \textstyle\sum_{i=1}^m y_{rj}\lambda_j + \varphi^{-1}(\alpha)\sigma^o_r \le y_{ro},\ r = 1,\dots,s \\ &\quad x_{ij}\lambda_j - \varphi^{-1}(\alpha)\sigma^I_{ij} \ge \mathrm{x}_{io}\theta,\ i = 1,\dots,m,\ j = 1,\dots,n \\ &\quad \lambda_j \ge 0,\ j = 1,\dots,n,\end{aligned} \tag{18}$$

Using the aforementioned property of normal distribution, and replacing non-negative variables w^I_{ij} and w^O_{rj}, respectively, by $\sigma^I_{ij}(\lambda_j, \theta)$ and $\sigma^O_{ij}(\lambda)$ we obtain the following deterministic models which are quadratic programs:

$$\begin{aligned}&\theta^U_o = \text{Min}\ \theta \\ &s.t.\ \textstyle\sum_{i=1}^m y_{rj}\lambda_j + \varphi^{-1}(\alpha)\, w^o_r \ge y_{ro} \\ &\quad x_{ij}\lambda_j - \varphi^{-1}(\alpha)\, w^I_{ij} \le \mathrm{x}_{io}\theta \\ &\quad (w^o_r)^2 = \sum_{j=1, j\ne o}^n \lambda_j^2(\sigma^o_{rj})^2 + (\lambda_o - 1)^2(\sigma^o_{ro})^2 + 2\sum_{i=1, i\ne o}^n \sum_{j=1, j\ne o, i<j}^n \lambda_i\lambda_j Cov\,(\tilde{y}_{ri}, \tilde{y}_{rj}) + 2\sum_{j=1, j\ne o}^n (\lambda_o - 1)\lambda_j Cov\,(\tilde{y}_{rj}, \tilde{y}_{ro}) \\ &\quad (w^I_{ij})^2 = \lambda_j^2(\sigma^I_{ij})^2 + \theta^2(\sigma^I_{io})^2 + 2\lambda_j\theta\ Cov\ (x_{ij}, x_{io}) \\ &\quad \lambda_j, w^o_r, w^I_{ij} \ge 0,\end{aligned} \tag{19}$$

and

$$\begin{aligned}&\theta^L_o = \text{Max}\ \theta \\ &s.t.\ \textstyle\sum_{i=1}^m y_{rj}\lambda_j + \varphi^{-1}(\alpha)\, w^o_r \le y_{ro} \\ &\quad x_{ij}\lambda_j - \varphi^{-1}(\alpha)\, w^I_{ij} \ge \mathrm{x}_{io}\theta \\ &\quad (w^o_r)^2 = \sum_{j=1, j\ne o}^n \lambda_j^2(\sigma^o_{rj})^2 + (\lambda_o - 1)^2(\sigma^o_{ro})^2 + 2\sum_{i=1, i\ne o}^n \sum_{j=1, j\ne o, i<j}^n \lambda_i\lambda_j Cov\,(\tilde{y}_{ri}, \tilde{y}_{rj}) + 2\sum_{j=1, j\ne o}^n (\lambda_o - 1)\lambda_j Cov\,(\tilde{y}_{rj}, \tilde{y}_{ro}) \\ &\quad (w^I_{ij})^2 = \lambda_j^2(\sigma^I_{ij})^2 + \theta^2(\sigma^I_{io})^2 + 2\lambda_j\theta\ Cov\ (x_{ij}, x_{io}) \\ &\quad \lambda_j, w^o_r, w^I_{ij} \ge 0.\end{aligned} \tag{20}$$

4. A numerical example

In this section, our method is illustrated using a numerical example. In this example, there are four DMUs with one input and two outputs. We assume that all indexes are stochastic variables with normal distribution. The mean values of these variables are shown in the columns (2-4) of Table 1. Also, we assume that the input and outputs of all DMUs have the same variance

Table 1: The data and results of example

DMU	x_1	y_1	y_2	$[\theta^L, \theta^U]$	θ^M	Rank
1	1	0.5	4	[0.001,0.546]	0.357	2
2	2	2	5	[0.352,0.476]	0.433	1
3	5	2	1	[0.028,0.191]	0.134	3
4	6	1	3	[0.070,0.079]	0.076	4

which is $(\sigma_{ij}^I)^2 = (\sigma_o^O)^2 = 0.01$. To compute the results of models (19,20) we have chosen $\alpha = 0.55$ for which $\varphi^{-1} = 0.12$. Furthermore, we assume that the input and outputs for different DMUs are independent. This independence assumption then implies that $Cov\left(\tilde{x}_{ij}, \tilde{x}_{ik}\right) = Cov\left(\tilde{y}_{rj}, \tilde{y}_{rk}\right) = 0$.

Running the models (19,20) on these data, the intervals of scores of DMUs have been estimated which can be seen in the fifth column of Table 1. Then, the integrated scores of DMUs have been determined by solving the system (8) and depicted in the sixth column of the table. Finally, DMUs have been ranked according to their aggregated scores as can be seen in the last column of the table.

The interval score of the first DMU is $[0.001, 0.546]$, this means that the minimum and maximum possible efficiency scores of this DMU at the significance level $\alpha = 0.55$ are respectively $\theta_1^L = 0.001$ and $\theta_1^U = 0.546$. According to these values, this DMU has the worst performance from the pessimistic viewpoint and the best performance from the optimistic viewpoint. So, only one of these attitudes cannot portray the real position of this DMU, so we should use both attitudes. To this end, these values are aggregated to the score $\theta_1^M = 0.357$ by using the system (8), and so, DMU_1gain the second position of ranking. The score θ_1^M obviously depicts the relative efficiency of DMU_1 using both of its week and strong points. Hence, it is more logical and reliable than the efficiency scores which are based on only one of these attitudes.

5. Conclusion

The proposed method of this study has some advantages in comparison with the other ranking methods. In this approach, the weights of inputs and outputs are endogenous. Also, it is based on the both optimistic and pessimistic attitudes of DEA, so its results can be more reliable and equitable than the ranking methods which are based on only one of these attitudes. This model has high discrimination power, and can be easily used when the number of inputs and outputs is too high relative to the number of DMUs. Furthermore, one can get a full rank of all DMU using this approach. Finally, in the future researches, the presented method can be developed to rank the DMUs with ordinal data.

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