International Journal of
Operations Research

International Journal of Operations Research Vol. 12, No. 1, 007–015 (2015)

Negative Data in the Centralized Resource Allocation Model

GhasemTohidi[1] , Hamed Taherzadeh[1*]

[1] Department of mathematics, Islamic Azad University, Central Tehran Branch, Tehran, Iran.

Received September 2014; Revised February 2015; Accepted February 2015

Abstract — Data Envelopment Analysis (DEA) is a technique based on mathematical programming for evaluating the efficiency of homogeneous Decision Making Units (DMUs). In this technique, inefficient DMUs are projected onto a frontier that has been constructed from the best performing DMUs. Centralized Resource Allocation (CRA) is a method in which all DMUs are projected onto the efficiency frontier through solving just one DEA model. The CRA model is an appropriate method for evaluating the efficiency score of systems that contain a numbers of DMUs, such as banks, chain restaurants, and university departments. This paper focuses on developing the CRA model to include negative data, by using an Ideal Point for the system as a whole, based on semi- positive inputs and outputs. This concept, which will be discussed throughout the paper, is illustrated by two numerical examples.

Keywords —Data envelopment analysis, Centralized resource allocation, Ideal point, Negative data.

1. INTRODUCTION

Data Envelopment Analysis (DEA) is a non-parametric Linear Programming (LP)-based methodology for evaluating the efficiency score of a number of similarly processing Decision Making Units (DMUs). It was introduced by Farrell, (1957) and developed by Charnes et al., (1978).

Following the influential work of Farrell on productive efficiency, many studies have focused on various aspects of DEA, and there has been much literature dealing with the interface between research, economics and management.

Applications and discussions of DEA have mainly assumed that all the inputs and outputs of DMUs are necessarily positive. There are however many scenarios where this is not the case, such as in the analysis of financial statements (Smith, 1990 and Ferzo et al., 2003),and the rating of mutual funds (Murthi et al.,1997). Therefore the question of handling negative data has attracted the attention of many researchers. Traditionally, negative inputs or outputs have been dealt with by using efficiency applications that employ data transformation. All the values of a given variable can be added successfully to a positive large number. All negative data are therefore turned into positive data (Pastor, 1994, and Lovell, 1995). In spite of this transformation of negative data, this solution can have other implications (Seiford et al., 2002).

Negative data can be subdivided into two types. The first type includes negative data whose variables are measured on a ratio scale that has a natural zero (Portela et al., 2004). The second type includes negative data whose variables are measured on an interval or ordinal scale that can have any natural zero (Ueda et al., 1997). Portela et al. (2004) developed an important approach to deal with negative input and output. At the same time they introduced a Range Directional Distance Model (RDM), which is a non-oriented model that looks for input contraction and output expansion. The RDM model is based on the directional distance function (Chung et al., 1997). One of the advantages of the directional distance function is flexibility, due to the range of its direction vectors. The RDM model utilizes both unit and translation invariants, but it never identifies all sources of inefficiency.

In many real situations, there are cases in which all DMUs are under the control of a centralized Decision Maker (DM), who oversees DMUsand whose tendency is to increase the efficiency of the systems a whole, rather than improving each unit separately. Cases of this sort occur when all the units belong to the same organization (public and/or private), which provides the units with the necessary resources to obtain their outputs, such as bank branches, restaurant chains, hospitals, university departments, and schools. Thus, the DM's goal is to optimize the resource utilization of all DMUs across the total entity. Lozano and Villa, (2004) first introduced the concept of centralized resource allocation, presenting the envelopment and multiplier form of the BCC model. There are other similar studies, such as Korhonen et al., (2004), Du et al., (2010) and Asmild et al., (2009).

Korhonen et al., (2004) used a multiple-objective approach in order to optimize the efficiency of a given system, while Du et al., (2010) proposed another approach for optimization in the centralized scenario. Asmild et al., (2009) reformulated the centralized model proposed by Lozano & Vila (2004, 2005), and considered adjustments of inefficient units. Hosseinzadeh Lotfi et al., (2010) and Yu et al., (2013) reviewed different research engaged with centralized resource allocation.

* Corresponding author's email: H.taherzadeh@hotmail.com

This paper develops the centralized resource allocation model to include negative data. The idea behind this paper is to make an IP for the DM's overall system. The rest of this paper is structured as following: Section 2 focuses on the original model in centralized resource allocation and the RDM model with regard to negative data. In section 3, a centralized model is proposed that includes data that have a negative value. In section 4 the proposed model is analyzed using two examples. The last section sums up and draws conclusions.

2. BACKGROUND

2.1 RDM Model for Dealing with Negative Data

Portela et al., (2004) outlined one of the first DEA models for dealing with negative data. They proposed an approach for evaluating a number of DMUs when some input and/or some outputs are negative. Their model, which is called Range Direction Model (RDM), can yield a measure of efficiency similar to the radial models in traditional DEA models. In addition, the RDM model is a particular case of the generic directional distance model, which was introduced by Chambers et al.,(1996, 1998).

To start with, consider there are n $DMUs$ which are indexed by $j \in \{1,\ldots,n\}$ and the performance of each DMU is characterized by a production process of m inputs $\left(x_{ij};\ i=1,..,m\right)$ to yield s outputs $\left(y_{rj}; r=1,\ldots,s\right)$. To estimate a DEA efficiency score of a specific oth DMU the generic directional distance model is shown as follows:

$$\begin{aligned}
&\text{Max } \beta \\
&s.t. \\
&\sum_{j=1}^{n}\lambda_j x_{ij} \le x_{io} - \beta g_{xi} \qquad i=1,\ldots,m \\
&\sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro} + \beta g_{y_r} \qquad r=1,\ldots,s \\
&\sum_{j=1}^{n}\lambda_j = 1 \\
&g_{y_r} \ge 0, g_{xi} \ge 0, \lambda_j \ge 0, \qquad j=1,\ldots,n \;\; i=1,\ldots,m \;\; r=1,\ldots,s.
\end{aligned} \tag{1}$$

In model (1), (g_{xi}, g_{y_r}) referred to the range of possible improvement of evaluation unitsandcan be chosen arbitrarily, but a common choice is observed input and output levels of positive data, respectively. In a negative data scenario, model (1) had to be modified. Portela et al., (2004) adapted model (1) by defining IP. Mathematically, IP is defined as follows:

$$\begin{cases}\text{IP's input ith}=\left(\min_{1\le j\le n}\{x_{ij}\}\right) & i=1,2,\ldots,m \\ \text{IP's output rth}=\left(\max_{1\le j\le n}\{y_{rj}\}\right) & r=1,2,\ldots,s.\end{cases} \tag{2}$$

(g_{xi}, g_{y_r}) will be chosen regarding to under evaluation of DMU_o as follows:

$$\begin{cases} g_{io} = x_{io} - \min_{1\le j\le n}\{x_{ij}\} & i=1,\ldots,m. \\ g_{ro} = \max_{1\le j\le n}\{y_{rj}\} - y_{ro} & r=1,\ldots,s.\end{cases} \tag{3}$$

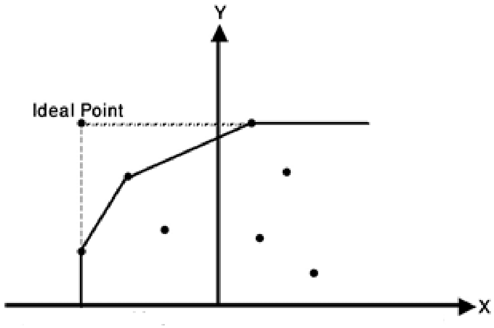

Figure 1: Ideal Point for one input and one output case

IP is shown in the Figure1. Therefore, the generic directional distance model has been modified to include negative data (Portela et al., 2004) as follows:

$$
\begin{aligned}
&\text{Max } \beta \\
&s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} - \beta R_{xi} && i = 1,\dots,m \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \beta R_{y_r} && r = 1,\dots,s \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1 \\
&\qquad \lambda_j \ge 0, && j = 1,\dots,n
\end{aligned}
\tag{4}
$$

where $R_{io} = x_{io} - \left(\min_{1\le j\le n}\{x_{ij}\}\right)$ and $R_{ro} = \left(\max_{1\le j\le n}\{y_{rj}\}\right) - y_{ro}$ are parameters in the model. The advantage of the RDM model (which can deal with negative data) over the generic directional distance model is that it uses a unit invariant, and it yields inefficiency scores between 0 and 1. Moreover, the RDM model has both translation and unit invariants.

2.2 Centralized Resource Allocation (CRA) Model

Performance evaluation is an important consideration for a DM for finding weaknesses in the system,in order to make improvements subsequently. Working with the usual DEA framework, the first phase of the CRA input-oriented (CRA-I) model developed by Lozano & Vila, (2004) evaluated the efficiency of a system using the following model:

$$
\begin{aligned}
&\min \ \theta \\
&s.t. \\
&\sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{jk} x_{ij} \le \theta \sum_{j=1}^{n} x_{ij} && i = 1,\dots,m \\
&\sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{jk} y_{sj} \ge \sum_{j=1}^{n} y_{rj} && r = 1,\dots,s \\
&\sum_{j=1}^{n} \lambda_{jk} = 1 && k = 1,\dots,n \\
&\lambda_{jr} \ge 0 && k, j = 1,\dots,n.
\end{aligned}
\tag{5}
$$

In the Phase II of the CRA model, additional reduction of any inputs or expansion of any output is followed. As is usual with the radial models in DEA, the Phase II of the CRA model for removing any possible input excesses and output shortfalls is formulated through retaining our knowledge of θ^* (obtained by the previous model) as follows:

$$
\begin{aligned}
&\max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} t_r^+ \\
&s.t. \quad \sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{jk} x_{ij} + s_i^- = \theta^* \sum_{j=1}^{n} x_{ij} && i = 1,\dots,m \\
&\qquad \sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{jk} y_{sj} - t_r^+ = \sum_{j=1}^{n} y_{rj} && r = 1,\dots,s \\
&\qquad \sum_{j=1}^{n} \lambda_{jk} = 1 && k = 1,\dots,n \\
&\qquad s_i^- \ge 0 && i = 1,\dots,m \\
&\qquad t_r^+ \ge 0 && r = 1,\dots,s \\
&\qquad \lambda_{jk} \ge 0 && k, j = 1,\dots,n.
\end{aligned}
\tag{6}
$$

Model (5) was formulated on the basis of two important aims. First, instead of reducing the inputs of each DMU, the aim is to reduce the total amount of input consumption of all the DMUs taken together. Second, after solving the problem in Phase II, all DMUs will be projected onto the efficiency frontier. It should be noted that the efficiency score of a whole system is more important than the efficiency score of any single unit in the centralized scenario. For that reason, the DM tries to reallocate resources in order to have more efficient system. Toward this end, some of the inputs can be transferred from one DMU to other DMUs. The improvement activity of DMU_o is defined as follows:

$$\begin{aligned} \overline{x_{io}} &= \sum_{j=1}^{n} \lambda_{jo}^{*} x_{ij} = \theta^{*} x_{io} - s_{i}^{-*} \quad && i = 1,\ldots,m \\ \overline{y_{ro}} &= \sum_{j=1}^{n} \lambda_{jo}^{*} y_{rj} = y_{ro} + t_{r}^{+*} \quad && r = 1,\ldots,s. \end{aligned} \tag{7}$$

The differences between the total consumption of the improved activity and the original DMUs can be previewed by the following relationship:

$$\begin{aligned} S_{i} &= \sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} \overline{x_{ij}} \geq 0 \quad && i = 1,\ldots,m \\ T_{r} &= \sum_{j=1}^{n} \overline{y_{ro}} - \sum_{j=1}^{n} y_{rj} \geq 0 \quad && r = 1,\ldots,s. \end{aligned} \tag{8}$$

The dual formulation of envelopment of the CRA input oriented model to find the common input and output weights that maximize the relative efficiency score of a virtual DMU with the average inputs and outputs can be written as follows:

$$\begin{aligned} \max \quad & \sum_{j=1}^{n} \sum_{r=1}^{s} u_{r} y_{rj} + \sum_{k=1}^{n} \zeta_{k} \\ s.t \quad & \sum_{j=1}^{n} \sum_{i=1}^{m} v_{i} x_{ij} = 1 \\ & \sum_{r=1}^{s} u_{r} y_{rj} - \sum_{i=1}^{m} v_{i} x_{ij} + \zeta_{k} \leq 0 \qquad && j,k = 1,\ldots,n \\ & u_{r} \geq 0 && r = 1,\ldots,s \\ & v_{i} \geq 0 && i = 1,\ldots,m. \end{aligned} \tag{9}$$

The above model has $n^2 + 1$ constraints and $m + s + n$ variables. Solving model (9) involves using the Common Set of Weights (CSW). It is worth mentioning that we can use this common set of weights to evaluate the absolute efficiency of each efficient DMU in order to rank them. The ranking adopts the common set of weights generated from model (9), which makes sense because a DM objectively chooses the common weights for the purpose of maximizing group efficiency. For instance, the government is interested in measuring the performance of DEA efficient banks. The government would determine one common set of weights based upon the group performance of DEA efficient banks.

3. Proposed Model

Before starting, we should define DM again. DM is someone who has a complete control over all of DMUs. In other words, DMUs are under DM's control and decisions. Accordingly, DM's inputs are all the inputs and DM's outputs are all the outputs, that is:

$$\begin{aligned} &\text{DM's Input ith} = \sum_{j=1}^{n} x_{ij} \quad (i = 1,\ldots,m) \\ &\text{DM's Output rth} = \sum_{j=1}^{n} y_{rj} (r = 1,\ldots,s) \end{aligned} \tag{10}$$

Since we are interested in evaluating the efficiency of a system, we make an IP for System (IPS) incorporating negative data. Finally, IPS is defined as follows:

$$\text{IPS:} \begin{cases} \text{Input ith} = \text{n.} \left(\min_{1 \leq j \leq n} \{ x_{ij} \} \right) & i = 1,\ldots,m \\ \text{Outputs rth} = \text{n.} \left(\max_{1 \leq j \leq n} \{ y_{rj} \} \right) & r = 1,\ldots,s. \end{cases} \tag{11}$$

In the above definition scalar n is the number of DMUs. The formula (11) means that IPS is the one whose DMUs are located in the IP. In other words, the ideal system is the one in which all DMUs are IP. Geometrically, Figure 2 portrays the situation of IPS in the typical case of one input and one output.
By considering the definition of IPS, Phase I of the RDM model can be reformulated in the centralized scenario as follows:

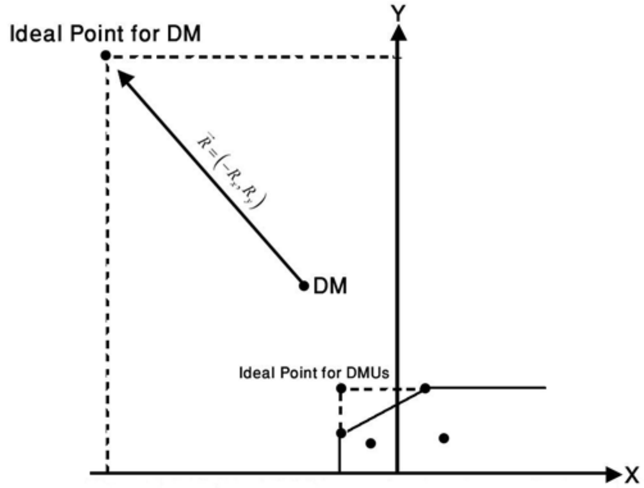

Figure2: DM's Ideal Point for one input and one output case

$$
\begin{aligned}
\text{RDMC:}\quad & \max\ \beta \\
\text{s.t.}\quad & \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij} \le \sum_{j=1}^{n}x_{ij} - \beta R_i && i=1,\ldots,m \\
& \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}y_{rj} \ge \sum_{j=1}^{n}y_{rj} + \beta R_r && r=1,\ldots,s \\
& \sum_{j=1}^{n}\lambda_{jk} = 1 && k=1,\ldots,n \\
& \lambda_{jk} \ge 0 && k,j=1,2,\ldots,n
\end{aligned} \tag{12}
$$

where $R_i = \sum_{j=1}^{n} x_{ij} - n\left(\min_{1\le j\le n}\{x_{ij}\}\right)$ and $R_r = n\left(\max_{1\le j\le n}\{y_{rj}\}\right) - \sum_{j=1}^{n} y_{rj}$. Model (12) involves $(n+1)$ variables and $(m+s+n)$ constraints and is feasible by $\left(\lambda_{jk} = 1 (j=k), \lambda_{jk} = 0 (j \ne k), \beta = 0\right)$. In Phase II, we solve the following linear programming model by retaining our knowledge concerning β^* :

$$
\begin{aligned}
& \max\ \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} t_r^+ \\
& \text{s.t.} \\
& \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}x_{ij} + s_i^- = \sum_{j=1}^{n}x_{ij} - \beta R && i=1,\ldots,m \\
& \sum_{k=1}^{n}\sum_{j=1}^{n}\lambda_{jk}y_{rj} - t_r^+ = \sum_{j=1}^{n}y_{rj} + \beta R_r && r=1,\ldots,s \\
& \sum_{j=1}^{n}\lambda_{jk} = 1 && k=1,\ldots,n \\
& \lambda_{jk} \ge 0 && k,j=1,2,\ldots,n
\end{aligned} \tag{13}
$$

The objective of Phase II is to find a solution that maximizes the sum of input excesses and output shortfalls while keeping $\beta = \beta^*$. Now, suppose that $(\beta^*, \lambda_{jk}^*(j,k=1,\ldots,n), s_1^{-*},\ldots,s_m^{-*}, t_1^{+*},\ldots,t_s^{+*})$ is the optimal solution, we thus define a formula for the improved activity $DMU_k (k=1,\ldots,n)$ via models (12) and (11) as follows:

$$
\begin{aligned}
\overline{x_{ik}} &= \sum_{j=1}^{n}\lambda_{jk}^* x_{ij} && i=1,\ldots,m \\
\overline{y_{rk}} &= \sum_{j=1}^{n}\lambda_{jk}^* y_{rj} && r=1,\ldots,s.
\end{aligned} \tag{14}
$$

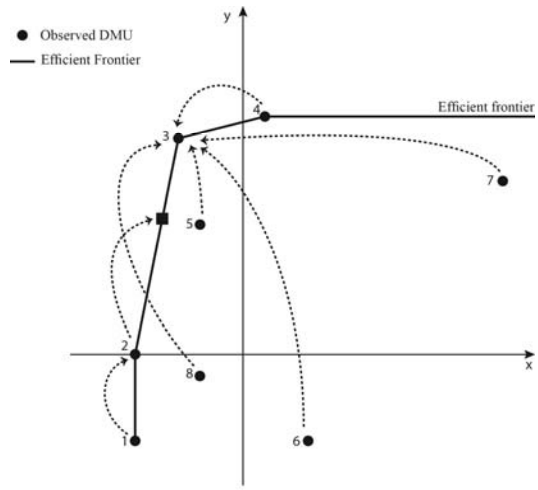

Figure 3: Geometric illustration of Example 1

Consequently, the projection point for the system can be obtained easily as follows:

$$\sum_{k=1}^{n} \overline{x_{ik}} = \sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{jk}^{*} x_{ij} \qquad i = 1,\dots,m$$
$$\sum_{k=1}^{n} \overline{y_{rk}} = \sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{jk}^{*} y_{rj} \qquad r = 1,\dots,s. \tag{15}$$

Table1: Data set of 8 DMUs

DMU	Input	Output	Projection point Input	Projection point Output	Reference set	Optimal Lambda
1	-5	-4	-5	0	DMU 2	$\lambda_2^{1*} = 1$
2	-5	0	-3.75	6.25	DMU 2,3	$\lambda_2^{2*} = 0.37, \lambda_3^{2*} = 0.63$
3	-3	10	-3	10	DMU 3	$\lambda_3^{3*} = 1$
4	1	11	-3	10	DMU 3	$\lambda_3^{4*} = 1$
5	-2	6	-3	10	DMU 3	$\lambda_3^{5*} = 1$
6	3	-4	-3	10	DMU 3	$\lambda_3^{6*} = 1$
7	12	8	-3	10	DMU 3	$\lambda_3^{7*} = 1$
8	-2	-3	-3	10	DMU 3	$\lambda_3^{8*} = 1$
Sum	1	24	-26.75	66.25		

Model (12) has one important property (the unit invariant) as well as the RDM model which is expressed and proved in the following theorem. It should be noticed that model (12) does not use a translation invariant.

Theorem 1: Model (12) utilizes a unit invariant.

Proof: Suppose that all levels of input i are multiplied by α_i, and of output r by β_r. This results in the following modified constraints of inputs: $\sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{kj}\alpha_i x_{ij} \le \sum_{j=1}^{n} \alpha_i x_{ij} - \beta\alpha_i R_i$ thatis equivalent to $\sum_{k=1}^{n}\sum_{j=1}^{n} \lambda_{kj} x_{ij} \le \sum_{j=1}^{n} x_{ij} - \beta R_i$ for each input. Proof for outputs can be proved in a way similar to the ones used for the input constraints.

■

In the next section, our proposed model with two numerical examples will be discussed. The first example is considered in one input and one output space due to geometrically interpreted results and the second one is the case of 13

Table 2: Data set of 13 DMUs

DMU	I1	I2	O1	O2	O3
DMU 1	1.03	-0.05	0.56	-0.09	-0.44
DMU 2	1.75	-0.17	0.74	-0.24	-0.31
DMU 3	1.44	-0.56	1.37	-0.35	-0.21
DMU 4	10.8	-0.22	5.61	-0.98	-3.79
DMU 5	1.3	-0.07	0.049	-1.08	-0.34
DMU 6	1.98	-0.1	1.61	-0.44	-0.34
DMU 7	0.97	-0.17	0.82	-0.08	-0.43
DMU 8	9.82	-2.32	5.61	-1.42	-1.94
DMU 9	1.59	0	0.52	0	-0.37
DMU 10	5.96	-0.15	2.14	-0.52	-0.18
DMU 11	1.29	-0.11	0.57	0	-0.24
DMU 12	2.38	-0.25	0.57	-0.67	-0.43
DMU 13	10.3	-0.16	9.56	-0.58	0

DMUs (Sharp et al., 2006).

4. Numerical illustration

4.1 Example 1

Consider a data set of 8 $DMUs$ each of which consumes one input to produce one output. Table 1 shows data for 8 DMUs and the results obtained in Phases I and II of the RDMC model. The efficiency score of the DM is 0.34. In other words, the inefficiency of the system is 0.66. By referring to Figure 3, all of the DMUs' projections are located on the efficiency frontier. Hence they are RDM efficient, and six DMUs are projected precisely onto DMU3. DMU2 is just projected onto the convex combination of DMU2 and DMU3. DMU1 is projected onto DMU2. As a result, in this small case of 8 DMUs, DMU3 is the most efficient DMU because 0.87 percent of units is projected onto DMU3. As expected, after projecting the DMUs the total consumption of the input is decreased from 1 to -26.75 and the total production of the output is increased from 24 to 66.25, which is favorable in management terms.

4.2 Example 2

Table 2 shows the data set from the notional effluent processing system extracted by Sharp et al., (2006). As can be seen, there are 13 DMUs, each focusing on one positive input (cost), one non-positive input (effluent), one positive output (saleable), and two non-positive outputs (Methane and CO2). Table 3 indicates the efficiency scores of $DMU_j (j = 1,..,13)$ measured by the RDM model and the overall efficiency score of the system, which is 0.11. IPS is introduced in the last row in Table 3 and can be interpreted as following: if DM wants to have the ideal system, he should decrease the total consumption of the first and second inputs from 50.61 to 12.61 and -4.33 to -30.16, respectively. Similar results can be obtained through outputs, requiring that the first, second and third output should increase from 30.17 to 124.28, -6.45 to 0 and -8.63 to 0, respectively. In a real case study, it is hard and sometimes impossible to increase one of the outputs from the current level of 30.17 to the best level of 124.28, but it could become one of the manager's future policypriorities. As can be seen from Table 4, there are five DMUs as a reference set, i.e. DMU2, DMU3, DMU7, DMU8 and DMU13. DMU1 and DMU2 are projected onto DMU13; DMU3 is projected onto the convex combination of DMU7 and DMU8; DMU4 is projected onto the convex combination of DMU3, DMU8 and DMU13. Other DMUs are projected onto DMU2. According to the results, we can identify DMU3 as the most efficient DMU, because the majority of DMUs are projected precisely onto DMU3 (about 79%). After projection, all the DMUs will be efficient on the RDM model. It is worth taking a look at the last row in Table 4. By using the CRDM model, the overall consumption of the first and second input are decreased from 50.61 to 46.28 and -7.27 to -4.33, respectively. Also, output 1, output 2 and output 3 are increased from 3.17 to 4.89, -6.45 to -5.72 and from -8.633 to -3.44, respectively.

Table3: RDM efficiency and Ideal point

DMUs	I1	I2	O1	O2	O3	RDM efficiency
DMU1	1.03	-0.05	0.56	-0.09	-0.44	0.9648
DMU2	1.75	-0.17	0.74	-0.24	-0.31	0.9181
DMU3	1.44	-0.56	1.37	-0.35	-0.21	1.0000
DMU4	10.8	-0.22	5.61	-0.98	-3.79	0.7352
DMU5	1.3	-0.07	0.49	-1.08	-0.34	0.9242
DMU6	1.98	-0.1	1.61	-0.44	-0.34	0.9708
DMU7	0.97	-0.17	0.82	-0.08	-0.043	1.0000
DMU8	9.82	-2.32	5.61	-1.42	-1.94	1.0000
DMU9	1.59	0	0.52	0	-0.37	0.9944
DMU10	5.96	-0.15	2.14	-0.52	-0.18	0.8595
DMU11	1.29	-0.11	0.57	0	-0.24	1.0000
DMU12	2.38	-0.25	0.57	-0.67	-0.43	0.8448
DMU13	10.3	-0.16	9.56	-0.58	0	1.0000
sum	50.61	-4.33	30.17	-6.45	-8.633	
Ideal point	0.97	-2.32	9.56	0	0	
DM's Ideal point	12.61	-30.16	124.28	0	0	

Table4: data projection

	xp(i1)	xp(i2)	yp(o1)	yp(o2)	yp(o3)	Optimal Lambda
DMU 1	10.3	-0.16	9.56	-0.58	0	$\lambda_{13}^{1*}=1$
DMU 2	10.3	-0.16	9.56	-0.58	0	$\lambda_{13}^{2*}=1$
DMU 3	5.72	-1.32	3.39	-0.8	-1.24	$\lambda_{7}^{3*}=0.46, \lambda_{8}^{3*}=0.56$
DMU 4	7	-0.59	6.05	-0.61	-0.33	$\lambda_{3}^{4*}=0.37, \lambda_{8}^{4*}=0.13, \lambda_{13}^{4*}=0.5$
DMU 5	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_{3}^{5*}=1$
DMU 6	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_{3}^{6*}=1$
DMU 7	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_{3}^{7*}=1$
DMU 8	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_{3}^{8*}=1$
DMU 9	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_{3}^{9*}=1$
DMU 10	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_{3}^{10*}=1$
DMU 11	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_{3}^{11*}=1$
DMU 12	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_{3}^{12*}=1$
DMU 13	1.44	-0.56	1.37	-0.35	-0.21	$\lambda_{3}^{13*}=1$
sum	46.28	-7.27	40.89	-5.72	-3.46	

5. Conclusion

There are many real situations where DMUs have negative data. The standard CRA model cannot be used for evaluating the efficiency score of systems, which involve units with negative data. This paper provides an approach to the problem of the centralized resource allocation model that takes account of negative data. Our proposed model shows that a new ideal point can be defined for the system and IPS. Moving forward in approaching the IPS, all the DMUs will be located on the efficiency frontier and overall consumption of inputs and production of outputs will be changed in an equitable way.

References

1. Ali, I, and Seiford, LM (1990).Translation invariance in data envelopment analysis. *Operations Research Letters*, 9:403–405.

2. Asmild, M., Paradi, J.C., and Pastor, J.T. (2009). Centralized resource allocation BCC models. *Omega* 37:40–49.
3. Charnes, A, Cooper, WW and Rhodes, E. (1978). Measuring efficiency of decision making units, *European Journal of Operational Research*, 2:429–444.
4. Chung, Y, Fare, R, and Grosskopf, S. (1997). Productivity and undesirable outputs: a directional distance function approach. *Journal of Environmental Management*, 51:229–240.
5. Du, J., Liang, L., Chen, Y., and Bi, G. (2010). DEA-based production planning. *Omega*, 38:105–112.
6. Fang, Lei., Zhang, C.-Q,. (2008). Resource allocation based on the DEA model. *Journal of the Operational Research Society*, 59 (8):1136–1141.
7. Farrell, M. (1957). The measurement of productive efficiency, Journal of the Royal Statistical Society Series A: General, 120 (3):253–281
8. Feroz, E., Kim, S., & Raab, R. (2003). Financial Statement Analysis: A Data Envelopment Analysis Approach, *Journal of the Operational Research Society*, 54 (1):48-58.
9. Hosseinzadeh Lotfi, F., Noora, A.A., Jahanshahloo, G.R., Geramia, J. and Mozaffari, M.R. (2010). Centralized resource allocation for enhanced Russell models. *Journal of Computational and Applied Mathematics*, 235 (1):1–10.
10. Korhonen, P., Syrjänen, M. (2004). Resource allocation based on efficiency analysis. *Management Science*, 50 (8):1134–1144.
11. Lovell CAK. (1995). Measuring the macroeconomic performance of the Taiwanese economy, *International journal of production Economics*, 39:165–178.
12. Lozano, S,. Villa, G. (2005). Centralized DEA models with the possibility of downsizing. *Journal of the Operational Research Society*, 56 (4):357–364.
13. Lozano, S., Villa, G. (2004). Centralized resource allocation using data envelopment analysis. *Journal of Productivity Analysis*, 22:143–161.
14. Murthi, B., Choi, Y. and Desai, P. (1997). Efficiency of Mutual Funds and Port-folio Performance Measurement: A Non-Parametric Approach, *European Journal of Operational Research*, 98 (2):408–418.
15. Pastor, J.T. (1994). *How to discount environmental effects in DEA: An application to bank branches*. Working paper No. 011/94, Depto. De Estadistica e Investigacion Operativa, Universidad de Alicante, Spain.
16. Seiford L.M, and Zhu, J,. (2002). Modeling undesirable factors in efficiency evaluation, *European Journal of Operational Research*, 142:16–20.
17. Sharp, J.A., Liu, W.B., and Meng, W. (2006). A modified slacks-based measure model for data envelopment analysis with 'natural' negative outputs and inputs. *Journal of Operational Research Society*, 57:1–6.
18. Portela, Silva MCA, Thanassoulis, E., and Simpson, G. (2004). Negative data in DEA: a directional distance approach applied to bank branches, *Journal of the Operational Research Society*, 55 (10):1111–1121.
19. Smith, P. (1990). Data Envelopment Analysis Applied to Financial Statements, *Omega*, 18 (2):131-138.
20. Ueda, T, and Hoshiai, Y. (1997). Application of component analysis for parsimonious summarization of DEA inputs and/or outputs, *Journals of Operational Research Society of Japan*, 40:466–478.
21. Yu, M.M., Chern, C.C., & Hsiao, B. (2013). Human resource rightsizing using centralized data envelopment analysis: evidence from Taiwan's airports, *Omega*, 41:119–130.