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Fuzzy Geometric Programming Approach in Multi-objective Multivariate Stratified Sample Surveys in Presence of Non-Response

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Abstract — In this paper, we have formulated the problem of non-response in multivariate stratified sample surveys as a Multi-Objective Geometric Programming problem (MOGPP). The fuzzy programming approach has described for solving the formulated MOGPP. The formulated MOGPP has been solved and the solution is obtained. The obtained solution is the dual solution corresponding to the multi-objective multivariate stratified sample surveys in presence of non-response. Afterward with the help of dual solution of formulated MOGPP and primal-dual relationship theorem the optimum allocation of sample sizes of respondents and non respondents are obtained. A numerical example is given to illustrate the procedure.

Keywords — geometric programming, fuzzy programming, multi-objective optimization, non-response, optimum allocation, multivariate stratified sampling

1. INTRODUCTION

In stratified sampling heterogeneous population is converted into a homogeneous population by dividing it into homogeneous stratum. The maximum precision will be obtained with the best choices of the sample sizes. The problem of optimum allocation in stratified random sampling for univariate population is well known in sampling literature; see for example Cochran (1977) and Sukhatme et al. (1984). In multivariate stratified sample survey the problem of non-response can appear when the required data are not obtained. The problem of non-response may occur due to the refusal by respondents or they are not at home making the information of sample inaccessible. The problem of non-response occurs in almost all surveys. The extent of non- response depends on various factors such as type of the target population, type of the survey and the time of survey. For dealing the problem of non-response the population is divided into two disjoint groups of respondents and non respondents. For the stratified sampling it may be assumed that every stratum is divided into two mutually exclusive and exhaustive groups of respondents and non respondents.

Hansen and Hurwitz (1946) presented a classical non-response theory which was first developed for the survey in which the first attempt was made by mailing the questionnaires and a second attempt was made by personal interview to a sub sample of the non respondents. They constructed the estimator for the population mean and derived the expression for its variance and also worked out the optimum sampling fraction among the non respondents. El-Badry (1956) further extended the Hansen and Hurwitz's technique by sending waves of questionnaires to the non respondent units to increase the response rate. The generalized El-Badry's approach for different sampling design was given by Foradari (1961). Srinath (1971) suggested the selection of sub samples by making several attempts. Khare (1987) investigated the problem of optimum allocation in stratified sampling in presence of non-response for fixed cost as well as for fixed precision of the estimate. Khan et al. (2008) suggested a technique for the problem of determining the optimum allocation and the optimum sizes of subsamples to various strata in multivariate stratified sampling in presence of non-response which is formulated as a nonlinear programming problem (NLPP). Varshney et al. (2011) formulated the multivariate stratified random sampling in the presence of non-response as a multi-objective integer nonlinear programming problem and a solution procedure is developed using lexicographic goal programming technique to determine the compromise allocation. Fatima and Ahsan (2011) address the problem of optimum allocation in stratified sampling in the presence of non-response.

Raghav et al. (2014) have discussed the various multi-objective optimization techniques in the multivariate stratified sample surveys in case of non-response

Geometric programming (GP) is a smooth, systematic and an effective non-linear programming method used for solving problems of sample surveys and engineering design that takes the form of convex programming. The convex

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programming problems occurring in GP are generally represented by an exponential or power function. GP has certain advantages over the other optimization methods because it is usually much simpler to work with the dual than the primal one. The degree of difficulty (DD) plays a significant role for solving a non-linear programming problem by GP method. Geometric Programming (GP) has been known as an optimization tool for solving the problems in various fields. Duffin, Peterson and Zener (1967) and also Zener (1971) have discussed the basic concepts and theories of GP with application in engineering in their books. Beightler, C.S., and Phililps, D.T., also published a famous book on GP and its application in (1976). Engineering design problems was also solved by Shiang (2008) and Shaojian et al. (2008) with the help of GP. Davis and Rudolph (1987) applied GP to optimal allocation of integrated samples in quality control.

Ahmed and Charles (1987) applied geometric programming to obtain the optimum allocations in multivariate double sampling. Maqbool et al. (2011), Shafiullah et al. (2013) have discussed the geometric programming approach for obtaining the optimum allocations in multivariate two-stage and three-stage sample surveys respectively.

In many real-world decision-making problems of sample surveys, environmental, social, economical and technical areas are of multiple-objectives problems. Multi-objective optimization problems differ from single-objective optimization. It is significant to realize that multiple objectives are often non-commensurable and in conflict with each other in optimization problems. The fuzzy goal is defined as the objective which can be obtained within exact target value. The multi-objective models with fuzzy objectives are more realistic than deterministic of it. The concept of fuzzy set theory was firstly given by Zadeh (1965). Later on, Bellman and Zadeh (1970) used the fuzzy set theory to the decision-making problem. Tanaka (1974) introduces the objective as fuzzy goal over the α-cut of a fuzzy constraint set and Zimmermann (1978) gave the concept to solve multi-objective linear-programming problem. Biswal (1992) and Verma (1990) developed fuzzy geometric programming technique to solve multi-objective geometric programming (MOGP) problem. Islam (2005, 2010) has discussed modified geometric programming problem and its applications and also another fuzzy geometric programming technique to solve MOGPP and their applications. Fuzzy mathematical programming has been applied to several fields.

In this paper, we have formulated the problem of non-response in multivariate stratified sample surveys as a multi-objective geometric programming problem (MOGPP). The fuzzy geometric programming approach has described for solving the formulated MOGPP and optimum allocation of sample sizes of respondents and non respondents are obtained. A numerical example is given to illustrate the procedure.

2. FORMULATION OF THE PROBLEM

In stratified sampling the population of N units is first divided into L non-overlapping subpopulation called strata, of sizes $N_1, N_2, ..., N_h, ..., N_L$ with $\sum_{h=1}^{L} N_h = N$ and the respective sample sizes within strata are denoted by $n_1, n_2, ..., n_h, ..., n_L$ with $\sum_{h=1}^{L} n_h = n.$

Let for the h^{th} stratum:

N_h : denote the stratum size.

$\overline{Y}_h$: Stratum mean.

S_h^2 : Stratum variance.

$W_h = \frac{N_h}{N}$: Stratum weight.

N_{h1} : be the sizes of the respondents.

$N_{h2} = N_h - N_{h1}$: be the sizes of non respondents groups.

n_h : Units are drawn from the h^{th} stratum. Further let out of n_h, n_{h1} units belong to the respondents group.

$n_{h2} = n_h - n_{h1}$: Units belong to the non respondents group.

$n = \sum_{h=1}^{L} n_h$: The total sample size.

A more careful second attempt is made to obtain information on a random subsample of size r_h out of n_{h2} non respondents for the representation from the non respondents group of the sample.

$r_h = \frac{n_{h2}}{k_h}$; $h = 1, 2, ..., L$: Subsamples of sizes at the second attempt to be drawn from n_{h2} non–respondent group of the h^{th} stratum. Where $k_h \geq 1$ and $\frac{1}{k_h}$ denote the sampling fraction among non respondents.

Since N_{h1} and N_{h2} are random variables hence their unbiased estimates are given as

$\hat{N}_{h1} = \dfrac{n_{h1} N_h}{n_h}$: Unbiased estimates of the respondents group.

$\hat{N}_{h2} = \dfrac{n_{h2} N_h}{n_h}$: Unbiased estimate of the non respondents group.

$\bar{y}_{jh1}$; $j=1,...,p$: denote the sample means of j^{th} characteristic measured on the n_{h1} respondents at the first attempt.

$\bar{y}_{jh2(r_h)}$; $j=1,...,p$: denote the r_h sub sampled units from non respondents at the second attempt.

Using the estimator of Hansen and Hurwitz (1946), the stratum mean $\bar{Y}_{jh}$ for j^{th} characteristic in the h^{th} stratum may be estimated by

$$\bar{y}_{jh(w)} = \frac{n_{h1}\bar{y}_{jh1} + n_{h2}\bar{y}_{jh2(r_h)}}{n_h} \tag{1}$$

It can be seen that $\bar{y}_{jh(w)}$ is an unbiased estimate of the stratum mean $\bar{Y}_{jh}$ of the h^{th} stratum for the j^{th} characteristic with a variance.

$$v(\bar{y}_{jh(w)}) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_{jh}^2 + \frac{W_{h2}^2 S_{jh2}^2}{r_h} - \frac{W_{h2}^2 S_{jh2}^2}{n_h} \tag{2}$$

where S_{jh}^2 is the stratum variance of j^{th} characteristic in the h^{th} stratum; $j=1,2,...,p,\ h=1,2,...,L.$ given as:

$$S_{jh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{jhi} - \bar{Y}_{jh}\right)^2$$

where y_{jhi} denote the value of the i^{th} unit of the h^{th} stratum for j^{th} characteristic. $\bar{Y}_{jh} = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{jhi}$: is the stratum mean of y_{jhi}. S_{jh2}^2 is the stratum variance of the j^{th} characteristic in the h^{th} stratum among non respondents, given by:

$$S_{jh2}^2 = \frac{1}{\hat{N}_{h2} - 1}\sum_{i=1}^{\hat{N}_{h2}}\left(y_{jhi} - \bar{Y}_{jh2}\right)^2,$$

$\bar{Y}_{jh2} = \frac{1}{\hat{N}_{h2}}\sum_{i=1}^{\hat{N}_{h2}} y_{jhi}$ is the stratum mean of y_{jhi} among non respondents. $W_{h2} = \frac{N_{h2}}{N_h}$ is stratum weight of non respondents in h^{th} stratum.

If the true values of S_{jh}^2 and S_{jh2}^2 are not known they can be estimated through a preliminary sample or the value of some previous occasion, if available, may be used. Furthermore, the variance of $\bar{y}_{j(w)} = \sum_{h=1}^{L} W_h \bar{y}_{jh(w)}$, (ignoring fpc) is given as:

$$V(\bar{y}_{j(w)}) = \sum_{h=1}^{L} W_h^2\, v(\bar{y}_{jh(w)}) = \sum_{h=1}^{L}\frac{W_h^2\left(S_{jh}^2 - W_{h2}S_{jh2}^2\right)}{n_h} + \sum_{h=1}^{L}\frac{W_h^2 W_{h2} S_{jh2}^2}{r_h} \tag{3}$$

where $\bar{y}_{j(w)}$ is an unbiased estimate of the overall population mean $\bar{Y}_j$ of the j^{th} characteristic and $V(\bar{y}_{jh(w)})$ is as given in Eqn.2.

Assuming a linear cost function the total cost C of the sample survey may be given as:

$$C = \sum_{h=1}^{L} c_{h0} n_h + \sum_{h=1}^{L} c_{h1} n_{h1} + \sum_{h=1}^{L} c_{h2} n_{h2}$$

where c_{h0} = the per unit cost of making the first attempt, $c_{h1} = \sum_{j=1}^{p} c_{jh1}$ is the per unit cost for processing the results of all the p characteristics on the n_{h1} selected units from respondents group in the h^{th} stratum in the first attempt and $c_{h2} = \sum_{j=1}^{p} c_{jh2}$ is the per unit cost for measuring and processing the results of all the p characteristics on the r_h units selected from the non respondents group in the h^{th} stratum in the second attempt. Also, c_{jh1} and c_{jh2} are per unit costs of measuring the j^{th} characteristic in first and second attempts respectively. As n_{h1} is not known until the first attempt has been made, the quantity $W_{h1} n_{h1}$ may be used as its expected value. The total expected cost $\hat{C}$ of the survey may be given as:

$$\hat{C} = \sum_{h=1}^{L} (c_{h0} + c_{h1} W_{h1}) n_h + \sum_{h=1}^{L} c_{h2} r_h \tag{4}$$

The problem therefore reduces to find the optimal values of sample sizes of respondents n_h^* and non-respondents r_h^* which are expressed as:

$$\left.\begin{array}{l} Min\left(f_{0j}\right) = \sum_{h=1}^{L} \frac{W_h^2\left(S_{jh}^2 - W_{h2} S_{jh2}^2\right)}{n_h} + \sum_{h=1}^{L} \frac{W_h^2 W_{h2} S_{jh2}^2}{r_h} \\ Subject\ \ to \\ \quad \sum_{h=1}^{L} (c_{h0} + c_{h1} W_{h1}) n_h + \sum_{h=1}^{L} c_{h2} r_h \le C_0 \\ \quad n_h,\ r_h \ge 0\ \ and\ \ n_h, r_h \text{ are integers} \end{array}\right\}, j = 1, 2, \ldots, p \tag{5}$$

3. MOGPP FORMULATION OF SAMPLE SURVEYS PROBLEM IN PRESENCE OF NON-RESPONSE

Geometric programming always transforms the primal problem of minimizing a "posynomial" subject to "posynomial" constraints to a dual problem of maximizing a function of the weights on each constraint. Posynomial functions can be defined as polynomials in several variables with positive coefficients in all terms and the power to which the variables are raised can be any real number.

The mathematical formulation of problem (5) can be rewritten as:

$$\left.\begin{array}{l} Min\ \ f_{0j} = \sum_{h=1}^{L} \frac{\psi_{1h}}{n_h} + \sum_{h=1}^{L} \frac{\psi_{2h}}{r_h} \\ Subject\ to \\ \sum_{h=1}^{L} C_h n_h + \sum_{h=1}^{L} C'_h r_h \le C_0 \\ n_h,\ r_h \ge 0\ \ and\ \ n_h, r_h\ are\ integers \end{array}\right\}, \ j = 1,2,..., p \tag{6}$$

where $C_h = (c_{h0} + c_{h1} W_{h1}),\ C'_h = c_{h2}$,

If q = 1, let the function ψ_{qh} be define as, $\psi_{1h} = W_h^2\left(S_{jh}^2 - W_{h2} S_{jh2}^2\right)$, and if q = 2, then $\psi_{2h} = W_h^2 W_{h2} S_{jh2}^2$, where q is the number of functions in objective function. The above expression (6) can be expressed in the standard Primal GPP as follows:

$$\left.\begin{array}{l} Max\ \ f_{0j}(n, r),\ \ j = 1,2,..., p \\ Subject\ \ \ f(n, r) \le 1 \\ n_h,\ r_h \ge 0,\ \ h = 1, 2, \cdots, L \end{array}\right\} \tag{7}$$

where $f_{0j}\left(n, r\right) = \sum_{h=1}^{L} \frac{\psi_{1h}}{n_h} + \sum_{h=1}^{L} \frac{\psi_{2h}}{r_h}, j = 1, 2, \ldots, p$ and $f\left(n, r\right) = \sum_{h=1}^{L} C_h n_h + \sum_{h=1}^{L} C'_h r_h$ are in the form of posynomial functions, where the posynomial function is given as:

$$f(n, r) = \sum_{h=1}^{L} \xi_{1h} \left[\prod_{h=1}^{L} n_h^{p_{11h}}\right] + \sum_{h=1}^{L} \xi_{2h} \left[\prod_{h=1}^{L} r_h^{p_{12h}}\right],\ \xi_{qh}, \ge 0, n_h, r_h \ge 0, q = 1,\ 2 \tag{8}$$

where ξ_{qh} are normalized constants. If q = 1, let the function ξ_{qh} be define as, $\xi_{1h} = \frac{C_h}{C_0}$ and if q = 2, then $\xi_{2h} = \frac{C'_h}{C_0}$, where q is the total number of functions in the constraint.

If q be the number of terms in the problem. Then the number of posynomial terms in objective function $f_{0j}(n, r)$ can be denoted by qh. For the above problem of sample surveys, q = 2 as n_h *and* r_h are two different variables corresponding to the h^{th} strata. Therefore, the total number of posynomial terms for the discussed problem will be 2h and h= 1, 2,…, L.

Similarly, the total numbers of posynomial terms corresponding to the primal constraint are denoted by 2h as n_h and r_h are two different variables and the exponents p_{0ih} *and* p_{1ih} are real constants corresponding to the objective functions and constraints functions respectively.

The dual form of standard Primal MOGPP which is stated in (7) can be given as:

$$\left.\begin{array}{ll} Max\ v_{0j}(w_{0i}^{*}) = \prod_{i=1}^{q}\prod_{h=1}^{L}\left\{\left(\frac{\psi_{ih}}{w_{0ih}}\right)^{w_{0ih}}\right\}\prod_{i=1}^{q}\prod_{h=1}^{L}\left\{\left(\frac{\xi_{ih}}{w_{1ih}}\right)^{w_{1ih}}\right\}\left(\sum_{i=1}^{q}\sum_{h=1}^{L}w_{1ih}\right)^{\sum_{i=1}^{q}\sum_{h=1}^{L}w_{1ih}} & (i) \\ Subject\ \sum_{i=1}^{q}\sum_{h=1}^{L}w_{0ih} = 1 & (ii) \\ \sum_{i=1}^{q}\sum_{h=1}^{L}p_{0ih}w_{0ih} + \sum_{i=1}^{q}\sum_{h=1}^{L}p_{1ih}w_{1ih} = 0 & (iii) \\ w_{0ih} \geq 0, w_{1ih} \geq 0,\ i = q\ and\ \ h = 1,\ldots,L & (iv) \end{array}\right\} j = 1,\ldots,p \qquad (9)$$

where $w_{0ih}'s\ \ and\ \ w_{1ih}'s$ are the dual variables corresponding to the objective functions and constraints functions.

The above formulated MOGPP (9) can be solved in the following two-steps:

Step 1: For the Optimum value of the objective function, the objective function always takes the form:

$$v_{0j}(w_{0i}^{*}) = \left(\frac{Coefficent\ \ of\ first\ term}{w_1}\right)^{w_1} \times \left(\frac{Coefficent\ \ of\ second\ \ term}{w_2}\right)^{w_2} \times \ldots\ldots \times \left(\frac{Coefficent\ \ of\ last\ \ term}{w_L}\right)^{w_L}$$

$$\left(\sum w's\, in\, the\ \ first\, constraints\right)^{\sum w's\, in\, the\ \ first\, constraints} \times \left(\sum w's\, in\, the\ \ last\, constraints\right)^{\sum w's\, in\, the\ \ last\, constraints}$$

The Multi-Objective objective function for our problem is:

$$\prod_{i=1}^{q}\prod_{h=1}^{L}\left\{\left(\frac{\psi_{ih}}{w_{0ih}}\right)^{w_{0ih}}\right\}\prod_{i=1}^{q}\prod_{h=1}^{L}\left\{\left(\frac{\xi_{ih}}{w_{1ih}}\right)^{w_{1ih}}\right\}\left(\sum_{i=1}^{q}\sum_{h=1}^{L}w_{1ih}\right)^{\sum_{i=1}^{q}\sum_{h=1}^{L}w_{1ih}} \qquad (10)$$

where $\xi_{1h} = \frac{C_h}{C_0}$ and $\xi_{2h} = \frac{C'_h}{C_0}.\xi_{qh}$ are normalized constants. $q = 1,2.\ ;\ h = 1,2,\ldots,L.$

Step 2: The equations that can be used for MOGPP for the weights are given below: $\sum_{i=1}^{q}\sum_{h=1}^{L}w_{0ih}$ in the objective function= 1(Normality condition ,see 9(ii)) and for each primal variable n_i and r_i having qh terms.

$$\sum_{i=1}^{q}\sum_{h=1}^{L}\left(w_{0ih}\text{ for each term in objective function }\right)\times\left(\text{exponent on } n_h \text{ and } r_h \text{ in objective function}\right)\times$$
$$\sum_{i=1}^{q}\sum_{h=1}^{L}\left(w_{1ih}\text{ for each term in constraints function }\right)\times\left(\text{exponent on } n_h \text{ and } r_h \text{ in constraints function}\right) = 0$$

(Orthogonality condition, see 9(iii))

and $w_{0ih} \geq 0, w_{1ih} \geq 0$ (Positivity condition, see 9(iv)).

4. FGP APPROACH IN SAMPLE SURVEYS IN PRESENCE OF NON- RESPONSE

The solution procedure to solve the problem (6) consists of the following steps:

Step 1: Solve the MOPP as a single objective problem using only one objective at a time and ignoring the others. These solutions are known as ideal solution.

Step 2: From the results of step-1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{cc} & \begin{array}{cccccc} f_{01}(n,r) & f_{02}(n,r) & \cdots & f_{0j}(n,r) & \cdots & f_{0p}(n,r) \end{array} \\ \begin{array}{c} (n^{(1)},r^{(1)}) \\ (n^{(2)},r^{(2)}) \\ \vdots \\ (n^{(j)},r^{(j)}) \\ \vdots \\ (n^{(p)},r^{(p)}) \end{array} & \begin{bmatrix} f_{01}^{*}(n^{(1)},r^{(1)}) & f_{02}(n^{(1)},r^{(1)}) & \cdots & f_{0j}(n^{(1)},r^{(1)}) & \cdots & f_{0p}(n^{(1)},r^{(1)}) \\ f_{01}(n^{(2)},r^{(2)}) & f_{02}^{*}(n^{(2)},r^{(2)}) & \cdots & f_{0j}(n^{(2)},r^{(2)}) & \cdots & f_{0p}(n^{(2)},r^{(2)}) \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ f_{01}(n^{(j)},r^{(j)}) & f_{02}(n^{(j)},r^{(j)}) & \cdots & f_{0j}^{*}(n^{(j)},r^{(j)}) & \cdots & f_{0p}(n^{(j)},r^{(j)}) \\ \vdots & \vdots & & \vdots & & \vdots \\ f_{01}(n^{(p)},r^{(p)}) & f_{02}(n^{(p)},r^{(p)}) & \cdots & .f_{0j}(n^{(p)},r^{(p)}) & \cdots & f_{0p}^{*}(n^{(p)},r^{(p)}) \end{bmatrix} \end{array}$$

Here $(n^{(1)},r^{(1)}),(n^{(2)},r^{(2)}),\cdots,(n^{(j)},r^{(j)}),\cdots,(n^{(p)},r^{(p)})$ are the ideal solutions of the objective functions

$$f_{01}(n^{(1)}, r^{(1)}),\ f_{02}(n^{(2)}, r^{(2)}), \cdots,\ f_{0j}(n^{(j)}, r^{(j)}), \cdots,\ f_{0p}(n^{(p)}, r^{(p)}) .$$

So $U_j = Max\left\{f_{01}(n^{(1)}, r^{(1)}),\ f_{02}(n^{(2)}, r^{(2)}),\ \cdots, f_{0p}(n^{(p)}, r^{(p)})\right\}$ and $L_j = f^*_{0j}(n^{(j)}, r^{(j)}),\ j=1,2,..., p$.

$\left[U_j\right.$ and L_j be the upper and lower bonds of the j^{th} objective function $\left.f_{0j}(n,r),\ j=1,2,..,p.\right]$

Step 3: The membership function for the given problem can be define as:

$$\mu_j\left(f_{0j}(n,r)\right) = \begin{cases} 0\ , & if\ \ f_{0j}(n,r) \geq U_j \\ \dfrac{U_j(n,r) -\ f_{0j}(n,r)}{U_j(n,r) - L_j(n,r)}, & if\ L_j \leq f_{0j}(n,r) \geq U_j,\ \ j = 1, 2,\ \ldots, p \\ 1,\ if\ f_{0j}(n,r) \leq L_j & \end{cases} \tag{11}$$

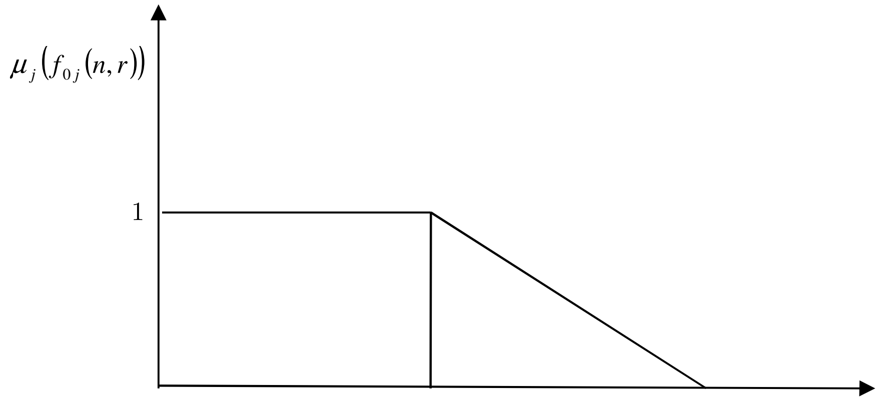

Figure 1: Membership function for minimization variance problem

The membership functions in Eqn. (11) i.e., $\mu_j\left(f_{0j}\left(n,r\right)\right),\ j = 1, 2, \cdots, p$. Therefore the general aggregation function can be defined as $\mu_{\tilde{D}}\left(n,r\right) = \mu_{\tilde{D}}\left\{\mu_1\left(f_{01}\left(n,r\right)\right), \mu_2\left(f_{02}\left(n,r\right)\right), \ldots, \mu_p\left(f_{0p}\left(n,r\right)\right)\right\}$.

The fuzzy multi-objective formulation of the problem can be defined as:

$$\left.\begin{array}{l} Max\ \mu_{\tilde{D}}(n,r) \\ \text{Subject to } \sum_{h=1}^{L} C_1 n_h + \sum_{h=1}^{L} C_2 r_h \leq 1; \\ n_h\ ,\ r_h \geq 0 \ \text{ and } \ n_h\ , r_h \text{ are integers,} \end{array}\right\}, j=1,2,..., p \tag{12}$$

The problem to find the optimal values of $(n^*,\ r^*)$ for this there are two types of fuzzy decision operators and they (1)

(i) Fuzzy decision based on max-min operator (like Zimmermann's approach (1978)). Therefore the problem (12) is reduced to the following problems according to max-min operator

$$\left.\begin{array}{l} Max\ \alpha \\ Subject\ to\ \ \mu_j\left(f_{0j}(n,r)\right) \geq \alpha \\ \sum_{h=1}^{L} C_1 n_h + \sum_{h=1}^{L} C_2 r_h \leq 1; \\ n_h\ ,\ r_h \geq 0. and \ \ n_h\ , r_h\ are \text{ integers,} \\ j=1,2,..., p,\ 0 \leq \alpha \leq 1. \end{array}\right\} \tag{13}$$

(ii) Convex-fuzzy decision based on addition operator (like Tewari et al. (1987)). Therefore the problem (12) is reduced according to max-addition operator as

$$\left.\begin{aligned}&\text{Max}\,\mu_D\left(n^*,r^*\right)=\sum_{j=1}^{p}\mu_j\left(f_{0j}(n,r)\right)=\sum_{j=1}^{p}\frac{U_j-\left(f_{0j}(n,r)\right)}{U_j-L_j}\\&\text{Subject to}\quad\sum_{h=1}^{L}C_1n_h+\sum_{h=1}^{L}C_2r_h\le 1;\\&\qquad 0\le\mu_j\left(f_{0j}(n,r)\right)\le 1,\ \ and\ \ n_h,\ r_h\ge 0,\\&\qquad n_h,r_h\ \text{are integers and}\ j=1,2,\cdots,p.\end{aligned}\right\}\tag{14}$$

The above problem (14) reduces to

$$\left.\begin{aligned}&\text{Max}\,\mu_D\left(n^*,r^*\right)=\sum_{j=1}^{p}\left\{\frac{U_j}{U_j-L_j}-\frac{\left(f_{0j}(n,r)\right)}{U_j-L_j}\right\}\\&\text{Subject to}\\&f_j(n,r)=\sum_{h=1}^{L}C_1n_h+\sum_{h=1}^{L}C_2r_h\le 1;\\&n_h,\ r_h\ge 0\ \text{and}\ \ n_h,r_h\ \text{are integers},\ j=1,2,...,p.\end{aligned}\right\}\tag{15}$$

The problem (15) maximizes if the function $\left\{\frac{\left(f_{0j}(n,r)\right)}{U_j-L_j}\right\}$ attain the minimum values. Therefore the problem (15) reduce into the problem (16) define as

$$\left.\begin{aligned}&Min\ \sum_{j=0}^{p}\left\{\frac{\left(f_{0j}(n,r)\right)}{U_j-L_j}\right\}\\&Subject\ \ to\\&f_j(n,r)=\sum_{h=1}^{L}C_1n_h+\sum_{h=1}^{L}C_2r_h\le 1;\\&n_h,\ r_h\ge 0\ and\ n_h,r_h\ \text{are integers},\end{aligned}\right\},\ j=1,2,...,p.\tag{16}$$

The problem (16) has been solved with the help of steps (1-2) discuss in section (3) and the corresponding solutions w_{0i}^* is the unique solution to the dual constraints, it will also maximize the objective function for the dual problem. Next, the solution of the primal problem will be obtained using primal-dual relationship theorem which is given below:
Primal-dual relationship theorem: If w_{0i}^* is a maximizing point for dual problem (9), each minimizing points

$$\left(n_1,n_2,n_3,n_4\ \text{and}\ r_1,r_2,r_3,r_4\right)$$

for primal problem (6) satisfies the system of equations:

$$f_{0j}(n,r)=\left\{\begin{aligned}&w_{0j}^*\,v\left(w^*\right), && j\in J[0],\\&\frac{w_{ij}}{v_L\left(w_{0i}^*\right)}, && i\in J[L],\end{aligned}\right\}\tag{17}$$

where L ranges over all positive integers for which $v_L\left(w_{0i}^*\right)>0$. The optimal values of respondents n_h^* and non-respondents r_h^* can be calculated with the help of the primal – dual relationship theorem (17).

5. NUMERICAL ILLUSTRATION

A numerical example is given to demonstrate the proposed method. The values of $S_{jh2}^2\ and\ S_{jh}^2$ and are practically unknown. Their values on some previous occasion may be used. It is assumed that the relative values of the stratum variances among the non respondents at the second attempt to the corresponding over all stratum variances are $\frac{S_{jh2}^2}{S_{jh}^2}=0.25$; h = 1, 2,…,L and j = 1,2,…, p. This ratio has been taken as 0.25 in the example for the sake of simplicity. Practically this ratio may vary from stratum to stratum and from characteristic to characteristic. Consider a population of size N = 3850 divided into four strata. The two characteristics are defined on each unit of the population and the population means are to be estimated. The available information is shown in the given table.

h	N_h	S_{1h}^2	S_{2h}^2	w_{h1}	w_{h2}	c_{h0}	c_{h1}	c_{h2}
1	1214	4817.72	8121.15	0.7	0.30	1	2	3
2	822	6251.26	7613.52	0.80	0.20	1	3	4
3	1028	3066.16	1456.4	0.75	0.25	1	4	5
4	786	6207.25	6977.72	0.72	0.28	1	5	6

Table 1: Data for four Strata and two characteristics

For solving MOGPP by using fuzzy programming, we shall first solve the two sub-problems:
Sub problem1: On substituting the table values in sub-problem 1, we have obtained the expressions given below:

$$\left.\begin{array}{l} Min\ f_{01} = \dfrac{456.3344}{n_1} + \dfrac{261.8965}{n_2} + \dfrac{209.5529}{n_3} + \dfrac{230.9097}{n_4} + \\ \quad \dfrac{11.10002688}{r_1} + \dfrac{2.75680566}{r_2} + \dfrac{3.492547875}{r_3} + \dfrac{4.866484}{r_4} \\ Subject\ \ to \\ \quad 0.00048n_1 + 0.00068n_2 + 0.0008n_3 + 0.00092n_4 + \\ \quad 0.0006r_1 + 0.0008r_2 + 0.001r_3 + 0.0012r_4 \le 1 \\ \quad n_h \ge 0\ ,\ r_h \ge 0\ ;\ \left(h = 1,2,\ldots,L\right) \end{array}\right\} \tag{18}$$

The dual of the above problem (18) is obtained as:

$$\left.\begin{array}{ll} max\ v(w_{0i}^*) = \left(\left(456.3344 / w_{01}\right)^{w_{01}}\right) \times \left(\left(261.8965 / w_{02}\right)^{w_{02}}\right) \times \left(\left(209.5529 / w_{03}\right)^{w_{03}}\right) & \\ \quad \times \left(\left(230.9097 / w_{04}\right)^{w_{04}}\right) \times \left(\left(11.100027 / w_{05}\right)^{w_{05}}\right) \times \left(\left(2.756806 / w_{06}\right)^{w_{06}}\right) & \\ \quad \times \left(\left(3.492548 / w_{07}\right)^{w_{07}}\right) \times \left(\left(4.866484 / w_{08}\right)^{w_{08}}\right) \times \left(\left(\dfrac{0.00048}{w_{11}}\right)^{w_{11}}\right) \times \left(\left(\dfrac{0.00068}{w_{12}}\right)^{w_{12}}\right) & (i) \\ \quad \times \left(\left(\dfrac{0.0008}{w_{13}}\right)^{w_{13}}\right) \times \left(\left(\dfrac{0.00092}{w_{14}}\right)^{w_{14}}\right) \times \left(\left(\dfrac{0.0006}{w_{15}}\right)^{w_{15}}\right) \times \left(\left(\dfrac{0.0008}{w_{16}}\right)^{w_{16}}\right) & \\ \quad \times \left(\left(\dfrac{0.001}{w_{17}}\right)^{w_{17}}\right) \times \left(\left(\dfrac{0.0012}{w_{18}}\right)^{w_{18}}\right) & \\ \quad \times ((\mathrm{w}_{11}+\mathrm{w}_{12}+\mathrm{w}_{13}+\mathrm{w}_{14}+\mathrm{w}_{15}+\mathrm{w}_{16}+\mathrm{w}_{17}+\mathrm{w}_{18})\hat{}(\mathrm{w}_{11}+\mathrm{w}_{12}+\mathrm{w}_{13}+\mathrm{w}_{14}+\mathrm{w}_{15}+\mathrm{w}_{16}+\mathrm{w}_{17}+\mathrm{w}_{18})); & \\ Subject\ \ to & \\ \quad w_{01} + w_{02} + w_{03} + w_{04} + w_{05} + w_{06} + w_{07} + w_{08} = 1;\ \text{(normality condition)} & (ii) \\ \quad \left.\begin{array}{l} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0; \\ -w_{03} + w_{13} = 0; \\ -w_{04} + w_{14} = 0; \\ -w_{05} + w_{15} = 0; \\ -w_{06} + w_{16} = 0; \\ -w_{07} + w_{17} = 0; \\ -w_{08} + w_{18} = 0; \end{array}\right\} \text{(orthogonality condition)} & (iii) \\ \quad \left.\begin{array}{l} w_{01}, w_{02}, w_{03}, w_{04}, w_{05}, w_{06}, w_{07}, w_{08} > 0; \\ w_{11}, w_{12}, w_{13}, w_{14}, w_{15}, w_{16}, w_{17}, w_{18} \ge 0 \end{array}\right\} \text{(positivity condition)} & (iv) \end{array}\right\} \tag{19}$$

For orthogonality condition defined in expression 19(iii) are evaluated with the help of the payoff matrix which is defined below

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{06} \\ w_{07} \\ w_{08} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \end{pmatrix} = \left\{ \begin{array}{l} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \\ -w_{05} + w_{15} = 0 \\ -w_{06} + w_{16} = 0 \\ -w_{07} + w_{17} = 0 \\ -w_{08} + w_{18} = 0 \end{array} \right\}$$

Solving the above formulated dual problem (19), we have the corresponding solution as:

$$w_{01} = 0.2311813, w_{02} = 0.2084537, w_{03} = 0.2022471, w_{04} = 0.2276698, w_{05} = 0.04031141,$$
$$w_{06} = 0.02319736, w_{07} = 0.02919185, w_{08} = 0.03774754, \text{ and } v(w^*) = 4.098446.$$

Using the primal dual- relationship theorem (17), we have the optimal solution of primal problem: *i.e.,* the optimal sample sizes of respondents and non respondents are computed as follows:

$$f_{0j}(n, r) = w_{0j}^* \, v(w_{0i}^*)$$

In expression (18), we first keep the r constant and calculate the values of n as:

$$f_{01}(n_1, r) = w_{01}^* \, v(w_{0i}^*) \qquad f_{02}(n_2, r) = w_{02}^* \, v(w_{0i}^*)$$
$$\frac{456.3344}{n_1} = 0.2311813 \times 4.098514 \qquad \frac{261.8965}{n_2} = 0.2084537 \times 4.098514$$
$$\Rightarrow n_1 \cong 482 \qquad \Rightarrow n_2 \cong 307$$
$$f_{03}(n_3, r) = w_{03}^* \, v(w_{0i}^*) \qquad f_{04}(n_4, r) = w_{04}^* \, v(w_{0i}^*)$$
$$\frac{209.5529}{n_3} = 0.2022471 \times 4.098514 \qquad \frac{230.9097}{n_4} = 0.2276698 \times 4.098514$$
$$\Rightarrow n_3 \cong 253 \qquad \Rightarrow n_4 \cong 247$$

Now, from the expression (13), we keep the n constant and calculate the values of r as:

$$f_{01}(n, r_1) = w_{01}^* \, v(w_{0i}^*) \qquad f_{02}(n, r_2) = w_{02}^* \, v(w_{0i}^*)$$
$$\frac{11.10002688}{r_1} = 0.04031141 \times 4.098514 \qquad \frac{2.75680566}{r_2} = 0.02319736 \times 4.098514$$
$$\Rightarrow r_1 \cong 67 \qquad \Rightarrow r_2 \cong 29$$
$$f_{03}(n, r_3) = w_{03}^* \, v(w_{0i}^*) \qquad f_{04}(n, r_4) = w_{04}^* \, v(w_{0i}^*)$$
$$\frac{3.492547875}{r_3} = 0.02919185 \times 4.098514 \qquad \frac{4.866484}{r_4} = 0.03774754 \times 4.098514$$
$$\Rightarrow r_3 \cong 29 \qquad \Rightarrow r_4 \cong 31$$

The optimal values and the objective function value are given below:

$$n_1^* = 482, n_2^* = 307, n_3^* = 253 \text{ and } n_4^* = 247;$$
$$r_1^* = 67, r_2^* = 29, r_3^* = 29 \text{ and } r_4^* = 31$$

and the objective value of the primal problem is 4.098514.

Sub problem1: On substituting the table values in sub-problem 2, we have obtained the expressions given below:

$$\left.\begin{array}{l} Min\ f_{02} = \dfrac{769.2353}{n_1} + \dfrac{318.9684}{n_2} + \dfrac{99.53584}{n_3} + \dfrac{259.5712}{n_4} + \\ \dfrac{18.7111296}{r_1} + \dfrac{3.35756232}{r_2} + \dfrac{1.658930625}{r_3} + \dfrac{5.47053248}{r_4}; \\ Subject\ to \\ \quad 0.00048n_1 + 0.00068n_2 + 0.0008n_3 + 0.00092n_4 + \\ \quad 0.0006r_1 + 0.0008r_2 + 0.001r_3 + 0.0012r_4 \le 1 \\ \quad n_h \ge 0\ ,\ r_h \ge 0\ ;\ (h = 1,2,\ldots,L) \end{array}\right\} \tag{20}$$

The dual of the above problem (20) is obtained as follows:

$$\left.\begin{array}{l} Max\ \ v(w_{0i}^*) = \left(\left(769.2353 / w_{01}\right)^{w_{01}}\right)\times\left(\left(318.9684 / w_{02}\right)^{w_{02}}\right)\times\left(\left(99.53584 / w_{03}\right)^{w_{03}}\right) \\ \quad \times\left(\left(259.5712 / w_{04}\right)^{w_{04}}\right)\times\left(\left(18.7111296 / w_{05}\right)^{w_{05}}\right)\times\left(\left(3.35756232 / w_{06}\right)^{w_{06}}\right) \\ \quad \times\left(\left(1.658930625 / w_{07}\right)^{w_{07}}\right)\times\left(\left(5.47053248 / w_{08}\right)^{w_{08}}\right)\times\left(\left(\dfrac{0.00048}{w_{11}}\right)^{w_{11}}\right) \\ \quad \times\left(\left(\dfrac{0.00068}{w_{12}}\right)^{w_{12}}\right)\times\left(\left(\dfrac{0.0008}{w_{13}}\right)^{w_{13}}\right)\times\left(\left(\dfrac{0.00092}{w_{14}}\right)^{w_{14}}\right)\times\left(\left(\dfrac{0.0006}{w_{15}}\right)^{w_{15}}\right) \\ \quad \times\left(\left(\dfrac{0.0008}{w_{16}}\right)^{w_{16}}\right)\times\left(\left(\dfrac{0.001}{w_{17}}\right)^{w_{17}}\right)\times\left(\left(\dfrac{0.0012}{w_{18}}\right)^{w_{18}}\right)\times \\ \quad ((\mathrm{w}_{11}+\mathrm{w}_{12}+\mathrm{w}_{13}+\mathrm{w}_{14}+\mathrm{w}_{15}+\mathrm{w}_{16}+\mathrm{w}_{17}+\mathrm{w}_{18})\hat{} \\ \quad (\mathrm{w}_{11}+\mathrm{w}_{12}+\mathrm{w}_{13}+\mathrm{w}_{14}+\mathrm{w}_{15}+\mathrm{w}_{16}+\mathrm{w}_{17}+\mathrm{w}_{18})); \qquad (i) \\ Subject\ to \quad w_{01} + w_{02} + w_{03} + w_{04} + w_{05} + w_{06} + w_{07} + w_{08} = 1; \quad \text{(normality condition)}\ (ii) \\ \qquad \left.\begin{array}{l} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0; \\ -w_{03} + w_{13} = 0; \\ -w_{04} + w_{14} = 0; \\ -w_{05} + w_{15} = 0; \\ -w_{06} + w_{16} = 0; \\ -w_{07} + w_{17} = 0; \\ -w_{08} + w_{18} = 0; \end{array}\right\} \text{(orthogonality condition)}\ (iii) \\ \left.\begin{array}{l} w_{01}, w_{02}, w_{03}, w_{04}, w_{05}, w_{06}, w_{07}, w_{08} > 0; \\ w_{11}, w_{12}, w_{13}, w_{14}, w_{15}, w_{16}, w_{17}, w_{18} \ge 0 \end{array}\right\} \text{(positivity condition)} \quad (iv) \end{array}\right\} \tag{21}$$

For orthogonality condition defined in expression 21(iii) are evaluated with the help of the payoff matrix which is defined below:

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$$
\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{06} \\ w_{07} \\ w_{08} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \end{pmatrix}
= \begin{Bmatrix} -w_{01} + w_{11} = 0 \\ -w_{02} + w_{12} = 0 \\ -w_{03} + w_{13} = 0 \\ -w_{04} + w_{14} = 0 \\ -w_{05} + w_{15} = 0 \\ -w_{06} + w_{16} = 0 \\ -w_{07} + w_{17} = 0 \\ -w_{08} + w_{18} = 0 \end{Bmatrix}
$$

Solving the above formulated dual problems, we have the corresponding solution as:

$$w_{01} = 0.2861167, w_{02} = 0.2192913, w_{03} = 0.1328703, w_{04} = 0.2300993, w_{05} = 0.04989059,$$

$$w_{06} = 0.02440339, w_{07} = 0.01917817, w_{08} = 0.03815034, \text{ and } \; v(w^*) = 4.510388.$$

The optimal values (n_h^*, r_h^*) of the sample sizes of the primal problems can be calculated with the help of the primal – dual relationship theorem (17) as we have calculated in the sub-problem 1are given as follows:

$$n_1^* = 596, n_2^* = 322, n_3^* = 166 \text{ and } n_4^* = 250;$$

$$r_1^* = 83, r_2^* = 30\ , r_3^* = 19 \text{ and } r_4^* = 32$$

and the objective value of the primal problem is 4.510388.

Now the pay-off matrix of the above problems is given below:

$$
\begin{array}{c}
\\
\left(n^{(1)}, r^{(1)}\right) \\
\left(n^{(2)}, r^{(2)}\right)
\end{array}
\begin{array}{c}
\begin{array}{cc} f_{01}(n,r) & f_{02}(n,r) \end{array} \\
\begin{bmatrix} 4.098446 & 4.703153 \\ 4.323975 & 4.510388 \end{bmatrix}
\end{array}
$$

The lower and upper bond of $f_{01}(n,r)$ and $f_{02}(n,r)$ can be obtained from the pay-off matrix

$$4.098446 \le f_{01}(n,r) \le 4.323975 \text{ and } 4.510388 \le f_{02}(n,r) \le 4.703153.$$

Let $\mu_1(n,r)$ and $\mu_2(n,r)$ be the fuzzy membership function of the objective function $f_{01}(n,r)$ and $f_{02}(n,r)$ respectively and they are defined as:

$$
\mu_1(n,r) = \begin{cases} 1 & , if \; f_{01}(n,r) \le 4.098446 \\ \dfrac{4.323975 - f_{01}(n,r)}{0.225529} & , if \; 4.098446 \le f_{01}(n,r) \le 4.323975 \\ 0 & , if \; f_{01}(n,r) \ge 4.323975 \end{cases}
$$

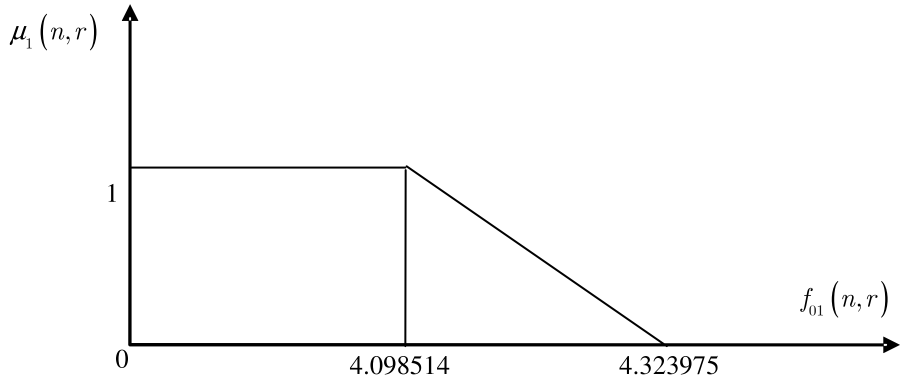

Figure 2: The figure illustrate the graph of the fuzzy membership function $\mu_1(n,r)$

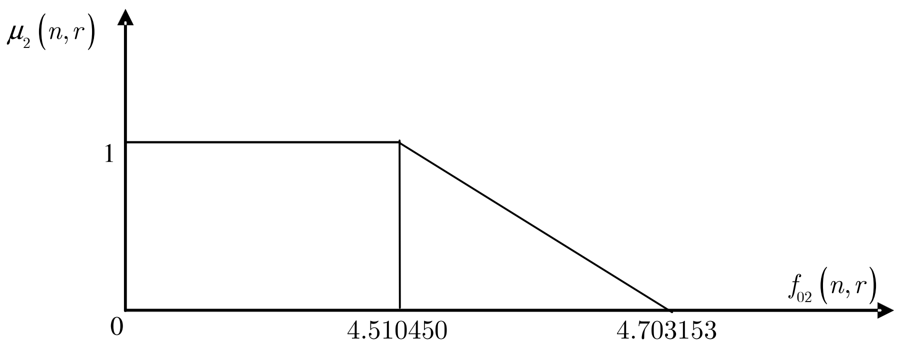

Figure 2: The figure illustrate the graph of the fuzzy membership function $\mu_2(n,r)$

$$\mu_2(n,r) = \begin{cases} 1 & , if\ Z_2(n,r) \le 4.510450 \\ \dfrac{4.703153 - Z_2(n,r)}{0.192703} & , if\ 4.510450 \le Z_2(n,r) \le 4.703153 \\ 0 & , if\ Z_2(n,r) \ge 4.703153 \end{cases}$$

On applying the max-addition operator, the MOGPP, the standard primal problem reduces to the crisp problem as:

$$\left.\begin{array}{l} \text{Maximize} \quad (\mu_1(n,r) + \mu_2(n,r)) \\ i.e \text{ Maximize } \left\{ \dfrac{4.323975 - f_{02}(n,r)}{0.225529} + \dfrac{4.703153 - f_{02}(n,r)}{0.192703} \right\} \\ i.e \text{ Maximize } \left\{ 43.5788 - \left(\dfrac{f_{01}(n,r)}{0.225529} + \dfrac{f_{02}(n,r)}{0.192703} \right) \right\} \\ \text{Subject to} \\ \quad 2.4\,n_1 + 3.4\,n_2 + 4n_3 + 4.6\,n_4 + 3\,r_1 + 4r_2 + 5\,r_3 + 6\,r_4 \le 5000 \\ \qquad n_h \ge 0\ ,\ r_h \ge 0\ ,\ h = 1,2,\dots,L \end{array}\right\} \tag{22}$$

In order to maximize the above problem, we have to minimize $\left(\dfrac{f_{01}(n,r)}{0.225529} + \dfrac{f_{02}(n,r)}{0.192703} \right)$, subject to the constraints as described below:

$$\left.\begin{array}{l} Min \left\{ \begin{array}{l} \dfrac{6015.1794}{n_1} + \dfrac{2816.4718}{n_2} + \dfrac{1445.6789}{n_3} + \dfrac{2370.8464}{n_4} + \\ \quad \dfrac{146.3152}{r_1} + \dfrac{29.6471}{r_2} + \dfrac{24.0947}{r_3} + \dfrac{49.9662}{r_4} \end{array} \right\} \\ Subject \text{ to } 0.00048\,n_1 + 0.00068n_2 + 0.0008\,n_3 + 0.00092n_4 + \\ \quad 0.0006\,r_1 + 0.0008\,r_2 + 0.001\,r_3 + 0.0012\,r_4 \le 1 \\ \qquad n_h \ge 0\ ,\ r_h \ge 0\ ;\ h = 1,2,\dots,L \end{array}\right\} \tag{23}$$

Degree of Difficulty of the problem (23) is = (16-(8+1) =7. Hence the dual problem of the above final formulated problem (23) is given as:

$$\left.\begin{array}{ll}
Max \quad v(w_{0i}^{*}) & =\left(\left(6015.1794 / w_{01}\right)^{w_{01}}\right)\times\left(\left(2816.4718 / w_{02}\right)^{w_{02}}\right)\times\left(\left(1445.6789 / w_{03}\right)^{w_{03}}\right) \\
& \times\left(\left(2370.8464 / w_{04}\right)^{w_{04}}\right)\times\left(\left(146.3152 / w_{05}\right)^{w_{05}}\right)\times\left(\left(16.0303 / w_{06}\right)^{w_{06}}\right) \\
& \times\left(\left(24.0947 / w_{07}\right)^{w_{07}}\right)\times\left(\left(49.9662 / w_{08}\right)^{w_{08}}\right)\times\left(\left(\frac{0.00048}{w_{11}}\right)^{w_{11}}\right) \\
& \times\left(\left(\frac{0.00068}{w_{12}}\right)^{w_{12}}\right)\times\left(\left(\frac{0.0008}{w_{13}}\right)^{w_{13}}\right)\times\left(\left(\frac{0.00092}{w_{14}}\right)^{w_{14}}\right) \\
& \times\left(\left(\frac{0.0006}{w_{15}}\right)^{w_{15}}\right)\times\left(\left(\frac{0.0008}{w_{16}}\right)^{w_{16}}\right)\times\left(\left(\frac{0.001}{w_{17}}\right)^{w_{17}}\right)\times\left(\left(\frac{0.0012}{w_{18}}\right)^{w_{18}}\right)\times \\
& ((\mathrm{w}_{11}+\mathrm{w}_{12}+\mathrm{w}_{13}+\mathrm{w}_{14}+\mathrm{w}_{15}+\mathrm{w}_{16}+\mathrm{w}_{17}+\mathrm{w}_{18})\hat{} \\
& (\mathrm{w}_{11}+\mathrm{w}_{12}+\mathrm{w}_{13}+\mathrm{w}_{14}+\mathrm{w}_{15}+\mathrm{w}_{16}+\mathrm{w}_{17}+\mathrm{w}_{18})); \qquad (i) \\
Subject\ to & w_{01}+w_{02}+w_{03}+w_{04}+w_{05}+w_{06}+w_{07}+w_{08}=1; \text{ (normality condition)} \qquad (ii) \\
& \left.\begin{array}{l}
-w_{01}+w_{11}=0 \\
-w_{02}+w_{12}=0; \\
-w_{03}+w_{13}=0; \\
-w_{04}+w_{14}=0; \\
-w_{05}+w_{15}=0; \\
-w_{06}+w_{16}=0; \\
-w_{07}+w_{17}=0; \\
-w_{08}+w_{18}=0;
\end{array}\right\} \text{(orthogonality condition)} \qquad (iii) \\
& \left.\begin{array}{l}
w_{01},w_{02},w_{03},w_{04},w_{05},w_{06},w_{07},w_{08}>0; \\
w_{11},w_{12},w_{13},w_{14},w_{15},w_{16},w_{17},w_{18}\geq 0
\end{array}\right\} \text{(positivity condition)} \qquad (iv)
\end{array}\right\} \qquad (24)$$

For orthogonality condition defined in expression 24(iii) are evaluated with the help of the payoff matrix which is defined below:

$$\begin{pmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{06} \\ w_{07} \\ w_{08} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \end{pmatrix}
=\left\{\begin{array}{l}
-w_{01}+w_{11}=0 \\
-w_{02}+w_{12}=0 \\
-w_{03}+w_{13}=0 \\
-w_{04}+w_{14}=0 \\
-w_{05}+w_{15}=0 \\
-w_{06}+w_{16}=0 \\
-w_{07}+w_{17}=0 \\
-w_{08}+w_{18}=0
\end{array}\right\}$$

After solving the formulated dual problem (24) using lingo software we obtain the following values of the dual variables which are given as:

$$w_{01}=0.2619838, \quad w_{02}=0.2133740, \quad w_{03}=0.1658174, \quad w_{04}=0.2277096, \quad w_{05}=0.04568250,$$
$$w_{06}=0.02374487, \quad w_{07}=0.02393363, \quad w_{08}=0.03775414 \text{ and } \quad v(w_{0i}^{*})=42.06568.$$

The optimal values (n_h^*, r_h^*) of the sample sizes of the primal problems can be calculated with the help of the primal – dual relationship theorem (17) as we have calculated in the sub-problem 1are given as follows:

$$n_1^* = 546,\ n_2^* = 314,\ n_3^* = 207 \text{ and } n_4^* = 248;$$
$$r_1^* = 76,\ r_2^* = 30,\ r_3^* = 24 \text{ and } r_4^* = 31$$

and the objective value of the primal problem is 42.06568.

6. CONCLUSION

This paper provides a profound study of fuzzy programming for solving the multi-objective geometric programming problem (MOGPP). The problem of non-response in multivariate stratified sample survey has been formulated as MOGPP and solution obtained. The obtained solution of MOGPP is dual solution corresponding to the problem of non-response in multivariate stratified sample surveys (primal problem). Therefore next, we obtained the optimum allocation of sample sizes of respondents and non respondents with the help of dual solutions MOGPP and primal-dual relationship theorem. To ascertain the practical utility of the proposed method in sample surveys problem in presence of non-response a numerical example is also given to illustrate the procedure.

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