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Economic order Quantity (EOQ) for Deteriorating Items with Non-instantaneous Receipt under Trade Credits

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***Abstract* —** This paper presents an inventory model with non-instantaneous receipt under the condition of permissible delay in payments. Certain items like volatiles deteriorate during production process. In this paper, we consider deterioration into two phases i.e. phase 1 and phase 2. The purpose of this paper is to determine the optimal replenishment policies under conditions of non-instantaneous receipt and permissible delay in payments. The replenishment rate is assumed to be greater than demand rate. Second order approximation have been used for in exponential terms for finding closed form solution of optimal order quantity, order cycle and order receipt period, so that the total profit per unit time is maximized. Some results have also been obtained. Numerical examples are presented to validate the proposed model. The sensitivity analysis of the solution with the variation of the parameters associated with the model is also discussed.

***Keywords* —** inventory; non-instantaneous receipt; deterioration; replenishment policies; trade credits; optimal policies

1. INTRODUCTION

The deterioration of goods is a realistic phenomenon in much inventory system. Maximum items or commodities undergo deterioration over time. The controlling of deteriorating items is a measure problem in any inventory system. Fruits, vegetables and food products suffer from depletion by direct spoilage while stored. Highly deteriorating items like volatile liquids such as alcohol, gasoline and turpentine undergo physical depletion over time through the evaporating process. Radioactive substances, electronic goods, photographic film, grains etc deteriorate through continuous loss of utility or potential with time. Hence, while developing an optimal inventory policy for such products, the loss of inventory due to deterioration cannot be ignored. The two researchers Ghare and Schrader (1963) were considered decaying inventory for a constant demand. Covert and Philip (1973) extended Ghare and Schrader's model for variable deterioration rate by assuming two parameters Weibull distribution function. Later, Shah and Jaiswal (1977) presented an order level inventory model for deteriorating items with a constant rate of deterioration. Aggarwal (1978) corrected and modified the error in Shah and Jaiswal's model (1977) by considering an order level inventory model and calculated the average inventory holding cost. Goyal and Giri (2001) developed a detailed review of deteriorating inventory. The models for these type products have been developed by Mishra (1975) , Chakrabarty et al. (1998), Hariga (1996) , Wee (1995) , Jalan et al. (1996) , Su et al. (2007) , Giri and Chaudhuri ((1997) , Teng et al. (2005), Hou (2006) , Dye et al. (2007) , Misra et al.(2011) , Jamal et al. (2000) , Li et al. (2010) , Hou and Lin (2009), Shah and Raykundaliya (2010), Tripathy and Mishra ((2011) , Tripathy and Pradhan (2012) ,Teng et al.(2005), Teng et al.(2011) , Chang et al. (2010), Yang et al.((2010), Sarkar et al. (2013) , Soni (2013) , Wang et al. (2014) , and Taleizadehaned Nematollahi (2014).

In today's business competition, it can be observed that suppliers offer a certain fixed credit period to settle the account for stimulating retailer's demand. The classical inventory management is almost concentrated on solving the optimal order quantity and reorder point but neglecting the type of payment. In the above models, authors/ researchers assumed that an entire order is received into inventory at one time (instantaneously). In real world the order quantity is frequently received gradually over time and the inventory level is depleted at the same time it is being replenished. Hence the more realistic assumption is the non-instantaneous receipt. Ouyang et al.(2004) developed an inventory system with non-instantaneous receipt under condition of permissible delay in payments. Sugapriya and Jeyaraman (2008) considered the economic production quantity for non-instantaneous deteriorating items allowing price discount with constant production and demand rate extending the facility of permissible delay in payments. Choi and Hwang (1986) developed a model determining the production rate for deteriorating items to minimize the total cost function over a finite planning horizon. Raafat (1985) extended Choi and Hwang (1986) model, given in Park (1983) to deal with a case in which the finished product is also subject to a constant rate of deterioration. Yang and Wee (2003) presented a multi- lot- size production inventory system for decaying items with constant demand and production rates. Ghiami et al. (2013) investigated a two-echelon supply chain model for deteriorating inventory in which the retailer's warehouse has a limited capacity. Ouyang and Cheng (2008) presented the

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optimal ordering with decaying items under permissible delay in payments, and considered two possible ways for retailer to pay off the loan. Li et al. (2014) developed a model for inventory game with permissible delay in payments. Ouyang and Chang (2013) explored the effects of the reworking imperfect quality item and trade credit on the EPQ model with imperfect production process and complete backlogging. Bhunia et al.(2014) developed an inventory model for single deteriorating item with two separate warehouses having different preserving facilities.

In this paper, we consider the order cycle [0, T] into two parts (i) inventory replenishment period and (ii) inventory depleted period. The deterioration is taken in both parts. In this study we develop an EOQ model with non-instantaneous receipt under trade credits. We then establish numerical solution for finding the optimal order quantity, order cycle and order receipt period so that the total relevant profit per unit time is maximized. Truncated Taylor's series expansion is used for finding closed form optimal solution. Numerical examples and sensitivity analysis is given to validate the proposed model. Three results have also been obtained from the optimal solution.

The rest of this paper is organized as follows. Section 2 is notation and assumption we adopt through this paper. In section 3 we develop mathematical models for the two different situations. Section 4 presents the determination of the optimal replenishment time with some results. In section 5 numerical examples are given to illustrate the proposed model followed by sensitivity analysis in section 6. Finally, we provide conclusion and future research direction in the last section 7.

2. NOTATIONS AND ASSUMPTION

2.1. Notations

The following notations are used throughout this manuscript to develop the proposed model:

s :ordering cost per order
h : unit holding cost per unit time excluding interest charges
c : purchase cost ($/ unit)
p : selling price ($/ unit)
Q : order quantity
θ : constant deterioration rate , $0 \leq \theta < 1$
$I_1(t)$: first phase (inventory replenishment) / inventory level at time 't'
$I_2(t)$: second phase (inventory depleted) / inventory level at time 't'
λ : per unit rate at which the order is received over time i.e. replenishment rate
D : per unit time rate which inventory is demanded
T :order cycle period
T_1 :order receipt period, $0 < T_1 < T$
m :permissible delay in settling the account
I_c :interest paid per dollar per unit time
I_d :interest earned per dollar per unit time
$Z_1(T)$:total profit per cycle for case I
$Z_2(T)$:total profit per cycle for case II
$T_1 = T_1^*$:optimal order receipt period for case I
$T_1 = T_1^{**}$:optimal order receipt period for case II
$T = T^*$:optimal cycle time for case I
$T = T^{**}$:optimal cycle time for case II
$Q^*(T^*)$:optimal order quantity for case I
$Q^*(T^{**})$:optimal order quantity for case II
$Z_1(T) = Z_1^*(T^*)$:optimal total profit per cycle for case I
$Z_2(T) = Z_2^*(T^{**})$:optimal total profit per cycle for case II
OC :ordering cost per order
HC : holding cost
IP : interest payable per cycle
IE_1 : interest earned per cycle for case I
IE_2 : interest earned per cycle for case II

2.2 Assumptions

In addition the following assumptions are being made throughout the manuscript:

1. Time horizon is infinite and lead time is zero.
2. Shortages are not allowed.
3. The inventory system under consideration deals with single item.
4. The replenishment rate λ, is finite and greater than demand rate D, i.e. $\lambda > D$.
5. Supplier offers a certain fixed period, m to settle the account.
6. Retailer would not consider paying the payment until receiving all items.
7. The order cycle period $[0, T]$ is divided into two phases (i) inventory replenished period (phase 1) (ii). Inventory depleted period (phase 2).
8. There is no replenishment or repair for a deteriorated item.

3. MATHEMATICAL FORMULATION

According to the assumption the order cycle $[0, T]$ is divided into two parts (i) inventory replenished period (ii) inventory depleted period. The two different cases are shown in the following fig 1. The change of inventory in the above two phases can be described as follows:

Phase 1. In this phase replenishment rate is greater than the demand rate, the inventory go up to maximum level (called order quantity). The rate of change of inventory at time 't', $\frac{dI_1(t)}{dt}$ is given by

$$\frac{dI_1(t)}{dt} = -\theta I_1(t) + (\lambda - D),\ \ 0 \le t \le T \tag{1}$$

With the boundary condition $I_1(0) = 0$

Phase 2. Replenishment is stopped and the inventory decreases due to demand and deterioration. The rate of change of inventory at time 't', $\frac{dI_2(t)}{dt}$ can be described by

$$\frac{dI_2(t)}{dt} = -\theta I_2(t) - D,\ \mathrm{T}_1 \le t \le T \tag{2}$$

With the boundary condition $I_2(T) = 0$.

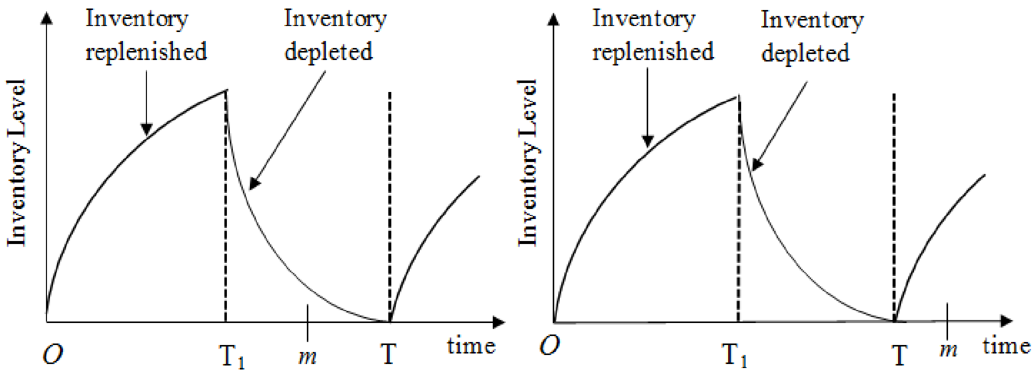

Fig. 1(a). $T_1 \le m \le T$ Fig.1(b) $T \le m$

Figure 1. Inventory level verses time.

The solution of (1) and (2) are respectively given by

$$I_1(t) = \frac{\lambda - D}{\theta}(1 - e^{-\theta t}),\quad 0 \le t \le T_1 \tag{3}$$

$$I_2(t) = \frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right),\quad T_1 \le t \le T \tag{4}$$

But the order quantity $Q = I_1(T_1) = I_2(T_1)$, from (3) and (4) we obtain

$$T_1 = \frac{1}{\theta}\log\left\{1 + \frac{D}{\lambda}\left(e^{\theta T} - 1\right)\right\} \tag{5}$$

We can obtain the total profit per unit time for the following two cases: (i) $T_1 \le m \le T$ and (ii) $T \le m$.

Case I: $T_1 \le m \le T$

This case is shown in Fig 1 (a). In this case, the total profit per cycle consists of sales revenue, ordering cost, holding cost, interest payable and interest earned. The components are calculated as follows:
(a). Sales Revenue

$$SR = p\left\{\int_0^{T_1}(\lambda - D)dt + \int_{T_1}^{T} Ddt\right\} = p\left\{\lambda T_1 + D\left(T - 2T_1\right)\right\} \tag{6}$$

(b) The ordering cost per order = s
(c) The holding cost during the interval [*0, T*] is given by

$$HC = h\left\{\int_0^{T_1} I_1(t)dt + \int_{T_1}^{T} I_2(t)dt\right\} = \frac{h}{\theta}\left[\left(\lambda - D\right)\left\{T_1 - \frac{1-e^{-\theta T_1}}{\theta}\right\} + D\left\{\frac{e^{\theta(T-T_1)}-1}{\theta} - (T - T_1)\right\}\right] \tag{7}$$

(d). The interest payable per cycle is given by

$$IP = cI_c\int_m^T I_2(t)dt = \frac{cI_cD}{\theta}\left\{\frac{e^{\theta(T-m)}-1}{\theta} - (T-m)\right\} \tag{8}$$

(e) The interest earned per cycle is given by

$$IE_1 = pI_d\int_0^m Dtdt = \frac{pI_dDm^2}{2} \tag{9}$$

Therefore, the total profit per unit time is given by

$$\begin{aligned} Z_1(T) &= \frac{1}{T}\left[SR - s - HC - IP + IE_1\right] \\ &= \frac{1}{T}\Bigg[p\left\{\lambda T_1 + D(T-2T_1)\right\} - s - \frac{h}{\theta}\left\{\left(\lambda - D\right)\left(T_1 - \frac{1-e^{-\theta T_1}}{\theta}\right) + \left(\frac{e^{\theta(T-T_1)}-1}{\theta} - (T-T_1)\right)\right\} \\ &\quad - \frac{cI_cD}{\theta}\left\{\frac{e^{\theta(T-m)}-1}{\theta} - (T-m)\right\} + \frac{DpI_dm^2}{2}\Bigg] \end{aligned} \tag{10}$$

Case 2. $T \le m$

This case is shown in fig.1 (b). In this case, the total profit per cycle consists of the sales revenue, ordering cost, holding cost and interest earned. Since cycle time is less than credit period, the retailer pays no interest and earns the interest during the period [*0, m*]. The interest earned in this case is given by

$$IE_2 = pI_d\left[\int_0^T Dtdt + (m-T)DT\right] = pI_dD\left(m - \frac{T}{2}\right)T \tag{11}$$

Total profit per unit time is given by

$$Z_2(T) = \frac{1}{T}\left[p\left\{\lambda T_1 + D\left(T - 2T_1\right)\right\} - s - \frac{h}{\theta}\left\{\left(\lambda - D\right)\left(T_1 - \frac{1-e^{-\theta T_1}}{\theta}\right) - \left(T - T_1\right)\right\} + pI_dD\left(m - \frac{T}{2}\right)T\right] \tag{12}$$

4. DETERMINATION OF OPTIMAL REPLENISHMENT TIME

Since it is difficult to handle above equations for finding the exact value of *T*, therefore, we make use of the second order approximation for the exponential and logarithm in equations (10), (12) and (5) , which follows as

$$e^{\theta(T-T_1)} \approx 1 + \theta(T - T_1) + \frac{\theta^2(T-T_1)^2}{2} \text{ and } e^{\theta(T-m)} \approx 1 + \theta(T-m) + \frac{\theta^2(T-m)^2}{2}.$$

Also for low deterioration rate, we can assume

$$e^{-\theta T} \approx 1 - \theta T + \frac{\theta^2 T^2}{2} \tag{13}$$

Hence, the total profit per unit time (from (10) and (12)) is approximated by

$$Z_1(T_1,T) \approx \frac{1}{T}\left[p\left\{\lambda T_1 + D(T-2T_1)\right\} - s - \frac{h}{2}\left(\lambda T_1^2 + DT^2 - 2DTT_1\right) - \frac{cI_c(T-m)^2}{2} + \frac{pI_dDm^2}{2}\right] \quad (14)$$

$$Z_2(T_1,T) \approx \frac{1}{T}\left[p\left\{\lambda T_1 + D(T-2T_1)\right\} - s - \frac{h}{2}\left(\lambda T_1^2 + DT^2 - 2DTT_1\right) + pI_dD\left(m - \frac{T}{2}\right)T\right] \quad (15)$$

Also from (5) we obtain

$$T_1 = \frac{DT}{\lambda}\left(1 + \frac{\theta kT}{2}\right) \quad (16)$$

where $k = 1 - \frac{D}{\lambda}$. Using (16) in (14) and (15), we obtain

$$Z_1(T) \approx pD + p(2k-1)D\left(1 + \frac{\theta kT}{2}\right) - \frac{s}{T} - \frac{hkDT}{2}\left\{1 + \frac{k(1-k)\theta^2T^2}{4}\right\} - \frac{cI_cD}{2}\left(T - 2m + \frac{m^2}{T}\right) + \frac{DpI_dm^2}{2T} \quad (17)$$

$$Z_2(T) \approx pD + p(2k-1)D\left(1 + \frac{\theta kT}{2}\right) - \frac{s}{T} - \frac{hkDT}{2}\left\{1 + \frac{k(1-k)\theta^2T^2}{4}\right\} + DpI_d\left(m - \frac{T}{2}\right) \quad (18)$$

Note that the purpose of this approximation is to obtain the unique closed form solution for the optimal value of T. By taking first and second order derivatives of $Z_1(T)$ and $Z_2(T)$ from (17) and (18), with respect to T, we obtain

$$\frac{dZ_1(T)}{dT} = \frac{2s + Dm^2(cI_c - pI_d)}{2T^2} - \frac{3k^2(1-k)D\theta^2T^2}{8} - \frac{hkD}{2} - \frac{cI_cD}{2} + \frac{pk(2k-1)D\theta}{2} \quad (19)$$

$$\frac{dZ_2(T)}{dT} = \frac{pk(2k-1)D\theta}{2} - \frac{s}{T^2} - \frac{3k^2(1-k)D\theta^2T^2}{8} - \frac{hkD}{2} - \frac{cI_cD}{2} - \frac{DpI_d}{2} \quad (20)$$

$$\frac{d^2Z_1(T)}{dT^2} = -\left[\frac{2s + Dm^2(cI_c - pI_d)}{T^3} + \frac{3k^2(1-k)D\theta^2T}{4}\right] < 0 \quad (21)$$

$$\frac{d^2Z_2(T)}{dT^2} = -\left[\frac{2s}{T^3} + \frac{3k^2(1-k)D\theta^2T}{4}\right] < 0 \quad (22)$$

From (21) and (22) it is clear that $Z_1(T)$ and $Z_2(T)$ both are concave function of T. It can be seen from the following graph:

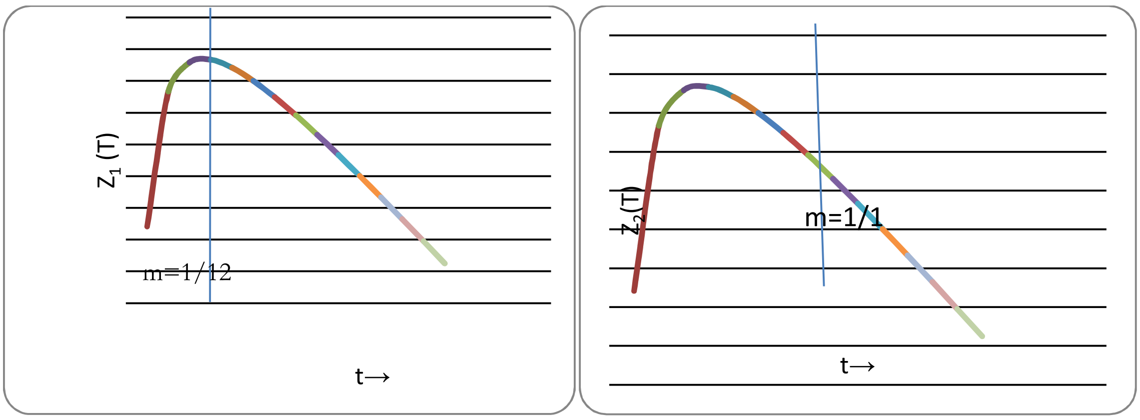

Fig. 2(a) Case 1: $m \le T$ Fig. 2(b) Case 2: $T \le m$

Figure 2. Graph between total profit $Z(T_1, T)$ and cycle time T .

The objective is to determine the optimal value of $T = T^*$ for case I which maximize the total profit per unit time $Z_1(T_1^*)$. The necessary condition for $Z_1(T)$ to be maximum at point $T = T^*$ is that $\frac{dZ_1(T)}{dT} = 0$. Solving $\frac{dZ_1(T)}{dT} = 0$, (after neglecting θ^2T^4 , since $\theta^2T^4 <<< 1$), we obtain

$$T = T^{*} = \sqrt{\frac{2s + D(cI_c - pI_d)m^2}{D\left\{hk + cI_c - pk(2k-1)\theta\right\}}} \quad (23)$$

The optimal value of $T = T^{**}$ is obtain by solving, $\frac{dZ_2(T)}{dT} = 0$, (after neglecting $\theta^2 T^4$, since $\theta^2 T^4 <<< 1$), we obtain

$$T = T^{**} = \sqrt{\frac{2s}{D\left\{hk + pI_d - pk(2k-1)\theta\right\}}} \quad (24)$$

From the above discussion we observe the following properties:

Result 1. Substituting (23) into inequality $m \le T^{*}$, we obtain

$$2s \ge D\left\{hk + pI_d - pk(2k-1)\theta\right\}m^2 \quad (25)$$

Result 2: Substituting (24) into inequality $T^{**} \le m$, we obtain

$$2s \le D\left\{hk + pI_d - pk(2k-1)\theta\right\}m^2 \quad (26)$$

Result 3: If $2s = D\left\{hk + pI_d - pk(2k-1)\theta\right\}m^2$, then

$$T^{*} = T^{**} = m \quad (27)$$

Note: It is not easy to find the exact relation between $T_1^{*} \le m \le T^{*}$ or $T_1^{*} \le m$, but we can numerically find out the relation.

The optimal economic order quantity for each case given by

$$Q(T) = Q^{*}(T^{*}) = DT^{*} = \sqrt{\frac{D\left\{2s + (cI_c - pI_d)m^2\right\}}{\left\{hk + cI_c - pk(2k-1)\theta\right\}}} \quad (28)$$

$$Q(T) = Q^{*}(T^{**}) = DT^{**} = \sqrt{\frac{2sD}{\left\{hk + pI_d - pk(2k-1)\theta\right\}}} \quad (29)$$

In classical *EOQ* model with non-instantaneous receipt, the retailer must pay the payment at the beginning of each cycle. Hence the classical optimal economic order

$$Q^{*} = \sqrt{\frac{2sD}{\left\{hk + cI_c - pk(2k-1)\theta\right\}}} \quad (30)$$

From the above discussion we obtain the following theorem.

Theorem: If

(i) $cI_c < pI_d$, then $Q^{*}(T^{*}) < Q^{*}$ and $Q^{*}(T^{**}) < Q^{*}$

(ii) $cI_c > pI_d$, then $Q^{*}(T^{*}) > Q^{*}$ and $Q^{*}(T^{**}) > Q^{*}$

(iii) $cI_c = pI_d$, *then* $Q^{*}(T^{*}) = Q^{*}(T^{**}) = Q^{*}$

Proof: It is obvious from (33), (34) and (35).

5. NUMERICAL EXAMPLES

Example 1. Let s = \$400/order, D = 5000 units / year, p = \$100/ unit, h = \$10/unit/year, λ = 10000 units/year, c = \$200/unit, θ = 0.05, I_c = \$0.15/\$/year, I_d = \$0.1/\$/year, m = 1/12 year $T = T^{*}$ = 0.0924104 years, $T_1 = T_1^{*}$ = 0.0488736 years, $Q(T) = Q^{*}(T^{*}) = 473.052$ units and $Z_1(T) = Z_1^{*}(T^{*})$ = \$ 496328.0, thus $T_1^{*} \le m \le T^{*}$, which proves case I.

Example 2. Let s = \$200/order, D = 5000 units / year, p = \$100/ unit, h = \$10/unit/year, λ = 10000 units/year, c = \$200/unit, θ = 0.05, I_c = \$0.15/\$/year, I_d = \$0.1/\$/year, m = 1/12 year $T = T^{**}$ = 0.0730297 year $T_1 = T_1^{**}$ = 0.0365482 years, $Q(T) = Q^{*}(T^{**}) = 365.1485$ units and $Z_2(T) = Z_2^{*}(T^{**})$ = \$ 499523.0, thus $T^{**} \le m$, which proves case II.

Example 3. Let s = \$400/order, D = 5000 units / year, p = \$200/ unit, h = \$10/unit/year, λ = 10000 units/year, c = \$100/unit, θ = 0.05, I_c = \$0.15/\$/year, I_d = \$0.1/\$/year, m = 1/12 year in this case $cI_c < pI_d$ then Q^{*} = 447.214 units ,

$Q^*(T^*) = 447.204$ units and $Q^*(T^{**}) = 400.00$, it is clear that $Q^*(T^*) < Q^*$ and $Q^*(T^{**}) < Q^*$, which proves the first part of the theorem i.e, $cI_c < pI_d$, then $Q^*(T^*) < Q^*$ and $Q^*(T^{**}) < Q^*$.

Example 4. Let s = \$400/order, D = 5000 units / year, p = \$100/ unit, h = \$10/unit/year, λ = 10000 units/year, c = \$200/unit, θ = 0.05, I_c = \$0.15/\$/year, I_d = \$0.1/\$/year, m = 1/12 year in this case $cI_c > pI_d$ then $Q^* = 381.385$ units, $Q^*(T^*) = 381.406$ units and $Q^*(T^{**}) = 516.398$ units it is clear that $Q^*(T^*) > Q^*$ and $Q^*(T^{**}) > Q^*$, which proves the second part of the theorem i.e. $cI_c > pI_d$, then $Q^*(T^*) > Q^*$ and $Q^*(T^{**}) > Q^*$.

Example 5. Let s = \$400/order, D = 5000 units / year, p = \$200/ unit, h = \$10/unit/year, λ = 10000 units/year, c = \$100/unit, θ = 0.05, I_c = \$0.2/\$/year, I_d = \$0.1/\$/year, m = 1/12 year in this case $cI_c = pI_d$, then $Q^* = 400$ units, $Q^*(T^*) = 400$ units and $Q^*(T^{**}) = 400$ units it is clear that $Q^*(T^*) = Q^*(T^{**}) = Q^*$,which proves the third part of the theorem i.e. $cI_c = pI_d$, then $Q^*(T^*) = Q^*(T^{**}) = Q^*$.

6. SENSITIVITY ANALYSIS

The effect of changing the parameters s , h , c , p, m, I_c , I_d , and θ, on the optimal replenishment policy are studied by assuming the values for s , h , c , p, and θ are all *400 , 10, 200, 100, 0.15,0.10, 1/12,* and *0.05* for case I and *200 , 10, 200, 100, 0.15, 0.1, 1/12*, and *0.05* for case II. The results are summarized in tables 1- 8.

The change in the values of parameters may happen due to variation or uncertainties in any decision – making situation. The sensitivity analysis will be very useful in decision making in order to examine the effect and variation of these changes. Using the above data, the sensitivity analysis of various parameters has been done. The results of sensitivity analysis are given in the following tables.

Case I:

s	$T_1 = T_1^*$	$T = T^*$	$Q^*(T^*)$	$Z_1^*(T^*)$
300	0.0430486	0.0860048	430.0240	497449.0
320	0.0437094	0.0873235	436.6175	497218.0
340	0.0443604	0.0886226	443.1130	496991.0
360	0.0450020	0.0899029	449.51450	496767.0
480	0.0486748	0.0972315	486.1575	495484.0
500	0.0492605	0.0983999	491.9995	495280.0

Table 1: Effect of ordering cost per order‘*s*’ on optimal replenishment policy.

c	$T_1 = T_1^*$	$T = T^*$	$Q^*(T^*)$	$Z_1^*(T^*)$
110	0.0488995	0.0976798	488.399	496374.0
120	0.0484631	0.96809	484.045	496367.0
130	0.0480768	0.0960383	480.1915	496360.0
140	0.0477325	0.0953514	476.757	496354.0
150	0.0474236	0.0947351	473.6755	496349.0

Table 2: Effect of purchase cost ‘*c*’ on optimal replenishment policy.

p	$T_1 = T_1^*$	$T = T^*$	$Q^*(T^*)$	$Z_1^*(T^*)$
50	0.0488749	0.0976307	488.1535	245336.0
90	0.0478455	0.955768	477.884	345715.0
110	0.0467935	0.0934778	467.389	446100.0
120	0.0457174	0.0913306	456.653	546491.0
130	0.0451698	0.0902378	451.189	596689.0
140	0.0446155	0.0891316	445.658	646888.0
150	0.0440542	0.0880115	440.0575	697089.0

Table 3: Effect of selling price ‘*p*’ on optimal replenishment policy.

h	$T_1 = T_1^*$	$T = T^*$	$Q^*(T^*)$	$Z_1^*(T^*)$
5	0.0480069	0.0958988	479.494	496916.0
6	0.0476414	0.0951695	475.8475	496797.0
7	0.0472841	0.0944566	472.283	496679.0
8	0.0469347	0.0937595	468.7975	496561.0
15	0.0446883	0.0892769	446.3845	495761.0
20	0.0432677	0.0864420	432.2100	495212.0
25	0.0419745	0.0838611	419.3055	494680.0

Table 4: Effect of unit holding cost '*h*' on optimal replenishment policy.

Case II

s	$T_1 = T_1^{**}$	$T = T^{**}$	$Q^*(T^{**})$	$Z_1^*(T^{**})$
150	0.0316478	0.0632456	316.228	500257.0
160	0.0326865	0.0653197	326.5985	500101.0
170	0.0336933	0.0673300	336.650	499950.0
180	0.0346710	0.069282	346.410	499804.0
190	0.0356219	0.0711805	355.9025	499661.0s
210	0.0374515	0.0748331	374.1655	499388.0
220	0.0383338	0.0765942	382.971	499225.0

Table 5: Effect of ordering cost per order'*s*' on optimal replenishment policy.

c	$T_1 = T_1^{**}$	$T = T^{**}$	$Q^*(T^{**})$	$Z_1^*(T^{**})$
150	0.0316478	0.0632456	316.228	499423.0
160	0.0326865	0.0653197	326.5985	499268.0
220	0.0383338	0.0765942	382.971	498422.0
230	0.0391961	0.0783156	391.578	498293.0
240	0.0400400	0.0800000	400.000	498167.0
250	0.0408665	0.0816497	408.2485	498043.0
260	0.0416766	0.0832666	416.333	497922.0

Table 6: Effect of purchase cost '*c*' on optimal replenishment policy.

p	$T_1 = T_1^{**}$	$T = T^{**}$	$Q^*(T^{**})$	$Z_1^*(T^{**})$
70	0.0408665	0.0816497	408.248	348018.0
110	0.0353866	0.0707107	353.5535	548926.0
120	0.0343291	0.0685994	342.997	599169.0
150	0.0316478	0.0632456	316.228	749925.0

Table 7: Effect of selling price '*p*' on optimal replenishment policy.

h	$T_1 = T_1^{**}$	$T = T^{**}$	$Q^*(T^{**})$	$Z_1^*(T^{**})$
4	0.0400400	0.080000	400.000	499167.0
5	0.0400400	0.080000	400.000	499167.0
6	0.0392617	0.0784465	392.2325	499068.0
7	0.0385270	0.0769800	384.900	498971.0
8	0.0378322	0.0755929	377.9645	498875.0
9	0.0371735	0.0742781	371.3905	498782.0
15	0.0338347	0.0676123	338.0615	498209.0
20	0.0316478	0.0632456	316.228	497842.0

Table 8: Effect of unit holding cost '*h*' on optimal replenishment policy.

The following inferences can be made from the result obtained.

(a). When ordering cost per order '*s*' increases, the optimal receipt T_1, optimal cycle time T, and optimal order quantity Q increases while total profit per cycle decreases. That is , the change in '*s*' will cause the positive change in optimal receipt period, optimal cycle time , and optimal order quantity while negative change in optimal total profit per cycle.

(b). When purchase cost '*c*' increases, the optimal receipt period T_1 , optimal cycle time , optimal order quantity *Q* and optimal total profit decreases. That is , the change in '*c*' will cause the negative the negative change in optimal receipt period T_1 , optimal cycle time T , optimal order quantity *Q* and optimal total profit decreases. That is , the change in '*c*' will cause the negative change in optimal receipt period T_1 , optimal cycle time T, optimal order quantity *Q* and optimal total profit .

(c). When the selling price '*p*' increases, the optimal receipt period T_1 , optimal cycle time T , optimal order quantity *Q* decreases wile optimal total profit decreases. That is, the change in '*p*' will cause the negative change in optimal receipt period T_1 , optimal cycle time *T,* optimal order quantity *Q* and positive change in optimal total profit .

(d). When the unit holding cost '*h*' increases optimal receipt period T_1 , optimal cycle time *T*, optimal order quantity *Q* and optimal total profit decreases. That is, the change in '*h*' will cause the negative change in optimal receipt period T_1 , optimal cycle time *T*, optimal order quantity *Q* and optimal total profit.

7. CONCLUSION AND FUTURE RESEARCH

This model incorporates some realistic features that are likely to be associated with some kinds of inventories like time features for goods and seasonal goods. This model is very useful in the retail business. This model can be used in domestic goods like green vegetable, milk, curd etc, fashionable cloths and other products. In this paper, we develop an economic order quantity model with non-instantaneous receipt under conditions of trade credits. Truncated Taylor's series expansion is used for finding closed form solution to find the optimal order cycle, optimal order quantity, optimal receipt period and optimal total profit. We have given the numerical formulation of the problem. From our results, we have also verified that the effects of various parameters in formulating optimal replenishment policy. The sensitivity of the solution to changes in the values of different parameters has been discussed. It is seen that the changes in various parameters are quiet sensitive and significant effects on the optimal solutions. Finally, numerical examples and sensitivity is presented to illustrate the theoretical results.

The model can be extended in several ways. For instance, we may extend the model for shortages and stock – dependent demand rate. Also, we could consider the demand as a function of inflation or selling price as well as time varying. Finally, we could generalize the model to allow quantity discount, time value of money and others.

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