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A Fuzzy Approach for a Multiobjective Selective Maintenance Problem

Sanam Haseen*, **Neha Gupta and Abdul Bari**

Department of Statistics and OR, Aligarh Muslim University, Aligarh-202002

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Abstract — In the field of reliability optimization often comes the problem of selective maintenance. A system with series and parallel components often have situations where it takes small breaks or intervals and start functioning again. These intervals give the experts time to replace or repair deteriorating components of the system which is termed as selective maintenance. For such system, a decision making problem may be formulated as to optimize the reliability taking into consideration the time or cost spent on components. Further, there are cases where the reliability of individual components, cost and time involved are not crisp instead there is imprecision or variability of data. In this paper we formulate this problem of selective maintenance into a multiobjective nonlinear programming problem with triangular fuzzy numbers and solve it using Fuzzy Programming Problem. Also, a numerical example with assumed data is used to illustrate the applicability of the formulated problem for certain degree of membership at a particular $\alpha-$ cut.

Keywords — Selective Maintenance, System Reliability, Triangular Fuzzy Number, Multiobjective Nonlinear Programming, Fuzzy Programming.

1. INTRODUCTION

Reliability is the probability of an item to perform consistently a required function or mission without failure for a stated condition or interval of time. In a system, reliability of individual component in that system plays an important role for its proper functioning. There are many systems which accomplish a sequence of operation with finite breaks at regular intervals. These breaks give an opportunity to repair or replace deteriorated components in the system so as to improve the system reliability that eventually improve the functioning of the whole mission. Such kind of system go through "selective maintenance".

The study of the selective maintenance in system was originally performed by Rice et al. (1998). They modeled a system of identical parallel series components and developed a decision making model to optimize. Cassady et al. (2001b) improved the selective maintenance by removing the structural restriction on subsystem and developing a general framework with binary state components. They further considered the components of the system to follow Wiebull distribution with multiple maintenance actions such as minimal repair and corrective replacement in Cassady et al. (2001a). Lust et al. (2009) discarded the conventional enumeration method and developed three new methods viz. a construction heuristic, a heuristic based on the adaptation of Tabu search, and an exact method based on a branch and bound procedure for various system configurations. Several other authors who have studied numerous reliability optimization techniques and selective maintenance policies in reliability improvement are Tillman et al. (1980), Kuo et al. (1987), Chern (1992), Wang (2002), Iyoob et al. (2006), Nourelfath and Ait-Kadi (2006), Rajagopalan and Cassady (2006), Nahas et al. (2008), Bartholomew-Biggs et al. (2009), Schneider et al. (2009), Liu and Huang (2010), Gupta et al. (2014) etc.

Now, many systems involve uncertainties and imprecision in data where the estimation of precise values of probabilities is very difficult. In such scenario fuzzy reliability is of much help. Singer (1990), considered the fuzzy reliability of both serial and parallel systems using an approximation of a fuzzy binary operation $\otimes$ with two L-R type fuzzy numbers. Cheng and Mon (1993) used the $\alpha-$ cut of a triangular fuzzy number to get the interval and find the fuzzy reliabilityof the serial system and the parallel system. Chen (1994) likewise omitted the statistical approach when he used fuzzy numbers to find the fuzzy reliability of both the serial system and the parallel system. In Mon and Cheng (1994), the authors used the $\alpha-$ cut of fuzzy number to derive a non-linear program of the fuzzy system in both the serial and parallel cases. Other researchers who have worked in this field are Park (1987), Rao and Dhingra (1992), Cai et al. (1993), Ravi et al. (2000), Khasawneh et al. (2002), Verma et al. (2004), Mahapatra and Roy (2006), and Ali et al. (2012, 2011) has done research on different fuzzy approaches in system reliability.

Yao et al. (2008) in their paper discussed the problem of fuzziness in system reliability. They justified that reliability of a system may fluctuate at point estimate during a time interval. Statistical confidence interval may be used instead of the point estimate which was further transferred into a triangular fuzzy number. This paper deals with a problem of system reliability in selective maintenance model. Two types of components (repairable and replaceable) are assumed to be involved.

* Corresponding author's email:sanam.haseen@gmail.com

The system consists of three groups of subsystems viz. X, Y and Z and reliability of each subsystem is needed to be maximized. The system runs after a fixed interval of time. In this limited interval of time, maintenance is performed keeping in view the fixed budget for repair and replacement of components. The reliability of individual components, cost and time involved are considered to be fuzzy in nature. Moreover, the fuzziness is defined by triangular fuzzy numbers (TFNs). The selective maintenance problem is formulated as a multiobjective nonlinear programming problem with fuzzy cost, time and reliability. The compromise allocation of repairable and replaceable components is then obtained using fuzzy programming approach. The problem is also illustrated in a numerical example with assumed data.

2. SOME PREREQUISITE

Few definitions of fuzzy sets must be quoted before solving the problem of system reliability in fuzzy selective maintenance model:

Fuzzy Set: Let A be the universe whose generic element be denoted by a. A fuzzy set $\widetilde{A}$ in A is a function $\widetilde{A} : A \to [0,1]$.

Fuzzy number: A fuzzy number $\widetilde{A}$ is a fuzzy set of the real line A whose membership function $\mu_{\widetilde{A}}(x)$ must posses the following properties with $-\infty \le a \le b \le c \le \infty$,

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & x \le a, \\ \mu_{\widetilde{L}}(x) & a < x \le b, \\ \mu_{\widetilde{U}}(x) & b < x \le c, \\ 0 & x > c, \end{cases} \tag{1}$$

where $\mu_{\widetilde{L}}(x) : \left[a, b \to [0,1]\right]$ is continuous and strictly increasing and $\mu_{\widetilde{U}}(x) : \left[b, c \to [0,1]\right]$ is continuous and strictly decreasing.

Membership function $\mu_{\widetilde{A}}(x)$: We frequently define function $\widetilde{A}$ and say that the fuzzy set A is characterized by its membership function $\mu_{\widetilde{A}} : A \to [0,1]$.

$\alpha -$ **cut**: $\alpha -$ cut of a fuzzy set $\widetilde{A}$ of A is a non-fuzzy set denoted by ${}^{\alpha}A$ defined by a subset of A is a non-empty $a \in A$, such that their membership functions exceed or equal to a real number $\alpha \in [0,1]$, that is,

$${}^{\alpha}A = \left[a : \mu_{\widetilde{A}}(a) \ge \alpha, \alpha \in [0,1], \forall a \in A\right]. \tag{2}$$

Triangular fuzzy number (TFN): A triangular fuzzy number can be completely specified by the threesome $\widetilde{R} = (a, b, c)$ with membership function.

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & x \le a, \\ \dfrac{x-a}{b-a} & a < x \le b, \\ \dfrac{c-x}{c-b} & b < x \le c, \\ 0 & x > c, \end{cases} \tag{3}$$

Now, to get a crisp value by $\alpha -$ cut operation, interval ${}^{\alpha}A$ may be obtained as follows $\forall \alpha \in [0,1]$.

$$\frac{{}^{\alpha}a - a}{b-a} = \alpha, \ \frac{c - {}^{\alpha}c}{c-b} = \alpha \tag{4}$$

From Eq. 4 we get

$${}^{\alpha}a = a + (b-a)\alpha,$$
$${}^{\alpha}c = c - (c-b)\alpha,$$

Thus

$$\begin{aligned} {}^{\alpha}A &= \left[{}^{\alpha}a, {}^{\alpha}c\right], \\ &= \left[a+(b-a)\alpha, c-(c-b)\alpha\right]. \end{aligned} \tag{5}$$

Arithmetic of Fuzzy Numbers

Let $\widetilde{A}_x$ and $\widetilde{A}_y$ be two TFN of set A and are such that
$\widetilde{A}_x = [a,b,c]$ and $\widetilde{A}_y = [d,e,f]$ with membership function

$$\mu_{\tilde{A}_x}(x) = \begin{cases} 0 & x \le a, \\ \dfrac{x-a}{b-a} & a < x \le b, \\ \dfrac{c-x}{c-b} & b < x \le c, \\ 0 & x > c, \end{cases}$$

$$\mu_{\tilde{A}_y}(x) = \begin{cases} 0 & y \le d, \\ \dfrac{y-d}{e-d} & d < y \le e, \\ \dfrac{f-y}{f-e} & e < y \le f, \\ 0 & y > f, \end{cases}$$

(i) Fuzzy Addition:

$\widetilde{A}_x + \widetilde{A}_y = \widetilde{A}_z$,

$\left[a,b,c\right]+\left[d,e,f\right]=\left[a+d,b+e,c+f\right]$.

Since, membership function of $A(+)B$ is also a TFN.

(ii) Fuzzy Subtraction:

$\widetilde{A}_x - \widetilde{A}_y = \widetilde{A}_z$,

$\left[a,b,c\right]-\left[d,e,f\right]=\left[a-d,b-e,c-f\right]$.

Since, membership function of $A(-)B$ is also a TFN.

(iii) Fuzzy Product:

$\widetilde{A}_x \cdot \widetilde{A}_y \approx \widetilde{A}_z$,

$\left[a,b,c\right]\cdot\left[d,e,f\right]=\left[a\cdot d,b\cdot e,c\cdot f\right]$.

Since, the membership function of $A(\cdot)B$ is slightly parabolic. The TFN so obtained is called the triangular approximation of the given fuzzy number (see Kaufmann and Gupta (1991)).

3. FUZZY SELECTIVE MAINTENANCE MODEL

We consider a system which needs to perform a sequence of identical missions after every given (fixed) period. The system consists of several subsystems. These subsystems are divided into three groups viz. X, Y and Z.

Assumptions for the model of system are given below

(i) Two types of components are subject to the system

a) Functioning of one type of component (say type I) is highly sensitive and hence deterioration of any such component leads to complete replacement.

b) Another type of component (say type II) in the system are at low risk and after deterioration they can be repaired and placed back.

(ii) Three groups X, Y and Z are connected in series

a) Group X has type I components each of which are connected in parallel in subsystems with 1 to s number of subsystems connected in series.

b) Group Y has type II components which are also connected in parallel in subsystems with $m-s$ number of subsystems connected in series.
c) Group Z has type I and type II components connected in series with reliability same as in group X and group Y respectively. The series of duo are connected in parallel in subsystem. There are $k-m$ number of subsystems connected in series.

(iii) It is known that all components are subject to repair and replacement prior to the next run.

The model is shown below (Figure 1) for better understanding.

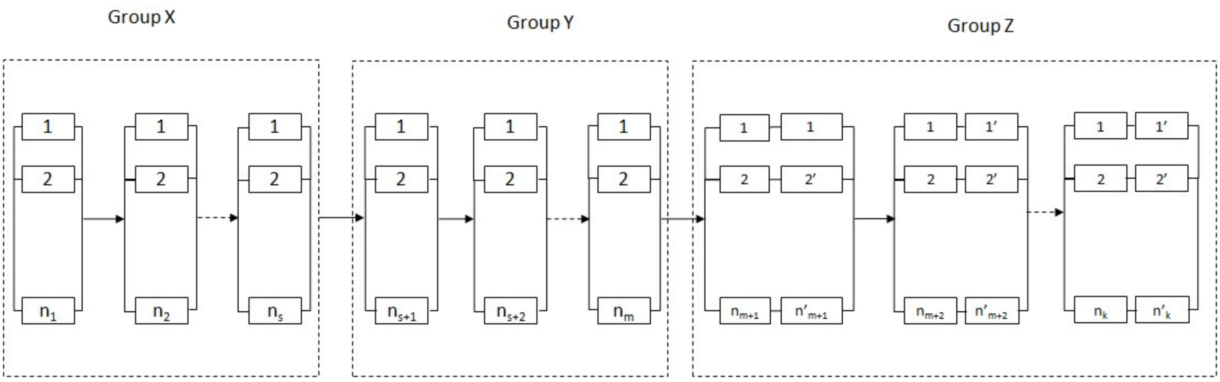

Figure 1. Selective Maintenance Model

Each component of the system has fuzzy reliability which is previously measured and given by triangular fuzzy number (see Yao et al. (2008)). The objective of the decision maker is to maximize the reliability of each group X, Y and Z improving the reliability of overall system. Group X of the system is a series arrangement of s subsystems and its reliability can be defined as

$$R_X = \prod_{i=1}^{s} 1-(1-\tilde{r}_i)^{n_i-a_i+d_i},\ i=1,\cdots,s. \tag{6}$$

Group Y consists of $m-s$ subsystems connected in series and the reliability can be defined as

$$R_Y = \prod_{i=s+1}^{m} 1-(1-\tilde{r}_i)^{n_i-a_i+d_i'},\ i=s+1,\cdots,m. \tag{7}$$

Again, group Z is a mixed model consists of $k-m$ subsystems with both series and parallel connections. The reliability is given by

$$R_Z = \prod_{i=m+1}^{k} 1-(1-\tilde{r}_i\tilde{r}_i')^{2n_i-a_i+d_i+d_i'},\ i=m+1,\cdots,k, \tag{8}$$

where

$n_i-a_i+d_i$ = number of components left at the start of next run in X group,

$n_i-a_i+d_i'$ = number of components left at the start of next run in Y group,

$2n_i-a_i+d_i+d_i'$ = number of components left at the start of next run in Z group,

$\tilde{r}_i$ = is the fuzzy reliability of the components that cannot be repaired,

$\tilde{r}_i'$ = is the fuzzy reliability of the components that can be repaired,

n_i = number of components in i^{th} subsystem,

a_i = number of failed components in i^{th} subsystem,

d_i = number of components in i^{th} subsystem that cannot be repaired and only replaced,

d_i' = number of components in i^{th} subsystem that can be replaced after repairing.

Now, while trying to maximize the reliability of the system the decision maker will also have to take care of the time limits and cost of repairable and replaceable components.

Let us consider before the next run, amount and time spent for replacing components are T_o and C_o respectively. And $T_o^{'}$ and $C_o^{'}$ for components that can be repaired.

Thus, the first two constraints are the time and cost constraint for components that can not be repaired and need replacement with a new one. The last two constraints are the time and cost constraint for components that can be repaired respectively.

$$\begin{aligned}
&\sum_{i=1}^{s} t_i d_i + \sum_{i=m+1}^{k} t_i d_i \le T_o,\\
&\sum_{i=1}^{s} c_i d_i + \sum_{i=m+1}^{k} c_i d_i \le C_o,\\
&\sum_{i=s+1}^{m} t_i' d_i' + \sum_{i=m+1}^{k} t_i' d_i' \le T_o',\\
&\sum_{i=s+1}^{m} c_i' d_i' + \sum_{i=m+1}^{k} c_i' d_i' \le C_o',
\end{aligned} \tag{9}$$

where

t_i is the time needed to replace a component,

t_i' is the time needed to repair and replace a component,

c_i is the cost per component that can not be repaired and need immediate replacement with a new one,

c_i' is the cost per component that can be repaired.

Now, we find that the cost and time of replaceable items are known but the cost and time of repairing a component depends on the type of repairing needed because of several diverse situations like uncertain judgements, unpredictable conditions or human error, etc, due to which it is not always necessary that we get a precise data. Such type of imprecise data can be very well handled by fuzzy numbers, thus we may consider that the cost and time of repair are triangular fuzzy numbers i.e., the component is subject to say three types of repairing.

Thus, the final multiobjective nonlinear programming problem (MONLPP) of the decision maker with fuzzy parameters will be as given below:

$$\left.\begin{aligned}
&\text{Maximize } R_X = \prod_{i=1}^{s} 1-(1-\tilde{r}_i)^{n_i-a_i+d_i},\ i=1,\cdots,s,\\
&\text{Maximize } R_Y = \prod_{i=s+1}^{m} 1-(1-\tilde{r}_i')^{n_i-a_i+d_i'},\ i=s+1,\cdots,m,\\
&\text{Maximize } R_Z = \prod_{i=m+1}^{k} 1-(1-\tilde{r}_i'')^{2n_i-a_i+d_i+d_i'},\ i=m+1,\cdots,k,\\
&\text{Subject to } \sum_{i=1}^{s} t_i d_i + \sum_{i=m+1}^{k} t_i d_i \le T_o,\\
&\qquad \sum_{i=1}^{s} c_i d_i + \sum_{i=m+1}^{k} c_i d_i \le C_o,\\
&\qquad \sum_{i=s+1}^{m} t_i' d_i' + \sum_{i=m+1}^{k} t_i' d_i' \le T_o',\\
&\qquad \sum_{i=s+1}^{m} c_i' d_i' + \sum_{i=m+1}^{k} c_i' d_i' \le C_o',\\
&\qquad 2 \le d_i, d_i' \le a_i \quad i=1,2,\cdots,k,
\end{aligned}\right\} \tag{10}$$

where $\tilde{r}_i'' = \tilde{r}_i \tilde{r}_i'$ is considered for simplicity. The last two constraints are the time and cost constraint of components that are subject to three kind of repairing.

4. CONVERTING FUZZY MODEL INTO AN EQUIVALENT CRISP MODEL

Let ${}^{\alpha}\left(\tilde{R}\right)$ be the $\alpha-$ cut of a fuzzy number $\tilde{R}$ defined by

$${}^{\alpha}\left(\tilde{R}\right) = \left\{ r \in \operatorname{supp}\left(\tilde{R}\right) \middle| \mu_{\tilde{R}}(r) \ge \alpha, \alpha \in [0,1) \right\}, \tag{11}$$

where $\operatorname{supp}(\widetilde{R})$ is the support of $\widetilde{R}$. Let ${}^{\alpha}(\widetilde{R})^{L}$ and ${}^{\alpha}(\widetilde{R})^{U}$ be the lower bound and upper bound of the $\alpha-$cut of $\tilde{R}$ respectively such that

$$ {}^{\alpha}(\widetilde{R})^{L} \le {}^{\alpha}(\widetilde{R}) \le {}^{\alpha}(\widetilde{R})^{U}. \tag{12}$$

Then, for a prescribed value of α, objectives $\widetilde{R}_X, \widetilde{R}_Y$ and $\widetilde{R}_Z$ to be maximized can be replaced by the upper bound of their respective $\alpha-$cuts, that is

$$\left.\begin{aligned} {}^{\alpha}(\widetilde{R}_X)^{U} &= \prod_{i=1}^{s}\left\{1-\left(1-{}^{\alpha}\{\widetilde{r}_i\}\right)^{n_i-a_i+d_i}\right\}^{U}, \\ {}^{\alpha}(\widetilde{R}_Y)^{U} &= \prod_{i=s+1}^{m}\left\{1-\left(1-{}^{\alpha}\{\widetilde{r}_i'\}\right)^{n_i-a_i+d_i'}\right\}^{U}, \\ {}^{\alpha}(\widetilde{R}_X)^{U} &= \prod_{i=m+1}^{k}\left\{1-\left(1-{}^{\alpha}\{\widetilde{r}_i'\}\right)^{2n_i-a_i+d_i+d_i'}\right\}^{U}. \end{aligned}\right\} \tag{13}$$

For inequality constraints with TFNs

$$\left.\begin{aligned} &\sum_{i=s+1}^{m}\widetilde{t}_i' d_i' + \sum_{i=m+1}^{k}\widetilde{t}_i' d_i' \le T_o', \\ &\sum_{i=s+1}^{m}\widetilde{c}_i' d_i' + \sum_{i=m+1}^{k}\widetilde{c}_i' d_i' \le C_o' \end{aligned}\right\} \tag{14}$$

can be rewritten as follows

$$\left.\begin{aligned} &\sum_{i=s+1}^{m}{}^{\alpha}\left(\widetilde{t}'\right)_i^{L} d_i' + \sum_{i=m+1}^{k}{}^{\alpha}\left(\widetilde{t}_i'\right)^{L} d_i' \le T_o', \\ &\sum_{i=s+1}^{m}{}^{\alpha}\left(\widetilde{c}_i'\right)_i^{L} d_i' + \sum_{i=m+1}^{k}{}^{\alpha}\left(\widetilde{c}_i'\right)^{L} d_i' \le C_o'. \end{aligned}\right\} \tag{15}$$

Therefore, the problem represented by (10) can be transformed into the following standard multiobjective nonlinear programming problem (MONLPP)

$$\left.\begin{aligned} &\text{Maximize } {}^{\alpha}(\widetilde{R}_X)^{U} = \prod_{i=1}^{s}{}^{\alpha}\left\{1-\left(1-{}^{\alpha}\{\widetilde{r}_i\}\right)^{n_i-a_i+d_i}\right\}^{U}, \\ &\text{Maximize } {}^{\alpha}(\widetilde{R}_Y)^{U} = \prod_{i=s+1}^{m}{}^{\alpha}\left\{1-\left(1-{}^{\alpha}\{\widetilde{r}_i'\}\right)^{n_i-a_i+d_i'}\right\}^{U}, \\ &\text{Maximize } {}^{\alpha}(\widetilde{R}_X)^{U} = \prod_{i=m+1}^{k}{}^{\alpha}\left\{1-\left(1-{}^{\alpha}\{\widetilde{r}_i'\}\right)^{2n_i-a_i+d_i+d_i'}\right\}^{U}, \\ &\text{Subject to } \sum_{i=1}^{s} t_i d_i + \sum_{i=m+1}^{k} t_i d_i \le T_o, \\ &\sum_{i=1}^{s} c_i d_i + \sum_{i=m+1}^{k} c_i d_i \le C_o, \\ &\sum_{i=s+1}^{m}{}^{\alpha}\left(\widetilde{r}'\right)_i^{L} d_i' + \sum_{i=m+1}^{k}{}^{\alpha}\left(\widetilde{r}_i'\right)^{L} d_i' \le T_o', \\ &\sum_{i=s+1}^{m}{}^{\alpha}\left(\widetilde{c}'\right)_i^{L} d_i' + \sum_{i=m+1}^{k}{}^{\alpha}\left(\widetilde{c}_i'\right)^{L} d_i' \le C_o'. \\ &2 \le d_i, d_i' \le a_i \quad i = 1, 2, \cdots, k, \end{aligned}\right\} \tag{16}$$

5. FUZZY PROGRAMMING APPROACH

NLPP(16) can be solved using fuzzy programming. For that, we first need to define the membership functions. The membership function for the objective functions of group X, Y and Z can be given by

$$\mu_j^{\alpha}\left(R_j\right) = \frac{\left[{}^{\alpha}\left(\tilde{R}_j\right)^{U} - {}^{\alpha}\left(R_j\right)^{-}\right]}{\left[{}^{\alpha}\left(R_j\right)^{O} - {}^{\alpha}\left(R_j\right)^{-}\right]}, \quad j = X, Y \text{ and } Z \tag{17}$$

where the aspired level ${}^{\alpha}\left(R_j\right)^{O}$ and highest acceptable level ${}^{\alpha}\left(R_j\right)^{-}$ are ideal and anti ideal solutions respectively, which can be obtained by solving each of the following problems independently

$${}^{\alpha}\left(R_X\right)^{-} = \min \prod_{i=1}^{s} 1 - \left[1 - \left\{r^{(1)} + \left(r^{(2)} - r^{(1)}\right)\alpha\right\}\right]^{n_i - a_i + d_i} \tag{18}$$

$${}^{\alpha}\left(R_X\right)^{O} = \max \prod_{i=1}^{s} 1 - \left[1 - \left\{r^{(3)} - \left(r^{(3)} - r^{(2)}\right)\alpha\right\}\right]^{n_i - a_i + d_i} \tag{19}$$

$${}^{\alpha}\left(R_Y\right)^{-} = \min \prod_{i=s+1}^{m} 1 - \left[1 - \left\{r'^{(1)} + \left(r'^{(2)} - r'^{(1)}\right)\alpha\right\}\right]^{n_i - a_i + d_i} \tag{20}$$

$${}^{\alpha}\left(R_Y\right)^{O} = \max \prod_{i=s+1}^{m} 1 - \left[1 - \left\{r'^{(3)} - \left(r'^{(3)} - r'^{(2)}\right)\alpha\right\}\right]^{n_i - a_i + d_i} \tag{21}$$

and

$${}^{\alpha}\left(R_Z\right)^{-} = \min \prod_{i=m+1}^{k} 1 - \left[1 - \left\{r''^{(1)} + \left(r''^{(2)} - r''^{(1)}\right)\alpha\right\}\right]^{2n_i - a_i + d_i} \tag{22}$$

$${}^{\alpha}\left(R_Z\right)^{O} = \max \prod_{i=m+1}^{k} 1 - \left[1 - \left\{r''^{(3)} - \left(r''^{(3)} - r'''^{(2)}\right)\alpha\right\}\right]^{2n_i - a_i + d_i} \tag{23}$$

where $r^{(1)}, r^{(2)}$ and $r^{(3)}$ are TFN of reliability of components. The Fuzzy Programming Problem (FPP) model of the problem can be explicitly formulated as follows:

$$\max \xi \tag{24}$$

$$\begin{aligned}
\text{Subject to } \ \xi &\le \frac{\left[{}^{\alpha}\left(\tilde{R}_j\right)^{U} - {}^{\alpha}\left(R_j\right)^{-}\right]}{\left[{}^{\alpha}\left(R_j\right)^{O} - {}^{\alpha}\left(R_j\right)^{-}\right]}, \\
&\sum_{i=1}^{s} t_i d_i + \sum_{i=m+1}^{k} t_i d_i \le T_o, \\
&\sum_{i=1}^{s} c_i d_i + \sum_{i=m+1}^{k} c_i d_i \le C_o, \\
&\sum_{i=s+1}^{m} \left\{t_i'^{(1)} + \left(t_i'^{(2)} - t_i'^{(1)}\right)\alpha\right\} d_i' + \sum_{i=m+1}^{k} \left\{t_i'^{(1)} + \left(t_i'^{(2)} - t_i'^{(1)}\right)\alpha\right\} d_i' \le T_o', \\
&\sum_{i=s+1}^{m} \left\{c_i'^{(1)} + \left(c_i'^{(2)} - c_i'^{(1)}\right)\alpha\right\} d_i' + \sum_{i=m+1}^{k} \left\{c_i'^{(1)} + \left(c_i'^{(2)} - c_i'^{(1)}\right)\alpha\right\} d_i' \le C_o', \\
&2 \le d_i, d_i' \le a_i \quad i = 1, 2, \cdots, k, \ \ j = X, Y, Z,
\end{aligned} \tag{25}$$

where ξ represent the deviation.

6. NUMERICAL EXAMPLE

A selective maintenance problem has been considered here with all the groups consisting of 3 subsystems. The total time and cost that can be spent on replaceable and repairable components are given by, $T_o = 75, C_o = 150, T_o' = 40, C_o' = 115$

Table 1: Numerical values of some parameters involved in selective maintenance model

	Group X			Group Y			Group Z		
Subsystems	1	2	3	4	5	6	7	8	9
n_i	7	5	8	8	10	12	8	7	10
a_i	3	2	4	3	4	5	12	8	11
t_i	6	10	7	×	×	×	6	10	7
$\tilde{t}_i$	×	×	×	$\tilde{3}$	$\tilde{4}$	$\tilde{3}$	$\tilde{3}$	$\tilde{4}$	$\tilde{3}$
c_i	16	12	13	×	×	×	16	12	13
$\tilde{c}_i$	×	×	×	$\tilde{8}$	$\tilde{7}$	$\tilde{8}$	$\tilde{8}$	$\tilde{7}$	$\tilde{8}$
$\tilde{r}_i$	$\widetilde{0.65}$	$\widetilde{0.55}$	$\widetilde{0.70}$	×	×	×	$\widetilde{0.65}$	$\widetilde{0.55}$	$\widetilde{0.70}$
$\tilde{r}'_i$	×	×	×	$\widetilde{0.70}$	$\widetilde{0.55}$	$\widetilde{0.60}$	$\widetilde{0.70}$	$\widetilde{0.55}$	$\widetilde{0.60}$

6.1 SOLUTION FOR FUZZY MODEL

The MONLPP formulated using (10) is

$$\text{Max } R_1 = 1-\left(1-\widetilde{0.65}\right)^{4+d_1} \times 1-\left(1-\widetilde{0.55}\right)^{3+d_2} \times 1-\left(1-\widetilde{0.70}\right)^{4+d_3}$$

$$\text{Max } R_2 = 1-\left(1-\widetilde{0.70}\right)^{5+d'_4} \times 1-\left(1-\widetilde{0.55}\right)^{6+d'_5} \times 1-(1-\widetilde{0.60})^{7+d'_6}$$

$$\text{Max } R_3 = 1-\left(1-\left(\widetilde{0.65}\times\widetilde{0.70}\right)^{4+d_7+d'_7}\right) \times 1-\left(1-\left(\widetilde{0.55}\times\widetilde{0.55}\right)\right)^{6+d_8+d'_8}$$

$$\times 1-\left(1-\left(\widetilde{0.70}\times\widetilde{0.60}\right)\right)^{9+d_9+d'_9}$$

$$\begin{aligned}
\text{s.t.}\quad & 6d_1 + 10d_2 + 7d_3 \le 75 \\
& 16d_1 + 12d_2 + 13d_3 \le 150 \\
& \tilde{3}d'_4 + \tilde{4}d'_5 + \tilde{3}d'_6 \le 40 \\
& \tilde{8}d'_4 + \tilde{7}d'_5 + \tilde{8}d'_6 \le 115 \\
& 6d_7 + 10d_8 + 7d_9 \le 75 \\
& 16d_7 + 12d_8 + 13d_9 \le 150 \\
& \tilde{3}d'_7 + \tilde{4}d'_8 + \tilde{3}d'_9 \le 40 \\
& \tilde{8}d'_7 + \tilde{7}d'_8 + \tilde{8}d'_9 \le 115 \\
& 1 \le d_i, d'_i \le n_i
\end{aligned}$$

The triangular fuzzy numbers are

$\widetilde{0.55} = (0.50, 0.55, 0.60), \widetilde{0.60} = (0.55, 0.60, 0.65), \widetilde{0.65} = (0.60, 0.65, 0.70), \widetilde{0.70} = (0.65, 0.70, 0.75),$

$\tilde{3} = (2,3,4), \tilde{4} = (3,4,5), \tilde{7} = (6,7,8), \tilde{8} = (6,8,10).$

After adding the $\alpha-$ cut the problem becomes

$$\text{Max } R_1^+ = 1-\left(1-\left(0.7-0.05\alpha\right)^{4+d_1}\right)\times 1-\left(1-\left(0.6-0.05\alpha\right)^{3+d_2}\right)\times 1-\left(1-\left(0.75-0.05\alpha\right)^{4+d_3}\right)$$
$$\text{Max } R_2^+ = 1-\left(1-\left(0.65-0.05\alpha\right)^{5+d_4'}\right)\times 1-\left(1-\left(0.60-0.05\alpha\right)^{6+d_5'}\right)\times 1-(1-\left(0.70-0.05\alpha\right)^{7+d_6'})$$
$$\text{Max } R_3^+ = 1-\left(1-\left(0.525-0.07\alpha\right)^{4+d_7+d_7'}\right)\times 1-\left(1-\left(0.360-0.0575\alpha\right)^{6+d_8+d_8'}\right)$$
$$\times 1-\left(1-\left(0.525-0.0675\alpha\right)^{9+d_9+d_9'}\right)$$

s.t. $\tilde{6}d_1+\widetilde{10}d_2+\ \tilde{7}d_3 \le 75$

$$\widetilde{16}d_1+\widetilde{12}d_2+\widetilde{13}d_3 \le 150$$
$$\left(2+\alpha\right)d_4'+\left(3+\alpha\right)d_5'+\left(2+\alpha\right)d_6' \le 40$$
$$\left(6+2\alpha\right)d_4'+\left(6+\alpha\right)d_5'+\left(6+2\alpha\right)d_6' \le 115$$
$$\tilde{6}d_7+\widetilde{10}d_8+\tilde{7}d_9 \le 75$$
$$\widetilde{16}d_7+\widetilde{12}d_8+\widetilde{13}d_9 \le 150$$
$$\left(2+\alpha\right)d_7'+\left(3+\alpha\right)d_8'+\left(2+\alpha\right)d_9' \le 40$$
$$\left(6+2\alpha\right)d_7'+\left(6+\alpha\right)d_8'+\left(6+2\alpha\right)d_9' \le 115$$
$$1 \le d_i, d_i' \le n_i$$

The ideal and anti-ideal solution for each individual objective function is given by

${}^{\alpha}\left(R_X\right)^O = 0.9969825,\ {}^{\alpha}\left(R_X\right)^- = 0.8829938,\ {}^{\alpha}\left(R_Y\right)^O = 0.9999357,$

${}^{\alpha}\left(R_Y\right)^- = 0.9923149,\ {}^{\alpha}\left(R_Z\right)^O = 0.9983845,\ {}^{\alpha}\left(R_Z\right)^- = 0.8864502,$

The final compromise allocation of the repairable and replaceable components at a prescribed level of α , i.e. $\alpha = 0.5$ is obtained after solving the above FPP.

The compromise allocation obtained after solving the FPP are $\overline{d}^c = (2,5,2,4,5,1)$ and $\overline{d}'^c = \left(4,6,4,4,7,2\right)$; $\rho = 6.38e^{-06}$, where $\overline{d}^c$ is $\left(d_1,d_2,d_3,d_7,d_8,d_9\right)$ and $\overline{d}'^c$ is $\left(d_4',d_5',d_6',d_7',d_8',d_9'\right)$.

6.2 SOLUTION FOR CRISP VALUES

The MONLPP formulated using (10) is

$$\text{Max } R_1 = 1-\left(1-0.65\right)^{4+d_1}\times 1-\left(1-0.55\right)^{3+d_2}\times 1-\left(1-0.70\right)^{4+d_3}$$
$$\text{Max } R_2 = 1-\left(1-0.70\right)^{5+d_4'}\times 1-\left(1-0.55\right)^{6+d_5'}\times 1-(1-0.60)^{7+d_6'}$$
$$\text{Max } R_3 = 1-\left(1-\left(0.65\times 0.70\right)^{4+d_7+d_7'}\right)\times 1-\left(1-\left(0.55\times 0.55\right)\right)^{6+d_8+d_8'}$$
$$\times 1-\left(1-\left(0.70\times 0.60\right)\right)^{9+d_9+d_9'}$$

s.t. $6d_1+10d_2+\ 7d_3 \le 75$

$$16d_1+12d_2+13d_3 \le 150$$
$$3d_4'+4d_5'+3d_6' \le 40$$
$$8d_4'+7d_5'+8d_6' \le 115$$
$$6d_7+10d_8+7d_9 \le 75$$
$$16d_7+12d_8+13d_9 \le 150$$
$$3d_7'+4d_8'+3d_9' \le 40$$
$$8d_7'+7d_8'+8d_9' \le 115$$
$$1 \le d_i, d_i' \le n_i$$

The ideal and anti-ideal solution for each individual objective function is given by

${}^{\alpha}\left(R_X\right)^O = 0.9952518,\ {}^{\alpha}\left(R_X\right)^- = 0.9516388,\ {}^{\alpha}\left(R_Y\right)^O = 0.9996763,$

${}^{\alpha}\left(R_Y\right)^- = 0.9948846,\ {}^{\alpha}\left(R_Z\right)^O = 0.9947687,\ {}^{\alpha}\left(R_Z\right)^- = 0.9169451,$

Now, the final compromise allocation of the components with crisp values of time, cost and reliability is obtained after solving the following FPP

$$\begin{aligned}
&\max \quad \rho' \\
&s.t \quad \rho' \le \frac{\left(\left(1-(1-0.65)^{4+d_1}\right)\times\left(1-(1-0.55)^{3+d_2}\right)\times\left(1-(1-0.70)^{4+d_3}\right)\right)-0.995252}{0.043613} \\
&\qquad \rho' \le \frac{\left(\left(1-(1-0.70)^{5+d'_4}\right)\times\left(1-(1-0.55)^{6+d'_5}\right)\times\left(1-(1-0.60)^{7+d'_6}\right)\right)-0.999676}{0.004792} \\
&\qquad \rho' \le \frac{\left(\left(1-\left(1-(0.65\times0.70)^{4+d_7+d'_7}\right)\right)\times\left(1-\left(1-(0.55\times0.55)\right)^{6+d_8+d'_8}\right)\times\left(1-\left(1-(0.70\times0.60)\right)^{9+d_9+d'_9}\right)\right)-0.994769}{0.077824}
\end{aligned}$$

$$\begin{aligned}
\text{s.t.}\quad & 6d_1+10d_2+\ 7d_3\le 75 \\
& 16d_1+12d_2+13d_3\le 150 \\
& 3d'_4+4d'_5+3d'_6\le 40 \\
& 8d'_4+7d'_5+8d'_6\le 115 \\
& 6d_7+10d_8+7d_9\le 75 \\
& 16d_7+12d_8+13d_9\le 150 \\
& 3d'_7+4d'_8+3d'_9\le 40 \\
& 8d'_7+7d'_8+8d'_9\le 115 \\
& 1\le d_i, d'_i\le n_i
\end{aligned}$$

The compromise allocation obtained after solving the FPP are $\overline{d}^{c}=(3,5,1,6,3,1)$ and $\overline{d}'^{c}=(3,5,3,1,7,3)$; $\rho=2.70e^{-07}$ where $\overline{d}^{c}$ is $(d_1,d_2,d_3,d_7,d_8,d_9)$ and $\overline{d}'^{c}$ is $(d'_4,d'_5,d'_6,d'_7,d'_8,d'_9)$.

7. SUMMARY

The optimal compromise objective values obtained after solving the formulated MONLPP (10) and crisp problem using FPP approach are shown in table 2

From table 2 it can be seen that the reliability of group X,Y and Z is much more when the cost and time are considered fuzzy in comparison to the crisp model. The MONLPPs are solved using LINGO-13 solver.

Table 2: Solution table for optimal compromise reliability using Fuzzy Programming Problem

	Group X	Group Y	Group Z	Trace Value
Reliability for Fuzzy Model	0.9957569	0.9998694	0.9999698	2.991968
Reliability for Crisp Model	0.9952518	0.9996763	0.9947687	2.989697

8. CONCLUSION

Many authors have studied system reliability in selective maintenance problems and have developed decision making problems to obtain the optimum allocation of the deteriorating components. In this paper we have considered the latest problem of imprecision in the data. The components of the system have fuzzy reliability. The cost and time required for the two types of components taken into consideration are also fuzzy in nature. Moreover, the decision maker has a hypothetical model with multiple objectives of maximizing reliability of different groups of subsystems. The objective of the decision maker is to simultaneously maximize the reliability of all the subsystems and find the optimal number of components required to be replaced or repaired after deterioration in the next run. Since the individual optimal solution of each group of subsystem might not happen to be optimal for another group, an optimal compromise allocation of repairable and replaceable components in the system is obtained. The problem is formulated as a multiobjective nonlinear programming problem with TFNs and solved using fuzzy programming problem at a prescribed level of $\alpha-$ cut. The problem is then compared with its crisp model and it can be seen in table 2, system reliabilty of the fuzzy model is found to be better over its alternative crisp model.

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