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Improvement for correct procedure of solution procedure on inventory models with trapezoidal type demand rate

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Abstract: Recently, a paper provided an improvement for a previous article to offer a new solution procedure for inventory model with trapezoidal type demand. The purpose of this paper is twofold. First, we will point out that the improvement did not provide any new results for the solution procedure. Second, we present a previous published finding to dramatically simplify the solution method. Our results will help practitioners understand the results of two previous papers.
Keyword — Inventory, deteriorating items, trapezoidal type demand rate.

1. INTRODUCTION

Hill (1995) is the first paper to develop inventory model with ramp type demand. Following his trend, there are 55 papers that have developed inventory models. We may classify them as follows. Zhou (2003), Agrawal and Banerjee (2011), and Skouri and Konstantaras (2013) studied for two-warehouse inventory model. Wu, (2001), Giri et al. (2003), Chen et al. (2006), Skouri et al. (2009), Skouri et al. (2011a), and Ahmed et al. (2013) developed for general deterioration rate. Cheng and Wang (2009), Cheng et al. (2011), Chung (2012), Chuang et al. (2013), Dem et al. (2014), Debata et al. (2015), Lin, K. P. (2013), Mehrotr (2013), and Zhao (2014) consider inventory models with trapezoidal type demand rate. Darzanou and Skouri (2011), Saha et al. (2013), Singh and Sharma (2013), and Sah and Sarmah (2015), examined model with two echelon supply chain. Hariga (1996), Wu et al. (1999), Deng et al. (2007), Hung (2011), Khedlekar et al. (2013), Konstantaras and Skouri (2009), Mandal, B. (2010), Mingbao and Bixi (2010), and Chauhan and Singh (2015), considered for general deteriorating items. Jaggi et al. (2015), Kumar et al. (2012), and Mahata and Goswami (2009) constructed models under fuzzy environment. Zhou et al. (2004), Lin, S. W. (2011), and Roy and Chaudhuri (2011), investigated inventory system with a finite time horizon. The other papers can be describes as Abdul and Murata (2011a) with set-up adjustment, and Chauhan and Singh (2014) for discounted cash flow, Andriolo et al. (2013) with sustainability, Diponegoro and Sarker (2002), and Diponegoro and Sarker (2007) with batch sizes, Banerjee and Sharma (2010) with option to change the market, Abdul and Murata (2011a) under unknown time horizon, Goyal et al. (2013) for ameliorating items, Lin et al. (2013) for a demand independent inventory model, Abdul and Murata (2011c) under inflation, Sharma (2009), and Lin, Y. (2013) with pricing strategy, Lin et al. (2012) under stock-dependent consumption rate, Wou, (2010) with a stochastic demand, Skouri et al. (2011b), and Tung et al. (2014) under permissible delay in payments, Tung (2013) for negative exponentially distributed changing point. At last, Andriolo et al. (2014) is a review for Harris's inventory model.

In this article, we pay attention to Cheng and Wang (2009) that extended the ramps type demand rate of Hill (1995) to trapezoidal type demand rate. The trapezoidal type demand rate increases with time up to certain time and then ultimately stabilizes and becomes constant, and finally the demand rate approximately decreases to constant or zero, and then begins the next replenishment cycle. In practice, Cheng and Wang (2009) indicate that such type of demand rate is quite realistic and useful. Although the inventory model of Cheng and Wang (2009) is interesting, they derive the solution procedure consisting of three properties and one theorem to locate the optimal solution of their model such that their solution procedure seems rather complicated. So, the main purpose of this paper not only incorporates but also simplifies the solution procedure of Cheng and Wang (2009). Chung (2012) published a paper to provide a correct procedure of the solution procedure on inventory models with trapezoidal type demand rate proposed by Cheng and Wang (2009). The purpose of this paper is first to point out that Chung did not provide any new discovery such that his correct procedure already published in Cheng and Wang (2009). Second, we will recall the finding of Lin (2011) that already provided a new and simplified approach to improve the solution procedure of Cheng and Wang (2009).

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2. REVIEW OF CHENG AND WANG (2009) AND CHUNG (2012)

We will briefly review the findings of Cheng and Wang (2009) and Chung (2012). The detailed derivation, please refer to their original papers.
The trapezoidal type demand rate is defined as

$$D(t) = \begin{cases} a_1 + b_1 t, & 0 \le t \le \mu_1 \\ D_0, & \mu_1 \le t \le \mu_2 \\ a_2 - b_2 t, & \mu_2 \le t \le T \le a_2/b_2 \end{cases} \tag{1}$$

where μ_1 is the time point changing from linearly increasing demand to constant demand and μ_2 is the time point changing from constant demand to linearly decreasing demand, with $a_1 + b_1\mu_1 = D_0 = a_2 - b_2\mu_2$.
The inventory model discussed in this paper is the same as that of Cheng and Wang (2009). Using the notation and assumptions adopted by Cheng and Wang (2009), we have three cases to be explored as follows:

Case 1: $0 \le t_1 \le \mu_1$. Cheng and Wang (2009) revealed that the average total cost per unit time $C_1(t_1)$ on $[0, \mu]$ can be given by

$$C_1(t_1) = \frac{1}{T}[A_0 + c_1 D_{1T} + c_2 H_{1T} + c_3 B_{1T}], \tag{2}$$

where

$$D_{1T} = (\frac{a_1}{\theta} + \frac{b_1}{\theta^2})(e^{\theta t_1} - 1) + \frac{b_1}{\theta} t_1 e^{\theta t_1} - a_1 t_1 - \frac{b_1}{2} t_1^2, \tag{3}$$

$$H_{1T} = (\frac{a_1 + b_1 t}{\theta} + \frac{b_1}{\theta^3})(e^{\theta t_1} - 1) - \frac{a_1\theta - b_1}{\theta^2} t_1 - \frac{b_1}{2\theta} t_1^2, \tag{4}$$

and

$$B_{1T} = \frac{a_1}{2}(t_1 - \mu_1)(t_1 + \mu_1 - 2T) + \frac{b_1}{6}(2t_1^3 - 2\mu_1^3 + 3Tt_1^2) + \frac{a_1}{2}(\mu_2 - T)^2 + \frac{b_2}{6}(3T\mu_2^2 - T^3 - 2\mu_2^3)$$
$$+ \frac{D_0}{2}(\mu_1 - \mu_2)(\mu_1 + \mu_2 - 2T). \tag{5}$$

Equation (2) yields the first-order derivatives of $C_1(t_1)$ with respect to t_1 as follows:

$$\frac{dC_1(t_1)}{dt_1} = [(c_1 + \frac{c_1}{\theta})(e^{\theta t_1} - 1) + c_3(t - T)](a_1 + b_1 t_1). \tag{6}$$

Case 2: $\mu_1 \le t_1 \le \mu_2$. Cheng and Wang (2009) revealed that the average total cost per unit time $C_2(t_1)$ on $[\mu_1, \mu_2]$ can be given by

$$C_2(t_1) = \frac{1}{T}[A_0 + c_1 D_{2T} + c_2 H_{2T} + c_3 B_{2T}], \tag{7}$$

Where

$$D_{2T} = \frac{D_0}{\theta^2} e^{\theta t_1} - \frac{b_1}{\theta^2} e^{\theta\mu_1} + \frac{b_1}{\theta^2} - \frac{a_1}{\theta} - D_0 t_1 + \frac{b_1}{2}\mu_1^2, \tag{8}$$

$$H_{2T} = \frac{D_0}{\theta^2} e^{\theta t_1} - \frac{b_1}{\theta^3} e^{\theta\mu_1} + \frac{b_1}{\theta 3} - \frac{a_1}{\theta^2} - \frac{D_0}{\theta} t_1 + \frac{b_1}{2\theta}\mu_1^2, \tag{9}$$

And

$$B_{2T} = \frac{D_0}{2}(\mu_2 - t_1)^2 + \frac{a_2}{2}(T - \mu_2)^2 + \frac{b_2}{6}(3T\mu_2^3 - T^3 - 2\mu_2^3) + D_0(\mu_2 - t_1)(T - \mu_2). \tag{10}$$

Equation (7) yields the first-order derivatives of $C_2(t_1)$ with respect to t_1 as follows:

$$\frac{dC_2(t_1)}{dt_1} = \frac{D_0}{T}[(c_1 + \frac{c_2}{\theta})(e^{\theta t_1} - 1) + c_3(t - T)]. \tag{11}$$

Case 3: $\mu_1 \le t_1 \le T \le \frac{a_2}{b_2}$. Cheng and Wang (2009) revealed that the average total cost per unit time $C_3(t_1)$ on $[\mu_1, T]$ can be given by

$$C_3(t_1) = \frac{1}{T}[A_0 + c_1 D_{3T} + c_2 H_{3T} + c_3 B_{3T}], \tag{12}$$

where

$$D_{3T} = \frac{[\theta(a_2 - b_1 t_1) + b_2]e^{\theta t_1} - b_1 e^{\theta\mu_1} - b_2 e^{\theta\mu_2} + b_1 - a_1\theta]}{\theta^2} + \frac{b_0}{2}\mu_1^2 - a_2 t_2 + \frac{b_2}{2}(t_1^2 + \mu_2^2). \tag{13}$$

and

$$H_{3T} = \frac{[\theta(a_2 - b_1 t_1) + b_2]e^{\theta t_1} - b_1 e^{\theta\mu_1} - b_2 e^{\theta\mu_2} + b_1 - a_1\theta]}{\theta^3} + \frac{b_1}{2\theta}\mu_1^2 - \frac{a_2}{\theta}t_1 + \frac{b_2}{2\theta}(t_1^2 + \mu_2^2), \tag{14}$$

And

$$B_{3T} = \frac{a_2}{2}(T - \mu_1)^2 + \frac{b_2}{2}t_1^2(T - t_1) + \frac{b_2}{6}(t_1^3 - T^3)). \tag{15}$$

Equation (12) yields the first-order derivatives of $C_3(t_1)$ with respect to t_1 as follows:

$$\frac{dC_3(t_1)}{dt_1} = \frac{a_2 - b_2 t_1}{T}[(c_1 + \frac{c_2}{\theta})(e^{\theta t_1} - 1) + c_3(t_1 - T)]. \tag{16}$$

Cheng and Wang (2009) derived the annual total cost per unit time $C\left(t_1\right)$ on $\left[0, T\right]$ as follows

$$C\left(t\right) = \begin{cases} C_1\left(t_1\right), & if \quad 0 \le t_1 \le \mu_1 \\ C_2\left(t_1\right), & if \ \mu_1 \le t_1 \le \mu_2 \\ C_3\left(t_1\right), & if \quad \mu_2 \le t_1 \le T \end{cases} \tag{17}$$

Cheng and Wang derived that

$$\frac{d}{dt_1}C_1\left(t_1\right) = \frac{a_1 + b_1 t_1}{T} f\left(t_1\right) \tag{18}$$

$$\frac{d}{dt_1}C_2\left(t_1\right) = \frac{D_0}{T} f\left(t_1\right) \tag{19}$$

$$\frac{d}{dt_1}C_3\left(t_1\right) = \frac{a_2 - b_2 t_1}{T} f\left(t_1\right) \tag{20}$$

$0 < t_1 < T$ where $f\left(t_1\right) = \left(c_1 + \frac{c_2}{\theta}\right)\left(e^{\theta t_1} - 1\right) + c_3\left(t_1 - T\right)$ for $0 \le t_1 \le T$.

Cheng and Wang (2009) and Chung (2012) both agreed that

$$f\left(t_1\right) = 0 \tag{21}$$

has a unique solution for $0 < t_1 < T$. In Cheng and Wang (2009) the solution is denoted by t_1^* and in Chung (2012), the solution is denoted by t_f^*.

We recalled the Property 1 of Cheng and Wang (2009). They mentioned that $C_1\left(t_1\right)$ obtains its minimum at $t_1 = t_1^*$, if $t_1^* < \mu_1$, and on the other hand, $C_1\left(t_1\right)$ obtains its minimum at $t_1 = \mu_1$, if $t_1^* \ge \mu_1$.
To compatible with Cheng and Wang (2009) and Chung (2012), we will use the expression, t_f^*, for the solution of $f\left(t_1\right) = 0$.

3. DISCUSSION OF PROOF PROCEDURE OF CHENG AND WANG (2009) AND CHUNG (2012)

We can simply the derivation of Cheng and Wang (2009) as follows:

$$\text{optimal solution of } C_1(t_1) = \min\{t_f^*, \mu_1\}. \tag{22}$$

Chung (2012) asserted that the validity of Property 1 of Cheng and Wang (2009) is questionable. We quoted his explanation "it is concluded that t_f^* is only the optimal the optimal solution of $C_1(t_1)$ on $[0,T]$ but it is not necessary the optimal solution t_1^* (Remark. In Chung (2012), he used of t_1^* as the optimal solution for $C(t_1)$, but in Cheng and Wang (2009), t_1^* is used for the solution of $f(t_1)=0$) $C(t_1)$ on $[0,T]$."

We agreed the comment of Chung (2012) that t_f^* is the optimal solution for $C_1(t_1)$ on $[0,T]$, but the domain of $C_1(t_1)$ is $[0,\mu_1]$. However, Cheng and Wang (2009) already were aware that t_f^* may be not the optimal solution for $C_1(t_1)$.

Cheng and Wang (2009) already derived that the optimal solution of $C_1(t_1)$ is the smaller one of t_f^* and μ_1. Hence, the challenge proposed by Chung (2012) to the Property 1 of Cheng and Wang (2009) is invalid.
In Property 2 of Cheng and Wang (2009), they derived that

$$\text{optimal solution of } C_2(t_1) = mid\{t_f^*, \mu_1, \mu_2\}. \tag{23}$$

where $mid\{1,2,3\}=2$ and $mid\{1,2,2\}=2$ represents the middle number among the threes.

In Property 3 of Cheng and Wang (2009), they derived that

$$\text{optimal solution of } C_3(t_1) = \max\{t_f^*, \mu_2\}. \tag{24}$$

Similarly, the challenge of Properties 2 and 3 proposed by Chung (2012) are also invalid.
Chung (2012) used t_{11}^*, t_{12}^* and t_{13}^* to denote the optimal solution of $C_1(t_1)$, $C_2(t_1)$ and $C_3(t_1)$, respectively.
We quoted the Eq. (16) of Chung (2012) for later discussion,

$$C(t_1^*) = \min\{C_1(t_{11}^*), C_2(t_{12}^*), C_3(t_{13}^*)\} \tag{25}$$

We must point out that the correct process proposed in Section 5 of Chung (2012) is only rewritten the above findings of equations (22-24) as follows

(i) If $0 < t_f^* < \mu_1$, then Chung (2012) derived

$$t_{11}^* = t_f^*,\ t_{12}^* = \mu_1 \text{ and } t_{13}^* = \mu_2. \tag{26}$$

(ii) If $\mu_1 \le t_f^* < \mu_2$, then Chung (2012) derived

$$t_{11}^* = \mu_1,\ t_{12}^* = t_f^* \text{ and } t_{13}^* = \mu_2. \tag{27}$$

(iii) If $\mu_2 \le t_f^* < T$, then Chung (2012) derived

$$t_{11}^* = \mu_1,\ t_{12}^* = \mu_2 \text{ and } t_{13}^* = t_f^*. \tag{28}$$

We quoted from Chung (2012), "The above arguments (i), (ii) and (iii) complement the key shortcoming of that Chung and Wang (2009) do not explore Eq. (16) (Remark. The Eq (16) of Chung (2012) is the equation (25) of this paper) to concretely demonstrate $t_1^* = t_f^*$".

If we compared the results of equations (22-24) with equations (26-28) then it is apparently that the findings proposed by Chung (2012) are more tedious which contain three unnecessary expressions of t_{11}^*, t_{12}^* and t_{13}^*.

We recalled the Theorem 1 of Cheng and Wang (2009) that they derived t_f^* is the optimal solution for $C(t_1)$ and the minimum value is expressed as follows

$$\min C(t_1) = \begin{cases} C_1(t_f^*), & if \quad 0 < t_f^* \le \mu_1 \\ C_2(t_f^*), & if \quad \mu_1 \le t_f^* \le \mu_2 \\ C_3(t_f^*), & if \quad \mu_2 \le t_f^* < T \end{cases}. \tag{29}$$

We recalled the Theorem 1 of Chung (2012) then he derived that the optimal replenishment time of $C(t_1)$ is t_f^*.

If we compared the Theorem 1 of Cheng and Wang (2009) and the Theorem 1 of Chung (2012), then Cheng and Wang (2009) derived an explicit result. From the above discussion, we can claim that Chung (2012) did not provide any improvement for Cheng and Wang (2009).

4. THE PRESENT DEVELOPMENT FOR INVENTORY MODEL WITH A TRAPEZOIDAL TYPE DEMAND RATE

Chung (2012) did not pay attention to the open question proposed by Cheng and Wang (2009) why Deng et al. (2007) with ramp type demand rate and Cheng and Wang (2009) with trapezoidal type demand rate could have the same optimal solution?
In Lin (2011), motivated by equations (18-20), he rewrote them as a simplified expression as

$$\frac{d}{d\,t_1} C_j\left(t_1\right) = \frac{D\left(t_1\right)}{T} f\left(t_1\right) \tag{30}$$

and then he developed an inventory model for any positive demand such that the optimal solution is independent of the demand type.

Cheng and Wang (2009) must divide their inventory model into three different sub-domains of $\left[0, \mu_1\right]$, $\left[\mu_1, \mu_2\right]$ and $\left[\mu_2, T\right]$ because the differential equations have different expressions in three different sub-domains. Lin (2011) considered abstract demand in his development to obtain the total cost, $C\left(t_1\right)$, for the replenishment cycle $\left[0, T\right]$ and then he obtained

$$\frac{d}{d\,t_1} C\left(t_1\right) = \frac{D\left(t_1\right)}{T} f\left(t_1\right) \tag{31}$$

Based on the findings of $C\left(t_1\right)$, it is the synthesized results for $C_j\left(t_1\right)$ of $j = 1,2,3$ with different corresponding sub-domains. Therefore, there is only one objective function so there should have only one expression for the first derivative. Consequently, equations (18-20) has a unified expression of equation (30) is answered. Moreover, the open question raised in Cheng and Wang (2009), why two different inventory models, Deng et al. (2007) and Cheng and Wang (2009), could have the same optimal solution also has a logical explanation that this kind of inventory models should be developed without relating to the demand type.

Finally, Chung (2012) mentioned "in supply chain management" in his title. However, there is only one item and one replenishment cycle in his paper such that his title contained questionable descriptions.

5. CONCLUSION

We pointed out that Chung (2012) did not realize the results of Cheng and Wang (2009) and then he derived the same findings as them. Lin (2011) already provided a generalization of Cheng and Wang (2009) that explained (a) three objective functions with different sub-domains could have the same expression of the first derivation and (b) why different inventory models of ramp demand type and trapezoidal type demand rate could have the same optimal solution.

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