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An alternative method for multiple criteria decision-making models using intuitionistic fuzzy information

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Abstract: The purpose of this study is to extend aggregation method under uncertain environment and to provide an improved method to measure the accuracy membership of each alternative with additional information for multi-criteria decision making employing intuitionistic fuzzy sets. And this study has mentioned some problems in the methods in two previous published papers in which a numerical example is presented to depict the feasibility and effectiveness of the proposed method. Furthermore, a computer-based decision support system has been developed, which will make a decision more efficient and make computing and ranking results easier.
Keyword — Intuitionistic fuzzy sets, multiple criteria decision making (MCDM), fuzzy weighted average, ranking method

1. INTRODUCTION

An engineering decision is generally made through available data or information that are mostly unclear, inexact, and uncertain in nature. As a result, some methods are usually needed for the decision-making process to deal with insufficient data or information that are difficult to exactly describe. However, these subjective characteristics of the alternatives are generally ambiguous and need to be evaluated by a decision maker with insufficient knowledge or judgments. The nature of this kind of vagueness and uncertainty is unclear rather than random. Fuzzy set theory (FST) offers a possibility for dealing with this kind of data or information that involve the subjective characteristics of human nature in the decision-making process.

The theory of fuzzy sets was addressed by Zadeh (1965). It has been successfully employed in dealing with uncertain decision-making problems by Kickert (1978), Laarhoven and Pedrycz (1983), Yager (1987, 1988), Chen (1998), and Jae and Moon (2002). The principal characteristic of fuzzy sets is that: each element x in a universe of discourse a membership degree ranging between zero is assigned to the membership function and one and the non-membership degree equals one minus the membership degree, i.e., this membership degree combines the evidence for x and the evidence against x. The single number tells us nothing about the lack of knowledge. In real applications, however, the information of an object corresponding to a fuzzy concept may be lacking, i.e., the sum of the membership degree and the non-membership degree of an element in a universe corresponding to a fuzzy concept may be less than one. In FST, there are no means to combine the lack of knowledge with the membership degree. Hence, a possible solution is to employ intuitionistic fuzzy sets (hereinafter referred to as IFSs) addressed by Atanassov (1986). The concept of IFSs is an extension of Zadeh's fuzzy sets (1965). It provides us the possibility to develop unknown information by using an additional degree. Based on FST, vague sets are an extension of fuzzy sets and were proposed by Gau and Buehrer (1993), but Burillo and Bustince (1996) presented that the notion of vague sets coincided with that of IFSs. In accordance with vague set theory, new approaches are presented by Chen and Tan (1994) to deal with fuzzy MCDM problems. Other approaches are provided by Hong and Choi (2000). And an improved method was proposed by Ye (2007). The previous method considered the effect of an unknown degree (hesitancy degree) of the vague values on the degree of suitability to which each alternative satisfies the decision-maker requirements. They developed new functions to measure the degree of accuracy in the grades of membership of each alternative with respect to a set of criteria.

Based on IFSs with intuitionistic fuzzy interval value, Li (2005) examined the MCDM problem to deal with criterion and weight rating and developed several linear programming models to produce optimal weights for criteria. Investigations have been conducted by Lin et al. (2007) and Liu and Wang (2007). And Lin et al. (2007) had adopted Chen and Tan's method plus the unknown of 0.5 by Hong and Choi's method as a measureing mechanism of the accuracy function. And they had also referred to Li's method (2007) employing the linear programming method to gain the optimal weight. Liu and Wang (2007) have proposed a new method to improve Chen and Tan's methods

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and suggested that the decision-making process does not need to be divided into several steps and does not need to add more functions.

However, those methods whether based on vague sets theory or IFSs theory, they do not really handle the multiplication problem of two fuzzy interval value. That is, they did not cope with the multiplication of two fuzzy interval values of the weight and the criterion (e.g., $[w_j^\ell, w_j^u] \times [c_j^\ell, c_j^u]$, where w_j^ℓ and c_j^ℓ, w_j^u and c_j^u denote lower bound and upper bound of the jth weight and criterion, respectively). In accordance with vague sets theory, those papers employ the crisp weight to make multiplication operation with vague interval value in each criterion (e.g., crisp weight$\times[c_i^\ell, c_i^u]$). Moreover IFSs theory, Li (2005) and Lin et al. (2007) have supposed the weight owning fuzzy interval value, but they also employ a transferred technology that let fuzzy interval value of the weight transfers to crisp weight and make multiplication operation with a fuzzy interval value of each criterion.

Hence, we will concentrate on handling the multiplication operated problem of two fuzzy interval values and utilizing an improved method to obtain the boundaries of fuzzy interval value for each alternative using IFSs in this study. Then the fuzzy interval value of each alternative can be used as a proposed ranking method to provide the outcomes of final ranking for all alternatives.

The remainder of this study is organized as follows. The IFSs theory is briefly depicted in Section 2. The general features of an MCDM problem employing IFSs are formulated in Section 3. Some problems of Li (2005) and Lin et al. (2007) are pointed out in Section 4. Section 5 presents an improved method and the proposed ranking method to handle a multi-criteria fuzzy decision-making problem. A numerical example and discussions have been illustrated in Section 6. Finally, a computer-based information system is provided in Section 7 before the conclusion.

2. THE RELATIVE BASIC CONCEPTS OF INTUITIONISTIC FUZZY SETS AND ITS OPERATIONS

In this section, we will briefly explain the definitions, properties, and its operations of IFSs proposed by Atanassov (11986, 1999).

Let X be the universe of discourse, X = $\{x_1, x_2, x_3, ..., x_n\}$, with a generic element of X denoted by x_i. A is an IFS in X which is characterized by a membership function μ_A and a non-membership function v_A, defined as

$$A = \{< x_i, \mu_A(x_i), v_A(x_i) > | x_i \in X\}$$

where μ_A: X ∈ [0, 1], v_A: X ∈ [0, 1], and $\mu_A(x_i)$ is a lower bound on the grade of the membership of x_i derived from the evidence for x_i; $v_A(x_i)$ is a lower bound on the negation of x_i derived from the evidence against x_i and $0 \le \mu_A(x_i) + v_A(x_i) \le 1$. The grade of the membership of x_i in the IFS A is bounded to a subinterval [$\mu_A(x_i)$, 1 − $v_A(x_i)$] of [0, 1]. The intuitionistic fuzzy value [$\mu_A(x_i)$, 1 − $v_A(x_i)$] exhibits that the grade of the membership $\pi_A(x_i)$ of x_i may be unknown or uncertain. That is, $\pi_A(x_i) = 1 - \mu_A(x_i) - v_A(x_i)$. Thus, it is bounded by $\mu_A(x_i) \le \pi_A(x_i) \le 1 - v_A(x_i)$. From the viewpoint of practice, both types of sets, ($\mu_A(x_i)$, $v_A(x_i)$) and [$\mu_A(x_i)$, $1 - v_A(x_i)$], are distinct. The membership degree and non-membership degree defined in the IFSs are more reasonable than without any assumption on indeterminacy and are exactly independent. That the sum of the two degrees does not exceed one is the only constraint. When employing IFSs, an expert concentrates on the facts: advantages and disadvantages or pros and cons. However, each element with an interval that approximates the correct (but unknown) membership degree is assigned by interval-valued fuzzy sets. When utilizing fuzzy interval value, experts only focused on the lower and upper approximations. By reason of the equivalent mathematical structure, it is worthy of note that the results obtained in this research can also be developed analogously for interval-valued fuzzy sets. And what is more, the arithmetic operations of IFSs are defined as the following,

Definition 1. The equivalence operator of two IFSs, A and B, $A = B$ if and only if $A(x_i) = B(x_i)$, for all x_i in X.

Definition 2. The non-equivalence operator of two IFSs, A and B, $A \prec B$ if and only if $\mu_A(x_i) \le \mu_B(x_i)$ and $v_A(x_i) \ge v_B(x_i)$ for all x_i in X.

Definition 3. The intersection operator of two IFSs, A and B, written as $M = A \wedge B$. The membership and non-membership functions are $\mu_M(x_i)$ and $v_M(x_i)$, respectively, where ∀ $x_i \in$ X, $\mu_M(x_i)$ = min($\mu_A(x_i)$, $\mu_B(x_i)$) and $v_M(x_i)$ = max($v_A(x_i)$, $v_B(x_i)$).

Definition 4. The union operator of two IFSs, A and B, written as $Z = A \vee B$. The membership function and non-membership functions are $\mu_Z(x_i)$ and $v_Z(x_i)$, respectively, where $\forall x_i \in$ X, $\mu_Z(x_i) = \max(\mu_A(x_i), \mu_B(x_i))$ and $v_Z(x_i) = \min(v_A(x_i), v_B(x_i))$.

3. MCDM PROBLEM FORMULATION BY IFSS

Multi-criteria fuzzy decision-making problems usually include a set of n alternatives Z = {A1, A2, …, An}. These alternatives are to be evaluated with a set of m criteria C = $\{c_1, c_2, \ldots, c_m\}$, which are independent of each other. Herein, we assume that μ_{ij} and v_{ij} are the degree of membership and the degree of non-membership of the alternative $A_i \in Z$, $i = 1, 2, \ldots, n$ with respect to the criteria $c_j \in C$, $j = 1, 2, \ldots, m$ to the fuzzy interval, respectively, where $\mu_{ij} \in [0, 1]$, $v_{ij} \in [0, 1]$, $\mu_{ij} + v_{ij} \le 1$. That is to say, the assessment of the alternative A_i with respect to the criteria c_j is an IFS. Denote $Z_{ij} = \{< A_i, \mu_{ij}, v_{ij} >\}$. In fact, each alternative evaluation bases on the closed interval $[\mu_{ij}, 1 - v_{ij}] = [\mu_{ij}, \mu_{ij} + \pi_{ij}] = [\mu_{ij}^{\ell}, \mu_{ij}^{u}]$, and $0 \le \mu_{ij}^{\ell} \le \mu_{ij}^{u} \le 1$ for all $A_i \in Z$ and $c_j \in C$.

Hence, the characteristics of the alternative Ai and criteria Cj can be shown as follows:

$$\text{Ai} = \{(C_1, [\mu_{iC_1}^{\ell}, \mu_{iC_1}^{u}]), (C_2, [\mu_{iC_2}^{\ell}, \mu_{iC_2}^{u}]), \cdots, (C_m, [\mu_{iC_m}^{\ell}, \mu_{iC_m}^{u}])\},\ 1 \le i \le n \text{ and } 1 \le j \le m.$$

As a result, the decision matrix is a n by m matrix with IFSs entries. And what is more, assume that there is a decision-maker who wants to choose a alternative which satisfies the criteria $C = \{C_j | j = 1, 2, \ldots, m\}$ or the criteria $D = \{D_s | s = 1, 2, \ldots, v\}$, then the requirement of decision makers is shown by the following two situations,

(1) Situation 1: It needs to satisfy all the criteria Ci as the following,

$\{C_1$ AND C_2 AND … AND $C_m\}$, or

(2) Situation 2: It satisfies all the criteria Ci, or all the criteria Di as follows,

$\{C_1$ AND C_2 AND … AND $C_m\}$ OR $\{D_1$ AND D_2 AND … AND $D_v\}$.

Then, the degrees that the alternative Ai satisfies or does not satisfy the decision-maker requirement can be measured by the evaluation function as discussed by Chen and Tan (1994), Hong and Choi (2000), and Ye (2007).

$$\begin{aligned}
E(U_i) &= ([\mu_{iC_1}^{\ell}, \mu_{iC_1}^{u}] \wedge [\mu_{iC_2}^{\ell}, \mu_{iC_2}^{u}] \wedge \cdots \wedge [\mu_{iC_m}^{\ell}, \mu_{iC_m}^{u}]) \text{ or } ([\mu_{iD_1}^{\ell}, \mu_{iD_1}^{u}] \wedge [\mu_{iD_2}^{\ell}, \mu_{iD_2}^{u}] \wedge \cdots \wedge [\mu_{iD_v}^{\ell}, \mu_{iD_v}^{u}]) \\
&= [\min(\mu_{iC_1}^{\ell}, \mu_{iC_2}^{\ell}, \ldots, \mu_{iC_m}^{\ell}), \max(\mu_{iC_1}^{u}, \mu_{iC_2}^{u}, \ldots, \mu_{iC_m}^{u})] \\
&\quad \text{or } [\min(\mu_{iD_1}^{\ell}, \mu_{iD_2}^{\ell}, \ldots, \mu_{iD_v}^{\ell}), \max(\mu_{iD_1}^{u}, \mu_{iD_2}^{u}, \ldots, \mu_{iD_v}^{u})] \\
&= [\max(\min(\mu_{iC_1}^{\ell}, \mu_{iC_2}^{\ell}, \ldots, \mu_{iC_m}^{\ell}), \min(\mu_{iD_1}^{\ell}, \mu_{iD_2}^{\ell}, \ldots, \mu_{iD_v}^{\ell})), \\
&\qquad \max(\max(\mu_{iC_1}^{u}, \mu_{iC_2}^{u}, \ldots, \mu_{iC_m}^{u}), \max(\mu_{iD_1}^{u}, \mu_{iD_2}^{u}, \ldots, \mu_{iD_v}^{u}))] \\
&= [\mu_{U_i}^{\ell}, \mu_{U_i}^{u}],
\end{aligned} \tag{1}$$

and the weights of the criteria $C = \{C_j | j = 1, 2, \ldots, m\}$ and $D = \{D_s | s = 1, 2, \ldots, v\}$, shown by the decision maker, are $W = \{w_j | j = 1, 2, \ldots, m\}$ and Q = $\{\eta_s | s = 1, 2, \ldots, v\}$, respectively, where $w_j \in [0, 1]$, $\sum_{j=1}^{m} w_j = 1$, $\eta_s \in [0, 1]$ and $\sum_{s=1}^{v} \eta_s = 1$. Furthermore, it can perform along the types of weight to distinguish different calculation operations, and then the degree of suitability to which the alternative A_i satisfies the decision-maker requirements can be measured by the weighting function R, as the follows,

(1) If the weight is crisp value, then

$$R(U_i) = \max\{(C_1 \times w_1 + C_2 \times w_2 + \ldots + C_m \times w_m), (D_1 \times \eta_1 + D_2 \times \eta_2 + \ldots + D_v \times \eta_v)\} = \max\{([\mu_{iC_1}^{\ell}, \mu_{iC_1}^{u}] \times w_1 + [\mu_{iC_2}^{\ell}, \mu_{iC_2}^{u}] \times w_2 + \ldots + [\mu_{iC_m}^{\ell}, \mu_{iC_m}^{u}] \times w_m), ([\mu_{iD_1}^{\ell}, \mu_{iD_1}^{u}] \times \eta_1 + [\mu_{iD_2}^{\ell}, \mu_{iD_2}^{u}] \times \eta_2 + \ldots + [\mu_{iD_v}^{\ell}, \mu_{iD_v}^{u}] \times \eta_v)\} = [\mu_{U_i}^{\ell}, \mu_{U_i}^{u}], \tag{2}$$

or

(2) If the weight is intuitionistic fuzzy interval value, let $w_j = [w_j^{\ell}, w_j^{u}]$ and $\eta_s = [\eta_s^{\ell}, \eta_s^{u}]$, then

$$\begin{aligned}
R(U_i) &= \max\{(C_1 \times w_1 + C_2 \times w_2 + \ldots + C_m \times w_m), (D_1 \times \eta_1 + D_2 \times \eta_2 + \ldots + D_v \times \eta_v)\} = \max\{([\mu_{iC_1}^{\ell}, \mu_{iC_1}^{u}] \times \\
&\quad [w_{C_1}^{\ell}, w_{C_1}^{u}] + [\mu_{iC_2}^{\ell}, \mu_{iC_2}^{u}] \times [w_{C_2}^{\ell}, w_{C_2}^{u}] + \ldots + [\mu_{iC_m}^{\ell}, \mu_{iC_m}^{u}] \times [w_{C_m}^{\ell}, w_{C_m}^{u}]), \\
&\quad ([\mu_{iD_1}^{\ell}, \mu_{iD_1}^{u}] \times [\eta_{D_1}^{\ell}, \eta_{D_1}^{u}] + [\mu_{iD_2}^{\ell}, \mu_{iD_2}^{u}] \times [\eta_{D_2}^{\ell}, \eta_{D_2}^{u}] + \ldots + [\mu_{iD_v}^{\ell}, \mu_{iD_v}^{u}] \times [\eta_{D_v}^{\ell}, \eta_{D_v}^{u}])\}. = [\mu_{U_i}^{\ell}, \mu_{U_i}^{u}],
\end{aligned} \tag{3}$$

where $R(U_i) \in [0, 1]$ and $1 \le i \le n$. If $R(U_i) = \Psi_i$ and ω is the largest value among all Ψ_i, then ω belonging to the alternative is the best choice.

4. BACKGROUND ARGUMENT

In this section, some problems from Li (2005) and Lin et al. (2007) will be discussed in this paper, and propose an improved method to deal MCDM problem with IFSs.

In accordance with investigations of Li (2005) and Lin et al. (2007), the optimal comprehensive value of each alternative Ai ∈ Z can be calculated via the following programming:

$$\max\left\{ z_i = \sum_{j=1}^{m} \theta_{ij} w_j \right\},$$

with constraints

$$\begin{cases} \mu_{ij}^{\ell} \le \theta_{ij} \le \mu_{ij}^{u}, i = 1,2,\dots,n;\ j = 1,2,\dots,m \\ w_j^{\ell} \le w_j \le w_j^{u} \\ \sum_{j=1}^{m} w_j = 1. \end{cases} \tag{4}$$

where θij j-th criterion of i-th alternative and wj denotes the weight for j-th criterion.

In order to solve Eq. (4), Li (2005) and Lin et al. (2007), this paper employ the linear programming method to solve the following two linear programming as a final interval score of each alternative Ai, denote $U_i = [z_i^{\ell}, z_i^{u}]$.

$$\min\left\{ z_i^{\ell} = \sum_{j=1}^{m} \mu_{ij}^{\ell} w_j \right\},$$

with constraints

$$\begin{cases} w_j^{\ell} \le w_j \le w_j^{u},\ j = 1,2,\dots,m, \\ \sum_{j=1}^{m} w_j = 1, \end{cases} \tag{5}$$

and

$$\max\left\{ z_i^{u} = \sum_{j=1}^{m} \mu_{ij}^{u} w_j \right\},$$

with constraints

$$\begin{cases} w_j^{\ell} \le w_j \le w_j^{u},\ j = 1,2,\dots,m, \\ \sum_{j=1}^{m} w_j = 1. \end{cases} \tag{6}$$

From the above programming, some problems in their investigations are mentioned as the following,

(1) It is invalid that the equation (14) or (15) of Li (2005) is derived from the equations (9) and (13) of Li (2005). It does not mean that they can be merged by the additional operation, although equations (9) and (13) have the same constraints.

Example 1, cited from Chuang et al. (2013)**:**

$$\max\ f(x) = 100(a+b) - a(x-2)^2$$

such that

$$\begin{cases} a > 0, \\ b > 0, \\ 0 \le x \le 10. \end{cases} \tag{7}$$

The optimal solution is $x^* = 2$.
and

$$\max\ g(x) = 100(a+b) - b(x-3)^2$$

Such that

$$\begin{cases} a > 0, \\ b > 0, \\ 0 \le x \le 10. \end{cases} \tag{8}$$

The optimal solution is $x^*= 3$.

If equations (7) and (8) are merged as the following equation (9),

$$\max\ h(x) = f(x) + g(x)$$

such that

$$\begin{cases} a > 0, \\ b > 0, \\ 0 \le x \le 10. \end{cases} \tag{9}$$

The $h(x)$ function has maximum point at $x^* = \dfrac{2a+3b}{a+b}$ that does not coincide with $x^* = 2$ for $f(x)$ or $x^* = 3$ for $g(x)$.

As a result, we infer from the above counterexample that it is questionable of merging two maximization problem into one.

(2) In addition, based on the examination of Li (2005), the equations (14) and (15) are very critical derivation for obtaining the optimal weight. However, the result of deviation is very weird. Because of the weighted capturing depends on the unknown or uncertain part of fuzzy interval value for each criterion. That is, if fuzzy interval value of the criterion unknown part is the greater, then the assigned weight is greater. This result is difficult to understand from the viewpoint of decision maker. On the whole, the weighted capturing depends on the importance of each criterion by experts' assumption or the decision-maker preference. Hence, it is meaningless to the derivative result of Li (2005).

(3) Li (2005) and Lin et al. (2007) have handled linear programming method by employing the Simplex method to capture the optimal weights for each criterion of an alternative. However, it is so complicated and unnecessary, because solving the equation of optimal weights only needs to adopt the intuitive adjustment technology. Its operations show: (a) the initialization is that each criterion gives the weight of lower bound, except maximized criterion rating; (b) the remaining weight has been assigned as the maximized criterion rating. If the weight of maximized criterion causes overflowing (over the upper bound), then the overflowed part will assign to the secondary bigger of criterion rating. Analogous iterative operation solves the optimal weight until no overflowing situation appears.

Now, an example is given to depict the procedures of our proposed intuitive adjustment mechanism.

Example 2: (This example is derived from Li (2005) and Lin et al. (2007))

$$\max\left\{ z = \frac{0.35w_1 + 0.47w_2 + 0.15w_3}{3} \right\}$$

s.t.

$$\begin{cases} 0.25\ (w_1^\ell) \le w_1 \le 0.75\ (w_1^u), \\ 0.35\ (w_2^\ell) \le w_2 \le 0.6\ (w_2^u), \\ 0.30\ (w_3^\ell) \le w_3 \le 0.35\ (w_3^u), \\ w_1 + w_2 + w_3 = 1. \end{cases}$$

Intuitive adjustment mechanism:

Step 1.	Sorting criterion rating	$j =$	c_2	c_1	c_3
			0.47 (max)	0.35	0.15
Step 2.	LB weight			0.25 (w_1^ℓ)	0.3 (w_3^ℓ)
Step 3.	reminded weight		0.45 ($\le w_2^u$)		

where LB denotes the lower bound weight. From Steps 1 to Step 3, it is easy to obtain optimal weight is (w_1, w_2, w_3) = (0.25, 0.45, 0.3).

(4) Based on Li (2005) and Lin et al. (2007), they assumed the same constraints such as $\sum_{j=1}^{m} w_j^\ell \le 1$, $\sum_{j=1}^{m} w_j^u \ge 1$, wi ∈ [0, 1], $w_j^\ell \le w_j \le w_j^u$ and $\sum_{j=1}^{m} w_j = 1$. However, from the viewpoint of decision makers, the constraints, $\sum_{j=1}^{m} w_j^\ell \le 1$ and $\sum_{j=1}^{m} w_j^u \ge 1$, can not always hold in practical applications. In the following, an example is given to demonstrate the practical operation. As a result, it ought to cancel the constraint of $\sum_{j=1}^{m} w_j^\ell \le 1$, and then to

devote the normalized mechanism of all weight as an improved method in section 5. We predict that $\sum_{j=1}^{m} w_j^{\ell} \le 1$ and $\sum_{j=1}^{m} w_j^{u} \ge 1$ are needed to guarantee the existence of the solution.

Example 3: The example of Hon et al. (1996) is adopted to evaluate the performance of teachers in higher education with a three-criterion index. The sum of all lower bound weights is greater than one. The example of Kao and Liu (1999) is to compute the competitiveness of manufacturing firms. The total number of the weighted terms is eighteen, and the sum of all lower bound weights is greater than one. Yet, the example of Vanegas and Labib (2001) is to choose the material problem. The weighted term is three, and the sum of all lower bound weights is greater than one.

From the observations in Example 3, the decision makers or experts should abandon the limitation of the weights, $\sum_{j=1}^{m} w_j^{\ell} \le 1$ and $\sum_{j=1}^{m} w_j^{u} \ge 1$, and then assign the weight through the practical requirement. Example 3 mentions that the method of Li (2005) can not apply to the 3 papers in Example 3. Hence, fuzzy weighted average (FWA) is proposed by previous papers as an efficient alternative method to deal MCDM problem with IFSs to get the optimal combination of weights for each criterion. Furthermore, with the help of linear programming software, fuzzy MCDM problems can be handled more efficiently through the linear programming method. Based on the investigation of Chang et al. (2006), the performance of utilizing FWA algorithm is superior to utilizing linear programming method. Thereby, the basic concept of fuzzy weighted average will be briefly introduced in next subsection. The proposed FWA algorithm is presented in section 5.2.

5. FUZZY WEIGHTED AVERAGE ALGORITHM AND A PROPOSED RANKING METHOD

In this section, the basic concept of fuzzy weighted average and FWA algorithm of Chang et al. (2006) have been introduced. A proposed ranking method has also been developed.

5.1. The concept of fuzzy weighted average

Based on previous papers, the FWA can be defined as the process via employing the fuzzy (criteria) ratings, cij, of alternatives Ai, i ∈ {1, 2, …, n} with respect to a set of criteria, attributes or factors j ∈ {1, 2, …, m}, the fuzzy weights or importance of these criteria as wj, for the objective function that aggregates the fuzzy weights and criteria ratings into the FNs Ai. It consists of fuzzy arithmetic and can be defined by

$$A_i = f(c_{i1},...,c_{im},w_1,...,w_m) = \frac{w_1 c_{i1} + w_2 c_{i2} + \cdots + w_m c_{im}}{w_1 + w_2 + \cdots + w_m} = \sum\nolimits_{j=1}^{m} w_j \cdot c_{ij} \Big/ \sum\nolimits_{j=1}^{m} w_j . \tag{10}$$

Besides, if the fuzzy weights wj and fuzzy criteria ratings cij have been denoted as follows

$$w_j = [w_j^{\ell}, w_j^{u}],\ c_{ij} = [c_{ij}^{\ell}, c_{ij}^{u}] . \tag{11}$$

Then Ai may be searched and attainable by

$$A_i = [z_i^{\ell}, z_i^{u}] = \left[\min_{\substack{c_{ij}^{\ell} \le c_{ij} \le c_{ij}^{u}, w_j^{\ell} \le w_j \le w_j^{u}, \\ j=1,...,m}} f\left(c_{i1},...,c_{im},w_1,...,w_m\right), \max_{\substack{c_{ij}^{\ell} \le c_{ij} \le c_{ij}^{u}, w_j^{\ell} \le w_j \le w_j^{u}, \\ j=1,...,m}} f\left(c_{i1},...,c_{im},w_1,...,w_m\right) \right]. \tag{12}$$

A unique combination of ci1, …, cim, w1, …, wm values can be found for the min{f} or z_i^{ℓ} and similarly for the max{f} or z_i^{u}, and avoid the employ of different endpoints of the parameters.

For equation (10), the following can also be obtained. The proof can be presented in Liou and Wang (1992) and Chang et al. (2006), due to the monotonicity of f with respect to all supports of cij.

$$\min_{\substack{c_{ij}^{\ell} \le c_{ij} \le c_{ij}^{u}, w_j^{\ell} \le w_j \le w_j^{u}, \\ j=1,\dots,m}} f(c_{i1},\dots,c_{im},w_1,\dots,w_m) = \min_{w_j \in \{w_j^{\ell}, w_j^{u}\}} f_{\mathrm{L}}(w_1,w_2,\dots,w_m), \tag{13a}$$

and

$$\max_{\substack{c_{ij}^{\ell} \le c_{ij} \le c_{ij}^{u}, w_j^{\ell} \le w_j \le w_j^{u}, \\ j=1,\dots,m}} f(c_{i1},\dots,c_{im},w_1,\dots,w_m) = \max_{w_j \in \{w_j^{\ell}, w_j^{u}\}} f_{\mathrm{R}}(w_1,w_2,\dots,w_m), \tag{13b}$$

where we can define

$$\begin{aligned} \min_{w_j \in \{w_j^{\ell}, w_j^{u}\}} f_{\mathrm{L}}(w_1,w_2,\dots,w_m) &= \min_{\substack{w_j^{\ell} \le w_j \le w_j^{u}, \\ j=1,\dots,m}} f(c_{i1}^{\ell},\dots,c_{im}^{\ell},w_1,\dots,w_m) \\ &= \min_{\substack{w_j^{\ell} \le w_j \le w_j^{u}, \\ j=1,\dots,m}} \frac{w_1 c_{i1}^{\ell} + w_2 c_{i2}^{\ell} + \dots + w_m c_{im}^{\ell}}{w_1 + w_2 + \dots + w_m}, \end{aligned} \tag{14a}$$

and

$$\begin{aligned} \max_{w_j \in \{w_j^{\ell}, w_j^{u}\}} f_{\mathrm{R}}(w_1,w_2,\dots,w_m) &= \max_{\substack{w_j^{\ell} \le w_j \le w_j^{u}, \\ j=1,\dots,m}} f(c_{i1}^{u},\dots,c_{im}^{u},w_1,\dots,w_m) \\ &= \max_{\substack{w_j^{\ell} \le w_j \le w_j^{u}, \\ j=1,\dots,m}} \frac{w_1 c_{i1}^{u} + w_2 c_{i2}^{u} + \dots + w_m c_{im}^{u}}{w_1 + w_2 + \dots + w_m}. \end{aligned} \tag{14b}$$

For z_i^{ℓ}, $c_{ij} = c_{ij}^{\ell}$ for all $j = 1, \dots, m$ and for z_i^{u}, $c_{ij} = c_{ij}^{u}$ for all $j = 1, \dots, m$ can be employed in the correct results of the A_i.
This is because the monotonicity of f exists with respect to all supports of cij in equation (10) and cij appears only once in equation (10). For the min{f} with $c_{ij} = c_{ij}^{\ell}$ and for max{f} with $c_{ij} = c_{ij}^{u}$ ∀i (i.e., boundaries of cij can be immediately determined from the boundaries of Ai. Hence, also, the optimal values of wi will be determined naturally to be boundaries of w1, …, wm by Liou and Wang (1992), but still they need to be searched.
Hence, if we define the initial evaluations for z_i^{ℓ} and z_i^{u} for further search of min{f_{L}} and max{f_{R}} as

$$\begin{aligned} \ell_0 &= \min_{w_j \in \{w_j^{\ell}, w_j^{u}\}} f_{\mathrm{L}}\left(w_1 = w_1^{\ell}, w_2 = w_2^{\ell}, \dots, w_m = w_m^{\ell}\right) \\ &= \min_{w_j^{\ell} \le w_j \le w_j^{u}} \frac{\beta_{\mathrm{L},0}}{\gamma_{\mathrm{L},0}} = \min_{w_j^{\ell} \le w_j \le w_j^{u}} \frac{w_1^{\ell} c_{i1}^{\ell} + w_2^{\ell} c_{i2}^{\ell} + \dots + w_m^{\ell} c_{im}^{\ell}}{w_1^{\ell} + w_2^{\ell} + \dots + w_m^{\ell}}, \end{aligned} \tag{15a}$$

and

$$\begin{aligned} \rho_0 &= \max_{w_j \in \{w_j^{\ell}, w_j^{u}\}} f_{\mathrm{R}}\left(w_1 = w_1^{\ell}, w_2 = w_2^{\ell}, \dots, w_m = w_m^{\ell}\right) \\ &= \max_{w_j^{\ell} \le w_j \le w_j^{u}} \frac{\beta_{\mathrm{R},0}}{\gamma_{\mathrm{R},0}} = \max_{w_j^{\ell} \le w_j \le w_j^{u}} \frac{w_1^{\ell} c_{i1}^{u} + w_2^{\ell} c_{i2}^{u} + \dots + w_m^{\ell} c_{im}^{u}}{w_1^{\ell} + w_2^{\ell} + \dots + w_m^{\ell}}, \end{aligned} \tag{15b}$$

the concept of FWA will now become the search and evaluation for replacement of $w_i = w_j^{\ell}$ by w_j^{u} in ℓ_0 and ρ_0 and improve ℓ_0 and ρ_0 toward min{f_{L}} and max{f_{R}}.

5.2 The Chang et al.'s FWA algorithm

The goal of the FWA algorithm is to assist the operation and reduce the complexity of the FWAs. In order to simplify the FWA, some FWA algorithms have been proposed such as Dong and Wong (1987), Liou and Wang (1992), Guh et al. (1996), Lee and Park (1997), Guu (2002) and Chang et al. (2006) with the aim of facilitating the operations. In this study, AFWA algorithm of Chang et al. (2006) has been adopted, because their FWA algorithm may be the fastest available FWA algorithm compared to other FWA algorithms. In addition, their algorithm utilizes an all-candidates' weights replacement policy in giving improved benchmark ℓp and ρq (adjusted from ℓ0 and ρ0) for improving ℓ0 and ρ0. Likewise, it can be illustrated for ρq, too. (For more detailed descriptions, see Chang et al. (2006)). In the following, the procedures of the AFWA algorithm have been depicted.

AFWA algorithm

Step 1) Discretize the range of membership [0, 1] into a finite number of intervals. Determine the fuzzy criteria ratings (c_{ij}) and the fuzzy weights (w_j).
Step 2) Calculate the initial benchmarks (ℓ_0 and ρ_0 of f_{L} and f_{R}):

$$\ell_0 := \beta_{\mathrm{L},0}/\gamma_{\mathrm{L},0} = \sum_{j=1}^{n} w_j^{\ell} c_j^{\ell} \Big/ \sum_{j=1}^{n} w_j^{\ell},$$

$\rho_0 := \beta_{R,0}/\gamma_{R,0} = \sum_{j=1}^{n} w_j^{\ell} c_j^{u} \Big/ \sum_{j=1}^{n} w_j^{\ell}$.

Sort c_j^{ℓ}'s and c_j^{u}'s in respectively nondecreasing orders. Let $I := \{1, 2, \ldots, n\}$, $I_0 := \{j \in I \mid c_j^{\ell} < \ell_0\}$, $J_0 := \{j \in I \mid c_j^{u} > \rho_0\}$, and $p = q = 1$.

Step 3) For $z_i^{\ell} = \min\{f_L\}$ of U_i:

3.1) Compute the improved benchmark ℓ_p: If $p = 1$, let $\ell_{p=1} := \beta_{L,1}/\gamma_{L,1} = \left(\beta_{L,0} + \sum_{j\in I_0} (w_j^{u} - w_j^{\ell}) c_j^{\ell}\right) \Big/ \left(\gamma_{L,0} + \sum_{j\in I_0} (w_j^{u} - w_j^{\ell})\right)$, else let $\ell_p := \beta_{L,p}/\gamma_{L,p} = \left(\beta_{L,p-1} - \sum_{j\in \Delta I_{p-1}} (w_j^{u} - w_j^{\ell}) a_i\right) \Big/ \left(\gamma_{L,p-1} - \sum_{j\in \Delta I_{p-1}} \left(w_j^{u} - w_j^{\ell}\right)\right)$.

3.2) Optimality test: Let $I_p := \{j \in I_{p-1} \mid c_j^{\ell} < \ell_p\}$ and $\Delta I_p := I_{p-1} \setminus I_p$. If $\Delta I_p = \varnothing$, $z_i^{\ell} = \min\{f_L\} = \ell_p$, and go to step 4). Else, let $p := p + 1$ and return to step 3.1).

Step 4) For $z_i^{u} = \max\{f_R\}$ of U_i:

4.1) Compute the revised benchmark ρ_q: If $q = 1$, let $\rho_{q=1} := \beta_{R,1}/\gamma_{R,1} = \left(\beta_{R,0} + \sum_{j\in J_0} (w_j^{u} - w_j^{\ell}) c_j^{u}\right) \Big/ \left(\gamma_{R,0} + \sum_{j\in J_0} \left(w_j^{u} - w_j^{\ell}\right)\right)$, else let $\rho_q := \beta_{R,q}/\gamma_{R,q} = \left(\beta_{R,q-1} - \sum_{j\in \Delta J_{q-1}} (w_j^{u} - w_j^{\ell}) c_j^{u}\right) \Big/ \left(\gamma_{R,q-1} - \sum_{j\in \Delta J_{q-1}} \left(w_j^{u} - w_j^{\ell}\right)\right)$.

4.2) Optimality test: Let $J_q := \{j \in J_{q-1} \mid c_j^{u} > \rho_q\}$ and $\Delta J_q := J_{q-1} \setminus J_q$. If $\Delta J_q = \varnothing$, $z_i^{u} = \max\{f_R\} = \rho_q$ and stop. Otherwise, increase q by one and return to step 4.1).

In the algorithm, the full displacement of $w_j = w_j^{u}$ $\forall j \in I_0$ and $\forall j \in J_0$ in ℓ_0 and ρ_0 gives an improved benchmark and the benchmark is then revised by the full displacement of $w_i = w_j^{u}$ to w_j^{ℓ} $\forall j \in \Delta I_{p-1}$ and $\forall j \in \Delta J_{q-1}$ in ℓ_p and ρ_q and constitutes the principal steps of the algorithm, 3.1) and 4.1), for min $\{f_L\}$ and max $\{f_R\}$. It stops only if the condition of optimality, $\Delta I_p = \varnothing$ and $\Delta J_q = \varnothing$.

Theorem. The minimum and maximum values in Eqs. (15*a*) and (15*b*) are always obtained by taking interval values of weight w_i equal to the extreme values. Then, the complexity of algorithm for calculation and comparison are $O(n\log n)$ and $O(n)$, respectively. The total number of possible function evaluation for min $\{f_L\}$ and max $\{f_R\}$ operators only requires a few numbers.
Roof. The proof of the theorem is given in Chang et al. (2006).

5.3 The proposed ranking method

In the literatures, some ranking methods have been developed and the reviews and comparison may be addressed in, e.g. Chen and Tan (1994), Hong and Choi (2000), Ye (2007). They proposed some techniques to handle multi-criteria fuzzy decision-making problems based on vague set theory. Moreover, some methods have also proposed by Li (2005), Lin et al. (2007) and Liu and Wang (2007) to handle MCDM problem employing IFSs.

This paper proposed a fair and easy method to compute the intuitionistic fuzzy interval value based on IFSs to rank all alternatives. The ranking operative procedures have been described as the following:

Let alternative $A_i = [\mu_{A_i}^{\ell}, \mu_{A_i}^{u}] = [\mu_{A_i}^{\ell}, 1 - v_{A_i}]$ be a intuitionistic fuzzy value, where $\mu_{A_i}^{\ell} \in [0, 1]$, $v_{A_i} \in [0, 1]$, $\mu_{A_i}^{\ell} + v_{A_i} \le 1$. Yet, $m_{A_i} = 1 - \mu_{A_i}^{\ell} - v_{A_i}^{\ell}$, $m_{A_i} \in [0, 1]$, where m_{A_i} denotes unknown. The score of Ai can be evaluated by the following score function and three rules.

Rule-1. The evaluated score function, *K*, is as follows:

$$K(E(A_i)) = \mu_{A_i}^{\ell} + \lambda m_{A_i} = \mu_{A_i}^{\ell} + \lambda(\mu_{A_i}^{u} - \mu_{A_i}^{\ell}), \tag{14}$$

where K(E(Ai)) ∈ [0, 1] , i = 1, 2, … , n and λ ∈ [0, 1]. The larger the value of K(E(Ai)), the higher the degree of suitability, which is, hence, more likely to satisfy the requirements of decision makers for alternative Ai.

In addition, considering a new parameter, λ, expressing the percentage of abstention from membership (or pro). When λ = 0, it shows that the decision-maker is the most pessimistic, because it can't obtain anything from the abstention part, when λ = 0.5, it shows that the decision-maker is fair and can obtain a half of the abstention and when $\lambda = 1$, it shows that the decision-maker is in the most optimistic situation, and can get complete support from the abstention part.

Rule-2. If Rule-1 produces the same value, then it will start the judgment mechanism by $\mu^{\ell}_{A_i}$. From decision-making intuition viewpoint, $\mu^{\ell}_{A_i}$ stands for the visual grade. Hence, $\mu^{\ell}_{A_i}$ is good to be the biggest, then the alternative is the best choice. In Rule-2 phase, if the alternatives have the same vague values, $\mu^{\ell}_{A_i}$, then Rule-3 has been started.

Rule-3. Similarly, the procedures of Rule-2, if Rule-2 produces same $\mu^{\ell}_{A_i}$, then it will start the judgment mechanism by $\mu^{u}_{A_i}$. From decision-making perspectives, the bigger the value of $\mu^{u}_{A_i}$, the smaller the value of the non-membership (or con). Hence, $\mu^{u}_{A_i}$ is good to be the biggest, and, hence, the alternative is the best choice.

Finally, in Rule-3, if the alternatives have the same intuitionistic fuzzy interval values, $\mu^{u}_{A_i}$, then those alternatives will have the same interval and, thus, produce the same rank.

6. NUMERICAL EXAMPLE AND DISCUSSIONS

An example drives from a case study by Wang and Elhag (2007) in Section 6.1. And in Section 6.2, some issues from the case study as well as our proposed method are raised.

6.1 An example as illustration

We quote an example by Wang and Elhag (2007). Consider the bridge risk assessment problem for the British Highway Agency. Besides, bridge risk assessment is often conducted to determine the priority or the optimal scheme of bridge structures maintenance. If there are five risk events, the bridge structure (BS) set can be denoted by U = {BS1, BS2, BS3, BS4, BS5}. If the four criteria, namely, safety (c1), functionality (c2), sustainability (c3) and environment (c4) are taken into consideration and criteria employing the intuitionistic fuzzy terms in independent manner, the degrees μij of membership and the degrees νij of non-membership for the bridge structure BSi, i = 1, 2, …, 5 satisfies the criterion cj, j = 1, 2, …, 4 can be obtained through statistical methods, respectively. The intuitionistic fuzzy matrix of criteria rating was can be derived as follows:

$((\mu_{ij}, v_{ij})_{5\times 4}) =$

	c_1	c_2	c_3	c_4
BS_1	(0.7, 0)	(0.3, 0.1)	(0.6, 0)	(0.1, 0.2)
BS_2	(0.6, 0)	(0.6, 0)	(0.3, 0.2)	(0.2, 0.3)
BS_3	(0.2, 0.1)	(0.7, 0)	(0.1, 0.3)	(0.1, 0.2)
BS_4	(0.1, 0.3)	(0.6, 0)	(0.6, 0)	(0.2, 0.1)
BS_5	(0.2, 0.1)	(0.1, 0.2)	(0.1, 0.4)	(0.1, 0.3)

$((\mu^{\ell}_{ij}, \mu^{u}_{ij})_{5\times 4}) =$

	c_1	c_2	c_3	c_4
BS_1	[0.7, 1.0]	[0.3, 0.9]	[0.6, 1.0]	[0.1, 0.8]
BS_2	[0.6, 1.0]	[0.6, 1.0]	[0.3, 0.8]	[0.2, 0.7]
BS_3	[0.2, 0.9]	[0.7, 1.0]	[0.1, 0.7]	[0.1, 0.8]
BS_4	[0.1, 0.7]	[0.6, 1.0]	[0.6, 1.0]	[0.2, 0.9]
BS_5	[0.2, 0.9]	[0.1, 0.8]	[0.1, 0.6]	[0.1, 0.7]

In a similar way, the degrees φj of membership and the degrees δj of non-membership for the four criteria cj ∈ C (j = 1, 2, 3, 4) to the fuzzy concept "importance" can be obtained, respectively. Namely,

$$((\varphi_j, \delta_j)_{1\times 4}) = ((0.7, 0), (0.4, 0.1), (0.1, 0.4), (0.1, 0.6)).$$

Hence, the weight of each criterion is as the following,

$$([w_j^{\ell}, w_j^{u}]_{1\times 4}) = ([0.7, 1.0], [0.4, 0.9], [0.1, 0.6], [0.1, 0.4]).$$

The following shall illustrate the proposed FWA algorithm's procedure by calculating the score of bridge structure BS1. The FWA algorithm yields the results as follows,

Step 1) Calculates the initial benchmarks as

$$\ell_0 = f_{\mathrm{L}}\left(w_1^{\ell}, w_2^{\ell}, w_3^{\ell}, w_4^{\ell}\right) = \frac{w_1^{\ell}\mu_{i1}^{\ell} + w_2^{\ell}\mu_{i2}^{\ell} + w_3^{\ell}\mu_{i3}^{\ell} + w_4^{\ell}\mu_{i4}^{\ell}}{w_1^{\ell} + w_2^{\ell} + w_3^{\ell} + w_4^{\ell}}$$

$$= \frac{(0.7\times 0.7) + (0.4\times 0.3) + (0.1\times 0.6) + (0.1\times 0.1)}{0.7+0.4+0.1+0.1} = \frac{0.68}{1.3} = 0.5230,$$

$$\rho_0 = f_{\mathrm{R}}\left(w_1^{\ell}, w_2^{\ell}, w_3^{\ell}, w_4^{\ell}\right) = \frac{w_1^{\ell}\mu_{i1}^{u} + w_2^{\ell}\mu_{i2}^{u} + w_3^{\ell}\mu_{i3}^{u} + w_4^{\ell}\mu_{i4}^{u}}{w_1^{\ell} + w_2^{\ell} + w_3^{\ell} + w_4^{\ell}}$$

$$= \frac{(0.7\times 1.0) + (0.4\times 0.9) + (0.1\times 1.0) + (0.1\times 0.8)}{0.7+0.4+0.1+0.1} = \frac{1.24}{1.3} = 0.9538,$$

and $I_0 = \{2, 4\}$ and $J_0 = \{1, 3\}$.

Step 2) For z_i^{ℓ}

2.1) Yields the revised benchmark,

$$\ell_1 = f_{\mathrm{L}}\left(w_1^{\ell}, w_2^{u}, w_3^{\ell}, w_4^{u}\right)$$

$$= \frac{0.68 + (0.9-0.4)\times 0.3 + (0.4-0.1)\times 0.1}{1.3 + (0.9-0.4) + (0.4-0.1)} = \frac{0.86}{2.1} = 0.4095.$$

2.2) Optimal test,

Because $I_1 = I_0 = \{2, 4\}$, $\Delta I_1 = \varnothing$ and step 2) stops.

Step 3) For z_i^{u}

3.1) Yields the revised benchmark,

$$\rho_1 = f_{\mathrm{R}}\left(w_1^{u}, w_2^{\ell}, w_3^{u}, w_4^{\ell}\right)$$

$$= \frac{1.24 + (1.0-0.7)\times 1.0 + (0.6-0.1)\times 1.0}{1.3 + (1.0-0.7) + (0.6-0.1)} = \frac{2.04}{2.1} = 0.9714.$$

3.2) Optimal test,

$J_1 = \{1, 3\}$. Since $J_1 = J_0 = \{1, 3\}$ and $\Delta J_2 = \varnothing$, stop the all process.

From the foregoing procedures of the proposed FWA algorithm performance, the optimal interval value of bridge structure (BS1) as $[z_1^{\ell}, z_1^{u}]$ = [0.4095, 0.9714]. The processes should be repeated for the other bridge structures. Finally, we can obtain BS2 = [0.4380, 0.9761], BS3 = [0.2416, 0.9333], BS4 = [0.2526, 0.9043] and BS5 = [0.1269, 0.8437].

When obtaining the result of BS1, we may employ equation (14) with three different parameters, $\lambda = 1$ (optimistic), $\lambda = 0.5$ (fair) and $\lambda = 0$ (pessimistic) in this alternative, respectively. Hence, it can get the following results,

(1) Case I: employing $\lambda = 1$ (optimistic).

Rule-1: $R(BS_1)$ = 0.4095+1×(0.9714−0.4095) = 0.9714. By Eq. (14) the same calculation, we get $R(BS_2)$ = 0.9761, $R(BS_3)$ = 0.9333, $R(BS_4)$ = 0.9043 and $R(BS_5)$ = 0.8437.

Hence, the order of quality is as $R(BS_2) \succ$ (means superior to) $R(BS_1) \succ R(BS_3) \succ R(BS_4) \succ R(BS_5)$. Hence, we can choose the bridge structure BS_2 to prioritize maintenance.

(2) Case II: employing $\lambda = 0.5$ (fair).

Rule-1: $R(BS_1)$ = 0.4095+0.5×(0.9714−0.4095) = 0.6904. By Eq. (14) the same calculation, we get $R(BS_2)$ = 0.7071, $R(BS_3)$ = 0.5874, $R(BS_4)$ = 0.5784 and $R(BS_5)$ = 0.4853. Obviously, the order of quality is as

$$R(BS_2) \succ R(BS_1) \succ R(BS_3) \succ R(BS_4) \succ R(BS_5).$$

The bridge structure BS_2 is the priority first. This ranking result is the same as Case I.

(3) Case III: employing $\lambda = 0$ (pessimistic).

Rule-1: $R(BS_1)$ = 0.4095+0×(0.9714−0.4095) = 0.4095. By Eq. (14) the same calculation, we get $R(BS_2)$ = 0.4380, $R(BS_3)$ = 0.2416, $R(BS_4)$ = 0.2526 and $R(BS_5)$ = 0.1269. Obviously, the order of quality is as

$$R(BS_2) \succ R(BS_1) \succ R(BS_4) \succ R(BS_3) \succ R(BS_5).$$

The bridge structure BS_2 is considered to prioritize maintenance.

According to the above three cases, we make a choice that the bridge structure BS_2 is prioritized because of maintenance considerations.

6.2 Analysis and discussions

Our analysis of the issues has provided the following:
(1) According to the Li (2005) and Lin et al. (2007) methods, linear programming software is employed to obtain the fuzzy interval values of the boundary for each alternative. But, employing the proposed FWA algorithm is more efficient than linear programming method which may be efficient but require linear programming software to handle fuzzy MCDM problems.
(2) Based on the practical case study, the improved method can provide a useful way to help decision makers to assess bridge risk conditions. The method is different from previous Li (2005) and Lin et al. (2007) methods for multi-criteria decision making due to the fact that the proposed method employs IFSs rather than fuzzy sets. In the future, the improved method maybe can be useful for the decision-making models to solve the assessment or selection problem employing IFSs.
(3) From the viewpoint of decision makers, the difference between adjacent ranking scores is very little, particularly, it appears in the alternatives of the first and second rank, further analysis is required to evaluate the values of two IFSs, because the decision-making process allows a little deficiency caused by the practical case study among uncertain environment and usage to achieve the optimal alternative.

7. THE COMPUTER-BASED DECISION SUPPORT SYSTEM

In real organizational environment, more and more decisions are made.
Utilizing vague sets into fuzzy multi-criteria decision-making methods to deal with imprecise, uncertain and vague decision-making situations may become a critical research issue in the current environment. The application of vague set in supporting decision making can provide a useful way to help decision makers to make their decisions efficiently. In order to make computing and ranking the results much easier and to increase the recruiting productivity, we have developed an information system called intuitionistic fuzzy sets decision support system (IFSsDSS) as shown in Figure 1. This prototyping system is developed by Visual Basic 2013 and ACCESS on an N-tier client-server architecture. In IFSsDSS, a decision maker needs to key in criteria, weight, logic gate, the score of each alternative on each criterion and his/her preferred parameter, λ, as illustrated in Figure 2. The system can compute the evaluation value for each alternative on each criterion by different preferred cases. The result is shown in Figure 3. The ranking score is the highest. Thus, the alternative is the best choice.

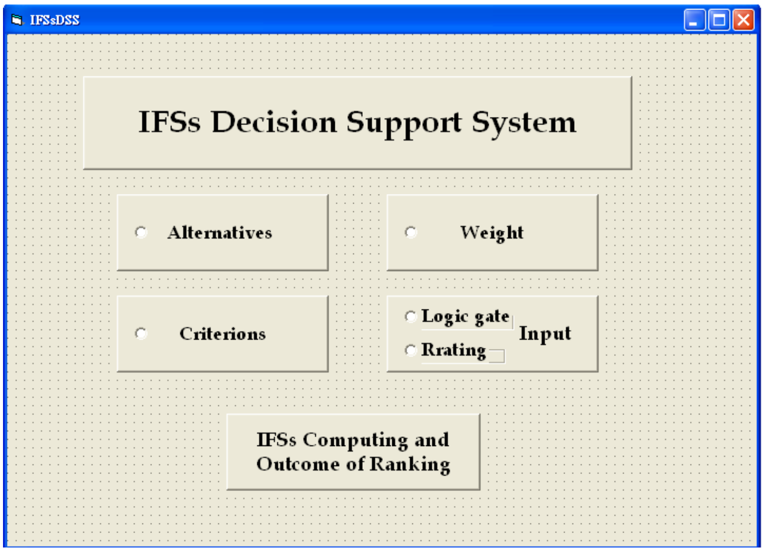

Figure 1: The functional interface of IFSsDSS

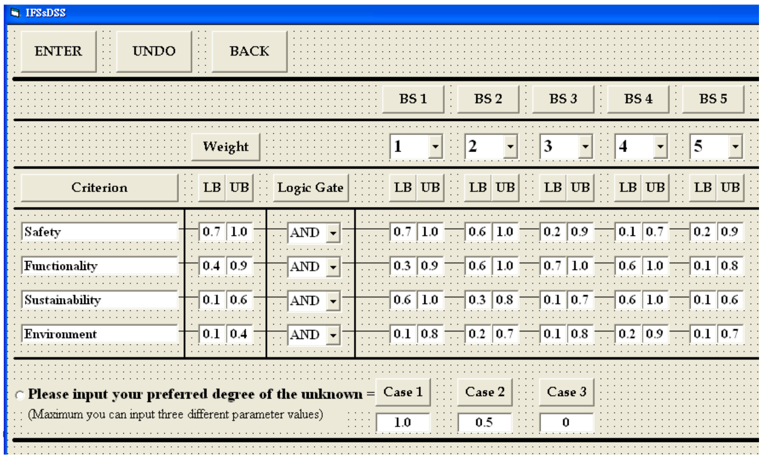

Figure 2: The input evaluation value of each alternative on each criterion

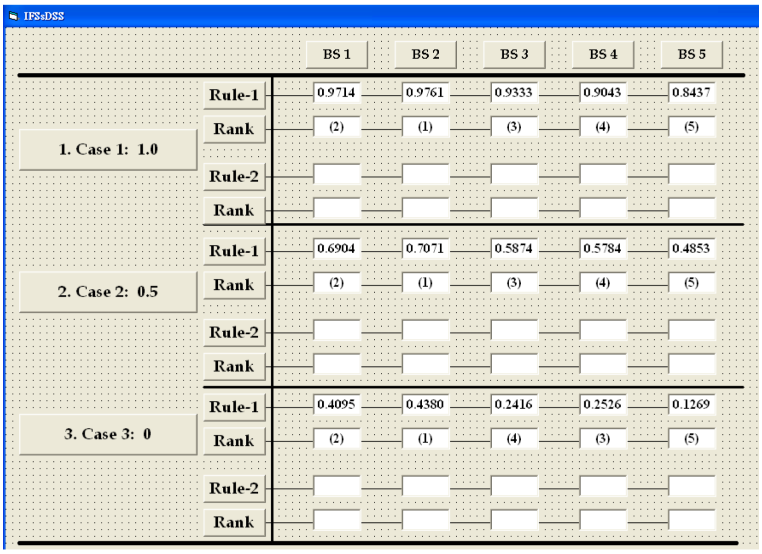

Figure 3: The outcomes of ranking by three different parameters, λ

8. CONCLUSION

Investigating the research of Li (2005) and Lin et al. (2007), some problems are pointed out and the complex procedures of the method of the multi-criteria fuzzy decision making has been modified by an improved method that measures the accuracy membership of each alternative with additional information for multi-criteria decision-making problem under an uncertain environment.Meanwhile, the proposed method provides a new viewpoint for the permeation of IFSs theory in different application areas. Furthermore, to make computing and ranking the results much easier and to increase the recruiting productivity, we have developed a computer-based IFSsDSS system to effectively assist decision makers to deal with problems of vague sets multi-criterion decision making. Future the proposed method may be extended to evaluate and study other practical cases of multi-criteria decision problems under the uncertain environment employing IFSs.

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