International Journal of
Operations Research

International Journal of Operations Research Vol. 13, No. 1, 001-012 (2016)

Performance optimization of the mini-slotted spectrum allocation strategy with imperfect sensing

Shunfu Jin[1], Wuyi Yue[2*], Jinliang Zhang[3] and Dandan Zhang[1]

[1]School of Information Science and Engineering, Yanshan University,

Key Laboratory for Computer Virtual Technology and System Integration of Hebei Province, Yanshan University

[2]Department of Intelligence and Informatics, Konan University

[3]National Center for Public Cultural Services, Ministry of Culture

Received August 2015; Revised January 2016; Accepted March 2016

Abstract **—**In this paper, in order to improve the normal throughput of secondary user packets and reduce the spectrum switching frequency in cognitive radio networks, a novel mini-slotted spectrum allocation strategy is proposed. Due to the mistake detection in practice, the secondary user packet and the primary user packet will occupy the spectrum simultaneously, i.e., a collision will occur on the spectrum. A heterogeneous discrete-time queueing model with possible collisions is established to model the system operation. Taking into account imperfect sensing results, the transition probability matrix is constructed. Applying the method of matrix geometric solution, performance measures in terms of the disruption rate of primary user packets, the normal throughput of secondary user packets, the spectrum switching rate and the average latency of secondary user packets are given. Numerical results are provided to verify the effectiveness of the proposed mini-slotted spectrum strategy. Finally, by trading off different system performance measures, a net benefit function is constructed, then the slot size is optimized.

Keywords **—**Spectrum Allocation Strategy, Mini-slot, Heterogeneous Queueing Model, Imperfect Sensing.

1. INTRODUCTION

Nowadays, with the rapid development of wireless communication technology, the demand for wireless spectrum increases gradually, the spectrum resource has become scarcer than ever before. However, recent studies show that as much as 90% of the time, most of the allocated spectrum is not used under the static spectrum assignment policy (Kuo, 1993). There are two types of users, namely primary users (PUs) and secondary users (SUs) in cognitive radio networks, the PUs have higher priority to the licensed spectrum, while the SUs are capable of sensing spectrum holes and utilizing them in an opportunistic way without causing harmful interference to the PUs (Wang, 2011, Nguyen, 2014, Zhao, 2007).

As an important function of cognitive radio networks, spectrum allocation strategy has drawn more and more attentions. Recently, a lot of researches on the performance of spectrum allocation strategies have been carried out from different view points. In (Soleimani, 2013), for the sake of decreasing the spectrum switches, with a Hidden Markov Model (HMM), the authors investigated the handoff procedure based on prediction approach. In (Tang, 2013), aiming to adapt the spectrum access behavior of PUs, the authors proposed an access strategy for SUs and analyzed the impact of the access strategy on the spectrum switches. In (Do, 2012), taking into account the cognitive radio network with a single SU and multiple PUs, the authors applied the M/G/1 preemptive priority queueing model to analyze the average waiting time of the SU. They also proposed an adaptive algorithm with the delay constraint. The researches mentioned above are based on the assumption of perfect spectrum sensing. However, due to the channel fading in a spectrum and the interference at the physical layer (Akyildiz, 2006), the sensing errors are inevitable in practice.

Some related works have been studied by considering the mistake detection and false alarm. In (Altrad, 2014), the authors applied the continuous-time Markov chain model to analyze the performance of opportunistic spectrum access under the imperfect sensing conditions. In (Ko, 2014), the authors provided a framework for the IEEE-802.11 Medium Access Control (MAC), and studied the tradeoff between the sensing time and throughput of SUs. Moreover, they also investigated the optimal sensing time by considering both the unsaturated and the saturated traffic conditions. Unfortunately, in these researches, the PUs and SUs are supposed to operate on the homogeneous unit of time.

For the purpose of making full use of the spectrum, some researches have been carried out based on a heterogeneous

* Corresponding author's email: yue@konan-u.ac.jp

structure. In (Bae, 2010), in order to improve the throughput, the authors proposed a modified spectrum allocated strategy, where the SUs were supposed to access the spectrum with a binary exponential backoff algorithm. According to the remaining time of the current slot, the winning SU packets with appropriate length will be transmitted. The main drawback of this work is that if the remaining time of the current slot is smaller than the length of the shortest SU packets, the remaining time of the current slot will be idle. In (Atmaca, 2013), considering that the PUs employed TDMA to access the spectrum, the SUs utilized slotted CSMA and accessed the spectrum when the slot was not occupied by PUs, the authors divided the idle time slot into contention period and data transmission period. They also analyzed the throughput for the two kinds of users and the overall network to evaluate the spectrum utilization. But if the SU queue is empty and there is an SU packet arrival after the contention period, the newly arriving SU packet will be not transmitted after the contention period.

To reuse the spectrum with higher throughput and decrease the expense of the spectrum switching in cognitive radio networks, in this paper, we propose a mini-slotted spectrum allocation strategy for SUs. Considering the imperfect spectrum sensing results in practice, we build a heterogeneous discrete-time queueing model with possible collisions. We also investigate and optimize the performance measures of the disruption rate of PU packets, the normal throughput of SU packets, the spectrum switching rate and the average latency of SU packets. In (Zhang, 2014), we presented a little results of this research in an early stage as a lecture note. However in this paper we present an analysis framework to derive the steady-state distribution of the queueing model. We also give some important performance measures in terms of in terms of the disruption rate of PU packets, the normal throughput of SU packets, the spectrum switching rate and the average latency of SU packets to investigate the stochastic behavior of the system. In addition, we provide numerical results with analysis and simulation to show the change trends of the performance measures. Moreover, we establish a reward-cost structure based function to optimize the slot size.

The rest of this paper is organized as follows. In Section 2, the system model for a mini-slotted spectrum allocation strategy is presented. In Section 3, the transition probability matrix is constructed and the stationary probability distribution of the three-dimensional Markov chain is computed. In Section 4, the formulas for the performance measures in terms of the disruption rate of PU packets, the normal throughput of SU packets, the spectrum switching rate and the average latency of SU packets are obtained. In Section 5, with the numerical results, the influence of the slot size and the imperfect sensing result on system performance are evaluated. In Section 6, the net benefit function is established, and the slot size is optimized. Finally, the conclusions are drawn in Section 7.

2. SYSTEM MODEL

2.1. A Mini-slotted Spectrum Allocation Strategy

In this paper, we propose a mini-slotted spectrum allocation strategy to utilize the spectrum resources more efficiently and reduce the spectrum switching frequency. Firstly, we present the mini-slotted allocation strategy used in the system as follows: time axis is divided into mini slots with fixed length, and several mini slots combine to constitute a slot. Then, we assume that the transmission of an SU packet is based on the mini slot, while the transmission of a PU packet is based on the slot. Moreover, we suppose that there are multiple PUs, multiple SUs and adequate licensed spectrums in a cognitive radio network.

In order to maximum the throughput of the SUs, we introduce a buffer for the SU packets, and the buffer is implemented with infinite length for simplicity. The SU packets are transmitted on a first come first service (FCFS) policy. However, for the purpose of minimizing the average delay of the PUs, no buffer is set for the PU packets.

Because a PU packet can access the spectrum at any slot boundary, so at the beginning instant of each slot, the SUs will sense the PUs' activity and then send the sensing results to the central controller. By synthesizing these sensing results, the central controller will allocate one of the idle spectrums for the SU packet queueing at the head of the SU buffer, then this SU packet will be transmitted on this spectrum. In this paper, we call the spectrum on which the SU packets are being transmitted as a tagged spectrum.

The working principle of the mini-slotted spectrum allocation strategy is presented in Figure 1.

As can be seen in Figure 1, throughout the transmission procedure of an SU packet, the SU will check whether the current mini slot is a slot boundary or not. If the current mini slot is not a slot boundary, the SU will ignore the PUs' activity and proceed the transmission. Otherwise, the SU will perform the spectrum sensing to find out whether the PU is active or not.

We note that the sensing errors are inevitable in practice. When a PU packet arrives at the system, but the SU is not aware this arrival, i.e., a mistake detection occurs, the SU packet and the arriving PU packet will occupy the spectrum simultaneously, it means that the SU packet will be collided with the arriving PU packet. After a mini slot, both of the collided packets will be dropped out of the system, namely, the collided packets are disrupted. The remaining SU packets in

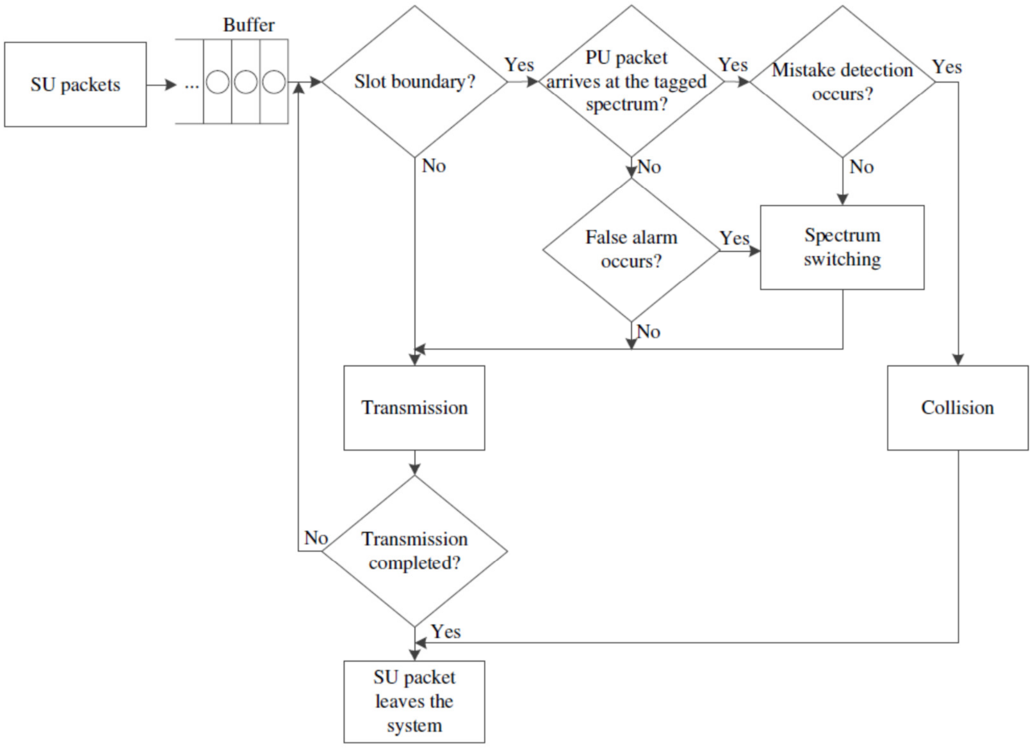

Figure 1: Working Principle of the Mini-slotted Spectrum Allocation.

the buffer will start the transmission from the next mini slot.

When SUs sense the channel via energy detection, two kinds of sensing errors, in terms of missed detection and false alarm, can possibly occur. Let t_s be the sensing time, f_s be the sensing frequency, γ be the signal-to-noise ratio (SNR) and σ be the variance of noise. Let f_m be the missed detection ratio and f_a be the false alarm ratio. In discrete time field, the missed detection ratio f_m and the false alarm ratio f_a are also called as the missed detection probability and the false alarm probability, respectively. Referencing (Liang, 2008), f_m and f_a are given as follows:

$$\begin{cases} f_m = 1 - Q\left(\left(\dfrac{\tau}{\sigma^2} - \gamma - 1\right)\sqrt{\dfrac{t_s f_s}{2\gamma + 1}}\right) \\ f_a = Q\left(\left(\dfrac{\tau}{\sigma^2} - 1\right)\sqrt{t_s f_s}\right) \end{cases} \tag{1}$$

where τ is the energy threshold used in channel sensing, and $Q(v)$ is the tail probability of the standard normal distribution given by

$$Q(v) = \frac{1}{\sqrt{2\pi}} \int_v^{\infty} \exp\left(-\frac{t^2}{2}\right) dt .$$

When a PU packet arrives at the system, and the SU being transmitted senses this arrival, the transmission of the SU packet will be preempted. Based on the spectrum condition in the cognitive radio network, the central controller will allocate one of idle spectrums for the preempted SU packet. We suppose that there are always available spectrums for use. The dispatch process that the central controller allocates idle spectrums is not considered in this paper. The preempted SU packet together with other SU packets in the buffer will be switched to the allocated available spectrum, and the preempted SU packet will queue at the head of the buffer. After the spectrum switching, the transmission of the preempted SU packet will be continued.

By contrary, when a PU packet does not arrive at the system, but the SU misjudges that there is a PU packet arrival, i.e., a false alarm occurs, the transmission of the SU packet will be preempted too. In this case, all the SU packets in the system will perform a spectrum switching, then the transmission of the preempted SU packet will be continued on the new spectrum. When a PU packet does not arrive at the system, and the SU being transmitted senses the spectrum condition correctly, the transmission of the SU packet will be continued on the current spectrum.

2.2. Model Description

With respect to the preemptive priority of PUs to exploit the spectrum, we regard PU packets and SU packets as two classes of customers, the tagged spectrum as a server. Considering the imperfect spectrum sensing results of SUs, the mini-slotted spectrum allocation strategy proposed in this paper can be described as a heterogeneous discrete-time queueing model with possible collisions.

We suppose that there are m mini slots in a slot, the mini slot boundaries in a slot are numbered as n $(n = 1, 2, 3, \ldots, m)$, while the slot boundaries are numbered as N $(N = 1, m+1, 2m+1, \ldots)$. For the mathematical clarify, an SU packet is supposed to arrive at the beginning instant of a mini slot, and depart at the end instant of a mini slot; a PU packet is supposed to arrive at the beginning instant of a slot, and departure at the end instant of a slot. In other words, an early arrival system with heterogeneous time structure is considered.

Referring to (Li, 2011, Jiao, 2012, Wang, 2013) and considering the digital nature of modern communications, the following assumptions are made in order to develop our analytical model.

- The arrival of an SU packet is assumed to follow Bernoulli process with arrival rate λ $(0 < \lambda < 1,\ \bar{\lambda} = 1 - \lambda)$, that is to say, in a mini slot, an SU packet arrives at the system with probability λ, no SU packet arrives at the system with probability $\bar{\lambda}$.
- The service time of an SU packet is assumed to follow geometric distribution with service rate μ $(0 < \mu < 1)$, it means that in a mini slot, the transmission of an SU packet will be completed successfully with probability μ, and continued with probability $\bar{\mu} = 1 - \mu$.
- The arrival of a PU packet is assumed to follow Bernoulli process with arrival rate α $(0 < \alpha < 1,\ \bar{\alpha} = 1 - \alpha)$, that is to say, in a slot, a PU packet arrives at the system with probability α, no PU packet arrives at the system with probability $\bar{\alpha}$.
- On the other hand, the arrivals and departures for these two kinds of packets are supposed to be independent. Moreover, we assume that the switching procedure is neglected.

We define the total number of SU packets in the system as the system level, the mini slot number in a slot as the system phase, the tagged spectrum condition as the system stage. We denote X_n as the system level at the instant n^+, Y_n as the system phase at the instant n^+, Z_n as the system stage at the instant n^+. $Z_n = 0$ represents the tagged spectrum is idle or being used by an SU packet, i.e., the spectrum is in a "normal" condition. $Z_n = 1$ represents that a PU packet and an SU packet appear at the tagged spectrum simultaneously, i.e., there is a collision on the spectrum and the spectrum is in a "disorder" condition. $\{X_n, Y_n, Z_n\}$ constitutes a three-dimensional Markov chain. The state space of this Markov chain is given as follows:

$$\Omega = \{(i, j, l) : i \geq 0, 1 \leq j \leq m, l = 0, 1\}.$$

3. SYSTEM ANALYSIS

3.1. Transition Probability Matrix

Since the SUs sense the spectrum at the boundary of a slot, the imperfect sensing results occur only at the beginning instant of a slot. We note that for the system level $X_n > 0$ and the system phase $Y_n = 1$, there are two stages: $Z_n = 0$ and $Z_n = 1$; otherwise, there is only one stage: $Z_n = 0$. We also note that whether or not there is an SU packet arrival in a mini slot, if the current system phase is $1 \leq Y_n \leq m - 1$, the next system phase will be increased to $Y_{n+1} = Y_n + 1$; if the current system phase is $Y_n = m$, the next system phase will be $Y_{n+1} = 1$.

Let $\mathbf{P}$ be the one step state transition probability matrix of the $\{X_n, Y_n, Z_n\}$, $\mathbf{P}(u, v)$ be the one step transition probability submatrix from the system level u to v. $\mathbf{P}(u, v)$ is all on the order of $(m+1) \times (m+1)$ and can be discussed as follows:

(1) If $u = 0$ and $v = 0$, it means that there is no SU packet arrival at the system with probability $\bar{\lambda}$ during the one step transition. Therefore, the one step transition probability submatrix $\mathbf{P}(0, 0)$ is given as follows:

$$\mathbf{P}(0,0)=\begin{pmatrix} 0 & 0 & & & & \\ & 0 & \bar{\lambda} & & & \\ & & 0 & \bar{\lambda} & & \\ & & & \ddots & \ddots & \\ & & & & 0 & \bar{\lambda} \\ 0 & \bar{\lambda} & 0 & \cdots & 0 & 0 \end{pmatrix}. \tag{2}$$

(2) If $u=0$ and $v=1$, it means that there is an SU packet arrival at the system with probability λ during the one step transition. Firstly, we discuss the transition for the system phase changing to $Y_{n+1}=1$ from $Y_n=m$. If a PU packet arrives at the beginning instant of a slot with probability α, and a mistake detection occurs with probability f_m, the system stage $Z_n=0$ will be changed to the system stage $Z_{n+1}=1$ with probability αf_m. There are two cases will make the system stage being fixed at $Z_{n+1}=0$: no PU packet arrives at the beginning instant of a slot with probability $\bar{\alpha}$; a PU packet arrives at the beginning instant of a slot with probability α, but no mistake detection occurs with probability $\bar{f}_m$ ($\bar{f}_m=1-f_m$), so the system stage will be fixed at $Z_{n+1}=0$ with probability $\bar{\alpha}+\alpha\bar{f}_m$. Therefore, the one step transition probability submatrix $\mathbf{P}(0,1)$ is given as follows:

$$\mathbf{P}(0,1)=\begin{pmatrix} 0 & 0 & & & & \\ & 0 & \lambda & & & \\ & & 0 & \lambda & & \\ & & & \ddots & \ddots & \\ & & & & 0 & \lambda \\ \lambda\alpha f_m & \lambda(\alpha\bar{f}_m+\bar{\alpha}) & 0 & \cdots & 0 & 0 \end{pmatrix}. \tag{3}$$

(3) If $u=1$ and $v=0$, there is no SU packet arrival at the system with probability $\bar{\lambda}$. For the system stage $Z_n=1$, the SU packet being collided with a PU packet has to leave the system during the one step transition. For the system stage $Z_n=0$, the transmission of the SU packet occupying the spectrum is completed successfully with probability μ during one step transition. Therefore, the one step transition probability submatrix $\mathbf{P}(1,0)$ is given by

$$\mathbf{P}(1,0)=\begin{pmatrix} 0 & 0 & \bar{\lambda} & & & \\ & 0 & \bar{\lambda}\mu & & & \\ & & 0 & \bar{\lambda}\mu & & \\ & & & \ddots & \ddots & \\ & & & & 0 & \bar{\lambda}\mu \\ 0 & \bar{\lambda}\mu & 0 & \cdots & 0 & 0 \end{pmatrix}. \tag{4}$$

(4) If $u=v\geq 1$, for the system stage $Z_n=1$, the collided SU packet has to leave the system and an SU packet arrives at the system with probability λ during the one step transition. For the system stage $Z_n=0$, there are two kinds of cases to be addressed: an SU packet arrives at the system with probability λ and the transmission of the SU packet occupying the spectrum is completed successfully with probability μ; no SU packet arrives at the system with probability $\bar{\lambda}$ and no SU packet departs from the system with probability $\bar{\mu}$. Similar to Item (2), when the system phase changes to $Y_{n+1}=1$ from $Y_n=m$, the stage $Z_n=0$ will change to $Z_{n+1}=1$ with probability αf_m, the stage $Z_n=0$ will be fixed at $Z_{n+1}=0$ with probability $\alpha\bar{f}_m+\bar{\alpha}$. By $\mathbf{A}_1$ we denote the one step transition probability submatrix $\mathbf{P}(u,u)$, $\mathbf{A}_1$ is then given as follows:

$$\mathbf{A}_1 = \begin{pmatrix} 0 & 0 & \lambda & & & \\ & 0 & \lambda\mu + \bar{\lambda}\bar{\mu} & & & \\ & & 0 & \lambda\mu + \bar{\lambda}\bar{\mu} & & \\ & & & \ddots & \ddots & \\ & & & & 0 & \lambda\mu + \bar{\lambda}\bar{\mu} \\ (\lambda\mu + \bar{\lambda}\bar{\mu})\alpha f_m & (\lambda\mu + \bar{\lambda}\bar{\mu})(\alpha\bar{f}_m + \bar{\alpha}) & 0 & \cdots & 0 & 0 \end{pmatrix}. \tag{5}$$

(5) If u $(u \geq 1)$ and $v = u + 1$, there is at most one SU packet arrival in a mini slot. When the system stage changes to $Z_{n+1} = 0$ from $Z_n = 1$, it is impossible for the system level to transfer from u to $(u+1)$. For the system stage $Z_n = 0$, there is an SU packet arrival at the system with probability λ and no SU packet departure from the system with probability $\bar{\mu}$ during the one step transition. Similar to Item (2), when the system phase changes to $Y_{n+1} = 1$ from $Y_n = m$, the system stage will change to $Z_{n+1} = 1$ from $Z_n = 0$ with probability αf_m, while the system stage will be fixed at 0 with probability $\alpha\bar{f}_m + \bar{\alpha}$. Let $\mathbf{A}_0$ be the one step transition probability submatrix $\mathbf{P}(u, u+1)$. $\mathbf{A}_0$ is given as follows:

$$\mathbf{A}_0 = \begin{pmatrix} 0 & 0 & & & & \\ & 0 & \lambda\bar{\mu} & & & \\ & & 0 & \lambda\bar{\mu} & & \\ & & & \ddots & \ddots & \\ & & & & 0 & \lambda\bar{\mu} \\ \lambda\bar{\mu}\alpha f_m & \lambda\bar{\mu}(\alpha\bar{f}_m + \bar{\alpha}) & 0 & \cdots & 0 & 0 \end{pmatrix}. \tag{6}$$

(6) For the case of u $(u \geq 2)$ and $v = u - 1$, we denote the one step transition probability submatrix $\mathbf{P}(u, u-1)$ as $\mathbf{A}_2$. Similar to Items (2) and (3), $\mathbf{A}_2$ is obtained as follows:

$$\mathbf{A}_2 = \begin{pmatrix} 0 & 0 & \bar{\lambda} & & & \\ & 0 & \bar{\lambda}\mu & & & \\ & & 0 & \bar{\lambda}\mu & & \\ & & & \ddots & \ddots & \\ & & & & 0 & \bar{\lambda}\mu \\ \bar{\lambda}\mu\alpha f_m & \bar{\lambda}\mu(\alpha\bar{f}_m + \bar{\alpha}) & 0 & \cdots & 0 & 0 \end{pmatrix}. \tag{7}$$

Combining Eqs. (2)-(7), the one step state transition probability $\mathbf{P}$ for the system is given by

$$\mathbf{P} = \begin{pmatrix} \mathbf{P}(0,0) & \mathbf{P}(0,1) & & & \\ \mathbf{P}(1,0) & \mathbf{A}_1 & \mathbf{A}_0 & & \\ & \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 & \\ & & \mathbf{A}_2 & \mathbf{A}_1 & \mathbf{A}_0 \\ & & & \ddots & \ddots & \ddots \end{pmatrix}. \tag{8}$$

From the structure of the one step state transition probability $\mathbf{P}$, we know that the stochastic process of $\{X_n, Y_n, Z_n\}$ is a quasi-birth-and-death (QBD) process.

3.2. Computation of the Stationary Probability Distribution

Let $\pi_{i,j,l}$ be the stationary probability distribution for the three-dimensional Markov chain $\{X_n, Y_n, Z_n\}$ being at the state (i, j, l), where $(i, j, l) \in \Omega$. $\pi_{i,j,l}$ can be given as follows:

$$\pi_{i,j,l} = \lim_{n\to\infty} P\{X_n = i, Y_n = j, Z_n = l\}, \quad i \geq 0, 1 \leq j \leq m, l = 0,1. \tag{9}$$

Let Π_i be the stationary probability vector for the system being at level i. Π_i can be given as follows:

$$\Pi_i = (\pi_{i,1,1}, \pi_{i,1,0}, \pi_{i,2,0}, \ldots, \pi_{i,m,0}). \tag{10}$$

Then, the stationary probability vector Π for the system can be given as follows:

$$\Pi = (\Pi_0, \Pi_1, \Pi_2, \ldots). \tag{11}$$

Let matrix $\mathbf{R}$ be the minimum non-negative solution of the following matrix quadratic equation:

$$\mathbf{R}^2\mathbf{A}_2 + \mathbf{R}\mathbf{A}_1 + \mathbf{A}_0 = \mathbf{R}. \tag{12}$$

From Eq. (12), we have

$$\mathbf{R} = (\mathbf{R}^2\mathbf{A}_2 + \mathbf{A}_0)(\mathbf{I} - \mathbf{A}_1)^{-1} \tag{13}$$

where $\mathbf{I}$ denotes an $(m+1)\times(m+1)$ unit matrix.

The reasonable approximation of $\mathbf{R}$ is obtained iteratively. When the spectral radius of the matrix $\mathbf{R}$ is less than 1, the system will achieve the stationary state.

Combining the system equilibrium equation and the normalization condition, we have

$$\begin{cases} (\Pi_0, \Pi_1)B[\mathbf{R}] = (\Pi_0, \Pi_1) \\ \Pi_0\, \boldsymbol{e} + \Pi_1(\mathbf{I} - \mathbf{R})^{-1}\boldsymbol{e} = 1 \end{cases} \tag{14}$$

where $\boldsymbol{e}$ is a one's column vector, matrix $B[\mathbf{R}]$ has the structure as follows:

$$B[\mathbf{R}] = \begin{pmatrix} \mathbf{P}(0,0) & \mathbf{P}(0,1) \\ \mathbf{P}(1,0) & \mathbf{A}_1 + \mathbf{R}\mathbf{A}_2 \end{pmatrix}. \tag{15}$$

Applying the matrix geometric solution method in (Zhao, 2015), we have

$$\Pi_i = \Pi_1\, \mathbf{R}^{i-1}, \quad i \geq 1. \tag{16}$$

Combining Eqs. (14)-(16), we can get the stationary probability distribution $\{\pi_{i,j,l} : i \geq 0, 1 \leq j \leq m, l = 0,1\}$ with numerical results.

4. PERFORMANCE MEASURES

In this section, we derive some performance measures to evaluate the mini-slotted spectrum allocation strategy in cognitive radio networks.

The disruption rate of PU packets is defined as the number of PU packets being collided with SU packets per mini slot. When a PU packet arrives at the slot boundary, but the SU packet being transmitted misjudges this arrival, the tagged spectrum will be in the "disorder" condition, i.e., the arriving PU packet is disrupted. Therefore, the disruption rate θ of PU packets is given as follows:

$$\theta = \pi_{i,1,1}. \tag{17}$$

The normal throughput ξ of SU packets is defined as the number of SU packets transmitted successfully per mini slot. An SU packet will be transmitted successfully except for being disrupted due to mistake detection. Therefore, the normal throughput ξ of SU packets is given as follows:

$$\xi = \lambda - \theta. \tag{18}$$

The spectrum switching rate ϕ is defined as the average number of spectrum switching per mini slot. When the transmission of the SU packet occupying the spectrum is preempted, the SU will switch to another spectrum. So the spectrum switching rate ϕ is given as follows:

$$\phi = \frac{\alpha\overline{f}_m + \bar{\alpha}f_a}{\alpha\overline{f}_m + \bar{\alpha}f_a + \bar{\alpha}\overline{f}_a}\sum_{i=1}^{\infty}\pi_{i,1,0} \tag{19}$$

where $\overline{f}_a = 1 - f_a$ and f_a is the false alarm ratio given by Eq. (1), $\overline{f}_m = 1 - f_m$ and f_m is the missed detection ratio given by Eq. (1), too.

The latency of an SU packet is defined as the time period in mini slots from the instant that an SU packet joins the system to the instant that SU packet leaves the system. With Little's law (Alfa, 2010), the average latency ω of SU packets is given as follows:

$$\omega = \sum_{i=0}^{\infty}\sum_{j=1}^{m}\frac{i\times(\pi_{i,j,0} + \pi_{i,j,1})}{\lambda}. \tag{20}$$

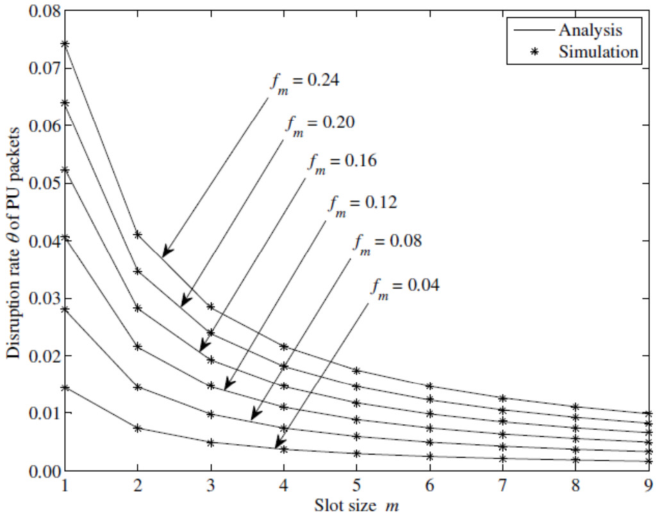

Figure 2: Disruption Rate θ of PU Packets vs. Slot Size m .

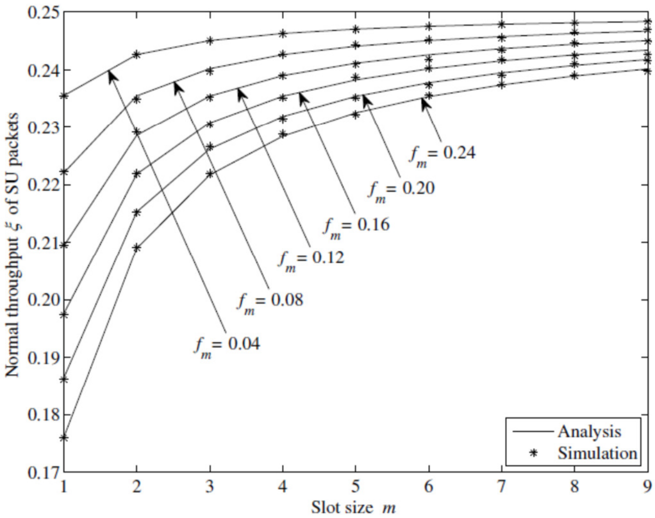

Figure 3: Normal Throughput ξ of SU Packets vs. Slot Size m .

5. NUMERICAL RESULTS

In this section, the influence of the system parameters on the system performance for the mini-slotted spectrum allocation strategy is evaluated quantitatively. Referring to (Zhao, 2014), the parameters are fixed as follows: $\lambda = 0.25$, $\mu = 0.4$, $\alpha = 0.6$ in numerical results.

Figure 2 illustrates the influence of the slot size m on the disruption rate θ of PU packets with different missed detection ratio f_m .

As can be seen in Figure 2, for the same missed detection ratio f_m , the disruption rate θ of PU packets will decrease with the enlargement of the slot size m . The reason is that the larger the slot size is, the less likely is that the transmission of the SU packet occupying the spectrum crosses the slot boundary, the disruption rate of PU packets will be lower. For the same slot size m , the disruption rate θ of PU packets will increase when the missed detection ratio f_m increases. This is because that the greater the missed detection ratio is, the more likely is that the SU packet and the arriving PU packet will occupy the spectrum simultaneously, namely, the higher the possibility is that the arriving PU packet will be disrupted, so the disruption rate of PU packets will be higher.

Taking the false alarm ratio $f_a = 0.08$ as an example, we show how the throughput ξ of the SU packets changes with respect to the slot size m for the different missed detection ratio f_m in Figure 3.

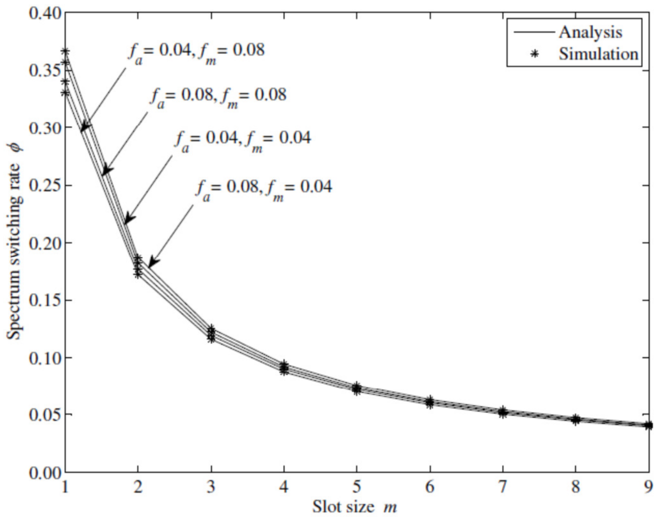

Figure 4: Spectrum Switching Rate ϕ vs. Slot Size m.

As can be seen in Figure 3, for the same missed detection ratio f_m, the normal throughput ξ of SU packets will increase with the enlargement of the slot size m. The reason is that as the slot size increases, there are more mini slots, during which the transmission of SU packets will not be disrupted, in a slot, then the higher the possibility is that an SU packet is transmitted successfully, so the normal throughput ξ of SU packets will be greater. For the same slot size m, the normal throughput ξ of SU packets will increase when the missed detection ratio f_m increases. This is because that the bigger the missed detection ratio is, the more likely is that an SU packet and a PU packet will be collided, i.e., more SU packets will be disrupted, this will certainly result in a lower normal throughput of SU packets.

Figure 4 describes the influence of the slot size m on the spectrum switching rate ϕ with different missed detection ratio f_m and false alarm ratio f_a.

From Figure 4, we observe that for the same missed detection ratio f_m and the same false alarm ratio f_a, the spectrum switching rate ϕ will decrease as the slot size m increases. The reason is that for a larger slot size, there are less potential preemption instants within a certain time period, which will reduce the spectrum switching frequency, i.e., the spectrum switching rate will be lower. For the same slot size m and the same missed detection ratio f_m, the spectrum switching rate ϕ will increase as the false alarm ratio f_a increases. This is because that the greater the false alarm ratio is, the more likely is that the SU will switch to another spectrum due to false alarm, so the spectrum switching rate will be higher. For the same slot size m and the same false alarm ratio f_a, the spectrum switching rate ϕ will increase when the missed detection ratio f_m decreases. The reason is that the smaller the missed detection ratio is, the more likely is that an SU packet will be preempted due to mistake detection, then the SU will more likely switch to another spectrum, so the spectrum switching rate will be higher.

We examine the influence of the slot size m on the average latency ω of SU packets for different missed detection ratio f_m in Figure 5.

From Figure 5, we find that for the same missed detection ratio f_m, the average latency ω of SU packets will increase as the slot size m increases. The reason is that the larger the slot size is, the more likely is that an SU packet will be transmitted successfully without disruption. The more the SU packets are transmitted successfully, the longer the average transmission time and the average waiting time will be, this will therefore result in a longer average latency of SU packets. For the same slot size m, the average latency ω of SU packets will decrease as the missed detection ratio f_m increases. This is because that the greater the missed detection ratio is, the more possible is that the spectrum being at the disorder state. Since the actual transmission time of the disrupted SU packet is shorter, so the average latency of SU packets will decrease.

Summarizing the numerical results shown in Figures 2-5, we find that from the view point of the normal throughput of SU packets and the spectrum switching rate, the proposal mini-slotted spectrum allocation strategy performs better than the conventional spectrum allocation strategy with homogeneous structure. On the other hand, from the perspective of the average latency of SU packets, we find that the system performance of the mini-slotted spectrum allocation strategy is

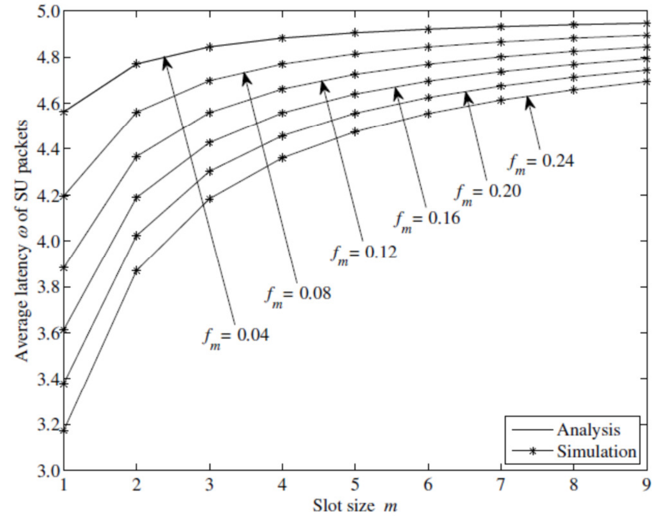

Figure 5: Average Latency ω of SU Packets vs. Slot Size m .

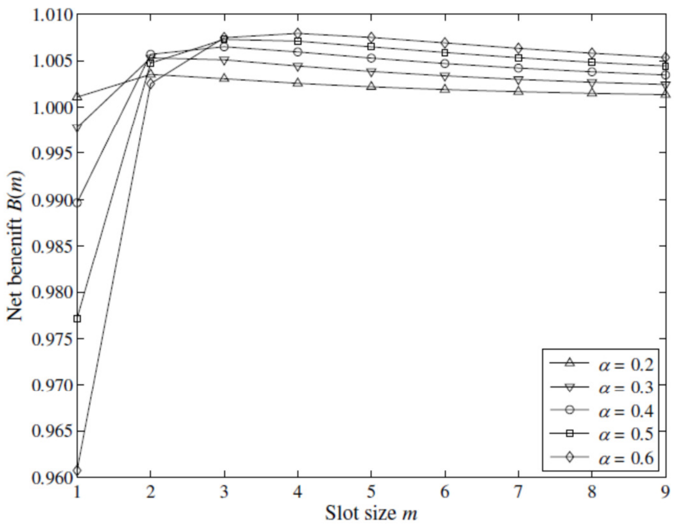

Figure 6: Net Benefit $B(m)$ vs. Slot Size m .

degraded a bit. That is to say, the proposed mini-slotted spectrum allocation strategy is more appropriated to the delay tolerance application networks. Moreover, we conclude that there is a tradeoff to be considered when setting the slot size in the mini-slotted spectrum allocation strategy.

6. PERFORMANCE OPTIMIZATION

In order to balance different performance measures, we construct a net benefit function $B(m)$ for the system as follows:

$$B(m) = r_1\xi - r_2\omega - r_3\phi \tag{21}$$

where r_1, r_2 and r_3 are supposed to be the reward for transmitting an SU packet successfully, the cost for a mini slot due to the latency of an SU packet and the expense for one spectrum switching, respectively.

Setting $r_1 = 60$, $r_2 = 2.75$ and $r_3 = 0.3$ as an example, we plot how the net benefit function $B(m)$ changes with respect to the slot size m for different arrival rate α of PU packets in Figure 6.

It can be seen in Figure 6, for all the arrival rates of PU packets, the net benefit functions $B(m)$ experience two stages. In the first stage, the net benefit function $B(m)$ will increase with the enlargement of the slot size m . During this stage, the main influence factors are the spectrum switching rate and the normal throughput of SU packets, the larger the slot size is, the less the spectrum switching rate is, and the higher the normal throughput of SU packets is, so the greater the net benefit will be. In the second stage, the net benefit function $B(m)$ will decrease as the slot size m increases. During this period, the average latency of SU packets is the dominant measure, the larger the slot size is, the longer the average latency

Table 1: Optimal Slot Size m^* and Maximum Net Benefit $B(m^*)$.

Arrival rate α of PU packets	Optimal slot size m^*	Maximum net benefit $B(m^*)$
0.2	2	1.0035
0.3	2	1.0053
0.4	3	1.0059
0.5	3	1.0073
0.6	4	1.0079

of SU packets is, so the lower the net benefit will be.

Conclusively, there is a maximum net benefit function when the slot size is set to the optimal value m^*, i.e., $m^* = arg\ max\{B(m)\}$. The optimal slot size m^* and the maximum net benefit $B(m^*)$ with different arrival rate α of PU packets are illustrated in Table 1.

7. CONCLUSIONS

In this paper, we proposed a novel mini-slotted spectrum allocation strategy in cognitive radio networks, the transmission of an SU packet is based on the mini slot, the transmission of a PU packet is based on the slot. Based on working principle of the proposed spectrum allocation strategy, we built a discrete-time queueing model with possible collisions. We constructed a three-dimensional Markov chain and derived the formulas for some important measures. From the numerical results, it is observed that the proposed mini-slotted spectrum allocation strategy can effectively improve the normal throughput of SU packets and decrease the spectrum switching frequency. Finally, considering the tradeoff between different performance measures, we optimized the slot size by maximizing the net benefit.

In this paper, the SU packets were supposed to access the spectrum without any constraint. In order to improve the response performance of SU packets, as a future work, we will consider to introduce an access threshold mechanism to the spectrum allocation strategy.

ACKNOWLEDGMENT

This work was supported in part by National Natural Science Foundation (No. 61472342), China and was supported in part by MEXT, Japan.

REFERENCES

1. Akyildiz, I. F., Lee, W., and Vuran, M. C. (2006). Next Generation/Dynamic Spectrum Access/Cognitive Radio Wireless Networks: A Survey. *Computer Networks*, 50: 2127--2159.
2. Alfa, A. S. (2010). *Queueing Theory for Telecommunications: Discrete Time Modeling of a Single Node System*. Springer.
3. Altrad, O. and Muhaidat, S. (2014). Opportunistic Spectrum Access in Cognitive Radio Networks under Imperfect Spectrum Sensing. *IEEE Transactions on Vehicular Technology*, 63: 920--925.
4. Atmaca, S. (2013). Improving TDMA Channel Utilization in Random Access Cognitive Radio Networks by Exploiting Slotted CSMA. *Wireless Personal Communications*, 71: 2417--2430.
5. Bae, Y. H., Alfa, A. S., and Choi, B. D. (2010). Performance Analysis of Modified IEEE 802.11-based Cognitive Radio Networks. *IEEE Communications Letters*, 14: 975--977.
6. Do, C. T., Tran, N. H., and Hong, C. S. (2012). Throughput Maximization for the Secondary User over Multi-channel Cognitive Radio Networks. *Proceedings of the International Conference on Information Networking*, pp. 65--69.
7. Jiao, L., Li, F. Y., and Pla, V. (2012). Modeling and Performance Analysis of Channel Assembling in Multichannel Cognitive Radio Networks with Spectrum Adaptation. *IEEE Transactions on Vehicular Technology*, 61: 2686--2697.
8. Ko, H., Lee, J., and Kim, C. (2014). The Optimal Spectrum Sensing Time for Maximizing Throughput of 802.11-based MAC Protocol for Cognitive Radio Networks under Unsaturated Traffic Conditions. *Wireless Personal Communications*, 77: 1397--1414.
9. Li, H. and Han, Z. (2011). Socially Optimal Queuing Control in Cognitive Radio Networks Subject to Service Interruptions: To Queue or not to Queue?. *IEEE Transactions on Wireless Communications*, 10: 1656--1666.
10. Liang, Y., Zeng, Y., Peh, E., and Hoang, A. (2008). Sensing-throughput Tradeoff for Cognitive Radio Networks. *IEEE Transactions on Wireless Communications*, 7: 1326--1337.
11. Liu, X., Evans, B., and Moessner, K. (2015). Energy-Efficient Sensor Scheduling Algorithm in Cognitive Radio Networks Employing Heterogeneous Sensors. *IEEE Transactions on Vehicular Technology*, 64: 1243--1249.
12. Nguyen, T. N., Pham, A. T., and Nguyen, V. T. (2014). Medium Access Control Design for Cognitive Radio Networks:

A Survey. *IEICE Transactions on Communications*, 97: 359--374.
13. Soleimani, M. T., Kahvand, M., and Sarikhani, R. (2013). Handoff Reduction Based on Prediction Approach in Cognitive Radio Networks. *Proceedings of the International Conference on Communication Technology*, pp. 319--323.
14. Tang, W., Zhou, J., and Yu, J. (2013). Modeling and Analysis of Opportunistic Spectrum Sharing with Considering Access Strategy. *Proceedings of the International Conference on Communications Circuits and Systems*, pp. 7--11.
15. Wang, B. and Liu, K. J. R. (2011). Advances in Cognitive Radio Networks: A Survey. *IEEE Journal on Selected Topics in Signal Processing*, 5: 5--23.
16. Wang, G., Zeng, Z., and Liu, M. (2013). Equilibrium Threshold Strategy in Observable Queueing Systems in Cognitive Radio Networks. *Proceedings of the International Conference on Measurement, Information and Control*, pp. 328-332.
17. Zhang, D., Jin, S., and Yue, W. (2014). Performance Analysis of the Spectrum Allocation Strategy with Mini-slot Structure. Memoirs of Konan University, *Intelligence and Informatics Series*, 7: 119--130.
18. Zhao, Q. and Sadler, B. M. (2007). A Survey of Dynamic Spectrum Access. *IEEE Signal Processing Magazine*, 24: 79--89.
19. Zhao, Y., Jin, S., and Yue, W. (2015). Adjustable Admission Control with Threshold in Centralized CR Networks: Analysis and Optimization. *Journal of Industrial and Management Optimization,* 11: 1393--1408.
20. Zhao, Y., Jin, S., and Yue, W. (2014). Performance Evaluation of the Centralized Spectrum Access Strategy with Multiple Input Streams in Cognitive Radio Networks. *IEICE Transactions on Communications*, 97: 334--342.