

A Fuzzy Goal Programming Approach for Channel Allocation Problem Using Linear and Non-Linear Membership Functions

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Abstract — Some companies produce and sell all of their products through their own internal channels and some use multiple external channels to distribute goods to consumers. Many authors have presented mathematical model for channel allocation problem and solved them. We consider a mixed integer fuzzy goal programming model for channel allocation problem with three different fuzzy goals. In this paper we extend a fuzzy goal programming approach by combining the weighted root power mean method of aggregation with linear and non linear membership function. Different models are generated using weighted root power mean method of aggregation and priority based solutions are achieved using different membership functions.

Keywords — Channel Allocation problem, Compromise fuzzy goal programming, multiobjective optimization, weighted goal programming

1. INTRODUCTION

Firms distribute their product through multiple distribution channels which include direct and indirect channels. The channel in which distribution function is grouped by a supplier and one or more of intermediaries, is known as an indirect channel. However, a direct channel distributes products from producers to the users directly without introducing any intermediaries. There may be several reasons for firms to adopt the multiple channel strategy such as manufacturing and information technologies have encouraged multiple channels. However, multiple channels for distribution of the products in the market arise many challenges and one of them is the allocation of products among multiple channels. An economic assessment model can be helpful for the firms to determine the most suitable allocation strategy for multiple channels.

Tsai et al (2008) have considered a channel allocation problem of largest steel maker in Taiwan. The steel maker company distributes steel coils using multiple distribution channels including indirect and direct channels to the users. The products made by a steel maker are highly customized as the specifications of products are defined at the time of placing an order. They are generally distributed directly from producer to their ultimate customer. Some standard products made by company can be sold through multiple distribution channels or directly to customers. In general, around 15% of the sale is of standard products distributed to consumer directly. While other sales accounted for 75% of the total sales which are carried out by intermediaries. Further, the steel market in Taiwan is increasing in favour of the producers due to the demand of steel supply. Therefore, steel makers can have dominant power in determining the allocation of steel products among multiple channels.

In Fig 1, a schematic graph showing the relation of the channel structure between the steel producers and to the customers is presented.

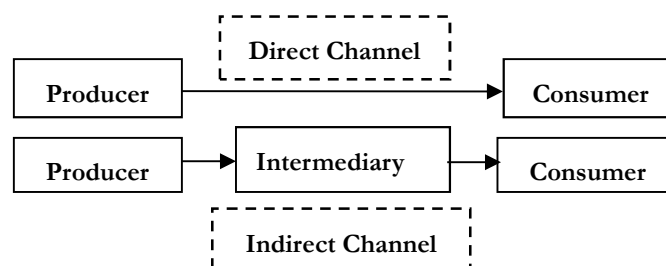


Figure 1: Representation of direct and indirect channels

It is shown that in a direct channel customers can buy their products from the steel maker directly without introducing any intermediates. On the other hand, in indirect channels, steel stockholders order a large quantity from the main producers with long lead time, and then break into small quantities and sell to the ultimate consumers with short lead times according

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to the customer's requirements [McAdam & Brown (2001), Potter et al (2004)].

As the process of channel allocation requires heavy capital amount and time, steel makers plan at the beginning of the year to make sure the facilities are available for allocation of the products among distribution channels. The process also includes a plan if the demand or supply changes accidentally. The allocation problem of capacity of the multiple channels or customers can also be addressed in many other industries where capacity expansion is costly and time-consuming. The producers generally have many considerations for channel allocation problem such as profit, future potential, rate of late lading and sales histories.

Corstjens and Doyle (1979) presented a solution to the channel selection problem referring to the manufacturer's selection of channels to serve designated end markets using geometrical programming model. Rangan (1987) introduced a mathematical model to maximize profits over several channel alternatives considering a number of basic distribution tasks and specified the optimum channel structure in terms of length, intensity, and levels of support for channel members. Moorthy (1988) developed a mathematical model to decide channel structure. Rangan and Jaikumar (1991) constructed an bi-objective optimization model for buying arrangement of intermediaries considering customer's procurement costs and manufacturer's profits simultaneously. Cachon and Lariviere (1999b) proposed that we can allocate supplier's scarce capacity among channel members by linear allocation, proportional allocation, and uniform allocation. Kumar et al (2003) present a fuzzy mixed integer goal programming vendor selection model that includes three different goals. Faez et al (2006) propose a case-based reasoning (CBR) approach for solving the vendor selection problem (VSP). A mixed integer programming model is employed to simultaneously consider suitable vendor selection and order allocation. Narsimhan (1980) proposed a fuzzy goal programming (FGP) technique to specify imprecise aspiration levels of the fuzzy goals. Paksoy and Pehlivan (2012) presented a fuzzy linear programming model for the optimization of the multi-stage supply chain model with triangular and trapezoidal membership functions. Conceicao (2012) present a case study in a complex and diversified multinational steel company using a deterministic formulation for a capacitated location problem based on a model discussed in the literature.

After the above discussion we conclude that the channel allocation problem for steel makers is a complex problem because some goals of the company should also be considered with uncertainty subject to various constraints. Li and Lai (2000) proposed a novel fuzzy compromise programming approach for multi-objective transportation problem using a weighted root power mean aggregation min operator. This model covers a wide spectrum of methods existing in multi criteria decision making problem. Zangiabadi and Maleki (2007) have proposed a powerful fuzzy goal programming approach for multiobjective transportation problem using linear and nonlinear membership function. In this paper weighted root power mean method of aggregation is integrated with both linear and non linear membership function and a powerful interactive fuzzy goal programming approach is proposed. A fuzzy mixed integer goal programming model of channel allocation problem presented by Tsai et al. (2008) has been used. Various priorities towards the objectives have been considered where priorities can be assigned in terms of weights on respective objectives. This paper is organised such that section 2 presents proposed Integrated FGP Approach. Section 3 consists of fuzzy mixed integer goal programming model of channel allocation problem and the application of proposed fuzzy goal programming approach. In section 4, numerical example has been discussed to illustrate the proposed approach. Section 5 consists of conclusion and future direction.

2. FUZZY GOAL PROGRAMMING APPROACH

Li and Lai (2000) presented a fuzzy compromise programming approach for multiobjective transportation problem with the characteristic feature that all objectives are synthetically considered by marginally evaluating individual objectives and globally evaluating all objectives. This fuzzy compromise programming can provide preferred compromise solution which is also non-dominated. The fuzzy compromise programming covers many approaches such as weighted sum method, quadratic programming method and Zimmermann's fuzzy programming approach. In this paper we are incorporating the exponential membership function in the weighted root power mean method of aggregation presented by Li and Lai (2000). The above integration result in the generation of different set of models with exponential membership function.

For each particular objective \tilde{Z}_i , $i = 1, 2, \dots, K$ in the presented model, we obtain two values L_i & U_i that can be assumed as lower and upper bound on the objective function. One of the major assumptions in fuzzy approach of solving mathematical programming problems in the literature involves the use of linear membership functions. A linear approximation is most commonly used because of its simplicity and efficiency. It is denoted by obtaining two points, the upper and lower levels of acceptability but sometimes fuzzy approach of solving mathematical programming problems in the literature involves the use of exponential membership function. Sometimes decision maker have to face conditions when he is worse off with respect to a goal and he want to have higher marginal rate of satisfaction with respect to that goal. Such behaviour is modelled using non linear membership function.

Linear Membership Function

Following triangular membership function can be employed to define the marginal evaluation mapping of the decision variable X .

$$\mu_i(x) = \begin{cases} 1 & \text{If } Z_i \leq L_i \\ 1 - \frac{L_i - Z_i}{L_i - U_i} & \text{If } L_i < Z_i < U_i \\ 0 & \text{If } Z_i \geq U_i \end{cases} \quad (1)$$

$\mu_i : X \rightarrow [0,1]$ for the objectives $\tilde{Z}_i, i = 1, 2, \dots, K$.

Exponential Membership Function

A non linear exponential membership function is defined as:

$$\mu_i(x) = \begin{cases} 1 & \text{If } Z_i \leq L_i \\ \frac{e^{-S\theta_i(x)} - e^{-S}}{1 - e^{-S}} & \text{If } L_i < Z_i < U_i \\ 0 & \text{If } Z_i \geq U_i \end{cases} \quad (2)$$

where $\theta_i(x) = \frac{L_i - Z_i}{L_i - U_i}, i = 1, 2, \dots, K, S$ is nonzero parameter which is prescribed by the decision maker. In most of the

practical situation these L_i & U_i can be viewed as an Ideal solution and tolerance limit on the ideal solution. Ideal solutions are usually obtained by solving series of objectives separately ignoring all other objectives.

Having defined marginal evaluation of X for all the objectives using these membership functions, the next step is to determine global evaluation of X with respect to all objectives. Thus a mapping $\Psi_i : X \rightarrow [0,1]$ defines us that a solution X satisfies objective functions up to what degree. However, in this paper we will consider the relative importance of all objective functions defined by preferences in terms of weights. Relative importance of objectives is usually given by a set of weights $w = (w_1, w_2, w_3, \dots, w_k)$, for which $\sum_{i=1}^K w_i = 1$. The aggregation operator employed here is the weighted root power mean operator $\Psi_w^{(\alpha)}$.

$$\Psi_w^{(\alpha)}(\mu_1, \mu_2, \mu_3, \dots, \mu_K) = \left(\sum_{i=1}^K w_i \mu_i^\alpha \right)^{1/\alpha}, \quad 0 < |\alpha| < \infty, \quad (3)$$

3. FUZZY MIXED INTEGER GOAL PROGRAMMING MODEL

Generally the channel allocation problem is considered to be a complex multi-objective decision making problem. Each objective involve in the problem may have an acceptable range of ideal value with different type of achievement levels. Therefore, the channel allocation problem is often modelled as a Fuzzy goal programming (FGP) problem. In this paper we have used the FGP model for channel allocation problem given by Tsai et al (2008).

3.1 Model Formulation

In In this problem it is assumed that the objective functions and constraints are linear and fuzziness exists in each objective. Price, unit cost, and Demand of the product and other relative parameters are all constant and known with certainty. Quantity discount is not considered. And it also assumed that only one kind of product is involved in this problem.

Before introducing the model formulation, following notations are described:

Z_i = Desirable achievement value for the i th fuzzy goal

p_i = lower bound of total distribution rating value

x_i = allocation quantity for channel i

c_i = unit cost of products sold to channel i

m_i = rate of end user claims that can be attributed to channel i

d_i = rate of late lading of channel i

D = aggregate demand of the product over a fixed planning period

S = aggregate supply of the product over a fixed planning period

P = lower bound of total distribution rating value

B_i = sales target allocated to each channel

U_i = upper bound of the distribution capacity available in channel i

F = lower bound of total flexibility in channel quota allocation

In the Fuzzy mixed integer goal programming model the three objectives will have trade-off values in the final decision. So the channel allocation problem has been formulated as a fuzzy mixed integer goal programming model (FMIGP) as follows:

Objectives

$$\text{Maximize } \sum_{i=1}^n (P_i x_i - c_i x_i) \cong \tilde{Z}_1 \quad (4)$$

$$\text{Minimize } \sum_{i=1}^n m_i x_i \cong \tilde{Z}_2 \quad (5)$$

$$\text{Minimize } \sum_{i=1}^n d_i x_i \cong \tilde{Z}_3 \quad (6)$$

Constraints

$$\sum_{i=1}^n x_i = D \quad (\text{When } D < S) \quad \text{or} \quad \sum_{i=1}^n x_i = S \quad (\text{When } D > S) \quad (7)$$

$$x_i \leq U_i \quad \text{for } i = 1, 2, 3 \quad (8)$$

$$\sum_{i=1}^n f_i x_i \geq F \quad (9)$$

$$\sum_{i=1}^n r_i x_i \geq P \quad (10)$$

$$P_i x_i \geq B_i, \text{ for } i = 1, 2, 3 \quad (11)$$

$$x_i \geq 0 \quad \text{and are integer for } i = 1, 2, 3$$

In the above formulation (4), (5) and (6) represents the objective of maximizing the profit of the allocation, minimizing the rate of end users claims and minimizing the rate of late lading. Constraint (7) ensures the overall demand of products, constraint (8) defines the maximum capacity of each channel and constraint (9) represents the flexibility of the channel quota allocation. The total compatibility rating is defined by equation (10), the sales target of each channel is described in (11). It is also assumed that all allocation quantities are nonnegative.

Symbol ‘ \cong ’ indicates the fuzziness of the goal in above formulation. It indicates that the objective value will be in the vicinity of the ideal level \tilde{Z}_i within the deviations signified by tolerance limit. The constraints of the model are assumed to be crisp. L_i & U_i are assumed as lower bound and upper bound on the objective function.

3.2 Single objective models Generation

Applying here the fuzzy goal programming from section 2, the mixed integer fuzzy multiobjective programming model presented above can be converted to mixed integer single objective programming model as follows:

Objective

$$\text{Maximize } \Psi_w^{(\alpha)}(\mu_1, \mu_2, \mu_3, \dots, \mu_K) = \left(\sum_{i=1}^K w_i \mu_i^\alpha \right)^{1/\alpha}, \quad 0 < |\alpha| < \infty,$$

Subject to

$$\mu_i \leq \mu_i'$$

$$\sum_{i=1}^n x_i \leq D \quad \text{or} \quad \sum_{i=1}^n x_i \leq S$$

$$x_i \leq U_i, \quad i = 1, 2, 3$$

$$\sum_{i=1}^n f_i x_i \geq F$$

$$\sum_{i=1}^n r_i x_i \geq P$$

$$P_i x_i \leq B_i, \quad i = 1, 2, 3$$

$$\tilde{Z}_1 \geq z_i, \quad i = 1, 2, 3$$

It is already proved in Li and Lai (2008) that this operator covers a wide range of aggregation operator used in multi criteria decision making, some of them are as follows:

1. For $\alpha = 1$, the single objective of proposed model converges into the weighted arithmetic mean aggregating operator as follows.

$$\text{Maximize } \Psi_w^{(\alpha)}(\mu_1, \mu_2, \mu_3, \dots, \mu_K) = \sum_{i=1}^K w_i \mu_i$$

$$\mu_i \leq \mu_i'$$

$$\sum_{i=1}^n x_i \leq D \quad \text{or} \quad \sum_{i=1}^n x_i \leq S$$

$$x_i \leq U_i, \quad i = 1, 2, 3$$

$$\sum_{i=1}^n f_i x_i \geq F$$

$$\sum_{i=1}^n r_i x_i \geq P$$

$$P_i x_i \leq B_i, \quad i = 1, 2, 3$$

$$\tilde{Z}_1 \geq z_i, \quad i = 1, 2, 3$$

2. For $\alpha = 2$, the single objective of proposed model converges into the weighted quadratic mean aggregating operator as follows.

$$\text{Maximize } \Psi_w^{(\alpha)}(\mu_1, \mu_2, \mu_3, \dots, \mu_K) = \left(\sum_{i=1}^K w_i \mu_i^2 \right)^{1/2}$$

$$\mu_i \leq \mu_i'$$

$$\sum_{i=1}^n x_i \leq D \quad \text{or} \quad \sum_{i=1}^n x_i \leq S$$

$$x_i \leq U_i, \quad i = 1, 2, 3$$

$$\sum_{i=1}^n f_i x_i \geq F$$

$$\sum_{i=1}^n r_i x_i \geq P$$

$$P_i x_i \leq B_i, \quad i = 1, 2, 3$$

$$\tilde{Z}_1 \geq z_i, \quad i = 1, 2, 3$$

3. For $\alpha = -\infty$, the single objective of proposed model converges into the conjunctive mean aggregating operator as follows:

$$\text{Maximize } \Psi_w^{(\alpha)}(\mu_1, \mu_2, \mu_3, \dots, \mu_K) = \min_{1 \leq i \leq K} \mu_i \quad (w_1 = w_2 = \dots = w_k = 1 / K)$$

$$\mu_i \leq \mu_i'$$

$$\sum_{i=1}^n x_i \leq D \text{ or } \sum_{i=1}^n x_i \leq S$$

$$x_i \leq U_i, \quad i = 1, 2, 3$$

$$\sum_{i=1}^n f_i x_i \geq F$$

$$\sum_{i=1}^n r_i x_i \geq P$$

$$P_i x_i \leq B_i, \quad i = 1, 2, 3$$

$$\tilde{Z}_1 \geq z_i, \quad i = 1, 2, 3$$

On solving these models we get a varying set of solution for the channel allocation problem. Priorities of decision maker can also be defined and a comparison can be made for these methods.

4. NUMERICAL RESULTS

4.1. Case study description

To verify the suitability of proposed model the data has been taken from Tsai et al (2008), presenting a real life situation of a steel company from Taiwan. Different channels have their unique market coverage and purchasing power. The channel profiles shown in Table 1 represent the data set for the profit ($p_i - c_i$), the rate of end user claims (m_i), the rate of late lading (d_i), channel capacities (U_i), channel quota flexibility on allocation (f_i) on a scale of 0 to 1, channel compatibility rating (r_i) on a scale of 0 to 1, and the sales target for the channels (B_i). Based on company policy, it is assumed that the lowest value on the flexibility of channel quota allocation is $F = fD$, and the lowest value of the compatibility rating is $P = rD$. For the aggregate demand of $D = 10$ million tons in this case, we can have $F = 10 \times 0.03 = 300$ thousand tons, and $P = 10 \times 0.96 = 9.6$ million tons, respectively.

Table 1: Data set from a steel company of Taiwan

| Channel Type | p_i (NTD) | c_i (NTD) | m_i (%) | d_i (%) | U_i (10,000 tons) | f_i | r_i | B_i (10,000 NTD) |
|----------------------------|----------------|----------------|-----------|-----------|------------------------|-------|-------|-----------------------|
| Direct Channels | 16000 | 10000 | 0.005 | 0.02 | 400 | 0.04 | 0.97 | 4000000 |
| Stockholders | 16500 | 10200 | 0.008 | 0.035 | 600 | 0.02 | 0.98 | 9000000 |
| Independent Intermediaries | 15800 | 10500 | 0.006 | 0.04 | 200 | 0.06 | 0.96 | 2000000 |

Here, we aim to present different solutions for different priorities towards objectives using model defined in section 3.

4.2 Formulation

For the above data we will have to obtain the marginal evaluation of solution for single objective. For each particular objective we assign two values L_i & U_i as its lower and upper bound respectively. For all the objectives, Ideal solutions (best solutions) are attained by solving each of them separately ignoring other objective with the same set of constraints. The tolerance limit of each of the fuzzy goal is assumed to be 5 billion NTD, 20 thousand tons and 100 thousand tons, respectively.

The problem is to determine a global evaluation of solution for all objectives. The aggregating operator here we are employing is weighted root power mean operator as follows:

$$\Psi_\alpha^w = \left(\sum_{i=1}^3 w_i \mu_i^\alpha \right)^{1/\alpha} \quad \left(0 < |\alpha| < \infty \right), \quad \sum_{i=1}^3 w_i = 1,$$

Linear Membership Function

The membership functions can be defined as follows:

$$\mu_1' = \frac{Z_1(x_i) - 5591100}{6091100 - 5591100} \text{ or}$$

$$\mu_1' = \frac{Z_1(x_i) - 5591100}{500000}$$

$$\mu_2' = \frac{8.765 - Z_2(x_i)}{8.765 - 6.765} \text{ or}$$

$$\mu_2' = \frac{8.765 - Z_2(x_i)}{2}$$

$$\mu_3' = \frac{42.25 - Z_3(x_i)}{42.25 - 32.25} \text{ or}$$

$$\mu_3' = \frac{42.25 - Z_3(x_i)}{10}$$

The final crisp formulation of can be stated as:

Objective

$$\text{Maximize } \Psi_\alpha^w = (w_1\mu_1^\alpha + w_2\mu_2^\alpha + w_3\mu_3^\alpha)^{1/\alpha}$$

Subject to

$$\mu_1 \leq ((16000 - 10000)x_1 + (16500 - 10200)x_2 + (15800 - 10500)x_3 - 5591100) / 500000$$

$$\mu_2 \leq (8.765 - (0.005x_1 + 0.008x_2 + 0.006x_3)) / 2$$

$$\mu_3 \leq (42.25 - (0.02x_1 + 0.035x_2 + 0.04x_3)) / 10 \quad x_1 + x_2 + x_3 = 1000$$

$$x_1 \leq 400, \quad x_2 \leq 600, \quad x_3 \leq 200$$

$$0.04x_1 + 0.02x_2 + 0.06x_3 \geq 30$$

$$0.98x_1 + 0.97x_2 + 0.96x_3 \geq 960$$

$$16000x_1 \leq 4000000$$

$$16500x_2 \leq 9000000$$

$$15800x_3 \leq 2000000$$

$$0 \leq \mu_1 \leq 1$$

$$0 \leq \mu_2 \leq 1$$

$$0 \leq \mu_3 \leq 1$$

$$x_i \geq 0, \text{ and are integers for } i = 1, 2, 3$$

Applying the integrated fuzzy goal programming technique with weighted root power mean Ψ_α^w as an aggregation operator with linear membership function, following results are generated.

Table 2: Solution for $\alpha = 1$

| w_i | μ_i | x_i | Z_i |
|-----------------|---------|-------|---------|
| $w_1 = 0.33333$ | 0.9676 | 327 | 6074900 |
| $w_2 = 0.33333$ | 1.0000 | 546 | 6.76500 |
| $w_3 = 0.33333$ | 1.0000 | 127 | 30.7300 |
| $w_1 = 0.6$ | 1.0000 | 273 | 6091100 |
| $w_2 = 0.2$ | 0.919 | 600 | 6.92700 |
| $w_3 = 0.2$ | 1.0000 | 127 | 31.5400 |

Table 3: Solution for $\alpha = 2$

| w_i | μ_i | x_i | Z_i |
|-----------------|---------|-------|---------|
| $w_1 = 0.33333$ | 0.9676 | 327 | 6074900 |
| $w_2 = 0.33333$ | 1.0000 | 546 | 6.76500 |
| $w_3 = 0.33333$ | 1.0000 | 127 | 30.7300 |
| $w_1 = 0.6$ | 0.9676 | 327 | 6074900 |
| $w_2 = 0.2$ | 1.0000 | 546 | 6.76500 |
| $w_3 = 0.2$ | 1.0000 | 127 | 30.7300 |

Table 4: Solution for $\alpha = -\infty$

| w_i | μ_i | x_i | Z_i |
|-------|---------|-------|---------|
| ----- | 0.9676 | 312 | 6074900 |
| | | 561 | 6.8100 |
| | | 127 | 30.9550 |

Non Linear Membership Function

If we use parameter $S=1$ then exponential membership function defined in section 2 can be written as

$$\mu_1' = \frac{e^{-(5591100-Z_1)/500000} - e^{-1}}{1 - e^{-1}}$$

$$\mu_2' = \frac{e^{-(Z_2-8.765)/2} - e^{-1}}{1 - e^{-1}}$$

$$\mu_3' = \frac{e^{-(Z_3-42.25)/10} - e^{-1}}{1 - e^{-1}}$$

The final crisp formulation can be stated as:

Objective

$$\text{Maximize } \Psi_\alpha^w = (w_1\mu_1^\alpha + w_2\mu_2^\alpha + w_3\mu_3^\alpha)^{1/\alpha}$$

Subject to

$$\mu_1 = \frac{e^{-(5591100-Z_1)/500000} - e^{-1}}{1 - e^{-1}}$$

$$\mu_2 = \frac{e^{-(Z_2-8.765)/2} - e^{-1}}{1 - e^{-1}}$$

$$\mu_3 = \frac{e^{-(Z_3-42.25)/10} - e^{-1}}{1 - e^{-1}}$$

$$x_1 + x_2 + x_3 = 1000$$

$$x_1 \leq 400$$

$$x_2 \leq 600$$

$$x_3 \leq 200$$

$$0.04x_1 + 0.02x_2 + 0.06x_3 \geq 30$$

$$0.98x_1 + 0.97x_2 + 0.96x_3 \geq 960$$

$$16000x_1 \leq 4000000$$

$$16500x_2 \leq 9000000$$

$$15800x_3 \leq 2000000$$

$$0 \leq \mu_1 \leq 1$$

$$0 \leq \mu_2 \leq 1$$

$$0 \leq \mu_3 \leq 1$$

$x_i \geq 0$, and are integers for $i = 1, 2, 3$

Table 5: Solution for $\alpha = 1$

| w_i | μ_i | x_i | Z_i |
|-----------------|---------|-------|---------|
| $w_1 = 0.33333$ | 1 | 273 | 6091100 |
| $w_2 = 0.33333$ | 1 | 600 | 6.927 |
| $w_3 = 0.33333$ | 1 | 127 | 31.54 |
| $w_1 = 0.6$ | 1 | 273 | 6091100 |
| $w_2 = 0.2$ | 1 | 600 | 6.927 |
| $w_3 = 0.2$ | 1 | 127 | 31.54 |

Table 6: Solution for $\alpha = 2$

| w_i | μ_i | x_i | Z_i |
|-----------------|---------|-------|---------|
| $w_1 = 0.33333$ | 1 | 273 | 6091100 |
| $w_2 = 0.33333$ | 1 | 600 | 6.927 |
| $w_3 = 0.33333$ | 1 | 127 | 31.54 |
| $w_1 = 0.6$ | 1 | 273 | 6091100 |
| $w_2 = 0.2$ | 1 | 600 | 6.927 |
| $w_3 = 0.2$ | 1 | 127 | 31.54 |

Table 7: Solution for $\alpha = -\infty$

| w_i | μ_i | x_i | Z_i |
|-------|---------|-------|---------|
| ----- | | 273 | 6091100 |
| | 1 | 600 | 6.927 |
| | | 127 | 31.54 |

Solutions are attained by proposed fuzzy goal programming approach using different membership function. For $\alpha = 1$ the weighted root power mean method converges into weighted sum method, for $\alpha = 2$ it converges into weighted quadratic method, for $\alpha = -\infty$ it converges into the conjunctive mean method. These approaches attain different solution for this problem and a variety of solution can be generated for channel allocation problem. It is observed that non linear membership function provides more satisfactory objective values than linear membership function.

5. CONCLUSION

In this paper we have discussed a priority based channel allocation problem having three different fuzzy goals. A fuzzy goal programming approach is also proposed by integrating weighted root power mean method of aggregation with linear and non linear membership function. Different single objective optimization model have been generated with a combination of membership function, in order to obtain the different set of solution. Different solutions are presented for different weights assigned to each goal for each generated model. It has been noted that the non linear membership function provides higher level of satisfaction for objective value than the linear membership function. The solution has a low rate of affection of weights on the taken example. In future, we plan to extend this fuzzy goal programming approach for the uncertain constraints in channel allocation problem.

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