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A Parametric Approach to Solve Bounded-Variable LFP by Converting into LP

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Abstract **—** In this paper, we have developed a new technique for solving bounded-variable linear fractional programming problems by converting into linear programming problems. In this technique, we have proposed to convert fractional objective function into linear objective function by making a relationship between the numerator and denominator with a parameter. A number of numerical examples are illustrated to demonstrate our technique. We have also developed a computer code by using a mathematical programming language AMPL and then present a comparison of our method with existing relevant methods.

Keywords **—** Linear Programming, Bounded variable, AMPL, Linear Fractional Programming, Objective Function.

1. INTRODUCTION

Linear fractional programming (LFP) with bounded-variables is a special kind of mathematical programming. It consists of an objective function which is the ratio of two linear functions and some linear constraints with bounded variables. From last three decades LFP problems are getting lots of attention due to its importance in modeling various decision processes in economics, management science, numerical analysis, stochastic programming, and decomposition algorithms (Stanchu-Minasian, 1997).

Hungarian mathematician Bela Martos (1964) first formulated LFP. Mesiter and Oettli (1967), Aggrawal and Sharma (1970) applied the idea of fractional programming to calculate the maximum transmission rate in an information channel. Bereanu (1964) applied the idea of LFP in stochastic programming problem. Under certain economic assumptions, Ziemba, Brooks-Hill and Parkance (1974) developed a fractional programming model for an investment portfolio.

There are many existing techniques for solving LFP problems. A. Charnes and W. W. Cooper (1973) developed a transformation technique for solving LFP problems by converting it into single linear programming problems (LP). W. Dinkelbach (Bajalinov, 2003) developed a parametric approach to solve LFP. Bitran and Novaes (1972) developed a method called updated objective function method to solve LFP by solving a sequence of linear programs only re-computing the local gradient of the objective function. Hasan and Acharjee (2011) proposed a technique for solving LFP by converting into LP. Das and Hasan (2012) developed a technique for solving bounded-variable LFP problems. Das, Hasan and Islam (2013) proposed another technique to solve bounded-variable LFP problems by converting into LP problems.

Bounded-variable LFP problems are more difficult to solve than LFP. In this paper, we develop a new technique for solving bounded-variable LFP. We use the idea of Dinkelbach's parametric transformation for solving LFP and then use the idea of the bounded valued simplex algorithm for solving LP problems to develop our technique. We use a mathematical programming language AMPL to develop a computer code according to our algorithm. There are many existing computer techniques that are developed by using MATHEMATICA. But our developed computer technique is easier to use and run. It is less time-consuming than other existing methods. To show that in this paper, we have made a comparison between other existing methods and our method.

The rest of the paper, we have organized as follows. In section 2, we discuss definition of LFP, bounded-variable LFP and a relation between LP, LFP and bounded-variable LFP. In section 3, we discuss briefly about some existing techniques. In section 4, we present our proposed method. In section 5, we present a computer code that we have developed by using a mathematical programming language AMPL. In section 6, we present some numerical examples and solve these by using our method. In section 7, we show a parallel representation of manual output and the AMPL output of the numerical examples. In section 8, we present a comparison between our method and existing methods. Finally, in section 9, we have drawn a conclusion about our work.

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2. SOME PREREQUISITE

In the current section, we present some basic definitions relevant to our work. First, we discuss about linear fractional programming (LFP). Then bounded-variable LFP and finally discuss a relation between linear programming (LP), LFP and bounded-variable LFP. We have discussed these as follows.

2.1 Linear Fractional Programming (LFP)

Problems of LFP arise when it becomes important to optimize the efficiency of some activities. Mathematically LFP problems can be defined as follows (Stanchu-Minasian, 1997).

$$\text{Maximize (or Minimize)} \quad z = \frac{\sum_{j=1}^{n} p_j x_j + \alpha}{\sum_{j=1}^{n} d_j x_j + \beta} \tag{1}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j (\leq, =, \geq) b_i, i = 1, \ldots\ldots, m \tag{2}$$

$$x_j \geq 0, j = 1, \ldots\ldots, n \tag{3}$$

Where $x_j, p_j, d_j \in \Re^n$, $a_{ij} \in \Re^m \times \Re^n$ is an $m \times n$ matrix $b_i \in \Re^m$ and $\alpha, \beta \in \Re$. It is assumed that $\sum_{j=1}^{n} d_j x_j + \beta \neq 0$. Equation (1) is representing the objective function which has to be maximized or minimized. It is the ratio of two linear functions. Equation (2) is representing the linear constraints and equation (3) is representing non-negative restrictions.

In the next section, we have discussed about bounded-variable linear fractional programming problem.

2.2 Bounded-variable LFP

Bounded-variable LFP is a special kind of LFP. Here decision variables are bounded above and below. Mathematically bounded-variable LFP is being formulated as follows (Stanchu-Minasian, 1997).

$$\text{Maximize (or Minimize)} \quad z = \frac{\sum_{j=1}^{n} p_j x_j + \alpha}{\sum_{j=1}^{n} d_j x_j + \beta} \tag{4}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j (\leq, =, \geq) b_i, i = 1, \ldots\ldots, m \tag{5}$$

$$l_j \leq x_j \leq u_j, j = 1, \ldots\ldots, n \tag{6}$$

From equation (6), we observe that decision variables x_j are bounded above by u_j and below by l_j.

LFP and bounded-variable LFP problems have a close relation to LP problems. Bounded-variable LFP problems can be converted into LP problems. In the next section, we have discussed briefly the relationship between LFP and bounded-variable LFP with LP.

2.3 Relation Between LP, LFP and Bounded-variable LFP

LFP problems and bounded-variable LFP problems can be solved by converting into LP problems. In the current section, we have discussed some conversion procedures of LFP and bounded-variable LFP into LP (Bajalinov, 2003) as follows.

Case-1: Consider the objective function presented in equation (1) or (4). If $d_j = 0, \beta = 1$ in either of these equations then the fractional objective function will be converted into a linear objective function i.e. equation (1) and (4) will be converted into a linear function as $z = \sum_{j=1}^{n} p_j x_j + \alpha$ (7)

Case-2: If $d_j = 0, \beta \neq 1$ then equation (1) and (4) will be converted into linear objective function as follows.

$$z = \sum_{j=1}^{n} \frac{p_j}{\beta} x_j + \frac{\alpha}{\beta} \tag{8}$$

Case-3: If $p_j = 0$ then equation (1) and (4) will become $z = \dfrac{\alpha}{\sum_{j=1}^{n} d_j x_j + \beta}$. Here $\sum_{j=1}^{n} d_j x_j + \beta$ is a linear function. Therefore equation (1) and (4) can be converted into linear function in this way.

In the next section, we have discussed some solution procedures of LFP and bounded-variable LFP problems.

3. LITERATURE REVIEW

There are many existing techniques to solve bounded-variable LFP and LFP problems. In the current section, we discuss about some existing techniques. We discuss about Das and Hasan's method, Charnes and Cooper's method, Hasan and Acharjee's method, Bitran and Novae's method and Das, Hasan, Islam's method briefly.

3.1 Das and Hasan's Method

H. K. Das and M. Babul hasan (2012) developed a technique for solving bounded-variable LFP problems. They extended the idea of bounded-variable simplex algorithm for solving LP problems to solve bounded-variable LFP problems. They also developed a computer code by using MATHEMATICA according to their algorithm. Their technique has been discussed briefly as follows.

Step-1: Converted bounded-variable LFP problem into standard form.

Step-2: Then performed $\Delta_j = z^2(p_j - z_j^1) - z^1(d_j - z_2^j)$ where $z^1 = \sum_{j=1}^{n} p_j x_j + \alpha$, $z^2 = \sum_{j=1}^{n} d_j x_j + \beta$ until $\Delta_j \leq 0$.

Step-3: Applied idea of bounded variable simplex algorithm.

Their technique is very similar to Swarup's method (1964) for solving LFP problems.

3.2 Charnes and Cooper's Method

Charnes and Cooper (1973) developed this method. They developed this technique for solving LFP problems by converting into LP problems. Their technique has been discussed briefly as follows.

Step-1: Considered the transformation $y = tx$ where $t \geq 0$

Step-2: Then converted LFP into two different LP problems.

Step-3: Optimal solution of LFP will be found if two of these LP problems have an optimal solution. If one of them is inconsistent or unbounded then LFP will have no optimal solution.

Demerits

- This method is only applicable for solving LFP problems not for bounded variable LFP problems.
- This technique is also very time-consuming because one has to solve two different LP problems.

3.3 Hasan and Acharjee's Method

M. Babul Hasan and Sumi Acharjee (2011) developed this technique. They proposed a transformation system to convert LFP into LP. We have discussed their technique as follows.

Step-1: In equation (1), denominator function must have to be positive *i.e.* $\sum_{j=1}^{n} d_j x_j + \beta > 0$.

Step-2: They considered $t = p_j - d_j g$, $y_j = \dfrac{x_j}{d_j x_j + \beta}$ and $g = \dfrac{\alpha}{\beta}$.

Step-3: Then the objective function of the LFP becomes $z = py + g$. They calculated variable values of LFP by $x_j = \dfrac{\beta y_j}{1 - d_j y_j}$.

Demerits

- Like Charnes and Coopers method, this technique is applicable for solving LFP problems only.
- Although this technique is very interesting but it is laborious and time consuming because one has to do a lots of calculations to convert LFP problem into single LP problem.

3.4 Bitran and Novae's Method

In this section, we have summarized Bitran and Novae's method. In (Bitran, 1972), they developed this. Assuming that the constraints set is nonempty and bounded and the denominator $\sum_{j=1}^{n} d_j x_j + \beta > 0$ for all feasible solutions, the authors proceeds as follows.

(i) Convert the LFP into a sequence of LP.
(ii) Then solve these LPs until two of them give identical solution.

3.5 Das, Hasan and Islam

Das, Hasan and Islam (2013) proposed a technique to solve bounded-variable LFP by converting into LP problem. They used Hasn and Acharjee's (2011) transformation idea to convert LFP into LP problem. Then they used idea of bounded-variable simplex method for solving LP problems. They developed a computer code by using MATHEMATICA. They demonstrated their technique by illustrating a number of numerical examples.

3.6 Dinkelbatch's Method

This is the most popular technique for solving LFP problems (Bajalinov, 2003). In this section, we discuss this method briefly as follows.

Step-1: Convert objective function (1) into a linear form as $z = (\sum_{j=1}^{n} p_j x_j + \alpha) - t(\sum_{j=1}^{n} q_j x_j + \beta)$

Step-2: Solve it by using the simplex algorithm.

Step-3: Solve by using simplex algorithm until the modified objective function gives zero value.

In the next section, we present algorithm of our proposed method. We have used the idea of Dinkelbatch's technique and the bounded-variable simplex algorithm's (Hillier, 2001) idea to develop this method.

4. PROPOSED METHOD

In this section, we have presented our proposed method. Consider the bounded-variable LFP presented in Section 2.2. We first convert the objective function into a linear form. Then substitute those variables whose lower bounds are not zero by using another variable. Then convert the whole problem into standard form. Then apply the idea of bounded-variable simplex method. We have discussed our method in the following steps.

Step-1: Convert the objective function presented in equation (4) into linear function as follows.

$$f^{(k)} = (\sum_{j=1}^{n} p_j x_j + \alpha) - t^{(k)}(\sum_{j=1}^{n} d_j x_j + \beta) \tag{9}$$

Where $-\infty \le t \le \infty$ and k is representing number of iterations.

Step-2: Consider $x_j = l_j + y_j$ in equation (6). Then the bounded-variable LFP problem will be converted into the following form where $g = \sum_{j=1}^{n} p_j l_j + \alpha - t^{(k)}(\sum_{j=1}^{n} d_j l_j + \beta)$, $\gamma_j = b_i - \sum_{j=1}^{n} a_{ij} l_j$, and $v_j = u_j - l_j$.

$$\text{Maximize (or Minimize) } f^{(k)} = \sum_{j=1}^{n} p_j y_j - t^{(k)} \sum_{j=1}^{n} d_j y_j + g \tag{10}$$

subject to

$$\sum_{j=1}^{n} a_{ij} y_j (\le, =, \ge) \gamma_i, i = 1, \ldots\ldots, m \tag{11}$$

$$0 \le y_j \le v_j, j = 1, \ldots\ldots, n \tag{12}$$

Step-3: Consider $t^{(k)} = \dfrac{\sum_{j=1}^{n} p_j (l_j + y_j^{(k)}) + \alpha}{\sum_{j=1}^{n} d_j (l_j + y_j^{(k)}) + \beta}$ and choose $y_j^{(k)} = 0$ initially *i.e.* when $k = 1$.

Step-4: Convert the problem into standard form presented by equations (10), (11), and (12).

Step-5: If any variable is at a positive lower bound then it should be substituted at its lower bound. Then apply idea of simplex method.

Step-6: Let $y_j^{(k)}$ be a non basic variable at zero level which is selected to enter the basic. Compute the following quantities. $\theta_1 = \min\left\{\frac{(y_B^*)_i}{a_{ij}}, a_{ij} > 0\right\}$, $\theta_2 = \min\left\{\frac{u_{i-}(y_B^*)_i}{-a_{ij}}, a_{ij} < 0\right\}$, and $\theta = \min(\theta_1, \theta_2, u_j)$. Here $(y_B^*)_i$ are basic variables.

Step-7: Now we have to do any one of the following three alternatives.

- If $\theta = \theta_1$ then $(y_B)_r$ leaves the solution and $y_j^{(k)}$ enters by using the regular row operation of simplex method.
- If $\theta = \theta_2$ then $(y_B)_r$ leaves the solution and $y_j^{(k)}$ enters. Then $(y_B)_r$ being non-basic at its upper bound and must be substituted by $(y_B)_r = u_r - (y_B)_r^{'}, 0 \le (y_B)_r^{'} \le u_r$.
- If $\theta = u_j$ then y_j is substituted at its upper bound difference $u_j - y_j^{'}$, while remaining non-basic.

Step-8: If $f^{(k)} = 0$ and all $\Delta_j \le 0$ then stop. Otherwise repeat steps 1 to 7. Here $E_j = \sum p_B a_{ij}$ and $\Delta_j = p_j - E_j$.

5. COMPUTER CODE

In the current section, we present a computer code. We develop this by using a mathematical programming language AMPL. Our code consists of three different parts. These are AMPL model file, AMPL data file and AMPL run file. We present AMPL model file below.

```
#----------------------------------------------------

#AMPL Model File

#----------------------------------------------------

param n;                                  # no of variables

param m;                                  # no of rows

param alpha;                              # numerator constant

param beta;                               # denominator constant

param lembda;                             # a parameter

param c{i in 1..n};                       # numerator's coefficients

param d{i in 1..n};                       #denominator's coefficients

param a{i in 1..m, j in 1..n};            # constraint's coefficients

param b{i in 1..m};                       # r.h.s constants

param lw{i in 1..n};                      # lower bounds

param up{i in 1..n};                      # upper bounds

var x{i in 1..n}>=0;                      # no of variables

maximize obv: sum{i in 1..n} (c[i]*x[i])+alpha-lembda*(sum{i in 1..n}(d[i]*x[i])+beta);

subject to cost{j in 1..m}: sum{i in 1..n} a[j,i]*x[i]<=b[j];

subject to limit{i in 1..n}: lw[i]<=x[i]<=up[i];

#--------------------------------------------------------------------------------
```

Due to the large volume of whole computer code we have presented AMPL model file here. If readers are interested then please contact with the authors.

6. NUMERICAL EXAMPLES

In this section, we present some numerical examples and apply our proposed method to solve these.

6.1 Numerical Example 1: This Example has taken from Erik B. Bajalinov (2003).

Maximize
$$z=\frac{x_1+3x_2+6}{2x_1+3x_2+12} \tag{13}$$
subject to
$$x_1+2x_2\ge 10 \tag{14}$$
$$2x_1+3x_2\le 60 \tag{15}$$
$$5\le x_1\le 15 \tag{16}$$
$$4\le x_2\le 30 \tag{17}$$

Solution: Suppose, $x_1=5+y_1$ and $x_2=4+y_2$ where $y_1\ge 0, y_2\ge 0$. Substituting these into the equations (13) to (17) and converting the objective function into linear form and converting the problem into standard form we obtain the followings.

Maximize
$$f^{(k)}=(y_1+3y_2+23)-t^{(k)}(2y_1+3y_2+34) \tag{18}$$
subject to
$$-y_1-2y_2+s_1=3 \tag{19}$$
$$2y_1+3y_2+s_2=38 \tag{20}$$
$$0\le y_1\le 10 \tag{21}$$
$$0\le y_2\le 26 \tag{22}$$

Here $s_1, s_2\ge 0$ are slack variables and $t^{(k)}=\dfrac{y_1^{(k)}+3y_2^{(k)}+23}{2y_1^{(k)}+3y_2^{(k)}+34}$. Initially let $y_i^{(1)}=0, i=1,2$. Now we have $t^{(1)}=\dfrac{0+3\times 0+23}{2\times 0+3\times 0+34}=\dfrac{23}{34}$ for the next iteration. Then equation (18) will be converted into the following form.

$$f^{(1)}=\frac{-6}{17}y_1+\frac{33}{34}y_2 \tag{23}$$

Solution steps for this problem have been presented into the following tables.

Table 1 : Initial Table for example 1

p_B ↓	$p_j\rightarrow$	$\frac{-6}{17}$	$\frac{33}{34}$	0	0			
	Basis ↓	y_1	y_2	s_1	s_2	y_B^*	θ_1	θ_2
0	s_1	-1	-2	1	0	3	-	∞
0	s_2	2	③	0	1	38	← $\frac{38}{3}$	∞
$E_j=\sum p_B a_{ij}$		0	0	0	0			
$\Delta_j=p_j-E_j$		$\frac{-6}{17}$	$\frac{33}{34}$ ↑	0	0	$f^{(1)}=0$		

Discussion: From Table-1, we observe that maximum value of Δ_j has obtained at fourth column. Therefore y_2 is the entering variable into the solution. It has been indicated by a vertical arrow in the table. Here $\theta_1=\frac{38}{3}, \theta_2=\infty$, and

$\theta = \min\{\theta_1, \theta_2, u_2\} = \min\{\frac{38}{3}, \infty, 26\} = \frac{38}{3}$. Therefore, s_2 will leave basis and y_2 will enter into that place. Although $f^{(1)} = 0$ here but since not all Δ_j are negative so we have to go for the next iteration. In the next table, we have presented the optimal solution.

Table 2 : Optimum table for example 1

$p_B \downarrow$	$p_j \rightarrow$	$\frac{-25}{36}$	$\frac{11}{24}$	0	0	
	Basis ↓	y_1	y_2	s_1	s_2	y_B^*
0	s_1	$\frac{1}{3}$	0	1	$\frac{2}{3}$	$\frac{85}{3}$
$\frac{11}{24}$	y_2	$\frac{2}{3}$	1	0	$\frac{1}{3}$	$\frac{38}{3}$
$E_j = \sum p_B a_{ij}$		$\frac{11}{36}$	0	0	$\frac{11}{72}$	
$\Delta_j = p_j - E_j$		-1	0	0	$-\frac{11}{72}$	$F(y) = \frac{209}{36}$

Discussion: Here $f^{(4)} = -\frac{25}{36}y_1 + \frac{11}{24}y_2 - \frac{209}{36} = F(y) - \frac{209}{36}$. From Table-2, we see that all Δ_j are negative and $f^{(4)} = \frac{209}{36} - \frac{209}{36} = 0$. Therefore, optimal solution has obtained. Optimal solution is $y_1 = 0, y_2 = \frac{38}{3}$. Therefore, $x_1 = 5, x_2 = \frac{50}{3}, z_{\max} = \frac{61}{72}$.

6.2 Numerical Example 2: This Example has taken from Erik B. Bajalinov (2003).

$$\text{Maximize} \quad z = \frac{5x_1 + x_2 + 10}{4x_1 + 2x_2 + 11} \tag{24}$$

subject to

$$5x_1 + x_2 + x_3 = 20 \tag{25}$$

$$4x_1 - x_2 + x_4 = 14 \tag{26}$$

$$2 \le x_1 \le 5 \tag{27}$$

$$4 \le x_2 \le 12 \tag{28}$$

$$0 \le x_3 \le 25 \tag{29}$$

$$0 \le x_4 \le 18 \tag{30}$$

Solution: Consider, $x_1 = 2 + y_1$ and $x_2 = 4 + y_2$ where $y_1 \ge 0, y_2 \ge 0$. Substituting these into the equations (24) to (30) and converting the objective function into linear form and converting the whole problem into standard form we obtain the following expressions.

$$\text{Maximize} \quad f^{(k)} = (5y_1 + y_2 + 24) - t^{(k)}(4y_1 + 2y_2 + 27) \tag{31}$$

subject to

$$5y_1 + y_2 + x_3 = 6 \tag{32}$$

$$4y_1 - y_2 + x_4 = 10 \tag{33}$$

$$0 \le y_1 \le 3 \tag{34}$$

$$0 \le y_2 \le 8 \tag{35}$$

$$0 \le x_3 \le 25 \tag{36}$$

$$0 \le x_4 \le 18 \tag{37}$$

Here $t^{(k)} = \dfrac{5y_1^{(k)} + y_2^{(k)} + 24}{4y_1^{(k)} + 2y_2^{(k)} + 27}$ and choose $y_i^{(1)} = 0, i = 1,2$ initially. Then equation (31) will be, $f^{(k)} = \frac{13}{9}y_1 - \frac{7}{9}y_2$. We have presented the initial calculations for this problem into the next table.

Table 3 : Initial Table for example 2

p_B $\downarrow$	$p_j \to$	$\frac{13}{9}$	$-\frac{7}{9}$	0	0			
	Basis $\downarrow$	y_1	y_2	x_3	x_4	y_B^*	θ_1	θ_2
0	x_3	$\textcircled{5}$	1	1	0	6	$\longleftarrow$ $\frac{6}{5}$	∞
0	x_4	4	-1	0	1	10	$\frac{5}{2}$	∞
$E_j = \sum p_B a_{ij}$		0	0	0	0	$f^{(1)} = 0$		
$\Delta_j = p_j - E_j$		$\frac{13}{7}\uparrow$	$-\frac{7}{9}$	0	0			

Discussion: From Table-3, we observe that y_1 is the entering variable into the basis and x_3 is the departing variable from the basis. Since not all Δ_j are negative so we have to go into the next iteration. We have presented optimum solution into the next table.

Table 4 : Optimum Table for example 2

p_B $\downarrow$	$p_j \to$	$\frac{65}{53}$	$-\frac{47}{53}$	0	0	
	Basis $\downarrow$	y_1	y_2	x_3	x_4	y_B^*
$\frac{65}{53}$	y_1	1	$\frac{1}{5}$	$\frac{1}{5}$	0	$\frac{6}{5}$
0	x_4	0	$-\frac{9}{5}$	$-\frac{4}{5}$	1	$\frac{26}{5}$
$E_j = \sum p_B a_{ij}$		$\frac{65}{53}$	$\frac{13}{53}$	$\frac{13}{53}$	0	$F(y) = \frac{73}{53}$
$\Delta_j = p_j - E_j$		0	$-\frac{60}{53}$	$-\frac{13}{53}$	0	

Discussion: Here $f^{(4)} = \frac{65}{53}y_1 - \frac{47}{53}y_2 - \frac{73}{53} = F(y) - \frac{73}{53}$. From Table-4, we see that all Δ_j are negative or equals to zero and $f^{(4)} = \frac{73}{53} - \frac{73}{53} = 0$. Therefore, optimal solution has obtained. Therefore $y_1 = \frac{6}{5}, y_2 = 0, x_3 = 0, x_4 = \frac{26}{5}$ and optimal solution of the bounded-variable LFP problem is $x_1 = \frac{16}{5}, x_2 = 4, x_3 = 0, x_4 = \frac{26}{5}, z_{\max} = \frac{50}{53}$.

7. AMPL OUTPUT

In this section, we solve numerical examples presented in Sction-6.1, and 6.2 by using our developed AMPL code. In Table-5, we present manual output and AMPL output of numerical examples. We also present CPU time used by our AMPL code. We have used "_ampl_time" command to determine these times.

Table 5 : Manual and AMPL outputs

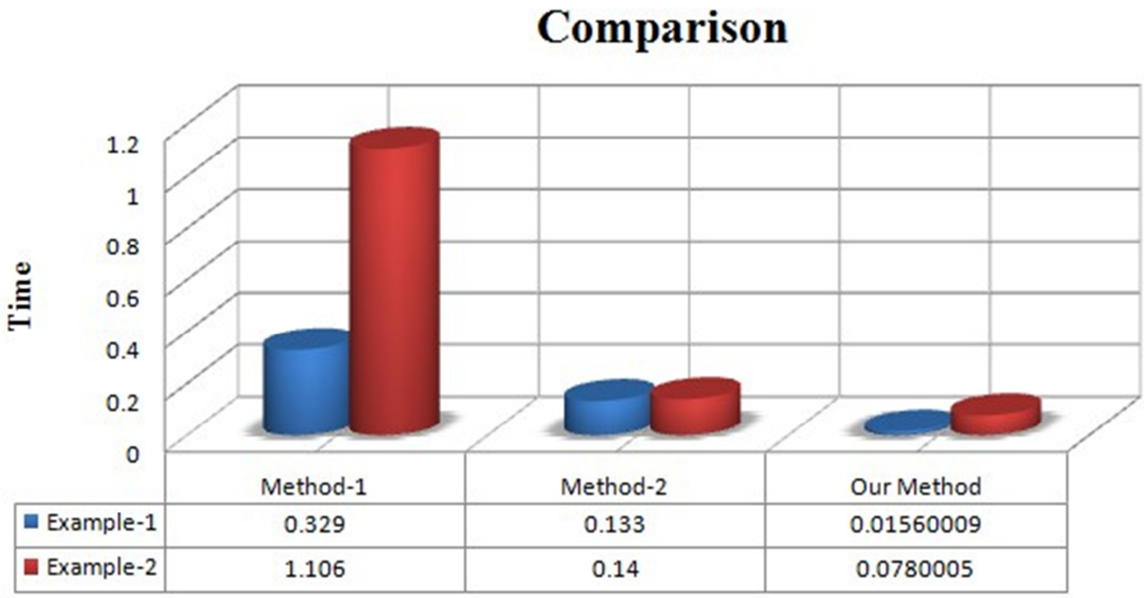
Numerical Examples	Manual Output	AMPL Output	Time Used (seconds)
Example 1	$Variables, x_1 = 5, x_2 = \frac{50}{3}$ $Objective, z_{\max} = \frac{61}{72}$	x [*] := 1 5 2 16.6667 $Objective, z_{\max} = 0.84722$	0.01560009
Example 2	$Variables, x_1 = \frac{16}{5}, x_2 = 4$ $x_3 = 0, x_4 = \frac{26}{5}$ $Objective, z_{\max} = \frac{50}{53}$	x [*] := 1 3.2 2 4 3 0 4 5.2 $Objective, z_{\max} = 0.94339$	0.0780005

Discussion: From parallel representation of manual output and AMPL output of examples, we observe that both the algorithm we developed and computer code we developed are giving same output. In AMPL output, some variable's value and objective function value have been presented in decimal form. For example, in Numerical example-2 fourth variable value is 5.2.

In the next section, we present a comparison between our method with other methods for solving bounded-variable LFP problems.

8. COMPARISON

In the current section, we make a comparison between our method with other two different methods for solving bounded-variable LFP problems. In Section 3.1, we present Das and Hasan's method (2012) and in Section 3.5 we present another method developed by Das, Hasan and Islam (2013). They used MATHEMATICA to develop their computer code and used "Timeused[ ]" command to determine CPU time. To find run time we use "_ampl_time_" command. We have used Intel(R) Pentium(R) Dual CPU B970 @2.30GHz~2.3GHz , Memory(RAM): 2.00GB, System type: 32-bit operating system. We develop a graphical comparison between these two methods and our developed method on the base of CPU time used by computer code. We present this comparison below.



	Method-1	Method-2	Our Method
🟦 Example-1	0.329	0.133	0.01560009
🟥 Example-2	1.106	0.14	0.0780005

Figure 1 : Comparison between existing methods and our method

In the above figure, we considered Das and Hasan's (2012) method as Method-1 and Das, Hasan and Islam's method (2013) as Method-2. We observe that Method-1 and Method-2 used 0.329 and 0.133 seconds respectively to solve example-1 and it takes 0.01560009 seconds to solve by our developed method. For example-2, Method-1 and Method-2 take 1.106 and 0.14 seconds respectively and 0.0780005 seconds by our method. Therefore we can conclude that our developed technique is easier and less time consuming than other techniques.

9. CONCLUSION

There are many existing techniques to solve bounded-variable LFP problems. In this paper, we developed a new technique to solve bounded-variable LFP. We used a parametric transformation to convert fractional objective function into linear form. We developed a computer code using a mathematical programming language AMPL. We solved numerical examples by our developed method. Then compared the results obtained by computer code. Finally, we made a comparison between our method with two other existing methods. Finally, found that our technique is easier and took less time than the other techniques.

REFERENCES

1. Aggarwal, S. P., Sharma, I. C. (1970). Maximization of the Transmission Rate of a Discrete, Constant Channel, *Unternehmensforschung*, 14, 152-155.
2. Bereanu, B. (1964). Programming De Risqué Minimal En Programmation Lineaire Stochastique, *C. R. Acad. Sci (Paris)*, 259 (5), 981-983.
3. Bajalinov, E. B. (2003). Linear Fractional Programming: Theory, Methods, Applications and Software, *Boston: Kluwer Academic Publishers*.
4. Bitran, G. R., and Novaes, A. G. (1972). Linear Programming with Fractional Objective Function, University of Sao Paulo, *Brazil*, 21, 22-29.
5. Charnes, A., Cooper, W.W. (1973). An Explicit General Solution in Linear Fractional Programming, *Naval Research Logistics Quarterly*, 20(3), 449-467.
6. Das, H. K., Hasan, M. B. (2012), A Proposed Technique for Solving Linear Fractional Bounded Variable Problems, *The Dhaka University Journal of Science*, 60(2), 223-230.
7. Das, H. K., Hasan, M. B., Islam, A. (2013). Solving Bounded Variable LFP Problems by Converting it into a Single LP Problem, *International Journal of Data Analysis and Information System*, 5 (2), 89-130
8. Hasan, M. B., Acharjee, S. (2011). Solving LFP by converting it into a Single LP, *International Journal of Operations Research*, 8(3), 1-14.
9. Hillier, F. S., Lieberman, G. J. (2001). Introduction to Operations Research, *MacGraw-Hill series in Industrial Engineering and Management Science*.
10. Martos, B. (1964). Hyperbolic Programming, *Naval Research and Logistics Quarterly*, 11, 135-155.

11. Meister, B., Oettli, W. (1967). On the Capacity of a Discrete, Constant Channel, *Information and Control*, 11(3), 341-351.
12. Stanchu-Minasian, I. M. (1997). Fractional Programming: Theory, Method, and Applications, *Kluwer Academic Publishers*.
13. Swarup, K. (1964). Linear Fractional Functional Programming, *Operations Research*, 13(6), 1029-1036.
14. Ziemba, W. T., Brooks-Hill, F. J., Parkance (1974). Calculations of Investment Portfolios with Risk Free Borrowing and Lending, *Management Science*, 21(2), 209-222.