International Journal of
Operations Research

International Journal of Operations Research Vol. 13, No. 2, 077–084 (2016)

Technical Note

On the robust optimization approach to closed-loop supply chain network design under uncertainty

Neda Makrooni, Maziar Salahi*

[1]Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Guilan, Rasht, Iran

Received June 2016; Revised June 2016; Accepted June 2016

Abstract — In this paper, we present a new formulation of the robust counterpart of the mixed integer programming model correspond to a closed loop supply chain problem given in [10]. Our formulation has significantly less constraints than the one in [10]. Finally, we compare the new model with the one in [10] for several examples showing that both of them give the same solution but the new one is faster.
Keywords —Closed loop supply chain, Robust optimization, Mixed integer program..

1. INTRODUCTION

Closed-loop supply chains are supply chain networks that include the returns processes and the manufacturer has the intent of capturing additional value and further integrating all supply chain activities [6]. It has attracted much attention in the last two decades both in industry and academia due to increased environmental concerns, government legislations and awareness of limited natural resource, see [2, 3, 7, 8, 9] and references therein. In this paper, we present an enhanced version of the uncertain model presented in [10] which is having much less constraints when demands, returns and transportation costs between facilities are considered as uncertain parameters. Moreover, on several examples it is shown that both models result to the same solution which also supports our theoretical foundation.

2. LITERATURE REVIEW

In this section we mention some research considering the concept of robust optimization in the closed loop supply chain, but in order to keep the paper shorter, the readers may see reference therein for further information. In the last two decades, the concept of robust optimization has been applied in many disciplines including supply chain due to its importance. In practice, supply chain models are characterized by different sources of technical and commercial uncertainties. Parameters such as customer demands, prices and facility capacities are quite uncertain. There are several studies which deal with uncertainty in supply chain management at different levels. In tactical level for example [14]. At the strategic level, there is a great deal of research in the facility location component of supply chain network design under uncertainty [5]. In [1] a multi-objective and robust optimization-based closed-loop supply chain design model is proposed, where in the objective function both expected total costs as well as carbon dioxide equivalents are minimized. In this study, customer demands and used-product return ratio are considered to be uncertain. In [11] a multi-product, multi-echelon, closed-loop logistic network model is considered in under uncertainty. Since these kind of networks are time consuming and costly project as well as a strategic and sensitive decision, thus the robust optimization approach is used to deal with uncertainty of demand and the return rate. In [12] a robust environmentally closed loop supply chain network is designed which includes multiple plants, collection centers, demand zones, and products, and consists of both forward and reverse supply chains. They have proposed a robust multi-objective mixed integer nonlinear programming model to deal with this network when cost parameters of the supply chain and demand fluctuations are subject to uncertainty. Most recently, in [13] the authors proposed a bi-objective optimization model for designing a closed loop supply chain network under uncertainty in which the total costs and the maximum waiting times in the

* Corresponding author's email: salahim@guilan.ac.ir (Maziar Salahi)

queue of products are considered to be minimized. Then they have introduced a new hybrid solution approach based on interval programming, stochastic programming, robust optimization approach, and fuzzy multi-objective programming. One may consult references in the above mentioned articles for further information on the applications of robust optimization in the context of closed loop supply chain.

3. NEW MODEL

In [10] the authors first have presented the following mixed integer linear program (MILP) formulation for a closed loop supply chain problem.

$$\begin{aligned}
\min \quad & \sum_{i\in I} f_i Y_i + \sum_{j\in J} g_j Z_j + \sum_{m\in M} h_m W_m + \sum_{k\in K}\sum_{i\in I} c_{ki} X_{ki} + \sum_{i\in I}\sum_{j\in J} a_{ij} U_{ij} + \sum_{j\in J}\sum_{m\in M} b_{jm} P_{jm} + \\
& \sum_{m\in M}\sum_{l\in L} e_{ml} Q_{ml} + \sum_{i\in I}\sum_{n\in N} v_{in} T_{in} + \sum_{l\in L} \pi_l \delta_l \\
\text{s. t.} \quad & \sum_{m\in M} Q_{ml} + \delta_l \ge d_l, \forall l \in L, \sum_{i\in I} X_{ki} = r_k, \forall k \in K, \\
& \sum_{j\in J} U_{ij} - (1-s)\sum_{k\in K} X_{ki} = 0, \forall i \in I, \sum_{n\in N} T_{in} - s\sum_{k\in K} X_{ki} = 0, \forall i \in I, \\
& \sum_{j\in J} P_{jm} - \sum_{l\in L} Q_{ml} = 0, \forall m \in M, \sum_{m\in M} P_{jm} - \sum_{i\in I} U_{ij} \le 0, \forall j \in J, \\
& \sum_{k\in K} X_{ki} \le Y_i cc_i, \forall i \in I, \sum_{i\in I} U_{ij} \le Z_j cr_j, \forall j \in J, \\
& \sum_{j\in J} P_{jm} \le W_m ce_m, \forall m \in M, \sum_{i\in I} T_{in} \le cd_n, \forall n \in N, \\
& Y_i, Z_j, W_m \in \{0,1\}, \forall i \in I, \forall j \in J, \forall m \in M, \\
& X_{ki}, U_{ij}, P_{jm}, Q_{ml}, T_{in}, \delta_l \ge 0, \forall i \in I, \forall j \in J, \forall k \in K, \forall l \in L, \forall m \in M,
\end{aligned} \tag{1}$$

where

Indices

- i Index of potential collection/inspection center locations $i = 1, 2, \cdots, I$
- j Index of potential recovery center locations $j = 1, 2, \cdots, J$
- m Index of potential redistribution center locations $m = 1, 2, \cdots, M$
- n Index of fixed points for disposal centers $n = 1, 2, \cdots, N$
- k Index of fixed locations of first market customer zones (returns) $k = 1, 2, \cdots, K$
- l Index of fixed locations of second market customer zones (demands) $l = 1, 2, \cdots, L$

Parameters

- d_l Demand of customer l for recovered products
- r_k Returns of used products from customer k
- s Average disposal fraction
- cc_i Capacity of handling returned products at collection/inspection i
- cr_j Capacity of handling recoverable products at recovery center j
- ce_m Capacity of handling recovered products at redistribution center m
- cd_n Capacity of handling scrapped products at disposal center n

Costs

- f_i Fixed cost of opening collection/inspection center i
- g_j Fixed cost of opening recovery center j
- h_m Fixed cost of opening redistribution center m

c_{ki} Shipping cost per unit of returned products from customer zone k to collection/inspection center i

a_{ij} Shipping cost per unit of recoverable products from collection/inspection center i to recovery center j

b_{jm} Shipping cost per unit of recovered products from recovery center j to redistribution center m

e_{ml} Shipping cost per unit of recovered products from redistribution center m to customer zone l

v_{in} Shipping cost per unit of scrapped products from collection/inspection center i to disposal center n

π_l Penalty cost per unit of non-satisfied demand of customer l

Variables

X_{ki} Quantity of returned products shipped from customer zone k to collection/inspection center i

U_{ij} Quantity of recoverable products shipped from collection/inspection center i to recovery center j

P_{jm} Quantity of recovered products shipped from recovery center j to redistribution center m

Q_{ml} Quantity of recovered products shipped from redistribution center m to customer zone l

T_{in} Quantity of scrapped products shipped from collection/inspection center i to disposal center n

δ_l Quantity of non-satisfied demand of customer l

$$Y_i = \begin{cases} 1 & \text{if a collection/inspection center is opened at location } i \\ 0 & \text{otherwise} \end{cases}$$

$$Z_j = \begin{cases} 1 & \text{if a recovery center is opened at location } j \\ 0 & \text{otherwise} \end{cases}$$

$$W_m = \begin{cases} 1 & \text{if a redistribution center is opened at location } m \\ 0 & \text{otherwise} \end{cases}$$

In the compact form, (1) is as follows

$$\begin{aligned} \min \;\; & fy + cx \\ s.\ t. \;\; & Ax \geq d, Hx = r, \\ & Nx = 0, Mx \leq 0, \\ & Bx \leq Cy \\ & y \in \{0,1\}, x \in \mathbb{R}^+ \end{aligned} \tag{2}$$

where vectors f, c, r and d correspond to fixed opening costs, transportation costs, returned products and demands, respectively. The matrices A, B, C, M and N are coefficient matrices of the constraints. Moreover, all binary decision variables are included into the vector y and all continuous decision variables are included into the vector x. Then they have presented the robust counterpart of it when demands (d), returns (r), and transportation costs (c) between facilities are considered as uncertain parameters and belong to the given boxes as below:

$$u_{Box} = \{\xi \in \mathbb{R}^n : |\xi_t - \overline{\xi}_t| \leq \rho G_t, t = 1, \ldots, n\}$$

where $\overline{\xi}_t$ is the nominal value of the ξ_t as t_{th} parameter of vector ξ (n-dimension vector) and the positive numbers G_t represent uncertainty scale and $\rho > 0$ is the uncertainty level. In order to write the robust counter part of (2) when $c \in u^c_{Box}, d \in u^d_{Box}, r \in u^r_{Box}$, they have used extreme points of the uncertain sets which lead to significantly larger MILP. In the sequel, we give an alternative equivalent form of the robust counterpart for (1) or (2) when uncertain parameters have exactly the same uncertainty sets as in [10] which is having much less constraints than the model in [10]. It should be noted that our proof for presenting the equivalent model is different than the one in [10] which leads to the improvement in the formulation. All other assumptions are as in [10].

Theorem 1. *The robust counterpart of (1) is equivalent to the following MILP.*

$$\begin{aligned}
\min \quad & z \\
\text{s. t.} \quad & \sum_{i\in I} f_i Y_i + \sum_{j\in J} g_j Z_j + \sum_{m\in M} h_m W_m + \sum_{k\in K}\sum_{i\in I}\left(\overline{c}_{ki} X_{ki} + \rho_c G^c_{ki} X_{ki}\right) + \sum_{i\in I}\sum_{j\in J}\left(\overline{a}_{ij} U_{ij} + \rho_a G^a_{ij} U_{ij}\right) + \\
& \sum_{j\in J}\sum_{m\in M}\left(\overline{b}_{jm} P_{jm} + \rho_b G^b_{jm} P_{jm}\right) + \sum_{m\in M}\sum_{l\in L}\left(\overline{e}_{ml} Q_{ml} + \rho_e G^e_{ml} Q_{ml}\right) + \sum_{i\in I}\sum_{n\in N}\left(\overline{\nu}_{in} T_{in} + \rho_\nu G^\nu_{in} T_{in}\right) + \\
& \sum_{l\in L} \pi_l \delta_l \le z \\
& \sum_{m\in M} Q_{ml} + \delta_l \ge \overline{d}_l + \rho_d G^d_l, \forall l \in L, \sum_{i\in I} X_{ki} \ge \overline{r}_k - \rho_r G^r_k, \forall k \in K \\
& \sum_{i\in I} X_{ki} \le \overline{r}_k + \rho_r G^r_k, \forall k \in K, \sum_{j\in J} U_{ij} - \left(1-s\right)\sum_{k\in K} X_{ki} = 0, \forall i \in I \\
& \sum_{n\in N} T_{in} - s\sum_{k\in K} X_{ki} = 0, \forall i \in I, \sum_{j\in J} P_{jm} - \sum_{l\in L} Q_{ml} = 0, \forall m \in M \\
& \sum_{m\in M} P_{jm} - \sum_{i\in I} U_{ij} \le 0, \forall j \in J, \sum_{k\in K} X_{ki} \le Y_i cc_i, \forall i \in I \\
& \sum_{i\in I} U_{ij} \le Z_j cr_j, \forall j \in J, \sum_{j\in J} P_{jm} \le W_m ce_m, \forall m \in M \\
& \sum_{i\in I} T_{in} \le cd_n, \forall n \in N, \\
& Y_i, Z_j, W_m \in \left\{0,1\right\}, \forall i \in I, \forall j \in J, \forall m \in M, \\
& X_{ki}, U_{ij}, P_{jm}, Q_{ml}, T_{in}, \delta_l \ge 0, \forall i \in I, \forall j \in J, \forall k \in K, \forall l \in L, \forall m \in M.
\end{aligned} \tag{3}$$

Proof. In the compact form (2), we know that

$$cx \le z - fy, \forall c \in u^c_{Box} \Leftrightarrow \max_{c\in u^c_{Box}} cx \le z - fy \tag{4}$$

From the definition of $c \in u^c_{Box}$, we have

$$\overline{c}_t - \rho_c G^c_t \le c_t \le \overline{c}_t + \rho_c G^c_t .$$

If we multiply it by nonnegative x_t, we get

$$\overline{c}_t x_t - \rho_c G^c_t x_t \le c_t x_t \le \overline{c}_t x_t + \rho_c G^c_t x_t .$$

Therefore, $\max_{c\in u^c_{Box}} c_t x_t = \overline{c}_t x_t + \rho_c G^c_t x_t$. Thus (4) is equivalent to

$$\overline{c}_t x_t + \sum_t \rho_c G^c_t x_t \le z - fy$$

which is the first constraint in theorem. The rest of uncertain constraints stay as they are in [10]. Thus we have (3).

As we see, the ten first set of constraints in robust model in [10] do not appear in our new model. The numbers of reduced constraints are $2\left|K\right|\left|I\right| + 2\left|I\right|\left|J\right| + 2\left|J\right|\left|M\right| + 2\left|M\right|\left|L\right| + 2\left|I\right| \mid N \mid$. Therefore, the new model has significantly less constraints than the model in [10]. Moreover, since MILPs are among NP-Hard problems, then any such improvement in the modeling is considered to be significant.

4. NUMERICAL EXAMPLES

In this section, we compare the two formulations on several examples. The first set of examples is exactly taken from [10] and the last example is a real example for a similar problem which is taken from [4]. For the first set of examples it is done as follow. Four test problems with different sizes are generated and for each size, the experiments are performed under three different uncertainty levels (i.e., $\rho = 0.2, 0.5, 1$) as given in Table 2. First, the robust and deterministic models are solved under nominal data, where nominal data are randomly generated as specified in Table 1. Then, under each uncertainty level, five random realizations are uniformly generated in the corresponding uncertainty set (i.e.

$\sim [\text{nominal value} - \rho G, \text{nominal value} + \rho G])$ to analyze the performance of the solutions obtained by the proposed robust and deterministic models. Both models are solved by ILOG CPLEX 12.5 optimization software and all tests are carried out on an Intel Core i5 computer with 6GB RAM. As we see in Table 2, the new model gives exactly the same results as the proposed robust model in [10] but in shorter time. This also verifies that the new model is exactly the old model but having significantly less constraints.

Table 1: The sources of random generation of the nominal data

Parameters	Corresponding random distribution
d_l	~Uniform (350,550)
r_k	~Uniform (450,650)
s	0.2
cc_i, ce_m	~Uniform (1500,2000)
cr_j	~Uniform (2000,3000)
cd_n	~Uniform (800,1000)
f_i	~Uniform (210000,2400000)
g_j	~Uniform (4500000,4900000)
h_m	~Uniform (160000,200000)
c_{ki} a_{ij} b_{jm} e_{ml} v_{in}	~Uniform (40,50)
π_l	~Uniform (4500,6000)

Table 2: The results of solving two robust formulations

Problem size $\lvert K\rvert * \lvert I\rvert * \lvert J\rvert * \lvert M\rvert * \lvert L\rvert * \lvert N\rvert$	Uncertainty level (ρ)	Objective function values under nominal data		Computational time under nominal data (s)		Objective function values under realizations	
		Robust (new)	Robust ([10])	Robust (new)	Robust ([10])	Robust (new)	Robust ([10])
	0.2	16,675,111	16,675,111	1.18	1.53	15,434,473	15,434,473
						15,436,829	15,436,829
						15,438,612	15,438,612
						15,435,935	15,435,935
						15,438,057	15,438,057
	0.5	16,684,328	16,684,328	1.59	1.69	15,451,689	15,451,689
						15,454,727	15,454,727
10*5*3*5*10*2						15,449,733	15,449,733
						15,457,435	15,457,435
						15,456,044	15,456,044
	1	17,558,395	17,558,395	1.71	2.01	17,559,853	17,559,853
						15,480,775	15,480,775
						15,460,027	15,460,027
						16,467,003	16,467,003
						15,494,274	15,494,274
	0.2	23,096,079	23,096,079	2.17	2.83	19,305,100	19,305,100
						19,397,794	19,397,794
						19,402,531	19,402,531
						19,395,216	19,395,216
						19,396,051	19,396,051
	0.5	23,104,863	23,104,863	2.25	2.30	19,432,178	19,432,178
						19,429,249	19,429,249
15*10*10*10*15*4						19,423,994	19,423,994
						19,426,139	19,426,139
						19,424,075	19,424,075
	1	23,119,505	23,119,505	2,21	3,11	19,454,485	19,454,485
						20,929,131	20,929,131
						27,213,365	27,213,365
						20,966,948	20,966,948
						19,459,889	19,459,889
	0.2	25,924,779	25,924,779	2.83	4.92	22,691,637	22,691,637
						22,694,297	22,694,297
						22,690,587	22,690,587
						22,695,276	22,695,276
						22,692,724	22,692,724
	0.5	25,943,370	25,943,370	2.79	4.11	22,729,938	22,729,938
						22,736,319	22,736,319
20*15*10*15*20*5						22,730,116	22,730,116
						22,723,799	22,723,799
						22,735,693	22,735,693
	1	25,973,572	25,973,572	3.49	3.90	25,471,285	25,471,285
						27,783,501	27,783,501
						22,796,731	22,796,731
						22,775,410	22,775,410
						22,804,609	22,804,609
	0.2	29,968,821	29,968,821	3.37	3.87	28,982,660	28,982,660
						28,985,424	28,985,424
						28,983,231	28,983,231
						28,980,521	28,980,521
						28,984,941	28,984,941
	0.5	29,997,546	29,997,546	3.61	4.52	29,023,417	29,023,417
						29,022,079	29,022,079
25*18*12*18*25*6						29,037,441	29,037,441
						29,035,418	29,035,418
						29,022,763	29,022,763
	1	30,036,319	30,036,319	2.87	3.57	32,648,685	32,648,685
						35,446,651	35,446,651
						32,860,931	32,860,931
						29,126,120	29,126,120
						29,116,612	29,116,612

The data for the last example given in Tables 3 to 5 are taken from [4]. We have again solved both formulations for this data and results are summarized in Table 6. As we see, again both formulations give the same results but the new one in shorter time.

Table 3: The demands and recycling quantities

Customers	demands of customers d_l	return products of customers r_k
c_1	500	100
c_2	700	154
c_3	800	168

Table 4: The capacity and fixed cost

Collection center		Redistribution center		Recycling center		Disposal center
Capacity	Fixed cost	Capacity	Fixed cost	Capacity	Fixed cost	Capacity
80	65	800	500	180	300	15
100	90	900	650	260	450	
120	100	1000	900			
150	140					

Table 5: shipping cost per unit of product

Shipping cost per unit of return product from customer zone k to collection center i

	i			
k	9	8	10	8
	7	9	10	12
	9	10	12	10

Shipping cost per unit of collection center i to recovery center j

	j	
i	10	13
	12	12
	13	14
	10	15

Shipping cost per unit of recovered product from recovery center j to redistribution center m

	m		
j	19	20	22
	23	19	24

Shipping cost per unit of recovered product from redistribution center m to customer zone l

	l		
m	5	6	8
	8	6	5
	9	4	5

Shipping cost per unit of scrapped product from collection center i to disposed center n

	n
i	5
	4
	5
	3

Penalty cost per unit of no satisfied demand of customer l

π_l
12
10
16

5. CONCLUSIONS

In this paper, a new formulation for the robust counterpart of the MIP model of closed loop supply chain problem given in [10] is presented which has significantly less constraints. Then on several examples, the two formulations are compared which shows both give the same results but the new formulation is faster.

Table 6: The results

Problem size $\|K\|*\|I\|*\|J\|*\|M\|*\|L\|*\|N\|$	Uncertainty level (ρ)	Objective function values under nominal data		Computational time under nominal data (s)		Objective function values under realizations	
		Robust (new)	Robust ([10])	Robust (new)	Robust ([10])	Robust (new)	Robust ([10])
$3*4*2*3*3*1$	0.2	37,514	37,514	1.98	2.35	37,554	37,554
	0.5	37,759	37,759	1.86	1.93	37,800	37,800
	1	38,165	38,165	1.96	2.30	38,231	38,231

ACKNOWLEDGMENT

The second author would like to thank University of Guilan for supporting this research. Part of this work was done when the second author was on sabbatical leave at the University of Waterloo.

REFERENCES

1. Altman, M. and Bogaschewsky, R. (2014). Environmentally conscious robust closed-loop supply chain design. *Journal of Business Economics,* 84(5): 613–637.
2. Ayres, R., Ferrer, G., Van Leynseele, T. (1997). Eco-efficiency, asset recovery and remanufacturing. *European Management Journal*, 15(5): 557—574.
3. Bloemhof, J.M., Corbett, C.J. (2010). Closed-loop supply chains: environmental impact. *Wiley Encyclopedia of Operations Research and Management Science.*
4. Chen, Y.T., Chan, F. T.S., Chung, S.H. And Niu, B. (2013). Closed-loop supply chain network optimization for Hong Kong cartridge recycling industry. *Proceedings of ISS & MLB*, 24-26.
5. Mark S. Daskin, Lawrence V. Snyder, Rosemary T. Berger. (2005). Facility location in supply chain design, *Logistics Systems: Design and Optimization*, 39–65
6. Guide Jr., V.D.R. and Van Wassenhove, L.N. (2006). Closed-Loop Supply Chains: An Introduction to the Feature Issue (Part 1), *Production and Operation Management*, 15(3): 345–350.
7. De Giovanni, P. (2014). Environmental collaboration in a closed-loop supply chain with a reverse revenue sharing contract. *Annals of Operations Research*, 220(1): 135-157.
8. Mohajeri, A. and Fallah, M. (2014). Closed-loop supply chain models with considering the environmental impact. *The Scientific World Journal*, Article ID 852529, 23 pages.
9. Paksoy, T., Ozceylan, E. (2014). Environmentally conscious optimization of supply chain networks. *Journal of the Operational Research Society*, 65(6): 855–872.
10. Pishvaee, M.S., Rabbani, M., Torabi, S.A. (2011). A robust optimization approach to closed-loop supply chain network design under uncertainty. *Applied Mathematical Modelling*, 35(2): 637-649.
11. Ramezani, M., Bashiri, M., Tavakkoli-Moghaddam, R. (2013). A robust design for a closed-loop supply chain network under an uncertain environment, *The International Journal of Advanced Manufacturing Technology*, 66(5): 825–843
12. Ruimin, MA., Lifei, YA., Maozhuxi J .Pei Yu R., and Zhihan, LV. (2015). Robust environmental closed-loop supply chain design under uncertainty,Chaos, Solitons Fractals, corrected proof.
13. Vahdani, B., Mohammadi, M. (2015). A bi-objective interval-stochastic robust optimization model for designing closed loop supply chain network with multi-priority queuing system. *International Journal of Production Economics*, 170(A): 67–87.
14. Van Landeghem, H., and Vanmaele, H. (2002). Robust planning: a new paradigm for demand chain planning. *Journal of Operations Management*, 20(6): 769–783.