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Transportation Problem with Uncertain Cost, Rough Demand and Supply

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Abstract — In a transportation problem, the cost of transportation from source to destination depends on many factors which may not be deterministic in nature. In this work, the unit cost of transportation is considered to be imprecise and the trapezoidal fuzzy cost is assigned to the model. Further the demand and supply are considered as uncertain due to lack of proper information. So the demand and supply are considered as rough variable in this proposed model. In the solution process of such a model, two solutions as pessimistic and optimistic solutions are obtained at certain trust level which suggests a range for the actual solution. Further, suitably converting the fuzzy cost as rough, the solution is obtained and comparison of results in both cases is made by taking one numerical example.

Keywords — Transportation problem, Generalized trapezoidal fuzzy number, Rough variable, Pessimistic value, Optimistic value.

1. INTRODUCTION

Transportation problem is one of the important linear programming problems. Many efficient methods have been developed to solve a transportation problem where the unit cost of transportation, demand and supply are all deterministic Dantzig (1963), Gass (1990). In many practical situations, the unit cost of transportation, demand and supply may not deterministic in nature. The cost is imprecise which depends upon many factors like road condition, traffic load causing delay in delivery, break down of vehicles, labor problem for loading and unloading, which may increase the cost. Also demand and supply are uncertain or unpredictable due to lack of proper information. All these values are uncertain which cannot be predicted beforehand due to insufficient information, unpredictable environment.

Transportation problem in various types of uncertain environment such as fuzzy, stochastic are studied by many researchers. Zimmermann (1978) has developed fuzzy linear programming which eventually used to develop several fuzzy optimization methods for solving transportation problem. Chanas et al. (1984) presented fuzzy linear programming model to solve transportation problem with crisp cost and fuzzy demand and supply. Dinagar and Palanivel (2009) investigate fuzzy transportation problem with the aid of trapezoidal fuzzy number. Kaur and Kumar (2011) have developed a new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers.

To deal with uncertainty, rough set theory as developed by Pawlak (1982) and uncertainty theory as developed by Liu (2004, 2007) can be successfully used. Liu (2004) has also proposed the concept of rough variable in uncertainty theory which is a measurable function from rough space to the set of real numbers. The rough variable can be used to find the solution of different kinds of problems under uncertain environment. Many researchers have used concept of rough variables in some optimization problems. Xu and Yao (2010) studied a two-person zero-sum matrix games with payoffs as rough variables. Youness (2006) introduced a rough programming problem considering the decision set as a rough set. Xu et al. (2009) proposed a rough DEA model to solve a supply chain performance evaluation problem with rough parameters. Xiao and Lai (2005) have considered power-aware VLIW instruction scheduling problem with power consumption parameters as rough variables. Kundu et al. (2013) proposed some solid transportation models with crisp and rough costs.

In this paper we have formulated transportation problems considering the unit cost of transportation from source to destination as trapezoidal fuzzy number with different membership values and the demand and supply as rough variables, where we get pessimistic and optimistic solution of the problem under certain level of trust (level of confidence). Further converting the trapezoidal fuzzy cost as rough cost, the problem is also solved to get other set of solution. Then both the solutions are compared.

The rest of the paper is organized as follows: In Section 2, we provide some definitions and properties of rough set, rough variable and generalized fuzzy number. In Section 3, we present our proposed transportation problems and model formulation is done. The solution procedure is discussed in Section 4. One numerical

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example is given in Section 5. The result interpretation is discussed in Section 6 and finally the conclusion is given in Section 7.

2. PRELIMINARIES

2.1. Rough set

Rough set theory developed by Pawlak (1982) is a mathematical tool for dealing with uncertain and incomplete data without any prior knowledge about the data. We deal only with the available information provided by the data to generate conclusion.

Let U be a finite non empty set called the universal and let R be a binary relation defined on U. Let R be an equivalence relation and $R(x)$ be the equivalence class of the relation which contains x. R shall be referred as indiscernibility relation. For any $X \subseteq U$, the lower and upper approximation of X is defined by

$$\underline{R}(X) = \{x \in U \ : \ R(x) \ \subseteq \ X\}, \ \overline{R}(X) \ = \ \{x \in U \ : \ R(x) \cap \ X \ \neq \ \phi\}.$$

The lower approximation $\underline{R}(X)$ is exact set contained in X so that the object in $\overline{R}(X)$ are members of X with certainty on the basis of knowledge in R, where the objects in the upper approximation $\overline{R}(X)$ can be classified as possible members of X. The difference between the upper and lower approximation of X will be called as R-boundary of X and is defined by $BN_R(X) = \overline{R}(X) - \underline{R}(X)$.

The set X is R-exact if $BN_R(X) \ = \ f$, otherwise the set is R - rough set.

2.2. Rough variable

The concept of rough variable is introduced by Liu (2004) as uncertain variable. The following definitions are based on Liu (2004).

Definition 1 : Let Λ be a non-empty set, A be an σ - algebra of subsets of Λ, Δ be an element in A and π be a nonnegative, real- valued, additive set function on A. Then, $(\Lambda, \Delta, A, \pi)$ is called a rough space.

Definition 2: A rough variable ξ on the rough space $(\Lambda, \Delta, A, \pi)$ is a measurable function from Λ to the set of real numbers $\Re$ such that for every Borel set B of $\Re$, we have $\{\lambda \in \Lambda \mid \ \xi(\lambda) \in \ \mathrm{B}\} \in A$. Then the lower and upper approximation of the rough variable ξ are defined as follows

$$\underline{\xi} = \{\xi(\lambda) \mid \lambda \in \Delta\} \ , \quad \overline{\xi} = \{\xi(\lambda) \mid \lambda \in \Lambda\} \tag{1}$$

Definition 3: Let $(\Lambda, \Delta, A, \pi)$ be a rough space. Then the upper and lower trust of event A is defined by

$$T\underline{r}(A) = \frac{\pi\{A\}}{\pi\{\Lambda\}} \text{ and } T\overline{r}(A) = \frac{\pi\{A \cap \Delta\}}{\pi\{\Lambda\}}$$

The trust of the event A is defined as

$$Tr(A) = \frac{1}{2}\left(T\underline{r}(A) + T\bar{r}(A)\right) \tag{2}$$

The trust measure satisfies the followings:

$$Tr(\Lambda) = 1, Tr(\phi) = 0$$
$$Tr(A) \leq Tr(B) \ \ where \ \ A \subseteq \ B$$
$$Tr(A) + Tr(A^c) = 1$$

Definition.4: Let ξ_1, ξ_2 are rough variables defined on the rough space $(\Lambda, \Delta, A, \pi)$. Then their sum and product are defined as

$$\begin{aligned}(\xi_1 + \xi_2)(\lambda) &= \xi_1(\lambda) + \xi_2(\lambda) \\ (\xi_1 \cdot \xi_2)(\lambda) &= \xi_1(\lambda) \cdot \xi_2(\lambda)\end{aligned} \tag{3}$$

Definition 5: Let ξ be rough variables defined on the rough space $(\Lambda, \Delta, A, \pi)$ and $\alpha \in (0,1]$ then

$$\xi_{\sup}(\alpha) = \sup\left\{r \middle| Tr\{\xi \geq r\} \geq \alpha\right\} \tag{4}$$

is called α-optimistic value of ξ.

$$\xi_{\inf}(\alpha) = \inf\left\{r \middle| Tr\{\xi \geq r\} \geq \alpha\right\} \tag{5}$$

is called α-pessimistic value of ξ.

Definition 6: The trust distribution $\phi : \ [-\infty, \infty] \to [0,1]$ of a rough variable ξ is defined by

$$\Phi(x) \;=\in \Lambda \Big| \quad Tr\ \{\ \lambda\xi(\lambda)\ \le\ x\} \tag{6}$$

Definition 7: The trust density function $f:\ \mathrm{R} \to [0,\infty)$ of a rough variable ξ is a function such that $f(x) = \int_{-\infty}^{\infty} \varphi(y)\,dy$ holds for all $x \in (-\infty, \infty)$, where ϕ is trust distribution of ξ.

If $\xi = ([\mathrm{a,b}], [c,d])$ be a rough variable such that $c \le a < b \le d$, then the trust distribution $\phi(x) = Tr\{\xi \le x\}$ is

$$\phi(x) = \begin{cases} 0 & if\ \ x \le c \\ \dfrac{x-c}{2(d-c)} & if\ \ c \le\ x \le\ a \\ \dfrac{[(b-a)+(d-c)]x + 2ac - ad - bc}{2(b-a)(d-c)} & if\ \ a \le x \le b \\ \dfrac{x+d-2c}{2(d-c)} & if\ \ b \le x \le d \\ 1 & if\ \ x \ge d \end{cases} \tag{7}$$

And the trust density function is defined as

$$f(x) = \begin{cases} \dfrac{1}{2(d-c)} & if\ c \le x \le a\ \ or\ \ b \le x \le d \\ \dfrac{1}{2(b-c)} + \dfrac{1}{2(d-c)} & if\ a \le x \le b \\ 0 & otherwise \end{cases} \tag{8}$$

In our work, we have considered the rough variable ξ as $\xi = ([a,b],[c,d])$ where $c \le a < b \le d$. α -optimistic value to $\xi = ([a,b],[c,d])$ is

$$\xi_{\sup}(\alpha) = \begin{cases} (1-2\alpha)d + 2\alpha c & if\ \alpha \le \dfrac{d-b}{2(d-c)} \\ 2(1-\alpha)d + (2\alpha(b-a)(d-c) & if\ \alpha \ge \dfrac{2d-a-c}{2(d-c)} \\ \dfrac{d(b-a)+b(d-c)-2\alpha(b-a)(d-c)}{(b-a)+(d-c)} & otherwise \end{cases} \tag{9}$$

α-pessimistic value to $\xi = \{[a,b],[c,d]\}$ is

$$\xi_{\inf}(\alpha) = \begin{cases} (1-2\alpha)c + 2\alpha d\ , & if\ \ \alpha \le \dfrac{a-c}{2(d-c)} \\ 2(1-\alpha)c + (2\alpha-1)d\ , & if\ \ \alpha \ge \dfrac{b+d-2c}{2(d-c)} \\ \dfrac{c(b-a)+a(d-c)+2\alpha(b-a)(d-c)}{(b-a)+(d-c)}, & otherwise \end{cases} \tag{10}$$

2.3 Fuzzy set

Zadeh (1965) first introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to the unit interval $[0,1]$ i.e. $A = \{(x,\ \mu_A(x)\ ;\ x \in X\}$, Here $\mu_A:\ X\ \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from $[0,1]$.

2.3.1 Generalized Trapezoidal fuzzy number

A fuzzy number $\tilde{A} = (a, b, c, d; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w\dfrac{(x-a)}{(b-a)}, & a \le x \le b \\ w, & b \le x \le c \\ w\dfrac{(x-d)}{(c-d)}, & c \le x \le d \\ 0\ , & otherwise \end{cases} \tag{11}$$

where, w is the confidence level.

2.3.1.1 Arithmetic operation

(a) Addition of two generalized trapezoidal fuzzy numbers $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)$, with different confidence levels generates a trapezoidal fuzzy number as follows

$$\begin{aligned} &\tilde{A}_3 = \tilde{A}_1 (+) \tilde{A}_2 = (a_3, b_3, c_3, d_3; w_3) \ \ where, \\ &a_3 = a_1 + a_2 \\ &b_3 = a_1 + a_2 + (b_1 - a_1) w_3 / w_1 + (b_2 - a_2) w_3 / w_2 \\ &c_3 = d_1 + d_2 - (d_1 - c_1) w_3 / w_1 - (d_2 - c_2) w_3 / w_2 \\ &d_3 = d_1 + d_2 \\ &and \quad w_3 = \min(w_1, w_2)\ ;\ w_1 \ne w_2 \end{aligned} \tag{12}$$

(b)

$$l\tilde{A}_2 = \begin{cases} (la_1, lb_1, lc_1, ld_1; w_1) & \text{if } l > 0 \\ (ld_1, lc_1, lb_1, la_1; w_1) & \text{if } l < 0 \end{cases} \tag{13}$$

These definitions are due to Chakraborty and Guha (2010)

3. DESCRIPTION OF THE PROBLEM AND MODEL FORMULATION

3.1 Description of the Problem:

In a transportation problem, the cost of transportation from source node to destination node depends on many factors like fuel price, labor charge, charges for halting due to traffic , road conditions etc. which are unpredictable in nature. In this work we have taken the cost of transportation as fuzzy trapezoidal numbers to deal with the factors which are imprecise in nature.

Further, more the distance between the source node and destination node, more the unpredictable nature of the factors. Hence, the confidence level decreases. In this work we have taken different confidence level for different fuzzy costs which are dependent on the distance. For increase in one unit of distance (which is one tenth of the difference between the maximum and minimum distance), the confidence level is decreased by 0.05. If d is the distance and α is the confidence level, then

$$\alpha(d) = \begin{cases} 0.95 & if \quad d = m \\ 0.95 - 0.05k & if \quad d = m + ku \end{cases} \tag{14}$$

where $u = \dfrac{M-m}{10}$, $k = 1, 2, 3, \cdots$, $M =$ maximum distance, $m =$ minimum distance .

Secondly, in transportation problem, there are other two constraints like demand and supply which are uncertain in nature and cannot be predicted before. These can be estimated with the available information which is also not complete. In this work to deal with this situation, demand and supply are taken as rough variables.

If $D = ([a,b],[c,d])$ is the rough demand of a particular node. We can say $[a,b]$ is the lower approximation and $[c,d]$ is the upper approximation of the demand which can be set with the available information (taking sample survey, consulting experts, considering the past available records etc.)

3.2 Transportation cost is fuzzy trapezoidal number, demand and supply are rough

In our model, the cost of transportation is taken as fuzzy trapezoidal number $\tilde{c}_{ij} = \left(c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4; w_{ij}\right)$, where $w_{ij}\left[0,1\right]$ is the level of confidence.

The supply available in i th node is rough in the form of

$$s_i = \left(\left[a_i^1, a_i^2\right], \left[a_i^3, a_i^4\right]\right), \quad where\ a_i^3 \le a_i^1 < a_i^2 \le a_i^4$$

The demand of the jet node is rough in the form of

$$d_j = \left(\left[b_j^1, b_j^2\right], \left[b_j^3, b_j^4\right]\right), \quad where\ b_j^3 \le b_j^1 < b_j^2 \le b_j^4$$

and α being the trust level.

Let

$$\begin{aligned} \xi_{\sup}^i &= \alpha - optimistic\ value\ of\ s_i. \\ \eta_{\sup}^j &= \alpha - optimistic\ value\ of\ d_j. \\ \xi_{\inf}^i &= \alpha - pessimistic\ value\ of\ s_i. \\ \eta_{\inf}^j &= \alpha - pessimistic\ value\ of\ d_j. \end{aligned}$$

The various models are given below:

3.2.1 α -Optimistic model

$$\begin{aligned} &Minimize \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\ x_{ij} \\ &Subject\ to \\ &\sum_{j=1}^{n} x_{ij} \le \xi_{\sup}^i \ , \quad i = 1,2,\ \dots\ ,m \\ &\sum_{i=1}^{m} x_{ij} \ge \eta_{\sup}^j \ , \quad j = 1,2,\ \dots.\ ,n \\ &x_{ij} \ge 0 \end{aligned} \tag{15}$$

3.2.2 α - Pessimistic model

$$\begin{aligned} &Minimize \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\ x_{ij} \\ &Subject\ to \\ &\sum_{j=1}^{n} x_{ij} \le \xi_{\inf}^i \ , \quad i = 1,2,\ \dots\ ,m \\ &\sum_{i=1}^{m} x_{ij} \ge \eta_{\inf}^j \ , \quad j = 1,2,\ \dots.\ ,n \\ &x_{ij} \ge 0 \end{aligned} \tag{16}$$

3.3 Transportation cost, demand and supply are rough variables

The generalized trapezoidal fuzzy number represents as $\tilde{c}_{ij} = \left(c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4\ ; w_{ij}\right)$ can be converted to rough variable as

$$c_{ij} = \left(\left[c_{ij}^2, c_{ij}^3\right], \left[c_{ij}^1, c_{ij}^4\right]\right), \ where\ c_{ij}^1 \le c_{ij}^2 < c_{ij}^3 \le c_{ij}^4$$

Such that, $\left[c_{ij}^2, c_{ij}^3\right]$ is the lower approximation and $\left[c_{ij}^1, c_{ij}^4\right]$ is the upper approximation of the unit transportation cost from source i to destination j. The objective function becomes a rough variable of the form

$$Z = \left(\left[Z^2, Z^3\right], \left[Z^1, Z^4\right]\right)$$

where $Z^r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r\ x_{ij}$, $r = 1,2,3,4$.

The optimal value of the objective function with rough cost can be obtained from the following models.

3.3.1 α - Optimistic Model

$$
\begin{aligned}
& Minimize \quad Z^r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r \, x_{ij} \\
& Subject\ to \\
& \qquad \sum_{j=1}^{n} x_{ij} \le \xi_{\sup}^{i} \quad , \quad i = 1,2, \ldots ,m \\
& \qquad \sum_{i=1}^{m} x_{ij} \ge \eta_{\sup}^{j} \quad , \quad j = 1,2, \ldots ,n \\
& \qquad x_{ij} \ge 0
\end{aligned}
\tag{17}
$$

3.3.2 α - Pessimistic Model

$$
\begin{aligned}
& Minimize \quad Z^r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^r \, x_{ij} \\
& Subject\ to \\
& \qquad \sum_{j=1}^{n} x_{ij} \le \xi_{\inf}^{i} \quad , \quad i = 1,2, \ldots ,m \\
& \qquad \sum_{i=1}^{m} x_{ij} \ge \eta_{\inf}^{j} \quad , \quad j = 1,2, \ldots ,n \\
& \qquad x_{ij} \ge 0
\end{aligned}
\tag{18}
$$

3.4 The optimal range of rough objective function with trust level α

The optimum objective value lies within the range $\left[Z_o, Z_p\right]$ with trust level at least α where, Z_p is α-pessimistic and Z_o is α-optimistic values with trust value α .These value can obtained from (9) and (10)

$$
Z_p = \begin{cases}
\left(1-2\alpha\right)Z^1 + 2\alpha Z^4, & if \quad \alpha \le \dfrac{Z^2 - Z^1}{2\left(Z^4 - Z^1\right)} \\
2\left(1-\alpha\right)Z^1 + \left(2\alpha - 1\right)Z^4, & if \quad \alpha \ge \dfrac{Z^3 + Z^4 - 2Z^1}{2\left(Z^4 - Z^1\right)} \\
\dfrac{Z^1\left(Z^3 - Z^2\right) + Z^2\left(Z^4 - Z^1\right) + 2\alpha\left(Z^3 - Z^2\right)\left(Z^4 - Z^1\right)}{\left(Z^3 - Z^2\right) + \left(Z^4 - Z^1\right)}, & otherwise
\end{cases}
\tag{19}
$$

and

$$
Z_o = \begin{cases}
\left(1-2\alpha\right)Z^4 + 2\alpha Z^1, & if \quad \alpha \le \dfrac{Z^4 - Z^3}{2\left(Z^4 - Z^1\right)} \\
2\left(1-\alpha\right)Z^4 + \left(2\alpha - 1\right)Z^1, & if \quad \alpha \ge \dfrac{2Z^4 - Z^2 - Z^1}{2\left(Z^4 - Z^1\right)} \\
\dfrac{Z^4\left(Z^3 - Z^2\right) + Z^3\left(Z^4 - Z^1\right) - 2\alpha\left(Z^3 - Z^2\right)\left(Z^4 - Z^1\right)}{\left(Z^3 - Z^2\right) + \left(Z^4 - Z^1\right)}, & otherwise
\end{cases}
\tag{20}
$$

4. SOLUTION PROCEDURE

4.1 Solution procedure where the unit cost of transportation is fuzzy, demand and supply are rough

4.1.1 Procedures for α - optimistic solution

Step 1: set the trust (confidence) level α

Step 2: Find the α - optimistic value of rough demand using equation (9)

Step 3: Find the α - optimistic value of rough supply using equation (9)

Step 4: Formulate the model as (3.2.1)

Step 5: Find the initial basic feasible solution by using generalized fuzzy Vogel's approximation method designed by Kaur and Kumar (2011)

Step 6: Test the optimality by using generalized fuzzy modified distribution method designed by Kaur and Kumar (2011).

Step7: Find the optimal solution by using equation (12) and equation (13).

4.1.2 Procedures for α - pessimistic solution

Step 1: Set the trust (confidence) level α
Step 2 Find the α - pessimistic value of rough demand using equation (10)
Step 3: Find the α - pessimistic value of rough supply using equation (10)
Step 4: Formulate the model as (3.2.2)
Step 5: Find the initial basic feasible solution by using generalized fuzzy Vogel's approximation method designed by Kaur and Kumar (2011).
Step 6: Test the optimality by using generalized fuzzy modified distribution method designed by Kaur and Kumar (2011).
Step 7: Find the optimal solution by using equation (12) and equation (13).

4.2 Solution procedure where the unit cost of transportation, demand and supply are rough

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4.2.1 Procedures for α - optimistic solution

Step 1: Set the trust (confidence) level α
Step 2: Find the α - optimistic value of rough demand using equation (9)
Step 3: Find the α - optimistic value of rough supply using equation (9)
Step 4: Formulate the model as (3.3.1)
Step 5: Find the initial basic feasible solution by using Vogel's approximation method.
Step 6: Test the optimality by using distribution method.
Step 7: Find the optimal solution.

4.2.2 Procedures for α - pessimistic solution

Step 1: Set the trust (confidence) level α
Step 2: Find the α - pessimistic value of rough demand using equation (10)
Step 3: Find the α - pessimistic value of rough supply using equation (10)
Step 4: Formulate the model as (3.3.2)
Step 5: Find the initial basic feasible solution by using Vogel's approximation method.
Step 6: Test the optimality by using modified distribution method.
Step 7: Find the optimal solution.

5. NUMERICAL EXAMPLE

There are three sources A, B, C and three destinations X, Y, Z. The distance matrix corresponding to the transportation problem is given as

Table 1 Distance matrix

	X	Y	Z
A	100	200	280
B	220	250	130
C	300	180	290

5.1 Problems with unit transportation costs as generalized trapezoidal fuzzy numbers; demand and supply are rough variables:

Consider a problem with three sources (i = 1, 2, 3) and three destinations (j = 1, 2, 3). The unit transportation costs are trapezoidal fuzzy variables, the availability at each source, demands of each destination are rough variables which is given in the table.

Table 2 Fuzzy transportation cost

	X	Y	Z
A	(5, 9, 12, 15; 0.95)	(18, 22, 25, 28; 0.7)	(28, 33, 35, 40; 0.5)
B	(25, 28, 32, 35; 0.65)	(25, 28, 32, 35; 0.6)	(9, 12, 15,22; .9)
C	(35, 38, 42, 45; 0.5)	(18, 22, 25, 28; 0.75)	(28, 33, 35, 40; 0.5)

Supply and Demands are in rough variable as follows

Demand: a_1=([32, 36] [30, 40]), a_2 = ([38, 44] [35, 45]) , a_3= ([22, 27] [20, 30])

Supply: b_1=([28, 32] [26, 33]), b_2 = ([34, 39] [32, 42]), b_3= ([30, 36] [27, 40])

5.1.1 α-Optimistic Solution

The cost value in trapezoidal fuzzy number and α-optimistic Supply and Demands with α =0.8are

$$\xi^1_{sup}=32.6,\ \xi^2_{sup}=38.38,\ \xi^3_{sup}=22.67,\ \eta^1_{sup}=28.3,\ \eta^2_{sup}=34.67 \text{ and } \eta^3_{sup}=30.7$$

Then α-optimistic solution is x_{11}=28.29 , x_{12}= 4.31, x_{22}=7.69, x_{23}=30.69, x_{32}=22.67 and Minimum Z = (1095.55 ,1338.79,1838.73, 2124.12 ; 0.6).

5.1.2 α-pessimistic solution

The cost value in trapezoidal fuzzy number and α-pessimistic supply and demands with α = 0.8 are

$$\xi^1_{inf}=36,\ \xi^2_{inf}=42.88,\ \xi^3_{inf}=26.67,\ \eta^1_{inf}=31.35,\ \eta^2_{inf}=38.67 \text{ and } \eta^3_{inf}=35.62$$

The α-pessimistic solution is x_{11}=31.35 , x_{12}= 4.65, x_{22}=7.26, x_{23}=35.62, x_{32}=26.67 and Minimum Z = (1222.59, 1496.09,2061.57, 2381.95 ; 0.6)

5.2 Problems with unit transportation cost, demand and supply as rough

Consider a problem with three sources (i = 1,2,3) and three destinations (j = 1,2,3).The unit transportation cost, the supply at each source, demands of each destination are rough variables. The fuzzy transportation cost depicted in Table 2 is suitably modified to rough cost as discussed in 3.3. The rough demand and supply are remain same as above

Table 3: Rough transportation cost

	X	Y	Z
A	([9,12][5,15])	([22,25][18,28])	([33,35][28,40])
B	([28,32][25,35])	([28,32][25,35])	([12,15][9,12])
C	([38,42][35,45])	([22,25][18,28])	([33,35][28,40])

Supply and Demands are in rough variable:

a_1= ([32,36][30,40]), a_2 = ([38,44] [35,45]), a_3= ([22,27][20,30])
b_1= ([28,32][26,33]), b_2 = ([34,39] [32,42]), b_3= ([30,36][27,40])

5.2.1 α-Optimistic Solution

The cost value is rough and α-optimistic Supply and Demands with α =0.8 are

$$\xi^1_{sup}=32.6,\ \xi^2_{sup}=38.38,\ \xi^3_{sup}=22.67,\ \eta^1_{sup}=28.3,\ \eta^2_{sup}=34.67 \text{ and } \eta^3_{sup}=30.7$$

Then α-optimistic solution is x_{11}=28.29 , x_{12}= 4.31, x_{22}=7.69, x_{23}=30.69, x_{32}=22.67 and minimum Z = ([1431.77,1720.41][1095.55,2124.12])

5.2.2 α-pessimistic solution

The cost value is rough and α-pessimistic supply and demands with α = 0.8 are

$$\xi^1_{inf}=36,\ \xi^2_{inf}=42.88,\ \xi^3_{inf}=26.67,\ \eta^1_{inf}=31.35,\ \eta^2_{inf}=38.67 \text{ and } \eta^3_{inf}=35.62$$

Then α-pessimistic solution is x_{11}=31.35 , x_{12}= 4.65, x_{22}=7.26, x_{23}=35.62, x_{32}=26.67 and minimum Z = ([1601.91,1925.82][1222.59,2384.95])

6. RESULT INTERPRETATION

6.1 Optimal solution when the unit transportation cost is fuzzy trapezoidal number

Using the proposed method the minimum total fuzzy transportation cost in α-optimistic is (1095.55 ,1338.79,1838.73, 2124.12 ; 0.6), which can be physically interpreted as follows:

(i) According to the decision maker the minimum transportation cost will be greater than Rs1095.55 and less than Rs 2124.12.

(ii) The overall level of satisfaction of the decision maker about the statement that the minimum total transportation cost will be Rs 1338.79 – 1838.73 is 60%.

So the solution can be represented by

$$\mu_{\tilde{A}} = \begin{cases} \dfrac{0.6\left(x-1095.55\right)}{\left(1338.79-1095.55\right)}, & 1095.55 \leq x < 1338.79 \\ 0.6 & 1338.79 \leq x < 1838.73 \\ \dfrac{0.6\left(x-2124.12\right)}{\left(1838.73-2124.12\right)}, & 1838.73 \leq x < 2124.12 \\ 0 & otherwise \end{cases}$$

Let x represents the minimum transportation cost then the overall level of satisfaction of the decision maker is

Similar interpretation can be made for α-pessimistic solution.

6.2 Optimal Solution when the unit transportation cost is rough

The α - optimistic solution is ([1431.77,1720.41][1095.55,2124.12]) which is obtained as rough. Using the equation (19) and equation (20) at confidence level $\alpha = 0.6$. The solution is transformed to the interval form as [1538.41, 1628.56], which signifies that the solution lies between Rs 1538.41 and Rs 1628.56 .

Similar interpretation can be made for α-pessimistic solution.

6.3 Result Comparison:

When the level of satisfaction of the decision maker is 60%.The comparison result between the unit transportation cost value in trapezoidal fuzzy number and the unit transportation cost in rough are given below

Table 3 Result

Method	Interval solution when the unit cost is trapezoidal fuzzy number	Interval solution when the unit cost is rough
α -Optimistic	[1338.79, 1838.73]	[1538.41, 1628.56]
α -Pessimistic	[1496.09, 2061.57]	[1721.90, 1823.22]

7. CONCLUSION

In this work, solution to transportation problem is attempted with different conditions. The demand and supply are considered to be rough to deal with the uncertainty. In the first case, the unit cost of transportation is taken as trapezoidal fuzzy number due to the impreciseness of different factor which determines the cost. These solutions set are obtained as α-optimistic and α-pessimistic solution. In the second case, the trapezoidal fuzzy numbers which determine the cost of transportaxtion are suitably converted to rough variable and again two sets of solution as α-optimistic and α-pessimistic solutions are obtained with 60% level of confidence. A comparison of this solution is given and it is found that in all two sets of solutions, the solutions obtained considering the cost as rough gives a better result than considering the cost as fuzzy trapezoidal number. In future work, trust distribution can be used to predict the solution as compromise between α-optimistic and α-pessimistic solutions.

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