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A Model for Uncertain Multi-objective Transportation Problem with Fractional Objectives

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Abstract: Fractional programming problems take into account the situations where the decision maker is interested to maximize or minimize the ratios of some functions rather than a simple function. Fractional programming modeling approach has a lot of scope in dealing with the transportation planning decision problems. This paper presents a model for transportation problem with multiple fractional objectives involving uncertain parameters. In order to make the model more realistic, we have considered the case when there exists more than one fractional objective. All the parameters involved in the proposed model viz. objective function coefficients, availabilities and demands are assumed to be uncertain. Moreover, an equivalent deterministic model is also presented. Fuzzy goal programming approach is discussed as the solution approach for reaching the compromise solution. A numerical example is also given to illustrate the model more clearly.
Keyword — Fractional programming problem, Transportation problems, Uncertain variables, Uncertain programming, Fuzzy goal programming, Membership function

1. INTRODUCTION

Transportation is an important aspect which remains as an essential part in the study domain of logistics and operations management. Transportation problems are concerned in determining an optimal strategy for distributing a commodity from a group of supply centers, called sources, to various receiving centers, called destinations. The main objectives of the problem underlies in minimizing the parameters such as cost, time, deterioration of items and taxes etc. Transportation problem was first studied by Hitchcock (1941). A special case of transportation planning model is, when there is a need to optimize any problem with objective that is in fractional form. The kind of models having fractional objective function lies under the category of fractional programming problems.

Fractional programming is an important part of management decision making which involves rational optimization. Ratio of technical or economical terms often represents the efficiency of any system. Fractional programming has its applicability in a large number of problems which are widely spread over the literature. Charnes and Cooper (1962) gives a transformation in which they converted a linear fractional program with one ratio to a linear program. Later, Corban (1973) gives an approach on programming with fractional linear objective function. Dür et al. (2007) presented an Algorithm called dynamic multistart improving Hit and Run and applied it to the class of Fractional Optimization problem. Pramanik and Dey (2011) presented a fuzzy goal programming approach for bi-level linear fractional programming problem with a single decision maker both at the upper level and lower level. Pal et al. (2011) solved linear goal programming to multi-objective fractional programming. Saad et al. (2012) gives the solution procedure for multi-objective integer linear fractional programming problems with uncertain data. Metaheuristics approaches have also been used in last few years to solve fractional programming problems. Sameeullah et al. (2008) presented a genetic algorithm based method to solve linear Fractional Programming problem. Calvete et al. (2009) developed a genetic algorithm for the class of bi-level problems in which both level objective functions are linear fractional and the common constraint region is a bounded polyhedron. Pal et al. (2013) used Particle Swarm Optimization algorithm to solve fractional programming problems.

Linear fractional programming is widely applied in various fields, such as finance, production planning, and transportation problem (Ibaraki and Schaible 1983; Stancu-Minasian 1997, Lara and Stancu-Minasian 1999). Stancu-Minasian (2017) has written a bibliography on fractional programming problems. Fractional objective functions serves as a tool of performance measures in transportation problems. Fractional Transportation problems are concerned with shipping the commodities from various sources to destinations along with a goal to maintain good relationships between some crucial parameters. These crucial parameters of transportation problems may occur in the form such as; actual transportation cost / total standard transportation cost, shipping cost/ preferred route,

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total return/ total investment etc. Fractional transportation Problems are therefore transportation problems along with the ratio optimization of crucial parameters where the ratios are taken as objective functions. The fractional transportation problem was originally given by Swarup (1966) and since then it is widely implemented in logistics and supply chain management for cost minimization and services improvement. Later, several researchers have shared their views in this field like Gupta et al. (1993) presented a paradox in linear fractional transportation problems with mixed constraints. Kanchan et al. (1981) investigated transportation techniques in linear plus linear fractional programming. Khurana and Arora (2006) considered a transportation problem with an objective function as the sum of a linear and linear fractional function with restricted and enhanced flow and presented an algorithm to solve such problems. Sivri et al. (2011) proposed a solution approach for solving the transportation problem with the linear fractional objective function. Joshi and Gupta (2011) investigated the transportation problem with fractional objective function when the demand and supply quantities are varying. Jadhav and Doke (2016) studied fractional transportation problem considering cost and profit coefficients as fuzzy parameters. Liu (2016) also studied fractional transportation problem with fuzzy parameters.

In real life problems, the need to tackle transportation problems with multiple objectives arises as there may be several objective functions related to a single transportation problem. The objectives are generally conflicting and have different units and measuring scale. Multi-objective Transportation problems are thus formulated, as it is very difficult to combine these conflicting objectives into one overall utility function. Lee and Moore (1973) applied goal programming to find a solution for a multi- objective transportation problem. Zimmermann (1978) applied the fuzzy set theory concept with some suitable membership function to solve multi-objective transportation problems. Bit et al. (1992) used α - cut for a k-objective transportation problem which was fuzzified by fuzzy numbers to obtain a transportation problem in the fuzzy sense expressed in linear programming form. El-Wahed and Lee (2006) presented an interactive fuzzy goal programming approach to determine the preferred compromise solution for the multi-objective transportation problems. Zangiabadi and Maleki (2007) proposed a fuzzy goal programming approach to determine an optimal compromise solution for the multi-objective transportation problem by assuming that each objective function has a fuzzy goal. Still there is a need to develop more effective models for the fractional transportation problems that can deal with uncertainty.

Uncertainty generally prevails in many decision making problems, as the various parameters related to the problem are not certain and are treated as uncertain variables. Uncertain variables are a mixture of uncertainty and randomness. Uncertain measure is used to measure the degree of truth of an uncertain event. Liu (2007) first developed uncertainty theory. Later, Liu (2010) redefined the theory of uncertainty. Since then researchers have applied uncertainty theory in various fields. Gao (2011) studied shortest path problem with the arc length as uncertain parameter. Gao (2012) applied uncertainty theory to model single facility location problem. Zhang and Peng (2012, 2013) proposed uncertain programming model for Chinese postman problem with uncertain weights. Han et al. (2014) proposed the maximum flow problem of uncertain network. Ke et al. (2015) proposed uncertain random programming model for project scheduling problem. Liu (2015) presented uncertain multi-objective programming and uncertain goal programming.

Transportation problems may be considered as an uncertain problem, due to the presence of several uncertain factors leading to an immediate change in various parameters. Changes in transportation problems may occur due to change in sales, change in weather and road conditions, and change in preferences leading to uncertain demand, supply and cost parameters. Models considering uncertainty are thus developed to tackle transportation problems with several uncertain parameters, treated as uncertain random variables. Sheng and Yao (2012a) studied fixed charge transportation problems in uncertain environment. Later Sheng and Yao (2012b) proposed transportation problem model with cost and demands as uncertain parameters. Cui and Sheng (2013) developed an uncertain programming model for solid transportation problem. Mou et al. (2013) investigated transportation problems with uncertain truck times and unit costs. Guo et al. (2015) proposed a transportation problem with uncertain costs and random supplies. Yang et al. (2015) studied the concept of type-2 uncertain variables and applied it to solid transportation problems. Dalman (2016) studied STP having multi-items to be transported, under uncertain conditions. Zhang et al. (2016) developed an algorithm for solving fixed charge solid transportation problem in uncertain environment.

The uncertain programming models as compared to crisp models are considered as very complex models due to the presence of uncertainty. As per the knowledge of authors, no model has been presented till date for the fractional transportation problems with uncertain parameters. In this paper, we have proposed a multi-objective transportation problem with fractional objective functions, considering all the parameters, viz. objective functions coefficients, demands and availabilities as uncertain. Computationally, to reach the optimal solution for uncertain fractional transportation problem is very complicated. To deal with such complications, a crisp equivalent model is also discussed. Furthermore, some mathematical properties of the proposed model are discussed. As fractional transportation problem is widely applicable in logistic industries, hence the proposed approach is very beneficial to tackle difficult logistic problems.

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The rest of the paper is organized as follows. In section 2, we have reviewed some results and definitions in uncertainty theory and fractional programming problems. In section 3, the basic form of uncertain programming model is discussed. The uncertain multi-objective fractional transportation planning model is presented in section 4. An equivalent deterministic model for the proposed uncertain model is given in subsection 4.2 under section 4. For solving the multi-objective model obtained in section 5, fuzzy goal programming is discussed as a solution methodology. In section 5, the linear membership function, Fuzzy goal model and stepwise algorithm is described. In section 6, a numerical example is given to understand the applicability of the proposed model. Data set for the problem, model for the given problem and analysis of results is done under section 6. Finally, some conclusions are drawn regarding the developed model in section 7.

2. PRELIMINARIES

2.1. **Uncertainty Theory**

A lot of surveys showed that some imprecise quantities behave neither like randomness nor like fuzziness. This provides a motivation to invent another mathematical tool to model those imprecise quantities. In order to do so, an uncertainty theory was founded and became a branch of axiomatic mathematics (Liu, 2010). This section comprise of some basic concepts and results in uncertainty theory (Liu (2007, 2009, 2010, 2012)).

Definition 1 Assume Γ to be a non-empty set, and τ be a σ - algebra on Γ. Each element Λ in the σ - algebra τ is called an event. Set function P is called an uncertain measure if the following four axioms are satisfied by P.

Axiom 1: (Normality Axiom**)** $\mathrm{P}\left\{ \Gamma \right\} = 1.$

Axiom 2: (Duality Axiom) $\mathrm{P}\left\{ \Lambda \right\} + \mathrm{P}\left\{ \Lambda^c \right\} = 1$ for any $\Lambda \in \tau.$

Axiom 3: (Subadditivity Axiom) For every countable sequence of events $\left\{ \Lambda_i \right\} \in \tau,$ we have

$$\mathrm{P}\left\{ \bigcup_{i=1}^{\infty} \Lambda_i \right\} \le \sum_{i=1}^{\infty} \mathrm{P}\left\{ \Lambda_i \right\} \tag{1}$$

The triplet $\left(\Gamma, \tau, \mathrm{P}\right)$ is called an uncertainty space.

Axiom 4: (Product Axiom) (Liu, 2009) Let $\left(\Gamma_k, \tau_k, \mathrm{P}_k\right)$ be uncertainty spaces for $k = 1,2,3,\ldots$ Then the product uncertain measure P is an uncertain measure satisfying

$$\mathrm{P}\left\{ \prod_{k=1}^{\infty} \Lambda_k \right\} = \bigwedge_{k=1}^{\infty} \mathrm{P}_k \left\{ \Lambda_k \right\} \tag{2}$$

Where Λ_k are arbitrary events from τ_k , $\forall\ k = 1,2,3,\ldots$ respectively.

Definition 2 Let T be an index set and let $\left(\Gamma, \tau, \mathrm{P}\right)$ be an uncertainty space. An uncertain process is a measurable function from $\mathrm{T} \times \left\{\Gamma, \tau, \mathrm{P}\right\}$ to the set of real numbers, i.e., for each $t \in \mathrm{T}$ and any Borel set *B* of real numbers, the set

$$\left\{ \gamma \in \Gamma \,\middle|\, \mathrm{X}_t(\gamma) \in \mathrm{B} \right\}$$

is an event.

Definition 3 Let $\xi_1, \xi_2, \ldots, \xi_n$ be n independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2,\ \Phi_n$ respectively. If the function $f\left(x_1, x_2, \ldots, x_n\right)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then $\xi = f\left(\xi_1, \xi_2, \ldots, \xi_n\right)$ is an uncertain variable with an inverse uncertainty distribution.

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$$\Psi^{-1}\left(r\right) = f\left(\Phi_1^{-1}\left(r\right), \Phi_2^{-1}\left(r\right), \ldots, \Phi_m^{-1}\left(r\right), \Phi_{m+1}^{-1}\left(1-r\right), \ldots, \Phi_n^{-1}\left(1-r\right)\right) \tag{3}$$

The expected value of an uncertain variable is an average of the uncertain variable in the sense of uncertain measure. This is called the operational law of uncertain variables.

Definition 4 The expected value of an uncertain variable ξ is defined by

$$E\left[\xi\right] = \int_0^{\infty} \mathrm{P}\left\{\xi \geq x\right\} dx - \int_{-\infty}^{0} \mathrm{P}\left\{\xi \geq x\right\} dx \tag{4}$$

provided that at least one of the two integral is finite. And the variance of ξ is defined by

$$V\left[\xi\right] = E\left[\left(\xi - e\right)^2\right] \tag{5}$$

Definition 5 For a normal uncertain variable $N(e,\sigma)$, the inverse uncertain variable is

$$\Phi^{-1}(\alpha) = e + \frac{\sigma\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha} \tag{6}$$

2.2 Fractional Programming

In real life situation we came across such situations where it is needed to maximize or minimize a objective function which involves the ratio of two functions. The problems which involve ratio optimization are commonly called fractional programs. The term fractional programming was first introduced by Charnes & Cooper (1962). In this section some well known definitions and properties (Bajalinov, 2013) are reviewed.

The most general mathematical form of fractional linear programming is given as:

$$Q(x) = \frac{C(x)}{D(x)} = \frac{\sum_{j=1}^{n} c_j x_j}{\sum_{j=1}^{n} d_j x_j} \tag{7}$$

Which must be maximized (or minimized) subject to the constraints

$$\bar{x} \in S = \left\{ \bar{x} \in \bar{R} \middle| \bar{A}\,\bar{x} \leq = \geq \bar{b}, \bar{x} \geq 0 \right\} \tag{8}$$

Where $c_j, d_j \in R_n$ and $\bar{A} \in R_{n\times m}$, $\bar{b} \in R_m$ and S is assumed to be non-empty, convex and compact in R_n and further we also assumed that $\bar{x} \in S$ and $D(x) > 0 \ \forall \bar{x} \in S$.

Definition 6 For a Multi-Objective Fractional Transportation Problem

$$Q(x) = \frac{C_1(x)}{D_1(x)}, \frac{C_2(x)}{D_2(x)}, \ldots, \frac{C_n(x)}{C_n(x)} \Big| \, x \in S \tag{9}$$

a point $x^* \in S$ is said to be weakly efficient for problem (12) if and only if there is no $x \in S$ such that

$$\frac{C_i(x)}{D_i(x)} > \frac{C_i(x^*)}{D_i(x^*)}, \ \forall i = 1, \ldots, p \tag{10}$$

Definition 7 A point $x^* \in S$ is said to be strongly efficient for problem (12) if and only if there is no $x \in S$ such

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that

$$\frac{C_i(x)}{D_i(x)} \geq \frac{C_i(x^*)}{D_i(x^*)},\ \forall i = 1,\ldots,n \tag{11}$$

3. UNCERTAIN PROGRAMMING - THE BASIC FORM

The type of mathematical programming which involves uncertain variables is termed as Uncertain Programming. It is assumed that x is a decision vector and ξ is an uncertainty vector. Since the objective function $f(x,\xi)$ is uncertain in nature, hence we cannot minimize it directly we may minimize its expected value i.e.,

$$\underset{x}{Min}\, E\left[f\left(x,\xi \right) \right] \tag{12}$$

Since, we cannot define a crisp feasible set for the uncertain constraints $g_j\left(x,\xi\right) \leq 0,\ j = 1,2,\ldots,p$. In this case, it is naturally desired that the uncertain constraints hold with confidence levels $\alpha_1,\alpha_2,\ldots,\alpha_p$. Then, we obtained the following set of chance constraints,

$$P\left\{ g_j\left(x,\xi \right) \leq 0 \right\} \geq \alpha_j,\ j = 1,2,\ldots,p. \tag{13}$$

In order to obtain the minimum expected value subject to a set of chance constraints, Liu (2010) proposed the following uncertain programming model

$$\underset{x}{Min}\, E\left[f\left(x,\xi\right)\right] \tag{14}$$

Sub to:

$$\mathrm{P}\left\{g_j\left(x,\xi\right) \leq 0\right\} \geq \alpha_j, \qquad j = 1,2,\ldots,p \qquad (15)$$

Theorem 1 (Liu, 2009) Assume that $\xi_1,\xi_2,\ldots,\xi_n$ are independent uncertain variables with regular uncertainty distributions $\Phi_1,\Phi_2,\ldots,\Phi_n$, respectively. We also assume that f is strictly increasing with respect to $\xi_1,\xi_2,\ldots,\xi_m$, and strictly decreasing with respect to $\xi_{m+1},\xi_{m+2},\ldots,\xi_n$. Also, the constraint function g_j is strictly increasing and strictly decreasing with respect to $\xi_1,\xi_2,\ldots,\xi_{m_j}$ and $\xi_{m_j+1},\xi_{m_j+2},\ldots,\xi_n$ respectively for $j = 1,2,\ldots,p$. The crisp equivalence model for the uncertain model (17-18) is as follows,

$$\underset{x}{Min} \int_0^1 f(x,\Phi_1^{-1}(t),\ldots,\Phi_m^{-1}(t),\Phi_{m+1}^{-1}(1-t),\ldots,\Phi_n^{-1}(1-t))\ dt \tag{16}$$

Sub to:

$$g_j(x,\Phi_1^{-1}(\alpha_j),\ldots,\Phi_{m_j}^{-1}(\alpha_j),\Phi_{m_j+1}^{-1}(1-\alpha_j),\ldots,\Phi_n^{-1}(1-\alpha_j)),\quad \forall\, j = 1,2,\ldots p \tag{17}$$

4. MULTI-OBJECTIVE FRACTIONAL PROGRAMMING TRANSPORTATION PROBLEM WITH UNCERTAINTY

Fractional programming problem deals with the situation when the objective function(s) is(are) given in the form of ratios of two linear or non-linear functions, that are to be optimized under any given set of constraints. Some cases of objectives for transportation problems belonging to the fractional programming category may include the minimization of total actual cost/standard cost, actual time/standard time, actual deterioration/standard deterioration, risk assets/capital etc. The fractional programming problem with linear functions is called linear fractional transportation problem. In most of the real world cases, the decision maker has to encounter more than one objective under any common set of constraints. Keeping in view the involvement of several objectives, in this

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paper we are proposing a multi-objective transportation problem with fractional objectives. All the parameters in the proposed model are assumed to be uncertain.
For modeling the multi-objective fractional transportation problem, the following assumptions such as index set, decision variable and parameters are considered.

Nomenclature	
Index set	
i	Index for sources, for all $i = 1,2,...,m$
j	Index for destinations, for all $j = 1,2,...,n$
K	Index for objectives, for all $K = 1,2,...,k$
Decision variable	
x_{ij}	Amount of product transported from source i to destination j
Parameters	
C_K	Coefficient vector for the numerator of the K^{th} objective
D_K	Coefficient vector for the denominator of the K^{th} objective
$c_{ij}^{(K)}$	Elements of the set C_K for the K^{th} objective
$d_{ij}^{(K)}$	Elements of the set D_K for the K^{th} objective
$\overline{x}$	Set of the decision variables x_{ij}
a_i	Amount of the product available at source i
b_j	Demand of the product at destination j
$\tilde{a}_i$	Uncertain availability at source i
$\tilde{b}_j$	Uncertain capacity of destination j
α_i	Confidence level for 1st constraint
β_j	Confidence level for 2nd constraint
$\xi_{ij}^{(K)}$	Uncertain coefficient for the numerator of the K^{th} objective
$\delta_{ij}^{(K)}$	Uncertain coefficient for the denominator of the K^{th} objective
$\Phi_{\xi_{ij}^{(K)}}$	Uncertainty distribution for independent uncertain variable $\xi_{ij}^{(K)}$
$\Phi_{\delta_{ij}^{(K)}}$	Uncertainty distribution for independent uncertain variable $\delta_{ij}^{(K)}$
$\Phi_{\tilde{a}_i}$	Uncertainty distribution for independent uncertain variable $\tilde{a}_i$
$\Phi_{\tilde{b}_j}$	Uncertainty distribution for independent uncertain variable $\tilde{b}_j$

4.1 Uncertain Model:

The fractional programming model (18-21), presented below is a general model for multi-objective fractional transportation problem.

$$Min \quad Z_K(\overline{x}) = \frac{C_K(\overline{x})}{D_K(\overline{x})} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{(K)} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{(K)} x_{ij}}, \qquad K = 1,2,...,k \tag{18}$$

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Sub to:

$$\sum_{j=1}^{n} x_{ij} \le a_i, \qquad i = 1,2,\ldots,m \tag{19}$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j, \qquad j = 1,2,\ldots,n \tag{20}$$

$$x_{ij} \ge 0, \qquad i = 1,2,\ldots,m, \qquad j = 1,2,\ldots,n \tag{21}$$

The constraint equation (19) ensures the limitation on the products to be transported because of some finite availability. Constraint (20) is the demand constraint that ensures the fulfillment of the demand of the product at each destination.

All the quantities involved in the model (18-21) are assumed to be known and certain. But some sort of uncertainty always exists in the real world situations. To overcome this problem of the above model (18-21), we now assume all the parameters to be independent and uncertain. The new model thus formed, assuming all the parameters as uncertain, can be said as, an uncertain multi-objective fractional transportation model. The presence of uncertain variables makes the model complex, so the operation of direct minimization cannot be done. That gives rise to the study of some techniques that can deal with the uncertainty. Expected-constrained programming by Liu (2009) is one among them. In this technique the minimization (or maximization) of expected value of the objective function is done under the given chance constraints. Here, we are dealing with a fractional objective function, which is simply a ratio of two independent functions. Hence in the case of fractional objectives, instead of taking the expected value of the objective function, we can take expectation of the numerator and denominator functions separately. The following model is the expected-constrained programming model for model (18-21).

$$Min\ Z_K = \frac{E\left[\sum_{i=1}^{m}\sum_{j=1}^{n} \xi_{ij}^{(K)} x_{ij}\right]}{E\left[\sum_{i=1}^{m}\sum_{j=1}^{n} \delta_{ij}^{(K)} x_{ij}\right]}, \qquad K = 1,2,\ldots,k \tag{22}$$

Sub to:

$$P\left\{\sum_{j=1}^{n} x_{ij} - \tilde{a}_i \le 0\right\} \ge \alpha_i, \qquad i = 1,2,\ldots,m \tag{23}$$

$$P\left\{\tilde{b}_j - \sum_{i=1}^{m} x_{ij} \le 0\right\} \ge \beta_i, \qquad j = 1,2,\ldots,n \tag{24}$$

$$x_{ij} \ge 0, \qquad i = 1,2,\ldots,m, \qquad j = 1,2,\ldots,n \tag{25}$$

4.2 Equivalent Crisp Model

Mathematically, there arise a lot of complexities in solving the uncertain fractional programming problem. To overcome these complexities, we have discussed a crisp equivalent model for the above proposed fractional programming problem (22-25).

Let us assume that, the functions $\Phi_{\xi_{ij}^{(K)}}$, $\Phi_{\delta_{ij}^{(K)}}$ and $\Phi_{\tilde{b}_j}$ are strictly increasing with respect to the parameters $\xi_{ij}^{(K)}$, $\delta_{ij}^{(K)}$ and $\tilde{b}_j$ respectively. Also, the function $\Phi_{\tilde{a}_i}$ is strictly decreasing with respect to $\tilde{a}_i$. The equivalent crisp model has now been obtained, using the definitions given in section 3 and theorems.

$$Min\ Z_K = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \int_0^1 \Phi^{-1}_{\xi_{ij}^{(K)}}}{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \int_0^1 \Phi^{-1}_{\delta_{ij}^{(K)}}}, \qquad K = 1,2,\ldots,k \tag{26}$$

Sub to:

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$$\sum_{j=1}^{n} x_{ij} - \Phi_{\tilde{a}_i}^{-1}\left(1-\alpha_i\right) \le 0, \qquad i = 1,2,\ldots,m \tag{27}$$

$$\Phi_{\tilde{b}_j}^{-1}\left(\beta_j\right) - \sum_{i=1}^{m} x_{ij} \le 0, \qquad j = 1,2,\ldots,m \tag{28}$$

$$x_{ij} \ge 0, \qquad i = 1,2,\ldots,m, \qquad j = 1,2,\ldots,n \tag{29}$$

The model (26-29) above is a deterministic multi-objective fractional programming model, and thus can be solved by applying any of the existing technique. In this paper, we have used fuzzy goal programming (FGP) as the solution technique.

5. FUZZY GOAL PROGRAMMING AS A SOLUTION METHOD

In this section, a fuzzy approach by Zimmermann (1978) for multi-objective linear programming problems is discussed.

5.1 Membership Function

For any fuzzy set A defined as $\left\{x, \mu_A(x) \mid x \in A\right\}$ where $\mu_A : A \to [0,1]$ is defined as the membership function of A, whereas $\mu_A(x)$ is the degree of membership to which x belongs to A. From among the various existing categories of membership functions in literature, such as linear, hyperbolic, piecewise-linear etc. In this paper, linear membership functions are adopted for computational simplicity. For the minimization objective functions, the membership grade $\mu_A(x)$ characterizing the objectives $Z_K(x)$ decrease linearly from 1 at worst values (Z_K^U) to 0 at best values (Z_K^L). Then the corresponding linear membership function $\mu_K(Z_K(x))$ is defined as

$$\mu_K(Z_K(x)) = \begin{cases} 0 & ;\ Z_K(x) \ge Z_K^0 \\ \dfrac{Z_K(x) - Z_K^0}{Z_K^1 - Z_K^0} & ;\ Z_K^0 \ge Z_K(x) \ge Z_K^1 \\ 1 & ;\ Z_K(x) \le Z_K^1 \end{cases} \tag{30}$$

Figure 1 below illustrates the shape of the linear membership function defined above.

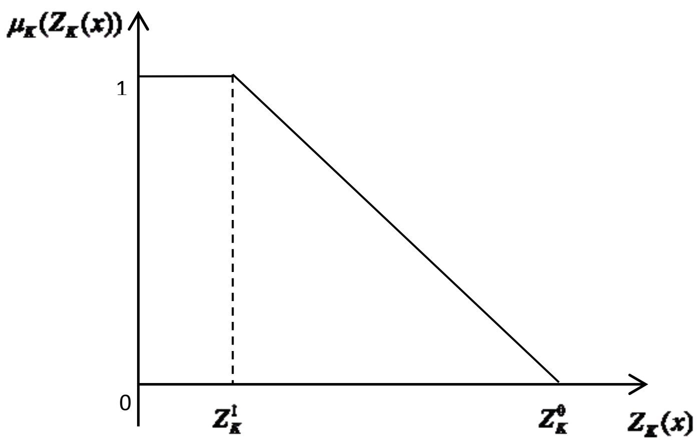

Figure 1. Linear membership function

5.2 Fuzzy Goal Programming Model

Using the above linear membership functions $\mu_K(Z_K(x))$, $K = 1, 2, \ldots, k$, and the fuzzy decision of Bellman and Zadeh (1970), the max-min multi-objective programming model can be given as

$$Max \quad \underset{K=1,2,\ldots,k}{Min} \left\{ \mu_K(Z_K(x)) \right\} \tag{31}$$

Sub to

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \qquad i = 1, 2, \ldots, m \tag{32}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j, \qquad j = 1, 2, \ldots, n \tag{33}$$

$$x_{ij} \geq 0, \qquad i = 1, 2, \ldots, m, \qquad j = 1, 2, \ldots, n \tag{34}$$

Now using Max-Min operator, the Zimmerman (1978) model for the proposed fractional programming transportation problem can be given as

$$Max \quad \lambda \tag{35}$$

Sub to:

$$\lambda \leq \mu_K(Z_K(x)), \qquad K = 1, 2, \ldots, k \tag{36}$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \qquad i = 1, 2, \ldots, m \tag{37}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j, \qquad j = 1, 2, \ldots, n \tag{38}$$

$$0 \leq \lambda \leq 1 \tag{39}$$

$$x_{ij} \geq 0, \qquad i = 1, 2, \ldots, m, \qquad j = 1, 2, \ldots, n \tag{40}$$

where λ is an auxiliary variable.

5.3 Stepwise Algorithm for Fuzzy Goal Programming

To solve multi-objective fractional transportation problem using Fuzzy goal programming approach, the following steps can be followed;

Step 1. Solve the multi-objective fractional transportation problem for each of the objectives individually at a time and ignoring others, as a simple single objective fractional transportation problem.

Step 2. Compute the values of each individual objectives that are derived in step 1.

Step 3. For each individual objective find the best (Z_K^1) and the worst (Z_K^0) values that corresponds to the set of solutions. Where (Z_K^1) and (Z_K^0) are the highest and lowest tolerance level for K^{th} fuzzy goal.

Step 4. Define membership function $\mu_K(Z_K)$ for K^{th} objective function, as defined in the above in equation (30). Construct the Fuzzy goal programming model using an auxiliary variable λ as defined in (35-40).

Step 5. Solve the crisp model so formed in Step 4.

The solution so obtained at Step 5 will be the compromise optimal solution for the multi-objective fractional transportation problem.

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6. NUMERICAL EXAMPLE

To demonstrate the proposed model, let us consider the following example of a transportation problem with three fractional objectives.

6.1 Problem Description and Data

For any logistic organization or firm, let the objectives are to minimize the actual cost/ standard cost, actual time/ standard time and actual deterioration/ standard deterioration involved in transporting any item(s) from various source points to several destinations. The restrictions that are being active are the limited availability at the source points and the minimum demand that is to be satisfied at the destinations. All the variables involved are assumed to be normal uncertain variable with parameter $N(\mu,\sigma)$.

The total actual cost (C_A) and the standard cost (C_S) with Normal distribution parameters $N(\mu_{ij}^{(C_A)},\sigma_{ij}^{(C_A)})$ and $N(\mu_{ij}^{(C_S)},\sigma_{ij}^{(C_S)})$ are:

$$C_A = \begin{bmatrix} (20,2) & (18,1.5) & (18,2) & (13,1) \\ (19,1) & (13,1) & (16,1.5) & (18,2) \\ (15,2) & (11,2) & (17,1) & (12,1) \\ (14,1.5) & (14,1.5) & (16,2) & (13,1.5) \end{bmatrix} C_S = \begin{bmatrix} (18,1) & (16,2) & (19,1) & (12,2) \\ (20,1.5) & (15,1.5) & (15,1) & (18,1.5) \\ (16,2) & (12,2) & (15,2) & (10,1.5) \\ (13,2) & (12,1) & (16,1) & (14,1) \end{bmatrix}$$

The total actual time (T_A) and the standard time (T_S) with Normal distribution parameters $N(\mu_{ij}^{(T_A)},\sigma_{ij}^{(T_A)})$ and $N(\mu_{ij}^{(T_S)},\sigma_{ij}^{(T_S)})$ are:

$$T_A = \begin{bmatrix} (30,2) & (34,1) & (34,1.5) & (34,1) \\ (26,1.5) & (24,2) & (31,1) & (29,1) \\ (21,1.5) & (20,2) & (26,1.5) & (29,1.5) \\ (21,1.5) & (22,1) & (25,2) & (31,1.5) \end{bmatrix} T_S = \begin{bmatrix} (28,1) & (33,2) & (32,1) & (35,1) \\ (25,1.5) & (25,1.5) & (32,1.5) & (28,1) \\ (22,1) & (20,1.5) & (25,2) & (28,1.5) \\ (21,1) & (20,1) & (26,2) & (30,2) \end{bmatrix}$$

The total actual deterioration (D_A) and the standard deterioration (D_S) with Normal distribution parameters $N(\mu_{ij}^{(D_A)},\sigma_{ij}^{(D_A)})$ and $N(\mu_{ij}^{(D_S)},\sigma_{ij}^{(D_S)})$ are:

$$D_A = \begin{bmatrix} (40,2) & (34,1.5) & (37,1) & (28,2) \\ (38,1) & (28,1) & (37,1.5) & (40,1.5) \\ (42,1) & (39,1) & (30,1.5) & (41,1) \\ (29,2) & (38,1) & (32,2) & (32,1.5) \end{bmatrix} D_S = \begin{bmatrix} (38,1) & (35,2) & (35,1.5) & (32,1) \\ (38,1) & (30,1.5) & (34,1.5) & (36,1) \\ (40,2) & (36,1) & (32,1) & (37,2) \\ (33,1) & (35,1.5) & (32,1) & (32,1.5) \end{bmatrix}$$

The random supplies with Normal distribution parameters $N(\mu_i,\sigma_i)$ are:

$$a_i = \begin{bmatrix} (25,1.5) & (30,1.5) & (32,2) & (28,2) \end{bmatrix}$$

The random demands with Normal distribution parameters $N(\mu'_j,\sigma'_j)$ are:

$$b_j = \begin{bmatrix} (10,1.5) & (14,1) & (22,1) & (18,1) \end{bmatrix}$$

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6.2 Model for the Given Problem

The uncertain multi-objective fractional programming model for this problem is given as

$$Min\ Z_1 = \frac{C_A.\overline{x}}{C_S.\overline{x}} = \frac{\sum_{i=1}^{4}\sum_{j=1}^{4} x_{ij}\mu_{ij}^{(C_A)}}{\sum_{i=1}^{4}\sum_{j=1}^{4} x_{ij}\mu_{ij}^{(C_S)}} \tag{41}$$

$$Min\ Z_2 = \frac{T_A.\overline{x}}{T_S.\overline{x}} = \frac{\sum_{i=1}^{4}\sum_{j=1}^{4} x_{ij}\mu_{ij}^{(T_A)}}{\sum_{i=1}^{4}\sum_{j=1}^{4} x_{ij}\mu_{ij}^{(T_S)}} \tag{42}$$

$$Min\ Z_3 = \frac{D_A.\overline{x}}{D_S.\overline{x}} = \frac{\sum_{i=1}^{4}\sum_{j=1}^{4} x_{ij}\mu_{ij}^{(D_A)}}{\sum_{i=1}^{4}\sum_{j=1}^{4} x_{ij}\mu_{ij}^{(D_S)}} \tag{43}$$

Sub to:

$$\sum_{j=1}^{4} x_{ij} - \left[\mu_i + \frac{\sigma_i\sqrt{3}}{\pi}\ln\frac{1-\alpha_i}{\alpha_i}\right] \le 0, \qquad i = 1,2,3,4 \tag{44}$$

$$\left[\mu'_j + \frac{\sigma'_j\sqrt{3}}{\pi}\ln\frac{\beta_j}{1-\beta_j}\right] - \sum_{i=1}^{4} x_{ij} \le 0, \quad j = 1,2,3,4 \tag{45}$$

$$x_{ij} \ge 0, \qquad i = 1,2,3,4, \qquad j = 1,2,3,4, \tag{46}$$

Where α_i and β_j are confidence level for the chance constraints.

The Model given above is deterministic in nature, hence any multi-objective technique for attaining the compromise optimal solution can be applied.

6.3 Results and Analysis

The fuzzy goal programming approach described in section 5 is applied to reach the compromise optimal solution for the given multi-objective problem with fractional objectives. Following the stepwise algorithm for solving through Fuzzy goal programming approach, first of all individual best and worst values for each objective are obtained. The best and worst values so obtained acts as the upper and lower tolerances. Table 1 represents the lower and upper tolerances for the three objectives for a set of values of α_i and β_j.

Table 1 Individual best and worst values for different confidence levels

Confidence Level (α_i,β_j)	Objective I		Objective II		Objective III	
	Lower tolerance Z_1^0	Upper tolerance Z_1^1	Lower tolerance Z_2^0	Upper tolerance Z_2^1	Lower tolerance Z_3^0	Upper tolerance Z_3^1
0.1, 0.1	1.144201	0.9139062	1.065432	0.9616064	1.093462	0.9004874
0.2, 0.2	1.144220	0.9138967	1.065471	0.9616013	1.093478	0.9004074
0.3, 0.3	1.144232	0.9138853	1.065496	0.9615980	1.093497	0.9003568

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0.4, 0.4	1.144241	0.9138716	1.065515	0.9615955	1.093520	0.9003167
0.5, 0.5	1.144250	0.9138544	1.065533	0.9615932	1.093548	0.9002809
0.6, 0.6	1.144257	0.9138321	1.065550	0.9615909	1.093586	0.9002461
0.7, 0.7	1.144266	0.9138012	1.065567	0.9615886	1.093637	0.9002093
0.8, 0.8	1.144276	0.9137536	1.065588	0.9615858	1.093717	0.9001658
0.9, 0.9	1.144290	0.9136617	1.065618	0.9615817	1.093872	0.9001031

The values of Lower and Upper tolerances Z_K^0 and Z_K^1 respectively follow a trend along with the change in the confidence level. With increase in the value of α_i and β_j, the lower tolerances increase and the upper tolerances decrease for each objective function.
Now the fuzzy goal programming model with the attained individual lower and upper tolerances is given as

$$Max \quad \lambda \tag{47}$$

Sub to

$$\frac{Z_K(x) - Z_K^0}{Z_K^1 - Z_K^0} \geq \lambda \qquad \forall K = 1,2,3 \tag{48}$$

$$\sum_{j=1}^{4} x_{ij} - \left[\mu_i + \frac{\sigma_i\sqrt{3}}{\pi}\ln\frac{1-\alpha_i}{\alpha_i}\right] \leq 0, \qquad i = 1,2,3,4 \tag{49}$$

$$\left[\mu_j' + \frac{\sigma_j'\sqrt{3}}{\pi}\ln\frac{\beta_j}{1-\beta_j}\right] - \sum_{i=1}^{4} x_{ij} \leq 0, \quad j = 1,2,3,4 \tag{50}$$

$$x_{ij} \geq 0, \qquad i = 1,2,3,4, \qquad j = 1,2,3,4, \tag{51}$$

On solving the problem so formed, Compromise solution for the original uncertain problem is obtained. Table 2 represents the compromise optimal solutions and their corresponding actual and standard costs for different values of the confidence levels.

Table 2 Compromise optimal values with actual and standard values of objective parameters

Confidence Level (α_i, β_j)	Objective I			Objective II			Objective III		
	Actual cost (C_A)	Standard cost (C_S)	Z_1^*	Actual time (T_A)	Standard time (T_S)	Z_2^*	Actual det. (D_A)	Standard det. (D_S)	Z_3^*
0.1, 0.1	1042.58	1083.29	0.96241	1981.22	2026.92	0.97745	2196.56	2334.18	0.94104
0.2, 0.2	1074.54	1116.38	0.96252	2041.12	2086.59	0.97821	2264.74	2406.36	0.94114
0.3, 0.3	1095.78	1138.38	0.96258	2080.95	2126.24	0.97869	2310.07	2454.35	0.94121
0.4, 0.4	1113.21	1156.43	0.96262	2113.62	2158.76	0.97908	2347.27	2493.69	0.94128
0.5, 0.5	1129.19	1172.98	0.96266	2143.58	2188.59	0.97943	2381.37	2529.79	0.94133
0.6, 0.6	1145.18	1189.56	0.96269	2173.57	2218.43	0.97977	2415.52	2565.90	0.94139
0.7, 0.7	1162.61	1207.63	0.96272	2206.26	2250.95	0.98014	2452.75	2605.26	0.94146
0.8, 0.8	1183.89	1229.70	0.96274	2246.17	2290.62	0.98059	2498.21	2653.28	0.94155
0.9, 0.9	1215.91	1262.94	0.96276	2306.26	2350.32	0.98125	2566.67	2725.56	0.94170

The values of the fractional objectives as given in table 2, can be seen increasing with increase in the values of confidence levels α_i and β_j from 0.1 to 0.9 both. The actual and the standard values for all the three objectives i.e. cost, time and deterioration has also increased with the confidence levels. The decision maker may define the confidence level according to situation of the concerned firm or organization.

The compromise optimal transportation scheme for the goods is presented in table 3. A sensitivity analysis has been carried out to check the effects of the change of the confidence level α_i and β_j.

Table 3. Compromise optimal scheme for the transportation of goods

Confidence Level (α_i, β_j)	Compromise optimal transportation policy (x^*)
0.1, 0.1	x_{13} =5.3699, x_{14} =14.8137, x_{22} =28.1836, x_{41} =8.1836, x_{43} =15.4192, x_{44} =1.9754
0.2, 0.2	x_{13} =5.8193, x_{14} =15.0346, x_{22} =28.8540, x_{41} =8.8540, x_{43} =15.4167, x_{44} =2.2013
0.3, 0.3	x_{13} =6.1199, x_{14} =15.1796, x_{22} =29.2996, x_{41} =9.2996, x_{43} =15.4131, x_{44} =2.3534
0.4, 0.4	x_{13} =6.3705, x_{14} =15.2943, x_{22} =29.6648, x_{41} =9.6648, x_{43} =15.4060, x_{44} =2.4822
0.5, 0.5	x_{13} =6.5971, x_{14} =15.4029, x_{22} =30.0000, x_{41} =10.0000, x_{43} =15.4029, x_{44} =2.5971
0.6, 0.6	x_{13} =6.8282, x_{14} =15.5070, x_{22} =30.3352, x_{41} =10.3352, x_{43} =15.3952, x_{44} =2.7165
0.7, 0.7	x_{13} =7.0826, x_{14} =15.6178, x_{22} =30.7004, x_{41} =10.7004, x_{43} =15.3843, x_{44} =2.8492
0.8, 0.8	x_{13} =7.3971, x_{14} =15.7489, x_{22} =31.1460, x_{41} =11.1460, x_{43} =15.3669, x_{44} =3.0151
0.9, 0.9	x_{13} =7.8789, x_{14} =15.9375, x_{22} =31.8164, x_{41} =11.8164, x_{43} =15.3320, x_{44} =3.2734

7. CONCLUSIONS

The main contribution of this paper is to develop a multi-objective fractional transportation planning decision model via uncertainty theory. Considering the vagueness among the parameters, that mostly exist in real life situations, we have assumed all the parameters as independent uncertain variables. As, the models with uncertain variables are often complex to deal with, hence the concept of expected constraint programming is applied to develop the model. An equivalent crisp or deterministic model is also discussed for the proposed transportation model. Fuzzy goal programming technique (max-min approach) is discussed to solve the obtained multi-objective fractional transportation model. At last, to demonstrate the applicability of the presented model a numerical illustration is also given. The results and findings are discussed in detail and the sensitivity analysis for the variation in the confidence levels is also given. As fractional programming has a lot of scope in the modeling of a wide range of real life situations, like information theory problem, inventory operations problem, investments allocation problems, repartition problems etc. Hence, in future the proposed uncertain multi-objective model can be extended to deal with the above mentioned real life problems.

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