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Pricing Strategy with Exponential Declining Demand Using Preservation Cost

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Abstract: The inventory model of seasonal products is developed in which the deterioration controlled by investing some cost of item preservation technology. In diminishing market, price sensitiveness analyzed with an exponentially declining demand. The object is to optimize the total profit per unit time, optimal selling price, length of replenishment cycle and preservation technology investment. We have shown that there exist an optimal selling price, and length of replenishment cycle that maximize the total profit. Also, we have proved the total profit per unit time is a concave function of the preservation technology investment. Finally, the model is illustrated by numerical example and fallowed by sensitive analysis.
Keyword — inventory, exponentially declining demand, preservation cost, deterioration, price, replenishment cycle.

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1. INTRODUCTION

Items incur a gradual loss in quality or quantity over time is called deterioration, and such type of item is called deteriorating items. Most of the existing inventory model in the literature assumes that items can be stored indefinitely to meet the future demands. However, certain types of commodities either deteriorate or become obsolete in the course of time and hence it's unstable. For example, the commonly used goods like fruits, vegetable, meat, perfumes, alcohols, one kept in cold storage as they do not loss their quality. Hence the cold storage is a kind of preservation technology which incurred a cost or cost of preservation technology. In poultry form hen and duck are kept in hot temperature because in low temperature they are died. In this case heater or thermo-wave is treated as preservation technology and this cost is known preservation technology investment cost. In view of this we have designed an Inventory model by incorporating cost of preservation technology.

Mostly, in diminishing market it happens that when demand declines then managers have to put their efforts to uplift the sale through media and pricing policy. This conflict also becomes a source of motivation for the dynamic behavior based study of the inventory system. He and Huang (2013) studied a model of seasonal product whose deterioration rate can be controlled by investing on the preservation efforts. This model studied both the preservation technology investment and pricing strategies of deteriorating seasonal product. Zhang *et al.* (2014) consider a model of deteriorating items in which the demand is a linear function of selling price and the deterioration also controlled by using preservation technology investment. In this study they assume that the inventory system involves a single deteriorating item over an infinite planning horizon.

Blackburn and Scudder (2009) studied the optimal control of warehouse temperature under warehouse capacity constraints. They also proved that it is beneficial to share the inventory between supply chain members. Kouki *et al.* (2010) reveals that a continuous temperature control policy could be efficient to the inventory management. Khedlekar et al. (2014) formulated a production inventory model for deteriorating item with production disruption and analyzed the system under different situations. Chandel and Khedlekar (2013) designed an integrated inventory model to optimize the total expenditure of warehouse set-up. Shula *et al.* (2012) presented an inventory model for deteriorating item in which demand is both time and price dependent, holding cost is also a time varying parameter. If demand of products is less price sensitive, optimal profit will be more but permit less price setting. Khedlekar and Shukla (2012) applied the concept for logarithmic demand and simulated the result for various businesses. The out comes of the study is that 'β' is most significant parameter affects optimal profit and respective number of price settings.

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Chen and Zhang (2010) considered a three-echelon supply chain system which consists of suppliers, manufacturer and customers under demand disruptions. Furthermore, an improved analytical hierarchy process (AHP) studied to select the best supplier, based on quantitative factors such as the optimal long-term total cost obtained through the simulated annealing method under demand disruptions and qualitative factors such as quality and service. The objective is to minimize the total cost under different demand disruption scenarios. Roy and Choudhury (2011) introduced an economic production lot size model, where production rate depends on stock and selling price both. Khedlekar and Namdeo (2015) modeled the strategy for sold out the entire stock at the end of the season by reducing the price. Balkhi and Bakry (2009) considered a dynamic inventory model of deteriorating items in which production rate, demand and the deterioration rates are function of time. Both inflation and time value of money are taken into account. Choudhury and Mukherjee (2011) considered right tail area of the waiting time distribution as the parametric function of arrival rate and service rate to be estimate in the model. Widyadana and Wee (2007) developed a deteriorating inventory model by considering price dependent demand and apply the markdown policy to increase profit.

Giri *et al.* (2003) extended the economic lot scheduling problem where the production follows a normal distribution. Sarkar and Moon (2011) considered a classical EPQ model with stochastic demand under the effect of inflation. The model is described by considering a general distribution function. Recent contribution in some EOQ models is a source of esteem important like Shukla and Khedlekar (2015), He and He (2010) and Zhang *et al.* (2014). Chang and Dye (1999) presented an inventory model for deteriorating items with constant deterioration rate and time varying demand with the assumption that the shortages were partially backlogged and the backlogging rate was a decreasing function of the waiting time. Dye and Hsieh (2012) extended Hsu et al's (2010) model and formulated an inventory model with a time varying rate of deterioration. The main objective of this research is to find the optimal replenishment time and preservation technology investment simultaneously. Giri *et al.* (1996) formulated an EOQ model for deteriorating items with time varying demand and costs. Goswami and Chaudhury(1991) investigated an EOQ model for deteriorating items with constant deterioration rate, linearly changing demand rate over a fixed planning horizon. There is a lot of inventory literature about deteriorating items under different conditions like Alamri and Balkhi (2007), Chung and Huang (2007), Gupta et al. (2011), Musa and Sani (2012), Ouyang *et al.* (2005), Khedlekar *et al.* (2012, 2016), Kumar and Sharma (2012b) and so on.

In most of the preceding research it is assumed that the deterioration rate is constant. But deterioration rate can never be constant, it can be reduced by applying some technique of preservation. The preservation technology investment will lead to an additional cost that we have to bear. Here we assume that if we invest preservation cost 'u' then the reduced deterioration rate become (1- f(u)). Despite the deterioration rate, the demand rate is another important factor to consider. In most of the literature the demand is assumed to be a constant, but in all the situations demand can not be constant. It is very sensitive factor related to any business management. The market price is highly related to market demand. If the market price uplift, the market demand will decrease and whereas the lower selling price will raise the demand rate. Since the time also affects the demand, so here we assume the price sensitive, time dependent, exponential decline demand. Since the time horizon is taken infinite in this study so we need to determine the optimal time for replenishment. The main goal of this study is to determine the optimal selling price, the optimal length of replenishment cycle and optimal preservation technology investment simultaneously.

The rest of the paper is organized as follow: In section 2, the assumption and notations are presented. In section 3, mathematical model to maximize the total profit per unit time is established. In section 3, a proposition presented to show that 'p' is the optimal price and 'T' is the replenishment time which maximizes the profit function $\Pi(p,T,u)$. An another proposition presented to show that there exists a unique optimal preservation technology investment u^* that maximizes $\Pi(p,T,u)$. In section 4, numerical examples and sensitive analysis are carried out to illustrate the model. Finally, we draw a conclusion in section 5.

2. ASSUMPTIONS AND NOTATIONS

Here we assume that the inventory system contains a single deteriorating item over an infinite time horizon and there is no replenishment during the period under consideration but it occurs instantaneously. Here lead time is zero and shortages are not allowed. The proportion of reduced deterioration rate f(u) is considered to be a continuous, increasing and concave function of capital investment in preservation technology 'u', i.e. $f'(u) > 0$, $f''(u) < 0$ and $f(0) = 0$. Price sensitive, non negative and exponentially decline demand rate are considered in this model, where $\alpha \geq 0$, is an initial demand and $\beta \geq 0$, is a price sensitive parameter. In this model the basic assumptions are

- K Replenishment cost per order.

- c Purchasing cost per unit.
- p Market price per unit, where p.
- h Inventory holding cost unit per unit time.
- T Length of replenishment cycle, and at the end of that the inventory is zero.
- I(t) Inventory level at time t.
- Q Order quantity per cycle.
- D(p,t) Demand rate is function of price and time both.
- θ Deterioration rate.
- u Preservation technology investment per unit time to reduce the deterioration rate.
- $f(u)$ Proportion of reduced deterioration rate, where $0 \leq f(u) \leq 1$.
- $\Pi(p,T,u)$ Total profit per unit time.

3. MATHEMATICAL MODEL

Suppose an enterprise runs a single deteriorating item with the deterioration rate θ that faces the price-sensitive decline demand rate D(p,t). The proportion of reduced deterioration rate $f(u)$ exists when the enterprise invests the items preservation cost u. The inventory level $I(t)$ can be presented according as fig. 1 and formulated as

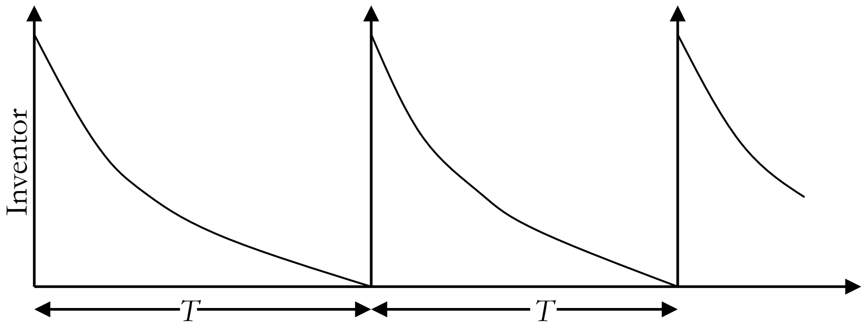

Figure 1. Graphical representation of the inventory system

$$\frac{\partial I(t)}{\partial t} + \theta(1-f(u))I(t) = -D(p,t), \text{ where } D(p,t) = \alpha e^{-at} - \beta p,\ 0 \leq t \leq T \tag{1}$$

By the boundary condition I(t) = 0, we get the inventory level

$$I(t) = \frac{-\alpha e^{-at}}{\theta(1-f(u))-a} + \frac{\beta p}{\theta(1-f(u))} + \frac{ae^{\theta(1-f(u))(T-t)-aT}}{\theta(1-f(u))-a} - \frac{\beta p e^{\theta(1-f(u))(T-t)}}{\theta(1-f(u))} \tag{2}$$

Therefore, the on hand inventory can be determined as

$$Q = \frac{\alpha e^{(\theta(1-f(u))-a)T} - 1}{\theta(1-f(u))-a} - \frac{\beta p e^{\theta(1-f(u))T} - 1}{\theta(1-f(u))} \tag{3}$$

The total profit Π(p,T,u) of the season could be formulated as

$$\begin{aligned}\Pi(p,T,u) = {} & Sales\ \mathrm{Re}venue\ (R) - Purcha\sin g\ Cost(C_p) - Ordering\ /\ \mathrm{Re}plenishment\ Cost(C_o) \\ & - Inventory\ Holding\ Cost(C_h) - \mathrm{Pr}eservation\ Cost(I_o)\end{aligned} \tag{4}$$

- **Sales Revenue -** The total revenue in time T formulated as
$$R = \frac{-\alpha p(e^{-aT}-1)}{a} - \beta p^2 T$$
- **Purchasing Cost -** The purchasing cost is
$$c_p = \frac{c\alpha e^{(\theta(1-f(u))-a)T} - 1}{\theta(1-f(u))-a} - \frac{c\beta p e^{\theta(1-f(u))T} - 1}{\theta(1-f(u))}$$

- **Holding Cost -** The formulation of the total inventory holding cost is
  $c_h = h\int_0^T I(t)\,dt$
- **Preservation Cost -** The preservation technology investment depends on the cycle length. For the cycle T, the preservation technology investment cost is
  $I_o = uT$ I_0
- **Replenishment Cost -** The replenishment or ordering cost is
  $C_o = K$

Hence, by equation (4), the total profit function per unit time is

$$\prod(p,T,u) = \frac{-\alpha p\left(e^{-aT}-1\right)}{aT} - \beta p^2 - u - \frac{K}{T} - \frac{c\alpha\left(e^{(x-a)T}-1\right)}{(x-a)T} + \frac{c\beta p\left(e^{xT}-1\right)}{xT}$$
$$-\frac{\alpha h\left(e^{-aT}-1\right)}{a(x-a)T} - \frac{h\beta p}{x} - \frac{h\alpha e^{-aT}\left(e^{xT}-1\right)}{x(x-a)T} + \frac{h\beta p\left(e^{xT}-1\right)}{x^2T} \tag{5}$$

Proposition 1. There exists unique p^* and T^* which maximizes the profit function $\Pi(p, T, u)$ for optimal value u^*.
Proof. The first and second partial derivatives of the profit function $\Pi(p, T, u)$ with respect to p are as follow

$$\frac{\partial\prod(p,T,u)}{\partial p} = \frac{-\alpha\left(e^{-aT}-1\right)}{aT} - 2\beta p + \frac{c\beta\left(e^{xT}-1\right)}{xT} - \frac{\beta h}{x} + \frac{\beta h\left(e^{xT}-1\right)}{x^2T} \tag{6}$$

$$R^* = \frac{\partial^2\prod(p,T,u)}{\partial p^2} = -2\beta\ . \tag{7}$$

Put the first order partial derivatives of the profit function Π(p, T, u) with respect to p equal to zero to find the value of p. for this value of p the second order derivative always be negative. Hence p is the global optimal that maximize the total profit. The first and second partial derivatives of the profit function Π(p, T, u) with respect to T are as follow, where x = θ(1 - f(u)) Put the first order partial derivatives of the profit function $\Pi(p, T, u)$ with respect to p equal to zero to find the value of p. for this value of p the second order derivative always be negative. Hence p is the global optimal that maximize the total profit. The first and second partial derivatives of the profit function Π(p, T, u) with respect to T are as follow, where $x = \theta(1 - f(u))$

$$\frac{\partial\prod(p,T,u)}{\partial T} = \left(\frac{\alpha p}{a} + \frac{\alpha h}{a(x-a)}\right)\left(\frac{ae^{-aT}}{T} + \frac{\left(e^{-aT}-1\right)}{T^2}\right) + \frac{K}{T^2} + \frac{\beta p}{x}\left(c + \frac{h}{x}\right)\left(\frac{xe^{xT}}{T} - \frac{e^{xT}-1}{T^2}\right)$$
$$-\frac{\alpha c}{x-a}\left(\frac{(x-a)e^{(x-a)T}}{T} - \frac{e^{(x-a)T}-1}{T^2}\right) - \frac{\alpha h}{x(x-a)}\left(\frac{ae^{-aT}+(x-a)e^{(x-a)T}}{T} + \frac{e^{-aT}-e^{(x-a)T}}{T^2}\right) \tag{8}$$

Put the first order partial derivatives of the profit function Π(p, T, u) with respect to T, equal to zero to find the value of T.

$$\tau = \frac{\partial^2\prod(p,T,u)}{\partial T^2} = -\frac{\alpha p}{a}\left(\frac{2\left(e^{-aT}-2\right)}{T^3} + \frac{2ae^{-aT}}{T^2} + \frac{a^2e^{-aT}}{T}\right)$$
$$+\frac{\beta p}{x}\left(c+\frac{h}{x}\right)\left(\frac{2\left(e^{xT}-1\right)}{T^3} - \frac{2xe^{xT}}{T^2} + \frac{x^2e^{xT}}{T}\right) - \frac{\alpha h}{a(x-a)}\left(\frac{2\left(e^{-aT}-1\right)}{T^3} + \frac{2e^{-aT}}{T^2} + \frac{a^2e^{-aT}}{T}\right) - \frac{2K}{T^3}$$
$$+\frac{\alpha h}{x(x-a)}\left(\frac{2\left(e^{-aT}-e^{(x-a)T}\right)}{T^3} + \frac{2\left(e^{-aT}+(x-a)e^{(x-a)T}\right)}{T^2} + \frac{a^2e^{-aT}-(x-a)^2e^{(x-a)T}}{T}\right)$$

$$-\frac{\alpha c}{x-a}\left(\frac{2\left(e^{(x-a)T}-1\right)}{T^{3}}-\frac{2(x-a)e^{(x-a)T}}{T^{2}}+\frac{(x-a)^{2}e^{(x-a)T}}{T}\right) \tag{9}$$

The second order partial derivatives of the profit function Π(*p*, *T*, *u*) with respect to *T* are also less and equal to zero. (see appendix 'A')

$$S^{*}=\frac{\partial^{2}\prod\left(p,T,u\right)}{\partial T\partial p}=\frac{\alpha e^{-aT}}{T}+\frac{\alpha\left(e^{-aT}-1\right)}{aT^{2}}+\frac{\beta ce^{xT}}{T}-\frac{\beta c\left(e^{xT}-1\right)}{xT^{2}}+\frac{\beta he^{xT}}{xT}-\frac{\beta h\left(e^{xT}-1\right)}{x^{2}T^{2}} \tag{10}$$

Now, we apply the condition of optimality of two variable for profit function Π(*p*, *T*, *u*). Since, $R^{*}{}_{\tau} - S^{*2} > 0$, and $R^{*} < 0$, ${}_{\tau} < 0$ for *p** and *T**(see appendix 'A'), So *p** and *T** both have the optimal value which maximize the profit function Π(*p*, *T*, *u*) for optimal value *u**.

Proposition 2. For nay given feasible (*p*, *T*), there exists a unique optimal preservation technology investment *u** that maximizes Π(*p*, *T*, *u*).

Proof. The first and second partial derivatives of the profit function Π(*p*, *T*, *u*) with respect to *u* are as follow

$$\frac{\partial\prod\left(p,T,u\right)}{\partial u}=-1+\frac{\alpha c\theta f'\left(u\right)e^{\left(\theta\left(1-f\left(u\right)\right)-a\right)T}}{\left(\theta\left(1-f\left(u\right)\right)-a\right)}-\frac{\alpha cf'\left(u\right)\left(e^{\left(\theta\left(1-f\left(u\right)\right)-a\right)T}-1\right)}{\left(\theta\left(1-f\left(u\right)\right)-a\right)^{2}\theta T}-\frac{\beta cpf'\left(u\right)e^{\theta\left(1-f\left(u\right)\right)T}}{\left(1-f\left(u\right)\right)}$$
$$+\frac{\beta cpf'\left(u\right)\left(e^{\theta\left(1-f\left(u\right)\right)T}-1\right)}{\theta\left(1-f\left(u\right)\right)T}-\frac{\alpha h\theta f'\left(u\right)\left(2e^{-aT}-1\right)}{a\left(\theta\left(1-f\left(u\right)\right)-a\right)^{2}T}$$

and

$$\frac{\partial^{2}\prod\left(p,T,u\right)}{\partial u^{2}}=-\frac{\alpha cf'(u)^{2}e^{(\theta(1-f(u))-a)T}}{(\theta(1-f(u))-a)^{3}T}G_{1}-\frac{\beta cpf'(u)^{2}e^{\theta(1-f(u))T}}{\theta(1-f(u))^{2}T}G_{2}+\frac{\alpha cf''(u)e^{(\theta(1-f(u))-a)T}}{\theta(\theta(1-f(u))-a)T}G_{3}$$

$$+\frac{\beta cpf''(u)e^{\theta(1-f(u))T}}{\theta(1-f(u))T}G_{4}-\frac{\alpha h\theta\left(e^{-aT}-1\right)}{a(\theta(1-f(u))-a)^{2}T}G_{5}$$

where,

$$G_{1}=\left(\theta^{4}(\theta(1-f(u))-a)+\left(1-\theta^{2}\right)T\right)\geq 0,\ \ \because\theta<1$$
$$G_{2}=\theta\left(1-\theta(1-f(u))T\right)T\geq 0,$$
$$G_{3}=\left(\theta^{2}-1\right)T\leq 0,$$
$$G_{4}=(1-f(u))(\theta T-1)\geq 0,$$
$$G_{5}=\left(f''(u)+\frac{2\theta f'(u)}{(\theta(1-f(u))-a)}\right)\geq 0,$$

Furthermore, $f'(u) > 0$ and $f''(u) < 0$, so the second derivative of Π(*p*, *T*, *u*) with respect to *u* is less to zero for any feasible (*p*, *T*). Thus there exists a unique *u** maximizing the profit Π(*p*, *T*, *u*).

4. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

Example 1. In this example, we consider the reduce deterioration rate is $f(u)=1-e^{-\gamma u}$, $\gamma > 0$, which is proposed by Dye and Hsieh (2012) and Zhang *et al.* (2014). The parametric values of the inventory system are as follow :

K = \$100 per order, c = \$5 per unit, a = 0.05, h = \$1 per unit per month, u = \$10 per unit time, θ = 0.01, α =100, β =5, γ = 0.05, ϵ = 0.0001. Then $R^*\tau - S^{*2}$ = 173877.40 > 0 and R^* = -10 < 0, τ = -17388.25 < 0, shows that the optimal selling price per unit is p^* = 12.95 and the optimal replenishment time T^* = 1.62659, both maximize the total profit per unit time $\Pi(p, T, u)$, that is $\Pi(p, T, u)$ = 198.20, and the order quantity Q^* = 51.18.

Now, we study the effects of changing the values of the system parameters on the p^*, T^*, $\Pi(p, T, u)$ and Q^*. The sensitive analysis is performed by changing each value of the parameter by +40%, +20%, -20%, and -40% , taking one parameter at a time and keeping the remaining parameters unchanged. The computational results are illustrated in table 1.

Table 1. Sensitive analysis with respect to major parameters

		-40%	-20%	0%	+20%	+40%
α	p	8.88	10.92	12.94	14.94	16.94
	Q	36	47	51	56	58
	T	3.311	2.209	1.626	1.346	1.1285
	Π	14	86	198	352	554
β	p	19.51	155.40	12.94	5799.53	233991.70
	Q	46	49	51	5893	87800
	T	1.183	1.408	1.626	7537.00	431734.60
	Π	486	304	198	8181	467005
γ	p	12.80	12.87	12.94	8314.77	477177.60
	Q	52	51	51	9840	736537
	T	1.622	1.624	1.626	10702.24	800509.60
	Π	200	199	198	814076	814076
h	p	12.92	12.78	12.94	13.10	12.98
	Q	51	52	51	49	50
	T	1.63	1.62	1.626	1.630	1.627
	Π	198	195	198	201	199

Based on the results in table 1, we observe the following fact

a. When initial demand α increases and the other parameter are fixed, then we observe that the optimal total profit per unit time $\Pi(p, T, u)$, the optimal selling price p^*, the optimal order quantity Q^* increases accordingly while the optimal replenishment time T^* decreases (see table 1). This means that when the scaling factor α increases, the market demand rate will increase, so we have to order more quantity per replenishment cycle and shorten the replenishment cycle to meet the increasing demand. Otherwise, we can increase the selling price to obtain more profit. Also the enterprise can also invest more preservation cost to reduce deterioration rate, so the enterprise can sell more products. Moreover, the scaling factor α is getting low then the enterprise close the order.

b. When the price sensitivity parameter β increases, the optimal $\Pi^*(p, T, u)$, p^* and Q^* Decrease while the optimal length of replenishment cycle will increase. It means that if the demand is going down then to coop- up this demand we have to decrease the optimal selling price or reduce the ordering quantity.

c. When the parameter γ increases, the total profit per unit time Π^*, the optimal order quantity Q^* decreases, while the optimal selling price p^* and the optimal replenishment time T^* will increases. So here we need to take the parameter γ as short as possible.

d. When the value of the holding cost per unit time h increases, it can be observed that the optimal selling price p^*, the optimal replenishment cycle T^* and the total profit $\Pi^*(p, T, u)$ increases, while the optimal order quantity Q^*decreases. It means that if we invest some extra cost to holding the items in our enterprise/ godown, then the stock will run for a long time and we have to order in small lots.

Example 2. In this example, the parameter are the same as that in example 1, except for deterioration rate θ. For the given value of θ(=0.01, 0.02, 0.03, 0.04, 0.05, 0.6), we find the corresponding optimal value p^*, Q^*,T^* and $\Pi^*(p, T, u)$ respectively. The computational results are shown in table 2.

Table 2. Sensitivity analysis with respect to θ.

θ	T	p	$\Pi^*(p, T, u)$	Q
0.01	1.6266	12.95	198.20	51
0.02	1.6064	12.55	203.00	54
0.03	1.5884	12.43	207.10	55
0.04	1.5732	12.38	213.25	55
0.05	1.5628	12.35	223.80	55
0.06	1.5627	12.34	244.80	55

We observe from table 2, that as the deterioration rate increases, the optimal replenishment cycle T^* and the optimal selling price p^* will decrease, i.e. we have to sell all goods in short time with a lower price because of deterioration. While the optimal order quantity Q^*will increase because due to the successive deterioration rate with a lower selling price the goods either deteriorate or sales out. And as the deterioration rate increases, the total profit per unit time Π^* will increase, this means that if the deterioration rate is big relatively, the enterprise will invest more funds into the preservation technology for reduce it.

5. CONCLUSION

In this paper we have studied the effects of item preservation technology on a deteriorating seasonal product in which demand is an exponentially decline function of price and time both. This paper developed theoretical results and proved that there exist unique optimal selling price, optimal length of replenishment cycle and optimal preservation technology parameter. Also we have shown there exist optimal selling price, optimal length of replenishment cycle for the optimal preservation technology investment. Numerical examples are provided to illustrate the proposed model and sensitivity analyses with respect to some key parameters are obtained to manage the system. Sensitivity analysis reveals that deterioration factor more affects the profit. In such situation it is advised to inventory management to invest more on preservation technology to reduce the loss in revenues. If the inventories incur high holding cost, then management could order in small lots. The sensitivity analysis also reveals that if demand is going down then inventory management has to decrease the optimal selling price accordingly and generate the excess demand.

For future research, one can consider the time dependent deterioration and allow the shortage. Also one can extend the model in multi-echelon supply chains and in fuzzy environment.

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REFERENCES

1. Alamri, A. A. and Balkhi, Z. T. (2007). The Effects of Learning and Forgetting on the Optimal Production Lot Size for Deteriorating Items with Time Varying Demand and Deterioration Rates. *International Journal of Production Economics*, 107:125-138.
2. Blackburn, J. and Scudder, G. (2009). Supply Chain Strategies for Perishable Products: The Case of Fresh Produce. *Production and Operations Management*, 18(2): 129-137.
3. Chandel, R.P.S. and Khedlekar U.K. (2013). A New Inventory Model with Multiple Warehouses. *International Research Journal of Pure Algebra*, 3(5): 192-200.
4. Chang, H. J. and Dye, C. Y. (1999). An EOQ Model for Deteriorating Items with Time Varying Demand and Partial Backlogging. *Journal of the Operational Research Society*, 50: 1176-1182.
5. Chen, X. and Zhang, J. (2010). Production Control and Supplier Selection under Demand Disruptions. *International Journal of Industrial Engineering and Management*, 3(3): 421-446.
6. Chung, K. J. and Huang, T. S. (2007). The Optimal Retailer's Ordering Policies for Deteriorating Items with Limited Storage Capacity under Trade Credit Financing. *International Journal of Production Economics*, 106: 127-145.

7. Dye, C. Y. and Hsieh, T. P. (2012). An Optimal Replenishment Policy for Deteriorating Items with Effective Investment in Preservation Technology. *European Journal of Operational Research*, 218: 106-112.
8. Giri, B. C., Goswami, A. and Chaudhuri, K., S. (1996). An Eoq Model for Deteriorating Items with Time Varying Demand And Costs. *Journal of the Operational Research Society*, 47: 1398-1405.
9. Giri, B. C., Moon, I. and Yun, W.Y. (2003). Scheduling Economic Lot Sizes in Deteriorating Production Systems. *Naval Research Logistics*, 50: 650-661.
10. Goswami, A. and Chaudhuri, K., S. (1991). An Eoq Model for Deteriorating Items with Shortage And Linear Trend in Demand. *Journal of the Operational Research Society*, 42:1105-1110.
11. Gupta, O. K., Shah, N. H. and Patel, A. R. (2011). An integrated deteriorating inventory model with permissible delay in payments and price-sensitive stock-dependent demand. *International Journal of Operational Research*, 11:425-442.
12. He, Y. and He, J. (2010). A Production Model for Deteriorating Inventory Items with Production Disruptions. *Discrete Dynamics in Nature and Society*, Volume 2010: 1-14.
13. He, Y. and Huang, H. (2013). Optimizing Inventory and Pricing Policy for Seasonal Deteriorating Products with Preservation Technology Investment. *Journal of Industrial engineering*, Volume (2013): 1-7.
14. Khedlekar, U.K. and Namdeo, A. (2015). An Inventory Model with Stock and Price Dependent Demand. *Bulletin of the Allahabad Mathematical Society*, 30(2): 2015.
15. Khedlekar, U.K. and Shukla, D. (2012). Dynamic Pricing Model with Logarithmic Demand. *Opsearch*, 50(1): 1-13.
16. Khedlekar, U.K. Shukla, D. and Chandel, R.P.S. (2014). Computational Study for Disrupted Production System with Time Dependent Demand. *Journal of Scientific and Industrial Research*, 73(5): 294-301.
17. Khedlekar, U.K. Shukla, D. and Namdeo, A. (2016). Pricing Policy for Declining Demand using Item Preservation Technology. *SpringerPlus*, 5(1957):1-11.
18. Kouki, C., Sahin, E., Jemai, Z. and Dallery, Y. (2013). Assessing the Impact of Perishability and the use of Time Temperature Technologies on Inventory Management. *International Journal of Production Economics*, 143(1): 72-85.
19. Kumar, R., and Sharma, S.K. (2012). Formulation of Product Replacement Policies for Perishable Inventory Systems using Queuing Theoretic Approach. *American Journal of Operational Research*, 2(4) (2012b): 27-30.
20. Musa, A. and Suni, B. (2012). Inventory Ordering Policies of Delayed Deteriorating Items under Permissible Delay in Payments. *International Journal of Production Economics*, 136: 75-83.
21. Ouyang, L. Y., Chang, C.T. and Teng, J. T. (2005). An EOQ Model for Deteriorating Items under Trade Credits. *Journal of the Operational Research Society*, 56: 719-726.
22. Roy, T. and Choudhary, K.S. (2011). An EPLS Model for a Variable Production Rate with Stock-Price Sensitive Demand and Deterioration. *Yugoslav Journal of Operations Research*, 21(2): 1-12.
23. Sarkar, B. and Moon, I. (2011). An EPQ Model with Inflation in an Imperfect Production System. *Applied Mathematics and Computation*, 217: 6159-6167.
24. Shukla, D. and Khedlekar, U.K. (2015). Inventory Model for Convertible Item with Deterioration. *Communication in Statistics- Theory and Methods*, 45(4): 1137-1147.
25. Shukla, D., Khedlekar, U.K. and Chandel, R.P.S. (2013). Time and Price Dependent Demand with Varying Holding Cost Inventory Model for Deteriorating Items. *International Journal of Operations Research and Information System*, 4(4): 75-95.
26. Widyadana, G.A. and Wee, H.M.. (2007). A Replenishment Policy for Item with Price Deoendent Demand and Deteriorating Under Markdown Policy. *Journal Teknik Industri*, 9(2): 75-84.
27. Zhang, J., Bai, Z. and Tang, W. (2014). Optimal Pricing Policy Deteriorating Items with Preservation Technology Investment. *Journal of Industrial and Management Optimization*, 10(4): 1261-1277.

Appendix 'A'

From the equation (9) we have,

$$\tau = \frac{\partial^2 \prod\left(p,T,u\right)}{\partial T^2}$$
$$= -\frac{\alpha p}{a}\varphi_1 - \frac{1}{T^3}\varphi_2 - \frac{\alpha c}{x-a}\varphi_3 - \frac{2e^{xT}}{T^3}\varphi_4 - \frac{2\alpha h e^{-aT}}{axT^3} - \frac{2e^{-aT}}{T^2}\varphi_5 - \frac{a^2 e^{-aT}}{T}\varphi_6 - \frac{\beta p}{x}\left(c+\frac{h}{x}\right)\varphi_7$$

where,

$$\varphi_1(a) = \left(\frac{2\left(e^{-aT} - 2\right)}{T^3} + \frac{2ae^{-aT}}{T^2} + \frac{a^2 e^{-aT}}{T} \right)$$

$$\varphi_1^{'}(a) = -a\,e^{-aT} = -a + a^2 T \geq 0, \qquad \because a > 0,\ \ T > 0$$

So, $\varphi_1(a)$ is an increasing function and $\varphi_1(0) = 0$, therefore $\varphi_1(a) \geq 0$, as $a > 0$.

and

$$\varphi_2(x) = 2K - \frac{\alpha h}{a(x-a)} \geq 0$$

Because $\varphi_2^{'}(x) = \frac{\alpha h}{a(x-a)^2} > 0$, so $\varphi_2(x)$ is an increasing function and $\varphi_2(\frac{\alpha h + 2a^2 K}{2aK}) = 0$,

therefore, $\varphi_2(x) \geq 0$ as $x > \left(\alpha h + 2a^2 K\right) \big/ 2aK$.

and

$$\varphi_3(x) = \left(\frac{2\left(e^{(x-a)T} - 1\right)}{T^3} - \frac{2\left(x-a\right)e^{(x-a)T}}{T^2} + \frac{\left(x-a\right)^2 e^{(x-a)T}}{T} \right)$$

$$\varphi_3^{'}(x) = \left(x-a\right)^2 e^{(x-a)T} \geq 0, \text{ as } a << 1.$$

Therefore, $\varphi_3(x)$ is an increasing function and $\varphi_3(a) = 0$, so $\varphi_3(x) \geq 0$, since $x > a$.

and

$$\varphi_4(x) = \frac{\alpha\, h\, e^{-aT}}{x(x-a)} - \frac{\beta\ p}{x}\left(c + \frac{h}{x}\right) \geq 0$$

Because

$$\varphi_4^{'}(x) = \frac{\alpha\, h\, e^{-aT}\left(2x-a\right)}{x^2(x-a)^2} + \beta\ p\left(\frac{h}{x^3} + \frac{1}{x^3}\left(c + \frac{h}{x}\right)\right) \geq 0, \text{ as } a << 1.$$

Therefore, $\varphi_4(x)$, is an increasing function. So there exits at least one smallest +ve value of x such that

$\varphi_4(x) = 0$.

$\varphi_4\left(x\right) \geq 0$ for all greater value of x.

and

$$\varphi_5(x) = \frac{\alpha\, h}{ax} - \alpha h x e^{xT} \leq 0$$

Because $\varphi_5^{'}(x) = -\frac{\alpha\, h}{ax^2} - \alpha h e^{xT}(xT+1) \leq 0$.

Therefore, $\varphi_5(x)$ is a decreasing function.

$$\varphi_6(x) = \frac{\alpha h}{ax} + \frac{\alpha h(x-a)\,e^{xT}}{x} \geq 0$$

because $\varphi_6^{'}(x) = \frac{\alpha h}{x}\left((x-a)Te^{xT} + e^{xT}\right) - \frac{\alpha h}{x^2}\left(\frac{1}{a} + (x-a)\,e^{xT}\right) \geq 0$, as $a << 1$.

Now, $\varphi_6(x)$ is an increasing function and $\varphi_6(a) = \frac{\alpha h}{a^2} > 0$, so $\varphi_6(x) \geq 0$ as $x \geq a$.

Similarly as $\varphi_3(x)$, we have

$$\varphi_7(x) = \left(\frac{2}{T^3} + \frac{2x\, e^{xT}}{T^2} - \frac{x^2\, e^{xT}}{T} \right) \geq 0.$$

Since $\phi_1(x)$, $\phi_2(x)$, $\phi_3(x)$, $\phi_4(x)$, $\phi_6(x)$, $\phi_7(x)$ all are greater and equal to zero and $\varphi_5(x)$ less than zero but $\varphi_5(x)$ is very less quantity for value of *a* and *T*.

Hence, $\tau = \dfrac{\partial^2 \prod(p,T,u)}{\partial T^2} < 0.$

Now, we already know

$$R = \frac{\partial^2 \prod(p,T,u)}{\partial\, p^2} = -2\beta < 0,$$

and $\left(\dfrac{\partial^2 \prod(p,T,u)}{\partial T^2}\right)\left(\dfrac{\partial^2 \prod(p,T,u)}{\partial p^2}\right)$ is big enough to $\left(\dfrac{\partial^2 \prod(p,T,u)}{\partial T\, \partial p}\right)^2$.

therefore, $R\tau - S^2 > 0$. This inequality are shown numerically in the given numerical example 1. So p^* and T^* both optimize the profit function $\Pi(p, T, u)$ for optimal value u^*.