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Dynamic Network Contraflow Evacuation Planning Problem with Continuous Time Approach

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Abstract: A number of efficient algorithms have been established to solve the evacuation problem modeled on dynamic network contraflow approach in discrete-time setting. The arcs are reversed with the consideration of constant transit time and arc capacities over a finite time horizon. In this paper, we consider dynamic network contraflow problem with continuous time setting and propose a strongly polynomial algorithm to solve the maximum dynamic network contraflow evacuation planning problem. Moreover, we propose a pseudo polynomial algorithm for the problem in which the arcs are reversed in any sub-interval of given time horizon.
Keyword — Route Planning, Continuous-time Network Flow, Evacuation Planning.

1. INTRODUCTION

Natural and human-created disasters have been not only causing massive destructions but motivating to a number of researchers to find efficient emergency management procedures also so that the destructions could be minimized. Besides the disasters, it becomes crucial in mass-meetings management and in mitigation of the traffic in a busy traffic hours. An evacuation planning problem asks to find an optimal evacuation plan in a realistic flow model where each evacuee is supposed to be evacuated in a minimal time period. This minimal time period is the lower bound that an evacuee needs.

Evacuation planning is an attempt of sending people and/or their logistics from a dangerous site (source) to a safe site (sink) as quickly as possible. Evacuation planning with lane reversal i.e. contraflow approach designed in discrete time model has been extensively considered in the literature. See, Dhamala (2015), Dhamala and Pyakurel (2013), Kim et al. (2008), Pyakurel and Dhamala (2015) and Rebennack et al. (2010). The contraflow approach reconfigurates the network identifying the ideal direction and reallocating the available capacity for each arc to minimize the evacuation time from source to sink. However, continuous time model naturally better reflects the real world behavior. In continuous time model, flow units can enter the network at any moment of time before the time horizon.

The dynamic network contraflow evacuation planning problem has been formulated as an integer programming formulation by Kim et al. (2008). Two heuristics, one: Greedy heuristic, which determines the condition of congestion and flips highly congested arc in a greedy manner and the other: Bottleneck Relief heuristic, which identifies the bottleneck and increases the capacity by contraflow to improve the maximum flow in each iteration, have also been investigated. The solution is based on empirical results. There exists analytical solution also for the problem which sends a maximum flow from a source to a sink in the two terminal case, see Rebennack et al. (2010). The solution with polynomial complexity has been investigated on both static and dynamic networks where the arc reversal ability has been adapted only once at very beginning of the time horizon for the dynamic case. It is crucial to reverse the arc direction not only at the beginning but at any interval of the time horizon also if the situation of sudden arc damage occurs so that the evacuees must be rerouted for evacuation. The task is more challenging in continuous time setting due to uncertainty of the interval of time over the time horizon when the arc is to be reversed.

The dynamic network flow problem in continuous time setting has been introduced in Philpott (1982). The amount of flow which enters the arc per time unit has been considered to be the flow function. The concept of cuts of source-sink over the time on the network with zero transit time as a solution procedure has been adopted in Anderson et al. (1982). Moreover, the procedure with arbitrary transit time has been extended in Philpott (1990). The amount of flow that arrives at the head of an arc during the unit interval of time beginning at any time step in continuous time setting is equal to the amount of flow that arrives at the head of that arc at that time step in discrete case. This idea has been adapted in Fleischer and Tardos (1998) to transform a feasible flow on a discrete approach

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into a feasible flow on the continuous time setting. Dynamic flow models in continuous time setting can also be found in Koch et al. (2011) and Hashemi and Nasrabadi (2012).

In this paper, we propose an efficient solution approach which produces optimal flow on the dynamic contraflow network in the continuous time setting.

The paper is organized as follows. In Section 2, the mathematical model of the problem is formulated. Section 3 contains the main contribution where a strongly polynomial algorithm on the dynamic contraflow network in which the arc is reversed at time zero is proposed. Moreover, a pseudo polynomial time algorithm for the problem in which the arc can be reversed in any sub-interval of time within the time horizon is also proposed. Section 4 concludes the paper.

2. PROBLEM FORMULATION

Optimization approach of the evacuation planning problem can be described on a network $\mathrm{N} = \left(\mathrm{V}, \mathrm{E}, \mathrm{c}_e, \tau_e, \mathrm{T}\right)$ where $V\left(\left|V\right| = n\right)$ is for the set of the nodes, $E\left(\left|E\right| = m\right)$ for the set of the arcs joining any two nodes, c_e for the flow capacity along $\mathrm{e} \in \mathrm{E}$, τ_e for the transit time i.e. the time a flow unit takes along the arc e and T for the time horizon. In particular, the source node and the sink node are denoted as s and d respectively. We denote $\vec{\mathrm{e}} \in E$ for an arc $\left(\mathrm{i}, \mathrm{j}\right)$ in which the flow unit is sent from the node i to the node j and $\overleftarrow{\mathrm{e}} \in E$ for an arc $.\left(\mathrm{j}, \mathrm{i}\right)$. in which the flow unit is sent from the node j to the node i for all $\mathrm{i}, \mathrm{j} \in \mathrm{V}$. Replacement of $\vec{\mathrm{e}}$ by $\overleftarrow{\mathrm{e}}$ is known as the arc reversal. For symmetric transit times, we write $\tau_{\vec{e}} = \tau_{\overleftarrow{e}}$ and the auxiliary dynamic network $\tilde{\mathrm{N}} = \left(\mathrm{V}, \tilde{\mathrm{E}}, \mathrm{c}_{\tilde{e}}, \tau_{\tilde{e}}, \mathrm{T}\right)$, where $\mathrm{c}_{\tilde{e}} = \mathrm{c}_{\vec{e}} + \mathrm{c}_{\overleftarrow{e}}$, $\tilde{\mathrm{E}} = \left\{\tilde{\mathrm{e}} = \vec{\mathrm{e}} \text{ or } \overleftarrow{\mathrm{e}}\right\}$ and $\tau_{\tilde{e}} = \tau_e$ if $\mathrm{e} \in \mathrm{E}$ and $\tau_{\tilde{e}} = \tau_{\overleftarrow{e}}$ otherwise. For detail see Example 1. Our network contains no loop and no holdovers at the node. It is note-worthy that evacuation planning problem has been first described in Ford and Fulkerson (1958, 1962). We consider the network on which the flow per time unit i.e. the flow function $\mathrm{f}\left(\mathrm{e}, \theta\right)$ is defined as $f : E \times \left[0, T\right] \rightarrow R^{+} \cup \left\{0\right\}$, where $\theta \in \left[0, \mathrm{T}\right)$. The flow function satisfies the flow conservation at node $\mathrm{i}, \mathrm{i} \in \mathrm{V}$ if

$$\sum_{e \in \delta^{+}\left(i\right)} \int_{0}^{T} f\left(e, \theta\right) d\theta = \sum_{e \in \delta^{-}\left(i\right)} \int_{0}^{T} f\left(e, \theta\right) d\theta; \forall \theta \in \left[0, T\right), \tag{1}$$

where $\delta^{+}\left(\mathrm{i}\right) = \left\{\left(\mathrm{j}, \mathrm{i}\right) \in \mathrm{E}\right\} \forall \mathrm{j} \in \mathrm{V}$ for the set of arcs heading towards node i and $\delta^{-}\left(\mathrm{i}\right)$ $= \left\{\left(\mathrm{i}, \mathrm{j}\right) \in \mathrm{E}\right\} \forall \mathrm{j} \in \mathrm{V}$ for the set of arcs leaving node i. The flow function satisfies the capacity constraints also i.e. $\mathrm{f}\left(\mathrm{e}, \theta\right) \leq \mathrm{c}_e, \forall \mathrm{e} \in \mathrm{E}$ and $\forall\ \theta \in \left[0, \mathrm{T}\right)$. An $\mathrm{s} - \mathrm{d}$ flow in which the flow function satisfies conservation constraint for any intermediate node $\mathrm{i} \in \mathrm{V} \setminus \left\{\mathrm{s}, \mathrm{d}\right\}$ and capacity constrain for all arcs at every time $\theta \in \left[0, \mathrm{T} - \tau_e\right)$ is said to be a feasible $s - d$ flow where τ_e is the transit time along the arc $\mathrm{e} \in \mathrm{E}$. The flow which obeys conservation constraint at each node $\mathrm{i} \in \mathrm{V}$ is commonly known as a circulation. Obviously, $\mathrm{f}\left(\mathrm{e}, \theta\right) = 0 \quad \forall \theta \notin \left[0, \mathrm{T} - \tau_e\right)$ and all flow units leave the network before the completion of time horizon T. Let $\mathbf{f}$ be the net flow value that leaves the source over all time steps or enters the sink over all time steps $\theta \in \left[0, \mathrm{T}\right)$.We can describe the net flow as follows.

$$\begin{aligned} \mathbf{f} &= \sum_{e \in \delta^{-}\left(s\right)} \int_{0}^{T} \mathrm{f}\left(e, \theta\right) d\theta - \sum_{e \in \delta^{+}\left(s\right)} \int_{0}^{T} \mathrm{f}\left(e, \theta\right) d\theta \\ &= \sum_{e \in \delta^{+}\left(d\right)} \int_{0}^{T} \mathrm{f}\left(e, \theta\right) d\theta - \sum_{e \in \delta^{-}\left(d\right)} \int_{0}^{T} \mathrm{f}\left(e, \theta\right) d\theta \end{aligned} \tag{2}$$

The net flow $\mathbf{f}$ is maximized for the maximum dynamic network contraflow problem. Let $\mathbf{f_d}$ and $\mathbf{f_c}$ denote the net flow for the discrete and the continuous time setting, respectively. Let the net flow **f** for the static case be $\mathbf{f_s}$.

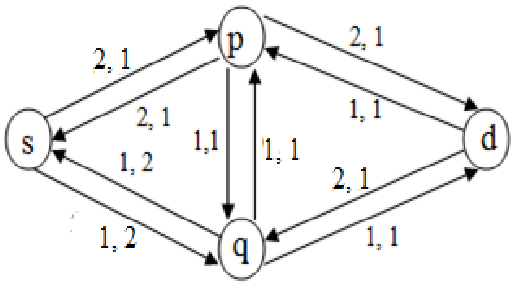

Figure 1. An evacuation network N.

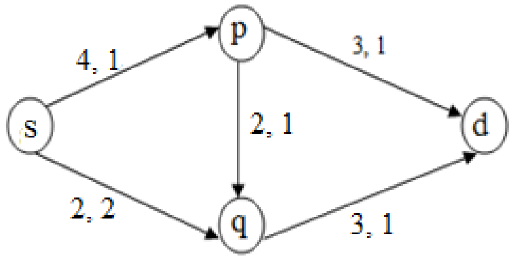

Figure 2. A contraflow reconfigured network $\tilde{N}$ of Fig 1.

Example 1. Consider a two terminal evacuation network as depicted in Fig 1. Line joining any two ovals (the node) is an arc for example a road. Here, arrow on the arc shows the direction of the flow. Node s is the dangerous place (source) that contains evacuees, d is the safe place (sink) that waits them with sufficient capacity and the remaining nodes are the intermediate nodes. The movement of the evacuees (possibly cars) is the flow. The first and the second numbers next to each arc are the arc capacity and the transit times respectively. For example, an arc between nodes s and p (directed towards p) has capacity 2 and transit time 1. That is, 2 vehicles can pass simultaneously through arc (s, p) within 1 unit time. A unit of time may be group of minutes or hours. A contraflow reconfiguration directed towards sink d of the network depicted in Fig 1 is shown in Fig 2. Capacities have been added but the transit time remains unaltered in each arc.

We use the notion of the natural transformation discussed by Fleischer and Tardos (1998) which states that the amount of flow that arrives at the node j through the arc e at time step θ in the discrete approach is equal to the amount of flow arriving at j through the arc e during the unit interval of time at the beginning of time step θ i.e. $\mathbf{f}_d(e, \theta) := \mathbf{f}_c(e, [\theta, \theta+1))$ for all $\theta \in \{0, 1, \ldots, T-1\}$. Flow movement on the arc with continuous time setting is shown in Fig 3 where the arc $e = (s, q)$ has a capacity $c_e = 2$ (after contraflow reconfiguration) and transit time $\tau_e = 2$ for each unit of flow. Each unit of flow entering the arc at time θ starts reaching at q at time $\theta + \tau_e$, that is, at time $\theta + 2$. The flow totally leaves the arc only at time $\theta + \tau_e + 1$ that is, at time $\theta + 3$. However, it can be considered that the same amount of flow completes its journey at time $\theta + 2$ in discrete time setting.

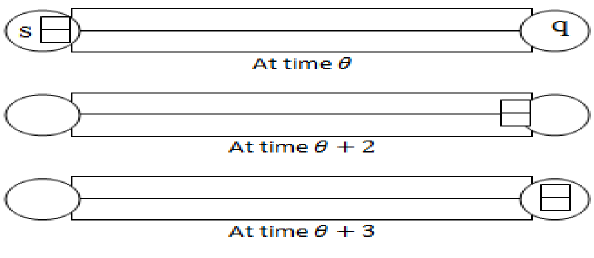

Figure 3. Continuous flow on arc having capacity 2 and transit time 2.

3. MAXIMUM DYNAMIC CONTRAFLOW

There exists a relationship between the maximum flow values of discrete and the continuous net flows while sending the evacuees from the source to the sink. The approach is based on chain decomposition. Let $\Gamma = \{\gamma_1, \gamma_2, \cdots, \gamma_r\}$ be the set of chains with static flow values $v(\gamma_1), v(\gamma_2), \cdots, v(\gamma_r)$ respectively. The static, the discrete and the continuous net flow values are $\mathbf{f_s} = \sum_{k=1}^{r} v(\gamma_k), \mathbf{f_d} = \sum_{k=1}^{r} v(\gamma_k).(T - \tau_{\gamma_k} + 1)$ and $\mathbf{f_c} = \sum_{k=1}^{r} v(\gamma_k).(T - \tau_{\gamma_k})$ respectively. Here, the term τ_{γ_k} stands for the transit time a unit flow takes along the chain γ_k. The natural transformation shows that the net flow value in continuous time setting does not exceed the case in discrete approach. Moreover, the maximum difference of the net flow between continuous and discrete time model within the given time horizon is maximum static flow.

We first consider the maximum dynamic contraflow problem with continuous time setting with arc reversal at time zero then the problem with the arc reversal in any sub-interval of time horizon T.

3.1 Arc Reversal at Time Zero

The problem with arc reversal at time zero in discrete model has been investigated in Rebennack et al. (2010). The problem has been solved with a strongly polynomial time algorithm. Maximum flow is obtained from the chain decomposition starting each chain flow at time zero and then adopting temporal repetition. There exists a temporally repeated flow in the continuous time setting also. Moreover, the flow is maximal due to the following lemma of Anderson and Philpott (1994).

Lemma 1. *[Anderson and Philpott (1994)] The temporally repeated flow with continuous time setting is maximal over the time horizon.*

The investigation of Ford and Fulkerson (1958), which states that a feasible flow on N is an equivalent feasible flow of the problem on the corresponding time expanded network, assures that the problem can be solved by converting the dynamic network N into the time expanded network N^T over the time horizon T. The time expanded network N^T is defined as $N^T = (V^T, E^T, c_e, \tau_e, T)$, where

$$V^T = \{i(\theta) : i \in V \text{ and } \theta \in \{0,1,\ldots,T-1\}\}$$

and

$$E^T = \{(i(\theta), j(\theta + \tau(i,j))) : i \neq j, i, j \in V \text{ and } \theta \in \{0,1,\ldots,T-1-\tau_{(i,j)}\}\}.$$

Lemma 2. *The maximum flow for two terminal case of the maximum dynamic contraflow problem with continuous time setting on N does not exceed the optimal flow for the corresponding time expanded network N^T.*

Proof: Every feasible flow on N is equivalent feasible flow of the maximum dynamic contraflow problem with continuous time setting on the corresponding time expanded network. Furthermore, the continuous time net flow $\mathbf{f_c}$ does not exceed the discrete time net flow $\mathbf{f_d}$.

Now we propose an algorithm say MDNCF-CT which can yield an optimal maximum flow on the dynamic network with arc reversal in continuous time setting. This is a modified algorithm designed for the discrete model in Rebennack et al. (2010).

Algorithm - 1 (Algorithm MDNCF-CT)

1. Transform the network $N = (V, E, c_e, \tau_e, T, \text{integer})$ into $\tilde{N} = (V, \tilde{E}, c_{\tilde{e}}, \tau_{\tilde{e}}, T)$ where $c_{\tilde{e}} = c_{\vec{e}} + c_{\overleftarrow{e}}$, $\tilde{E} = \{\tilde{e} = \vec{e} \text{ or } \overleftarrow{e}\}$ and $\tau_{\tilde{e}} = \tau_e$ if $e \in E$ and $\tau_{\tilde{e}} = \tau_{\overleftarrow{e}}$ otherwise.
2. Compute the discrete dynamic, temporally repeated flow on network $\tilde{N}$ for time horizon $T-1$.
3. Transform the discrete dynamic flow into continuous dynamic flow using the natural transformation $\mathbf{f_d}(e,\theta) = \mathbf{f_c}(e,[\theta,\theta+1))$ for all $\theta \in \{0,1,\cdots,T-1\}$.
4. Perform the flow decomposition into chain and cycle flows of the maximum flow obtained from step-3 and remove all cycle flows.

5. Arc $\overleftarrow{e} \in E$ is reversed if and only if the flow along arc $\vec{e} \in E$ is greater than c_e or if there is non-negative flow along arc $e \notin E$.
6. Obtain the maximum dynamic contraflow with continuous time setting for the given time horizon T.

The algorithm MDNCF-CT yields an optimal solution to the maximum dynamic contraflow problem with continuous time setting. The following is the proof of correctness of the algorithm.

Theorem 1. *The algorithm MDNCF-CT yields an optimal solution to the maximum dynamic contraflow problem with continuous time setting on the network N.*

Proof: The algorithm MDNCF-CT is a modified algorithm P-MDCF investigated in Rebennack et al. (2010). Thus the steps 1, 2 and 4 are clearly well defined. The flow decomposition breaks the optimal flow into chains from source to sink and into cycles with positive flows. These positive flows vanish in each cycles after cancelation and ensures that there is either a flow along arc (i,j) or (j,i), but never on both arcs. Hence the resulting flow from step 5 is a feasible flow with arc reversal for N. Step 3 is feasible since the natural transformation converts of a feasible $(T-1)$-horizon maximum dynamic flow in discrete time setting into a feasible T-horizon maximum dynamic flow in continuous time setting.

Since every feasible flow for the continuous time setting on $\tilde{N}$ is also feasible for the continuous time setting on N, the algorithm is correct for the feasible flow for the continuous time setting.

By the feasibility condition,

$$\tilde{N}_{MDF-CT_{opt}} \leq N_{MDCF-CT_{opt}}$$

where $N_{MDCF-CT_{opt}}$ and $\tilde{N}_{MDF-CT_{opt}}$ stand for the optimal value of the maximum dynamic flow on N and on $\tilde{N}$, respectively in the continuous time setting.

It is clear by Lemma 2 that the maximum static contraflow on N^T is not less than the maximum dynamic contraflow on N in continuous time setting. That is,

$$N_{MDCF-CT_{opt}} \leq N^T{}_{MSCF_{opt}},$$

where $N^T{}_{MSCF_{opt}}$ is the optimal value of the maximum static flow on N^T.

We have the fact that the maximum static contraflow problem on N^T is equivalent to the maximum static flow problem on $\tilde{N}^T$ where the arc set $\tilde{E}$ is defined as $\tilde{E} = \left\{\tilde{e} = \vec{e} \in E \text{ or } \overleftarrow{e} \in E\right\}$, $c_{\tilde{e}}$ is defined as $c_{\tilde{e}} = c_{\vec{e}} + c_{\overleftarrow{e}}$ and the transit time is $\tau_{\tilde{e}} = \tau_e$ if $e \in E$ and $\tau_{\tilde{e}} = \tau_{\overleftarrow{e}}$ otherwise.

Thus,

$$N^T{}_{MSCF_{opt}} = \tilde{N}^T_{MSF_{opt}}$$

where $\tilde{N}^T_{MSF_{opt}}$ stands for the optimal value of the maximum static contraflow on $\tilde{N}^T$.

By Lemma 1, the maximum flow in time expanded network $\tilde{N}^T$ can be obtained by temporally repeated chain flow of a static network $\tilde{N}$. That is, $\tilde{N}^T_{MSF_{opt}} = \tilde{N}_{MDF-CT_{opt}}$.

Hence, we have

$$\begin{aligned} N_{MDCF-CT_{opt}} &\leq N^T{}_{MSCF_{opt}} \\ &= \tilde{N}^T_{MSF_{opt}} \\ &= \tilde{N}_{MDF-CT_{opt}}. \end{aligned}$$

Therefore,

$$\tilde{N}_{MDF-CT_{opt}} = N_{MDCF-CT_{opt}}.$$

The following example illustrates the presented algorithm MDNCF-CT.

Example 2. Consider the evacuation network N of Fig 1 with time horizon $T = 4$. At first we transform the network as described in step 1 and get the transformed network $\tilde{N}$ as shown in Fig 2. With the aid of step 2 we obtain the discrete time maximum dynamic flow of value 9 in $\tilde{N}$ for time horizon $T - 1 = 3$. We calculate the flow in $\tilde{N}$ for each time step $\theta \in \{0,1,2,3\}$. At time steps 0 and 1 no flow reaches at sink. The chain $(s - p - d)$ carries 3 units of flow at sink at time $\theta = 2$ for the first time. The chains $(s - p - d)$, $(s - p - q - d)$ and $(s - q - d)$ respectively carry 3 units, 1 unit and 2 units of flow at sink at time $\theta = 3$. Now, step 3 converts this discrete time flow into continuous time in the following manner: Flow of value 3 at time $\theta = 2$ is considered as the continuous time flow of same amount in the time interval $[2,3)$. Similarly, the flow of value 3, 1 and 2 at time $\theta = 3$ are considered as the continuous time flow of same amounts in the time interval $[3,4)$. Thus summing up these flow values within time horizon $T = 4$ we get the continuous time dynamic flow of value 9. This flow value is the maximum dynamic flow in continuous time setting in $\tilde{N}$ for time horizon T. Eventually, the maximum dynamic contraflow in continuous time setting in N for given time horizon $T = 4$.

Theorem 2. *The Algorithm MDNCF-CT solves the maximum dynamic contraflow problem with continuous time setting in strongly polynomial time.*
Proof: Finding a temporally repeated flow is equivalent to solving a minimum cost flow problem. The algorithm due to Goldberg and Tarjan (1989) leads to a strongly polynomial time of order $O(n^2.m^3.\log n)$ for solving this problem. Let us denote it by $h_1(n,m)$. Since the natural transformation of $(T-1)$ -horizon discrete time maximum dynamic flow yields a T-horizon continuous time maximum dynamic flow, the time complexity of finding a temporally repeated continuous flow is also $h_1(n,m)$. Therefore, time complexity of Algorithm-1 is $O(h_1(n,m) + h_2(n,m)) + h_3(n,m))$ where $h_2(n,m) = O(n^2.\sqrt{m})$ and $h_3(n,m) = O(n.m)$ are the times required to solve the maximum static flow (MSF) problem and the flow decomposition respectively; which is strongly polynomial.

3.2 Arc Reversal at any Sub-interval of the Time Horizon

During evacuation we may encounter the situation with all of sudden blockage of road segments that causes obstacles for evacuees from being evacuated through the current route (chain). The model, allowing the arc reversal capability only once at time zero, cannot deal with such situation. In the following, we have tried to overcome this hurdle, if exists, by rerouting (as we have not considered the immediate road repairing after disaster) the flow unit (evacuee) that is currently traveling on the network by contraflow approach. In this model we do not restrict the arc reversal capability only at time zero but allow reversing, if necessary, in any time interval $[\theta,\theta+1)$ for all $\theta \in \{0,1,\ldots,T-1\}$ only once at the beginning and call it the generalized maximum dynamic contraflow problem with continuous time setting (G-MDNCF-CT). We have proposed solution procedure, Algorithm-2 (Algorithm G-MDNCF-CT) below, to solve this problem.

An arc reversal capability has been considered to be at each integer time points within given time horizon T for lexicographically maximum dynamic contraflow (LMDCF) problem in Pyakurel and Dhamala (2015). The LMDCF is the maximum dynamic contraflow that maximizes the flow in given priority of terminals. An algorithm to solve this problem has been given with polynomial time complexity $O(\delta \times (m \log n(m + n \log n))$ where m, n and δ are the numbers of arcs, nodes and terminals, respectively. However, their model is based on discrete time setting.

We define the term Last-Returns-First property (LRF property) for the situation in which an arc allows to return back for the last flow unit at the first, the second-last flow unit at the second and so on. An arc e is a dead arc at time θ if all or some flow units on it are blocked at time θ. An arc remains dead throughout the time horizon after the time of death if it is dead once as there is no immediate road repairing consideration in our model. Without loss of generality, we assume that the capacity of a dead arc is zero since our model does not allow partial contraflow. In particular, if a chain contains a dead arc at time θ we reverse only those necessary arcs of it which are directed towards the dead arc only once at the beginning of the time interval $[\theta,\theta+1)$. The flow units on every arc of the chain containing a dead arc should satisfy the LRF property while reversing its direction at the time if there are flow units traveling on it. This prevents two flow units (possibly cars) from being collided each other. How we handle such problem is explained in Example 3.

Algorithm - 2 (Algorithm G-MDNCF-CT)

1. Transform the network $N = \left(V, E, c_e, \tau_e, T, \text{integer}\right)$ into $\tilde{N} = \left(V, \tilde{E}, c_{\tilde{e}}, \tau_{\tilde{e}}, T\right)$ where $c_{\tilde{e}} = c_{\vec{e}} + c_{\overleftarrow{e}}$, $\tilde{E} = \left\{\tilde{e} = \vec{e} \text{ or } \overleftarrow{e}\right\}$ and $\tau_{\tilde{e}} = \tau_e$ if $e \in E'$ and $\tau_{\tilde{e}} = \tau_{\overleftarrow{e}}$ otherwise.
2. Compute the discrete dynamic, temporally repeated flow on network $\tilde{N}$ for time horizon $T-1$.
3. Transform the discrete dynamic flow into continuous dynamic flow using the natural transformation $\mathbf{f_d}\left(e, \theta\right) = \mathbf{f_c}\left(e, \left[\theta, \theta+1\right)\right)$ for all $\theta \in \left\{0, 1, \ldots, T-1\right\}$.
4. Perform the flow decomposition into chain and cycle flows of the maximum flow obtained from step-3 and remove all cycle flows.
5. Arc $\overleftarrow{e} \in E$ is reversed if and only if the flow along arc $\vec{e} \in E$ is greater than c_e, if there is non-negative flow along arc $e \notin E$ or arc e is dead at time θ.
6. Obtain the generalized maximum dynamic contraflow with continuous time setting for the given time horizon T.

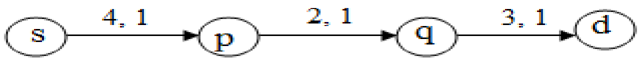

Figure 4. An s-d chain of the evacuation network of Fig 2.

Example 3. Consider an $s-d$ chain of the evacuation network (after contraflow reconfiguration) given in Fig 2 as depicted in Fig 4 with $\left(q, d\right)$ a dead arc at time $\theta+2$ among three arcs. Let us start to send any number of flow units less or equal to 2 from s at time θ and continue it until time $\theta+1$. The Flow units that entered the arc $\left(s, p\right)$ at time θ reaches at q at time $\theta+2$. But, at this time the arc $\left(q, d\right)$ is dead and flow cannot move towards sink d via current route (chain). Now, we reverse the direction of arc $\left(p, q\right)$ (i.e. from heading towards q to heading towards p) at the beginning of the time interval $[\theta+2, \theta+3)$ so that all the flow units that has entered on arc $\left(p,q\right)$ return back to p by satisfying LRF property. That is, the flow units that entered the arc at the last should return back at the first and so on.

In step 5 of Algorithm G-MDNCF-CT, direction of the arc is allowed to reverse only after satisfying the LRF-property if dead arc do exist and therefore flow in the arc does not travel in both directions at the same time. Other steps are similar to that of Algorithm MDNCF-CT and their feasibility have been discussed already in Theorem-1. Moreover, each direction reversal takes place only at the beginning of the time interval. However, the direction of the same arc may be reversed more than once within the given time horizon T in different time intervals. This can happen at most T times for at most m arcs. Therefore, step 5 of algorithm-2 can be carried out in at most $O\left(T.m\right)$ time, depending on T, that dominates the overall time bound of the algorithm. Thus we can state the following theorem:

Theorem 3. *Algorithm G-MDNCF-CT solves generalized maximum dynamic contraflow problem with continuous time setting in pseudo polynomial time.*

4. CONCLUDING REMARK

The importance and applicability of the idea of contraflow especially in evacuation planning problem has been increasing. The model of the problem and the solution approaches based on continuous time setting better reflects the real world situation. In this paper, we have considered the problem with two algorithms as solution procedures with strongly polynomial time complexity if the arcs are flipped only once at time 0 in continuous time setting. Furthermore, an algorithm, with pseudo polynomial time complexity under the consideration of flipping the arcs in any sub-interval of given time horizon, has also been investigated.

Investigation on the problem with partial contraflow and total chain flipping instead of flipping only the arcs would be an interesting research area in the future.

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