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Robust Multi-Objective Facility Location Model of Closed-Loop Supply Chain Network under Interval Uncertainty

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Abstract: In this paper, we consider a supply chain network that includes multiple plants, collection centers, demand markets, and products, where a multi-objective mixed integer programming model has been developed to minimize cost and maximize some environmental issues by Amin and Zhang (2013a). Due to the uncertainty of the demands and returns, the robust counterpart of the model is discussed under interval uncertainty. According to some numerical results, the percentage changes of robust and stochastic models are compared relative to deterministic models in different cases. The numerical results show that the robust model in comparison with the stochastic programming model gives a closer fit to the results of the deterministic model.
Keyword — Closed-loop supply chain, Mixed-integer linear programming, Multi-objective programming, Robust Optimization.

1. INTRODUCTION

Closed loop supply chain (CLSC) network aims at reducing the waste and generating profit for enterprises through integrating forward and reverse logistics. CLSC has been considered as one of the sustainable practice to push conventional open-loop systems to closed-loop ones by Elahi and Franchetti (2014). Reusing returned products by recycling process, prevents depleting resources, reduces environmental pollution and optimizes utility by Guide Jr. and Van Wassenhove (2006). Therefore, considering the process of handling the product returns in supply chain is of significant importance by Zareian Jahromi et al. (2014). Due to its various aspect importance such as environmental, limited resources, it is the focus of current research, Ayres et al (1997), Bloemhof and Corbett (2010), De Giovanni (2014), Mohajeri and Fallah (2014), Paksoy and Ozceylan (2014), and several mixed integer programming models are developed to optimize CLSC. However, due to the conflict between economic optimization and environmental protections, several research has considered multi-objective and goal programming approach for CLSC. For example, in Amin and Zhang (2013a), a mixed integer linear programming (MILP) model is proposed that minimizes the total cost. Besides, the model is extended to consider environmental factors by weighed sums and ε-constraint methods.

Since in a CLSC network, in particular, in reverse flow, some parameters might be uncertain, many researchers have used different approaches to deal with it. El-Sayed et al. (2010) developed an MILP model for a CLSC network in the presence of uncertainty in demand. Pishvaee et al. (2011) formulated an MILP model in the existence of uncertainty in demands, returns and transportation costs parameters based on Ben-Tal and Nemirovski approach (1998), (2000). An enhanced version of the uncertain model in Pishvaee et al. (2011) is given in Makrooni and Salahi (2016) that has much less constraints when demands, returns and transportation costs between facilities are uncertain. Salema et al. (2007) extended the reverse logistics model of Fleischmann et al. (2001) in order to take into account uncertainty in demands. Francas and Minner (2009) proposed a two-stage stochastic model to design a closed-loop network under uncertain demands and returns. Pishvaee et al. (2009) proposed a deterministic optimization model for a reverse logistics network. However, environmental factors have not been taken into account in the model. Lee and Dong (2009) proposed a two-stage stochastic programming model for CLSC network. They also developed a solution approach by simulated annealing. Pishvaee and Torabi (2010) developed a possibilistic mixed integer programming model to deal with uncertainty in CLSC configuration. Shi et al. (2010) proposed a model to maximize the profit of a remanufacturing system using a Lagrangian relaxation method. Wang and Hsu (2010) proposed an interval programming model where the uncertainty is in the form of fuzzy numbers. Shi et al. (2011) studied a production planning problem for a multi-product closed-loop system. The authors considered uncertain demands and returns by stochastic programming. Amin and Zhang (2013b) developed an optimization model under uncertain

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demands and decision environment for a CLSC. Vahdani et al. (2012) applied fuzzy multi-objective robust optimization to configure a CLSC network. Amin and Zhang (2013a) have investigated the impact of demands and returns uncertainties on the network configuration by stochastic programming (scenario-based). In this paper our focus is on the model of Amin and Zhang (2013a). We consider interval uncertainty in returns and demands and present the robust counterpart of the model in Amin and Zhang (2013a). Moreover, on several examples, the robust approach is compared with stochastic programming approach of Amin and Zhang (2013a) showing that the robust approach performs better.

2. PROBLEM DEFINITION

In this section, a general CLSC network is described that includes plants, collection centers, and demand markets (Fig. 1) Amin and Zhang (2013a). The plants can manufacture new products and remanufacture returned products. The products are sent to demand markets by plants. Then, the returned products are sent to collection centers, where collecting used products from demand markets, determining the condition of them by inspection and/or separation to find out whether they are recoverable or not. Then sending recoverable returns to the plants and unrecoverable returns to the disposal center. The goal is to know how many and which plants and collection centers should be open, and which products and in which quantities should be stock in them in order minimize the cost. Moreover, we assume that

- The network is for a single period.
- All returned products from demand markets are collected in collection centers.
- Locations of demand markets are fixed.
- Locations and capacities of plants and collection centers are known in advance

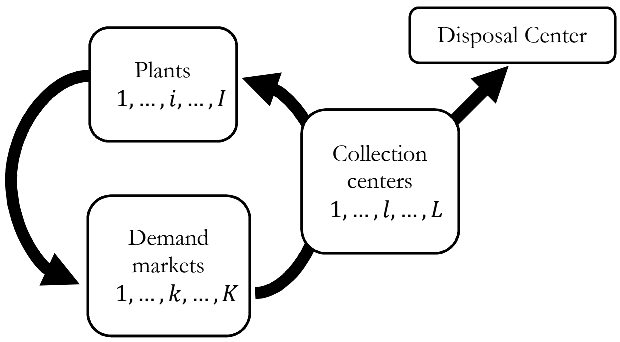

Figure 1. The closed loop supply chain network Amin and Zhang (2013a)

3. MATHEMATICAL MODEL

The network is formulated as an MILP problem, where sets, parameters and decision variables are defined as follows Amin and Zhang (2013a):

Sets

$I =$ set of potential manufacturing and remanufacturing plants locations $(1,\ldots,i,\ldots,I)$

$J =$ set of products $(1,\ldots,j,\ldots,J)$

$K =$ set of demand markets locations $(1,\ldots,k,\ldots,K)$

$L =$ set of potential collection centers locations $(1,\ldots,l,\ldots,L)$

Parameters

$A_j =$ production cost of product j

$B_j =$ transportation cost of product j per km between plants and demand markets

$C_j =$ transportation cost of product j per km between demand markets and collection centers

$D_j =$ transportation cost of product j per km between collection centers and plants

$O_j =$ transportation cost of product j per km between collection centers and disposal centers

$E_i =$ fixed cost for opening plant i

$F_l =$ fixed cost for opening collection center l

$G_j =$ cost saving of product j (because of product recovery)

$H_j =$ disposal cost of product j

$P_{ij} =$ capacity of plant i for product j

$Q_{lj} =$ capacity of collection center l for product j

$t_{ik} =$ the distance between location i and k generated based on the Euclidean method (t_{kl} and t_{li} are defined in the same way).

$t_l =$ is the distance between collection center l and disposal center

$d_{kj} =$ demand of customer k for product j

$r_{kj} =$ return of customer k for product j

$\alpha_j =$ minimum disposal fraction of product j

Variables

$X_{ikj} =$ quantity of product j produced by plant i for demand market k

$Y_{klj} =$ quantity of returned product j from demand market k to collection center l

$S_{lij} =$ quantity of returned product j from collection center l to plant i

$T_{lj} =$ quantity of returned product j from collection center l to disposal center

$Z_i =$ 1, if a plant is located and set up at potential site i, 0, otherwise

$W_l =$ 1, if a collection center is located and set up at potential site l, 0, otherwise

$$\min Z_1 = \sum_i E_i Z_i + \sum_l F_l W_l + \sum_i \sum_k \sum_j \left(A_j + B_j t_{ik}\right) X_{ikj} + \sum_k \sum_l \sum_j C_j t_{kl} Y_{klj} + \sum_l \sum_i \sum_j \left(-G_j + D_j t_{li}\right) S_{lij} + \sum_l \sum_j \left(H_j + O_j t_l\right) T_{lj}$$

$$s.t. \quad \sum_i X_{ikj} \geq d_{kj}, \forall k, j \tag{1}$$

$$\sum_l \sum_j S_{lij} + \sum_k \sum_j X_{ikj} \leq Z_i \sum_j P_{ij}, \quad \forall i \tag{2}$$

$$\sum_l Y_{klj} \leq \sum_i X_{ikj}, \quad \forall k, j \tag{3}$$

$$\alpha_j \sum_k Y_{klj} \leq T_{lj}, \quad \forall l, j \tag{4}$$

$$\sum_k \sum_j Y_{klj} \leq W_l \sum_j Q_{lj}, \quad \forall l \tag{5}$$

$$\sum_k Y_{klj} = \sum_i S_{lij} + T_{lj}, \quad \forall l, j \tag{6}$$

$$\sum_l Y_{klj} = r_{kj}, \quad \forall k, j \tag{7}$$

$$Z_i, W_l \in \left\{0, 1\right\}, \quad \forall i, l \tag{8}$$

$$X_{ikj}, Y_{klj}, S_{lij}, T_{lj} \geq 0, \quad \forall i, k, j, l \tag{9}$$

The objective function is minimization of the total cost. The first and second terms are the fixed costs of opening plants and collection centers, respectively. The third term is the production and transportation costs of the new products. The forth term is related to product recovery and transportation costs of returned products. The fifth term represents the total recovery and transportation costs of returned products from collection centers to plants. Finally, the sixth term is the disposal and transportation costs.

The constraint (1) guarantees that the total number of each manufactured product for each demand market is equal or greater than the demand. Capacity constraint of plants is given by (2). Constraint (3) indicates that forward flow is greater than reverse flow. Constraint (4) enforces a minimum disposal fraction for each product. Constraint (5) gives capacity constraint of collection centers. Constraint (6) tells that the quantity of returned products from demand market is equal to the quantity of returned products to plants and quantity of products in disposal center for each collection center and each product. Constraint (7) represents the returned products. Constraints (8) and (9) are the binary and non-negative decision variables.

Beside the total costs which is minimized, the author in Amin and Zhang (2013a) considered environmental issues in the model. To do so, new parameters are defined. M_{ij} is parameter of using environmental friendly materials by plant i to produce product j and N_{lj} is parameter of using clean technology by collection center l to process product j. Thus the second objective function can be written as follows:

$$\max Z_2 = \sum_i \sum_j M_{ij} \left(\sum_k X_{ikj} + \sum_l S_{lij} \right) + \sum_l \sum_j N_{lj} \left(\sum_k Y_{klj} + \sum_i S_{lij} + T_{lj} \right)$$

To solve this multi-objective optimization problem ε-constraint method is used in Amin and Zhang (2013a) as follow.

$$\min Z = \sum_i E_i Z_i + \sum_l F_l W_l + \sum_i \sum_k \sum_j \left(A_j + B_j t_{ik} \right) X_{ikj} + \sum_k \sum_l \sum_j C_j t_{kl} Y_{klj} + \sum_l \sum_i \sum_j \left(-G_j + D_j t_{li} \right) S_{lij} +$$
$$\sum_l \sum_j \left(H_j + O_j t_l \right) T_{lj}$$

$$s.t. \quad \sum_i \sum_j M_{ij} \left(\sum_k X_{ikj} + \sum_l S_{lij} \right) + \sum_l \sum_j N_{lj} \left(\sum_k Y_{klj} + \sum_i S_{lij} + T_{lj} \right) \geq \varepsilon \tag{10}$$

$$\sum_i X_{ikj} \geq d_{kj}, \quad \forall k, j \tag{11}$$

$$\sum_l \sum_j S_{lij} + \sum_k \sum_j X_{ikj} \leq Z_i \sum_j P_{ij}, \quad \forall i \tag{12}$$

$$\sum_l Y_{klj} \leq \sum_i X_{ikj}, \quad \forall k, j \tag{13}$$

$$\alpha_j \sum_k Y_{klj} \leq T_{lj}, \quad \forall l, j \tag{14}$$

$$\sum_k \sum_j Y_{klj} \leq W_l \sum_j Q_{lj}, \quad \forall l \tag{15}$$

$$\sum_k Y_{klj} = \sum_i S_{lij} + T_{lj}, \quad \forall l, j \tag{16}$$

$$\sum_l Y_{klj} = r_{kj}, \quad \forall k, j \tag{17}$$

$$Z_i, W_l \in \left\{ 0,1 \right\}, \quad \forall i, l \tag{18}$$

$$X_{ikj}, Y_{klj}, S_{lij}, T_{lj} \geq 0, \quad \forall i, k, j, l \tag{19}$$

4. ROBUST MODEL UNDER INTERVAL UNCERTAINTY

Uncertainty in demands and returns are major issues in a supply chain network. Thus it is beneficial to take them into account in the optimization model. In Amin and Zhang (2013a) the authors have used stochastic programming in order to include uncertainty in the presented model. However, here we discuss the robust counterpart of the presented model under interval uncertainty for demands and returns.

Consider the following deterministic linear optimization problem:

$$\begin{aligned} \min \quad & cx + d \\ s.t. \quad & Ax \le b \end{aligned}$$

Based on Ben-Tal and Nemirovski (1998, 2000), the related uncertain linear optimization problem that consists of a collection of linear optimization problems can be defined as follows:

$$\begin{aligned} &\min \quad cx + d \\ &s.t. \quad Ax \le b \\ &c, d, A, b \in U \end{aligned} \tag{20}$$

where U is the uncertainty set for uncertain data. A vector x is a robust feasible solution to problem (20) if it satisfies all realizations of the constraints from the uncertainty set U. Ben-Tal and Nemirovski (1999) defined the robust counterpart of problem (20) as follows

$$\min \left\{ \hat{c}\left(x\right) = \sup_{c,d\, A,b \in U} \left[cx + d\right] : Ax \le b, \forall c, d, A, b \in U \right\} \tag{21}$$

An optimal solution to problem (21) is the optimal robust solution of problem (20). Such a solution satisfies the constraints for all possible realizations of the data, and guarantees an optimal objective function value not worse than $\hat{c}\left(x^*\right)$. Problem (21) is a semi-infinite linear optimization problem and seems to be computationally intractable. Nevertheless, it turns out that for a wide variety of compact, convex uncertainty sets, the robust counterpart model is a tractable (polynomials solvable) convex optimization problem, usually a linear optimization or a conic quadratic problem (see Ben-Tal et al. (2009), Ben-Tal and Nemirovski (2000, 2002)).

Under box uncertainty, $\xi = \left\{\xi_{ij}\right\}_{i=1,..,m, j=1,...,n}$ is unknown but bounded in a box of the form

$$u_{Box} = \left\{ \xi_{ij} \in R : \left| \xi_{ij} - \overline{\xi}_{ij} \right| \le \rho G_{ij}, i = 1, \ldots, m, j = 1, \ldots, n \right\}$$

where $\overline{\xi}_{ij}$ is the nominal value of the ξ_{ij} and the positive numbers G_{ij} represent "uncertainty scale" and $\rho > 0$ is the "uncertainty level". A particular case of interest is $G_{ij} = \overline{\xi}_{ij}$, which corresponds to a simple case where box contains ξ_{ij} whose relative deviation from the nominal data is of size up to ρ.

To develop the robust counterpart of model (1), demands and returns are considered as uncertain parameters and it is assumed they belong to certain intervals. Thus constraint (11) in the uncertain case is as follows:

$$\sum_i X_{ikj} \ge d_{kj}, \quad \forall k, j, \quad such\ that\ d_{kj} \in u^d_{Box} = \left\{ d_{kj} \in R : \left| d_{kj} - \overline{d}_{kj} \right| \le \rho G^d_{kj}, \ \forall k = 1, \ldots, K, j = 1, \ldots, J \right\}$$

To have this inequality feasible for any d_{kj} in the given uncertainty set u^d_{Box} so as to immunize against infeasibility, it is sufficient to have

$$\sum_i X_{ikj} \ge \max_{d_{kj} \in u^d_{Box}} d_{kj}, \quad \forall k, j$$

This is further equivalent to

$$\sum_i X_{ikj} \geq \overline{d}_{kj} + \rho G^d_{kj}, \quad \forall k, j$$

However, for the returns constraints since it is in equality form in (17), thus first we relax it to the inequality one without losing anything. Since the model aims to minimize the returned products as it is a part of the objective function. Then analogous to the demands constraints, the uncertain version of returns constraints (17) with interval uncertainty is as follows:

$$\sum_l Y_{klj} \geq \overline{r}_{kj} + \rho G^r_{kj}, \quad \forall k, j$$

Therefore, the robust counterpart of model (10)-(19) under interval uncertainty for demands and returns is the following MILP problem:

$$\min Z = \sum_i E_i Z_i + \sum_l F_l W_l + \sum_i \sum_k \sum_j \left(A_j + B_j t_{ik}\right) X_{ikj} + \sum_k \sum_l \sum_j C_j t_{kl} Y_{klj} + \sum_l \sum_i \sum_j \left(-G_j + D_j t_{li}\right) S_{lij} +$$
$$\sum_l \sum_j \left(H_j + O_j t_l\right) T_{lj}$$
$$s.t. \quad \sum_i \sum_j M_{ij} \left(\sum_k X_{ikj} + \sum_l S_{lij}\right) + \sum_l \sum_j N_{lj} \left(\sum_k Y_{klj} + \sum_i S_{lij} + T_{lj}\right) \geq \varepsilon$$
$$\sum_i X_{ikj} \geq \overline{d}_{kj} + \rho G^d_{kj}, \quad \forall k, j$$
$$\sum_l \sum_j S_{lij} + \sum_k \sum_j X_{ikj} \leq Z_i \sum_j P_{ij}, \quad \forall i$$
$$\sum_l Y_{klj} \leq \sum_i X_{ikj}, \quad \forall k, j$$
$$\alpha_j \sum_k Y_{klj} \leq T_{lj}, \quad \forall l, j$$
$$\sum_k \sum_j Y_{klj} \leq W_l \sum_j Q_{lj}, \quad \forall l$$
$$\sum_k Y_{klj} = \sum_i S_{lij} + T_{lj}, \quad \forall l, j$$
$$\sum_l Y_{klj} \geq \overline{r}_{kj} + \rho G^r_{kj}, \quad \forall k, j$$
$$Z_i, W_l \in \{0,1\}, \quad \forall i, l$$
$$X_{ikj}, Y_{klj}, S_{lij}, T_{lj} \geq 0, \quad \forall i, k, j, l$$

5. COMPUTATIONAL EXPERIMENTS

In this section, on several randomly generated examples, we compare the robust optimization model with the stochastic programming approach used in Amin and Zhang (2013a). All optimization problems are solved using CPLEX 12.5. In Amin and Zhang (2013a) scenario analysis has been used to observe the effects of uncertainty. In this paper, at first like Amin and Zhang (2013a), deterministic model is being solved by using nominal data and the existing scenarios in Table 2. Nominal data are randomly generated using the random distributions specified in Table 1. In the deterministic model, ε is a very important parameter. In Amin and Zhang (2013a), the value of the objective function is evaluated for different ε values and it is observed that if the value of ε increases then the value of the objective function will increase too. According to Figure 5 in Amin and Zhang (2013a), for ε from 40,000 to 500,000, the changes of the objective function is not significant but for ε greater than 500,000 the objective function increases magnificently. Thus in this paper, we consider ε is 500,000. By considering the probability of accuracy of each scenario, the result of the stochastic model is obtained and listed in Table 2. In the last column of Table 2, the percentage changes of results from both deterministic and stochastic models relative to scenario 5 (base case) are obtained. As stated in Amin and Zhang (2013a), the results show that the stochastic programming model can gain flexible optimal CLSC configuration with the objective function near to the scenario 5 (base case).

Table 1. The sources of random generation of the nominal data Amin and Zhang (2013a)

Parameter	Corresponding random distribution
$I \times J \times K \times L \times U$	$4 \times 3 \times 5 \times 4 \times 9$
A_j	~uniform (13.5, 16.5)
B_j	~uniform (0.0131, 0.0160)
C_j	~uniform (0.0045, 0.0055)
D_j	~uniform (0.0027, 0.0033)
O_j	~uniform (0.0014, 0.0017)
G_j	~uniform (6.3, 7.7)
H_j	~uniform (2.25, 2.75)
α_j	~uniform (0.27, 0.33)
E_i	~uniform (4,500,000, 5,500,000)
F_l	~uniform (450,000, 550,000)
$t_{ik}, t_{kl}, t_{li}, t_l$	~uniform (0,100)
M_{ij}, N_{lj}	~uniform (0,1)
P_{ij}	~uniform (75,600, 92,400)
Q_{lj}	~uniform (30,600, 37,400)
d_{kj}	30000
r_{kj}	10000

Table 2. Scenario analysis.

	Scenarios	Demand	Return	Probability	Objective Value	Change %
Deterministic	1	33000	9000	0.075	18,906,118	6.1218
	2	27000	11000	0.075	16,731,719	-6.0833
	3	30000	11000	0.1	17,859,228	0.2455
	4	30000	9000	0.1	17,773,129	-0.2376
	5 (Base Case)	30000	10000	0.3	17,815,494	0.0000
	6	33000	10000	0.1	18,948,124	6.3576
	7	27000	10000	0.1	16,723,548	-6.1292
	8	33000	11000	0.075	18,992,876	6.6088
	9	27000	9000	0.075	16,720,837	-6.1444
Stochastic	10	Combination of nine scenarios			17,826,417	0.0613

In order to evaluate the robust model we consider two cases. In the first case, the number of demands and returned products are constant but in the second case these quantities are chosen randomly in an interval in order to consider the entire amounts scenario in this interval. We consider 5 uncertainty levels (ϱ=0.2,0.5,1,2,3) and we let $G^d_{kj} = 1$ and $G^r_{kj} = 1$.

First case

Deterministic and robust models are solved using nominal data provided in Table 1, and the results are listed in the third column of Table 3. Then under each uncertainty level, three random realizations are uniformly generated in the corresponding uncertainty set (i.e. $\left[\text{nominal value} - \varrho\text{G}, \text{nominal value} + \varrho\text{G}\right]$) to analyze the performance of the solutions obtained by the proposed robust and deterministic models. The results are given in the forth columns of Tables 3. The fifth column of Table 3 shows the percentage changes of the values of the objective function for deterministic and robust (fourth column) relative to the value of the objective function of the deterministic model under the nominal data.

Second case

In this case, it is supposed that nominal data for parameters with uncertainty d_{kj} and r_{kj} are randomly generated using the random distributions specified in the intervals (27000, 33000) and (9000, 11000), respectively and the nominal data for other parameters generated from Table 1. Similar to the method explained in the first case, deterministic and robust models are investigated under nominal data and realization and percentage changes are calculated. The results are obtained in Table 4.

The results obtained in Tables 3 and 4 show that increasing uncertainty level causes an increase in the value of the objective function for the robust model and results in an increase in the percentage of its changes relative to deterministic model under nominal data. For lower uncertainty levels, percentage changes of robust model in many cases is less than the stochastic model but with increasing the uncertainty level, the percentage of changes of robust model is more than stochastic model, because in stochastic model, moving far away from nominal data brings down the probability of accuracy of scenarios while in robust model, data are distributed uniformly around the nominal data.

Table 3. Summary of First case results under uncertain return and demands.

d_{kj}, r_{kj}	ρ	Objective Function Value under nominal data		Objective function values under realizations		Percentage change over the final objective function value under nominal data (%)	
		Deterministic	Robust	Deterministic	Robust	Deterministic	Robust
$d_{kj} = 30000$ $r_{kj} = 10000$	0.2	17,815,494	17,818,939	17,818,818	17,822,263	0.0187	0.0380
				17,813,953	17,817,398	-0.0086	0.0107
				17,813,147	17,816,591	-0.0132	0.0062
	0.5		17,824,106	17,815,305	17,823,917	-0.0011	0.0473
				17,823,466	17,832,078	0.0447	0.0931
				17,820,653	17,829,265	0.0290	0.0773
	1		17,832,718	17,802,966	17,820,189	-0.0703	0.0264
				17,812,948	17,830,171	-0.0143	0.0824
				17,825,892	17,843,115	0.0584	0.1550
	2		17,849,941	17,844,760	17,879,207	0.1643	0.3576
				17,825,951	17,860,398	0.0587	0.2521
				17,840,238	17,874,685	0.1389	0.3322
	3		17,867,164	17,863,793	17,915,464	0.2711	0.5611
				17,834,543	17,886,213	0.1069	0.3970
				17,840,799	17,892,469	0.1420	0.4321

Table 4. Summary of Second case results under uncertain return and demands.

d_{kj}, r_{kj}	ρ	Objective Function Value under Nominal data		Objective function values under realizations		Percentage change over the final objective function value under nominal data (%)	
		Deterministic	Robust	Deterministic	Robust	Deterministic	Robust
$d_{kj} \in (27000, 33000)$ $r_{kj} \in (9000, 11000)$	0.2	17,666,427	17,669,874	17,668,658	17,672,105	0.0126	0.0321
				17,667,825	17,671,272	0.0079	0.0274
				17,664,831	17,668,278	-0.0090	0.0105
	0.5		17,675,044	17,660,008	17,668,624	-0.0363	0.0124
				17,659,467	17,668,084	-0.0394	0.0094
				17,669,787	17,678,404	0.0190	0.0678

	1		17,683,660	17,670,873 17,681,938 17,664,239	17,688,106 17,699,171 17,681,472	0.0252 0.0878 -0.0124	0.1227 0.1853 0.0852
	2		17,700,893	17,651,009 17,685,209 17,644,566	17,685,475 17,719,675 17,679,032	-0.0873 0.1063 -0.1237	0.1078 0.3014 0.0714
	3		17,718,126	17,714,916 17,660,919 17,688,670	17,766,614 17,712,618 17,740,369	0.2745 -0.0312 0.1259	0.5671 0.2615 0.4185

Finally, to show the effect of minimum disposal fraction of product j (α_j), which is an important parameter related to reverse supply on the objective function, sensitivity analysis is performed. The three models, deterministic, robust and stochastic are evaluated for different values of α_j. Fig. 2 and 3 show the results for all three deterministic (under nominal data), robust and stochastic models. These two figures show that by increasing parameter (α_j), the value of the objective function for all the three models increases. Also it can be observed that the value of the objective function for deterministic and robust models are close to each other.

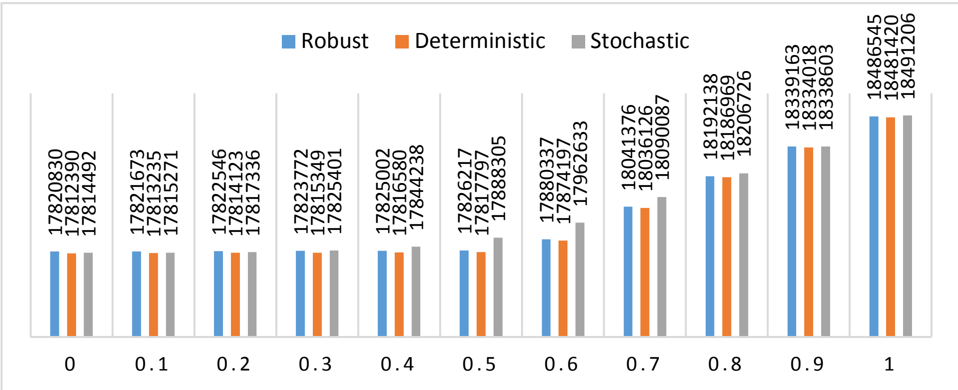

Figure 2. Sensitivity analysis of α_j for First case in deterministic (under nominal data), robust (with ϱ=0.5) and stochastic scenarios

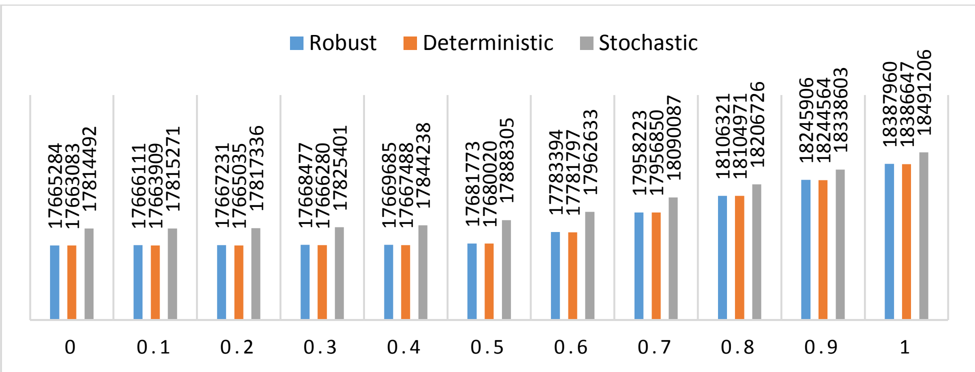

Figure 3. Sensitivity analysis of α_j for Second case in deterministic (under nominal data), robust (with ϱ=0.5) and stochastic scenarios.

6. CONCLUSION

In this paper, a mixed integer programming model for a closed loop supply chain network is considered in accordance with Amin and Zhang (2013a). Due to the existence of uncertainty in demand and return parameters, the robust counterpart of the model was presented. Then for some examples, the sensitiveness of the model relative to different uncertainty levels and parameter α_j was assessed and the obtained numerical results showed that the percentage of changes of results in robust models has been less than stochastic model in most cases while by increasing the level of uncertainty, this rate for stochastic model has been less. Also the numerical results showed that the value of the objective function for all three models with parameter α_j have a direct relation, it means that by increasing the quantity of the parameter α_j the value of the objective function for all the three models increases.

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