International Journal of
Operations Research

International Journal of Operations Research Vol. 14, No. 4, 169−176 (2017)

Effectiveness Comparison of Solution Approaches for Solving the Economic Lot Scheduling Problem

Yu-Jen Chang[1], and Ming-Jong Yao[2]*

[1]Department of Information Management,
Tunghai University, Taichung, Taiwan

[2]Department of Transportation and Logistics Management,
National Chiao-Tung University, Hsinchu, Taiwan

Abstract: The most popular solution approaches for solving the economic lot scheduling problem (ELSP) include the common cycle (CC) approach, the extended basic period (EBP) approach and the time-varying lot sizing (TVLS) approach where both the CC and EBP approaches are the basic period-based approaches. Some studies claimed that the TVLS approach is able to obtain better solutions than the basic period-based approaches though without the support of extensive numerical experiments. Therefore, we are motivated to compare the performance of the CC, EBP and the TVLS approaches in this study. Interestingly, our random experiments show no one serves as a dominant solution approach among the three approaches, but the settings of setup cost and setup time could affect their performance significantly. Our results may be valuable for the decision makers before they choose a solution approach for solving the ELSP.
Keyword — Lot, scheduling, inventory, production

1. INTRODUCTION

The Economic Lot Scheduling Problem (ELSP) has been studied for longer than fifty years and more than 100 related papers have been published in the literature (Moon et al., 2002). The ELSP may be applied in the production planning and inventory control for some industries such as plastics extrusion, metal stamping, textile manufacturing, bottling, printing and packing. The ELSP is well known as NP-hard (Hsu, 1983), and it is concerned with the scheduling of cyclical production of more (n 2) than two products on a single facility in equal lots over an infinite planning horizon, assuming stationary and known demands for each product. The objective of the ELSP is to determine the lot size and the schedule of production of each product so as to minimize the total cost incurred per unit time. The costs considered include the setup cost and inventory holding cost.

In the literature, the three most popular solution approaches for solving the ELSP could be the common cycle (CC) approach, the basic period (BP) approach, the extended basic approach (EBP) approach and the time-varying lot sizing (TVLS) approach. We present a brief review on these three approaches as follows.

The CC approach is a simple method that guarantees a feasible solution. It assumes that all products must be produced within the same cycle. Therefore, a common cycle time must be large enough to accommodate the production lots of all products. The solution of the CC approach can be viewed as the upper bound of the cost for the ELSP. One may refer to Hanssmann (1962) for the details on the CC approach. Jones and Inman (1989) proved that the CC approach obtains an optimal solution when the ratio of the demand rate to the production rate of each product is equal to each other.

When applying the BP approach, researchers (e.g., Bomberger, 1966 and Elmaghraby, 1978) use either dynamic programming or nonlinear integer programming to solve the ELSP. It restricts all products must be produced at the first period, but each of all products may or may not be produced at other periods. Then for product i, its cycle time Ti (or production frequency) is an integer multiplier ki of the basic period B, i.e., Ti = kiB. In general, the BP approach can find better solutions than the CC approach. Its disadvantage is that B must be large enough in order to produce all products at the first period. Therefore, there exists much idle capacity at other periods. Grznar and Riggle (1997) used the BP approach to provide an algorithm that can find optimal solutions for the ELSP.

The EBP approach is similar to the BP approach though the former allows the flexibility of scheduling the production of a product not being started at the first basic period. If the sum of the production time and setup time of all products that are produced within a period on a facility is less than or equal to the length of the basic period B, the cyclic schedule for a facility is feasible. Since the CC and BP approaches are special cases of the EBP approach, the EBP approach always obtains better solutions than the BP approach. However, it cannot ensure the feasibility of

*Corresponding author's e-mail: myaoie@gmail.com Tel: +886-3-5712121 ext. 57215. Fax: +886-3-5720844.

the schedule for a facility. To solve the ELSP, one needs not only to minimize the objective function value, but also to generate a feasible production schedule. Yao (2001) provided an effective heuristic, namely, Proc FT, to test the feasibility of a candidate solution (B, {ki}). Yao et al. (2003) compared Yao's (2001) heuristic with Park and Yun's (1987), Boctor's (1987) and Geng and Vickson's (1988) heuristics, and recommend Yao's (2001) heuristic as a preferred one. One may observe that the CC, BP and EBP approaches are all basic-period based. Theoretically, the EBP approach shall obtain better solutions than the CC and BP approaches since it enjoys the most flexibility in the production scheduling. However, when the utilization factor of a facility is closer to 1, the EBP approach often solves the same solution as the CC approach due to the difficulty of generating a feasible production schedule. Also, the EBP sometimes become unattractive due to its long run time for testing the feasibility of candidate solutions obtained during the search process. (Moon et al., 2002).

Maxewll (1964) proposed the TVLS approach that relaxed the restriction of equally spaced production lots. The TVLS approach allows the lot sizes of each product i to be different within a replenishment cycle T. It implies that the production frequencies in of all products are not the same in the replenishment cycle. Dobson (1987) proposed a heuristic based on the TVLS approach for solving the ELSP under PoT policy. Under the TVLS approach, PoT policy restricts that the production frequency xi of product i is a power-of-two integer, i.e., , for some non-negative integer wi. Dobson (1987) emphasized that the TVLS approach has two advantages: First, feasibility is no longer a problem since the TVLS approach is always able to find a feasible solution if a facility has idle time to setup. This property is particularly useful when the proportion of the sum of setup times is low or zero. It can avoid a feasibility checking problem as the EBP approach meets. Second, the utilization of a facility will be substantially more uniform than the basic period-based approaches. Also, under PoT policy, the TVLS approach is a more efficient solution approach than the BP and EBP approaches. The major difficulty for the TVLS approach arises from finding an optimal production sequence by minimizing the average total cost when the production frequencies {xi} of all products produced on a facility are known.

Zipkin (1991) extends Dobson's (1987) heuristic to find the production run times and machine idle times for each product for a given sequence. Moon et al. (2002) provided a GA as the solution approach to solve the ELSP using the TVLS approach under GI policy where GI policy restricts the production frequency of each product must be a positive integer. They proposed a GA to improve Dobson's heuristic by 1%, and they claimed that the TVLS approach finds better solutions than the CC and BP approaches (but, without the supports of numerical experiments). In Moon et al.'s (2002) GA, an individual represents a possible production sequence for all products, and product i is produced xi times in unequal lot sizes (and production times) in a known production sequence. Therefore, the GA must compute the time length of each production of each product for a production sequence represented in each individual.

Using the TVLS approach, Chang and Yao (2010) developed a hybrid GA approach to solve the ELSP with multiple identical facilities. Their numerical experiments showed that the quality of the solutions obtained from the TVLS approach is data-dependent. For some combination of parameter settings, the TVLS approach obtained better solutions than the EBP approach. However, surprisingly, the solutions obtained from the TVLS approach could be worse than the CC approach for some instances. Chang and Yao's (2010) results showed significant difference from those studies that advocated of the TVLS approach.

To the best of the authors' knowledge, no studies compared the TVLS approach with the basic period-based approaches for solving the ELSP in the literature. Therefore, we are motivated to investigate the effectiveness comparison in this study. Note that we engage ourselves in the comparison of the solution quality of three solution approaches for the ELSP rather than the aspect of their run time.

The rest of this paper is organized as follows. The second section presents the numerical experiments of this study and the discussions on our observations from the numerical experiments. We address our concluding remarks in the third section.

2. NUMERICAL EXPERIMENTS AND DISCUSSIONS

We would investigate the relationship between the settings of setup (times and costs) and the performance of three solution approaches, namely, the CC, the EBP and the TVLS approaches since Chang and Yao (2010) indicated that the setup costs and times of products are critical when solving the ELSP.

Before presenting our numerical experiments, we would like to introduce the common assumptions when applying the three approaches as follows.

1. The facility produces only one product at one time.
2. The capacity of the facility is affordable to meet the demand of all products.
3. The setup times and setup costs of products depend only on the product to be produced, but are independent of the sequence and the lot sizes of production lots.
4. The demand of each product is continuous.
5. The parameters (e.g., the demand rate, the production rate, the setup time, the setup cost and the holding

cost) of each product are known constants and invariant over time.

One may refer the mathematical model of the CC approach to Hanssmann (1962). This study adopted Dobson's (1987) heuristic when applying the TVLS approach. Note that Dobson's heuristic employs Power-of-Two (PoT) policy when determining the production frequency of each product. Also, we applied the GA approach proposed by Chang and Yao (2006) as solving the ELSP by the EBP approach.

We randomly generated the instances in our experiments using the ranges shown in Table 1. Therefore, the values of the parameters were generated from the uniform distributions $U\left[\text{mean} - \left(\text{range}/2\right), \text{ mean} + \left(\text{range}/2\right)\right]$. The data ranges in Table 1 refer to Carreno's (1990). We divided our numerical experiments into nine cases (combinations) of the setup costs and times as shown in Figure 1 and Table 2.

Table 1. The ranges of the parameters for a product

	Mean	Range
Production (units/day) (p_i)	14000.000	5000.000
Setup cost ($) ($a_i$)	200.000	400.000
Setup-time (days) (s_i)	0.280	0.440
Holding costs ($/unit-year) ($h_i$)	0.350	0.700
Demand (units/day) (d_i)	2500.000	4800.000
1 year = 240 days		

Source: Carreno (1990)

We refer it the case the setup costs (time) take *lower* values when the values of the setup cost (time) of a product fall in the range of (0, 200) ([0.03, 0.25]). On the other hand, we name it the situation the setup costs (time) take *higher* values when the values of the setup cost (time) of a product are in the range of [200, 400] ([0.31, 0.53]).

Table 2. The nine cases for the setup costs and times of products

Cases	The value range of setup cost	The value range of setup time
A	(0, 400]	[0.06, 0.50]
B1	(0, 200]	[0.06, 0.50]
B2	[200, 400]	[0.06, 0.50]
C1	(0, 400]	[0.03, 0.25]
C2	(0, 400]	[0.31, 0.53]
D1	[200, 400]	[0.03, 0.25]
D2	(0, 200]	[0.03, 0.25]
D3	(0, 200]	[0.31, 0.53]
D4	[200, 400]	[0.31, 0.53]

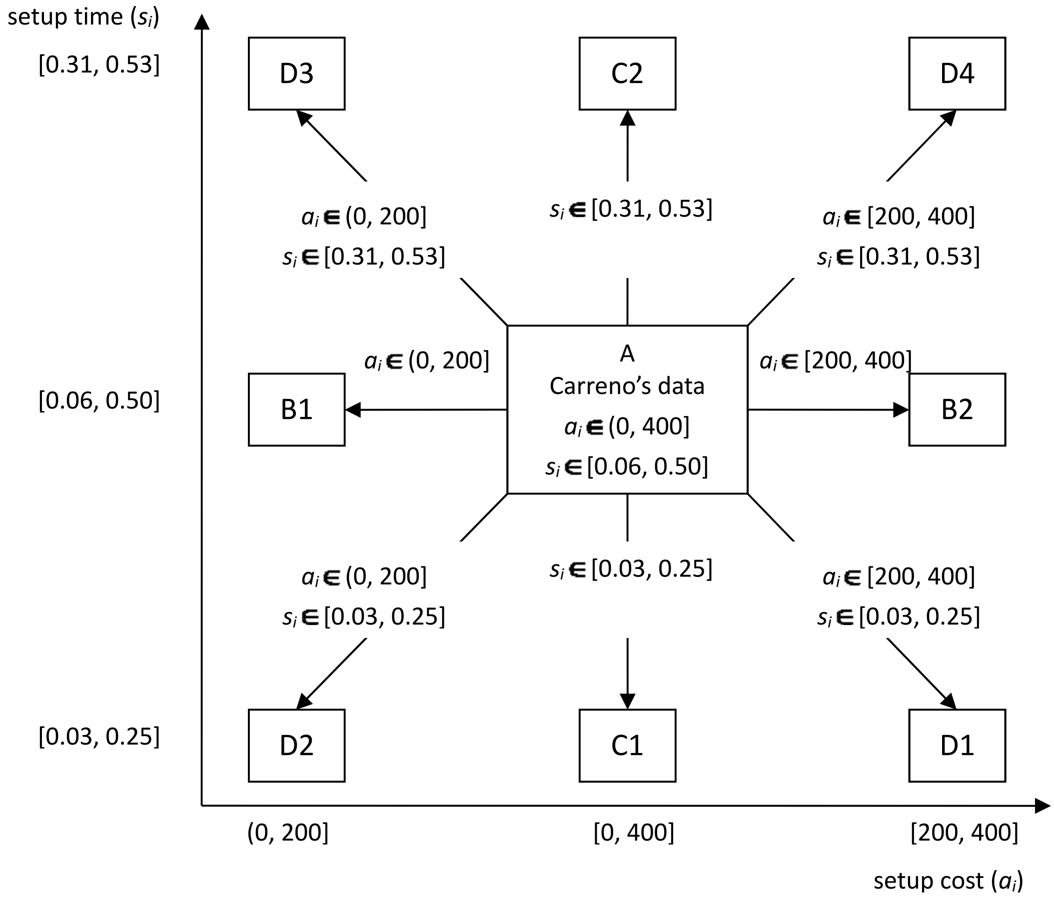

Figure 1. The settings of setup costs and setup times of products in the nine cases

Beside of the settings of the setup cost and time, we conduct our numerical experiments by taking into account of the utilization factor (UF) of a facility and the number of products as follows.

1. The utilization factor of a facility: We tested four levels of utilization factor, namely, [0.5, 0.6), [0.6, 0.7), [0.7, 0.8), and [0.8, 0.9] when randomly generating the instances in our numerical experiments.
2. The number of products: A facility produces a total of 10, 20 or 30 products in an instance of our experiments.

Therefore, there are a total of 108 cases of experimental settings in our experiments from the nine settings for the setup costs and times of products, four levels of utilization factor for a facility, three sizes for the number of products. We randomly generated 20 instances for each of the 108 cases above. And, we recorded the objective function value, i.e., the average total cost, for each instance after solving the 20 instances by the three ELSP solution approaches for each case. We used a lower bound from the sum of Economic Production Quantity (EPQ) as a benchmark and calculated the deviation (in percentage) from the lower bound for the performance comparison of the three ELSP solution approaches.

Table 3 summarizes the performance of three ELSP solution approaches for solving different cases in our numerical experiments. Also, Figure 2 graphically illustrates how the performance of three ELSP solution approaches varies with the levels of utilization factor and the number of products. We have the following observations from Figure 2.

1. Evidently, the EBP approach outperforms the CC approach for most of the cases. However, when the setup times of products take *higher* values (i.e., from the range of [0.31, 0.53],) the EBP and CC approaches often obtained the same solutions, and the proportion increases as the number of products increases. Obviously, the EBP approach is not able to take advantage when the setup times of products are high since the generating a feasible schedule becomes much more difficult as a period of (larger) setup time must lead the production lot of a product. When the setup costs and times of products take *lower* values (e.g. Case D3), the EBP and CC approaches have higher opportunity to get the same solutions at lower utilization levels as

the number of products increases. Such results are inconsistent with those presented in the literature. Researchers believe that the EBP and CC approaches will get the same solutions when the utilization level of a facility is higher, especially, close to 1.

2. The CC approach often outperforms the TVLS approach when the setup costs or the setup times of products take *higher* values. The TVLS approach often outperforms the EBP approach for the cases the setup costs or the setup times of products take *higher* values..
3. The parameters of setup cost and setup time could be equivalently important to the performance of the TVLS approach by observing the pair cases of (B1, B2) and (C1, C2).
4. Under the same settings of setups, the TVLS approach has *higher* opportunities to outperform the basic period-based approaches at *higher* utilization levels and the proportion increases as the number of products increases. One may have the observations by examining those cases of (B1, A, B2), and (C1, A, C2) in Figure 2.
5. Under the same settings of setups, the EBP and CC approach have higher opportunities to get the same solutions at higher utilization levels and the proportion increases as the number of products increases. This observation is more significant in the cases C2, B1 and D3 in Figure 2.

We have more interesting comments by summarizing our numerical results shown in Figure 2 as follows.

1. When neither the number of the products on a facility is small nor the utilization levels of a facility is low, the EBP approach may get same solution as the CC approach as either of the following conditions holds.
   (1) The setup costs take *lower* values.
   (2) The setup times take *higher* values.
2. When a facility shall produce a larger number of products, the TVLS approach may get better solutions than the EBP approach, as either one of the following conditions hold.
   (1) The setup costs take *lower* values.
   (2) The setup times take *higher* values.
3. When a facility shall produce a smaller number of products or have a lower level of utilization, the TVLS approach may get worse solutions than the CC approach, as either one of the following conditions hold.
   (1) The setup costs take *higher* values.
   (2) The setup times take *lower* values.

3. CONCLUSION

Most of the existing studies applied the basic period-based approaches when solving the ELSP. The TVLS approach serves as an alternative and becomes more popular in the recent years. However, no study compared the performance of the TVLS and the basic period-based approaches for solving the ELSP in the literature. This study intends to fulfill this gap, and we randomly generating the instances for our experiments by referring to Carreno's (1990). conduct extensive numerical experiments by comparing the three solution approaches for different settings of setup costs and setup times of products.

This study tested different settings of setup costs and setup times of products to generate randomly the instances in the experiments. Our results show that the TVLS approach outperforms both basic period-based approaches when either the setup costs take *lower* values or the setup times take *higher* values, specifically, when the number of products is large. However, the TVLS approach may get worse solutions than the CC approach, which usually serves as an upper bound for solving the ELSP, especially, when the setup costs take *higher* values or the setup times take *lower* values.

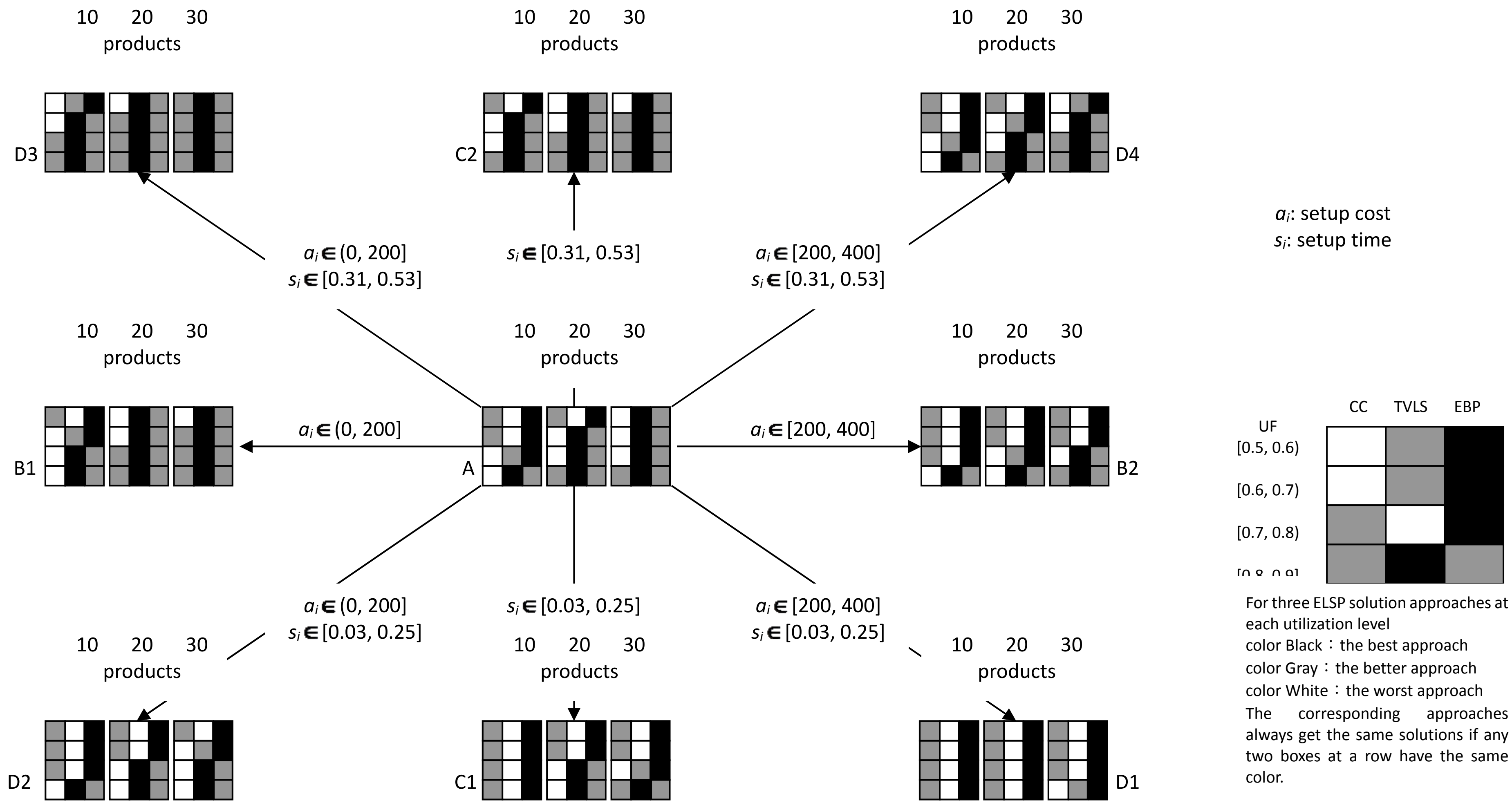

Figure 2. The performance of three ELSP solution approaches for the test cases

Table 3. The deviation (%) from the lower bound of three ELSP solution approaches for the test cases

			CC	TVLS	EBP		CC	TVLS	EBP		CC	TVLS	EBP
			Avg(%)	Avg(%)	Avg(%)		Avg(%)	Avg(%)	Avg(%)		Avg(%)	Avg(%)	Avg(%)
10 products	[0.5, 0.6)	D3	17.22	9.79	6.44	C2	18.85	25.85	5.18	D4	11.58	55.79	1.56
	[0.6, 0.7)		18.68	6.50	16.49		18.00	8.60	8.84		11.21	27.55	1.63
	[0.7, 0.8)		26.17	13.57	26.17		18.12	5.95	16.19		11.01	7.59	4.84
	[0.8, 0.9]		73.33	58.04	73.33		50.18	36.22	50.18		19.30	10.25	18.98
20 products	[0.5, 0.6)		17.89	3.86	17.30		19.98	5.13	10.61		12.36	21.89	1.81
	[0.6, 0.7)		26.89	10.26	26.89		17.30	3.75	15.39		11.27	6.94	3.20
	[0.7, 0.8)		52.68	32.77	52.68		25.12	11.28	25.12		12.92	3.71	11.87
	[0.8, 0.9]		130.96	103.58	130.96		64.17	46.19	64.17		39.56	25.99	39.56
30 products	[0.5, 0.6)		22.41	8.20	22.41		15.32	3.21	14.43		10.58	8.33	3.01
	[0.6, 0.7)		38.10	21.69	38.10		19.90	7.01	19.90		12.04	2.82	10.16
	[0.7, 0.8)		74.65	51.94	74.65		39.08	23.27	39.08		20.45	9.34	20.45
	[0.8, 0.9]		175.15	140.27	175.15		100.26	75.37	100.26		59.14	41.91	59.14
10 products	[0.5, 0.6)	B1	18.77	32.80	3.42	A	18.45	60.76	2.69	B1	12.00	107.11	1.46
	[0.6, 0.7)		18.27	12.90	8.49		17.78	31.26	3.16		11.08	74.08	1.64
	[0.7, 0.8)		18.20	6.36	14.72		19.30	13.80	11.72		10.97	30.63	2.24
	[0.8, 0.9]		33.02	20.86	32.91		21.85	11.20	19.83		10.87	5.54	7.36
20 products	[0.5, 0.6)		19.37	8.09	9.12		17.08	21.60	2.90		12.07	52.47	1.94
	[0.6, 0.7)		19.77	4.76	17.39		17.27	7.13	9.48		10.76	25.96	1.92
	[0.7, 0.8)		25.01	10.34	25.01		16.84	3.90	15.18		10.07	7.97	4.27
	[0.8, 0.9]		75.17	49.57	75.17		37.09	20.21	37.09		17.95	7.95	16.90
30 products	[0.5, 0.6)		17.22	3.13	14.47		16.58	8.19	8.69		10.68	32.50	2.27
	[0.6, 0.7)		20.42	6.51	20.42		17.23	3.40	14.20		10.42	12.83	2.95
	[0.7, 0.8)		36.77	18.81	36.77		21.08	7.60	20.84		11.22	3.56	9.29
	[0.8, 0.9]		85.63	58.33	85.63		52.21	32.51	52.21		26.23	12.23	26.23
10 products	[0.5, 0.6)	D2	19.28	106.19	1.73	C1	18.66	175.00	2.08	D1	11.53	302.09	1.42
	[0.6, 0.7)		17.72	62.69	2.08		16.63	119.73	2.87		11.82	211.34	1.54
	[0.7, 0.8)		15.91	33.34	5.16		17.16	43.13	3.13		11.08	121.15	1.48
	[0.8, 0.9]		17.36	7.60	13.21		14.46	18.07	7.13		10.56	44.63	2.19
20 products	[0.5, 0.6)		18.32	41.35	2.27		18.32	41.35	2.27		10.64	182.37	1.93
	[0.6, 0.7)		16.95	22.45	3.28		16.95	22.45	3.28		11.44	109.27	1.61
	[0.7, 0.8)		18.36	6.96	10.33		18.36	6.96	10.33		10.02	55.08	1.57
	[0.8, 0.9]		20.47	7.50	19.66		20.47	7.50	19.66		10.11	18.18	2.99
30 products	[0.5, 0.6)		16.65	24.77	2.40		16.66	47.34	2.22		10.83	124.08	2.24
	[0.6, 0.7)		17.47	8.03	6.72		17.04	22.17	2.79		10.75	73.76	2.26
	[0.7, 0.8)		16.43	3.55	12.81		17.03	8.31	5.42		10.03	31.99	2.07
	[0.8, 0.9]		29.66	14.85	29.39		16.60	4.61	16.06		9.96	6.61	5.81

REFERENCES

1. Boctor, F.F., "The G-group Heuristic for Single Machine Lot scheduling," *International Journal of Production Research*, **25**, 363-379 (1987).
2. Bomberger, E., "A Dynamic Programming Approach to a Lot Size Scheduling Problem," *Management Science*, **12**, 778-784 (1966).
3. Carreno, J.J., "Economic Lot Scheduling for Multiple Products on Parallel Identical Processors," *Management Science*, **36**, 348-358 (1990).
4. Chang, Y.J. and M.J. Yao, "Solving the Economic Lot Scheduling Problem with Multiple Facilities in Parallel Using the Time-varying Lot Sizes Approach", *Journal of Information and Optimization Sciences,* **31**(4), 809–835 (2010).
5. Dobson, G. "The Economic Lot Scheduling Problem: Achieving Feasibility Using Time-varying Lot Sizes," *Operations Research*, **35**, 764 – 771 (1987).
6. Elmaghraby, S.E., "The Economic Lot Scheduling Problem (ELSP): Review and Extension," *Management Science*, **24**, 587-597 (1978).
7. Geng, P.C. and R.G. Vickson, "Two Heuristics for the Economic Lot Scheduling Problem: An Experimental Study," *Naval Research Logistics,* **35,** 605-617 (1988).
8. Grznar, J. and C. Riggle, "An Optimal Algorithm for the Basic Period Approach to the Economic Lot Scheduling Problem," *Omega, International Journal of Management Science*, **25**, 355-364 (1997).
9. Hanssmann, F., *Operation Research in Production and Inventory Control*, Wiley, New York (1962).
10. Hsu, W.L. "On the general feasibility of scheduling lot sizes of several products on one machine," *Management Science*, **29**, 93-105 (1983).
11. Holland, J.H., *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor, MI (1975).
12. Jones, P. C. and R. B. Inman, "When is the Economic Lot Scheduling Problem Easy?" *IIE Transactions*, **21** (1), 11-20, (1989).
13. Maxwell, W.L., "The Scheduling of Economic Lot Sizes," *Naval Research Logistics Quarterly*, **11**, 98 - 124 (1964).
14. Moon, I., E.A. Silver, and S. Choi, "Hybrid Genetic Algorithm for the Economic Lot-Scheduling Problem," *International Journal of Production Research*, **40**, 809-824 (2002).
15. Park, K.S. and D.K. Yun, "Feasibility Test for Multi-Product Lot Size Scheduling on One Machine," *Policy and Information*, **11**, 101-108 (1987).
16. Yao, M.-J., "The Peak Load Minimization Problem in Cycle Production," *Computers & Operations Research*, **28**, 1441-1460 (2001).
17. Yao, M.J., S.E. Elmaghraby and Chen, I.-C., "On the Feasibility Testing of the Economic Lot Scheduling Problem Using the Extended Basic Period Approach," *Journal of the Chinese Institute of Industrial Engineering*, **20**, 435-448 (2003).
18. Zipkin, P. H., "Computing optimal lot sizes in the economic lot scheduling problem," *Operations Research*, **39**, 56-63 (1991).