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Portfolio Optimization Utilizing the Framework of Behavioral Portfolio Theory

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Abstract: Investors are assumed to be rational, however, empirical evidence shows otherwise. Investors categorize their investments into different mental accounts (MAs) with different biases. Behavioral portfolio theory (BPT) takes these behaviors and MAs into account when selecting for optimal portfolios. This study presents an aggregated portfolio optimization procedure using the framework of BPT. The procedure consists of three parts: return estimation, return weighting and MAs selection. Returns are estimated considering indexes that may reflect investor biases; Return estimates are then weighted according to SP/A theory to reflect investors' perception; Portfolios are then selected using an integrated optimization model where both the safety-first and risk-seeking MAs are considered. The estimation and weighting parameters used are based on market index forecasts. The back-test results show that the resulting aggregate portfolio and the embedded portfolios of the two MAs can outperform the market and mean-variance portfolio.
Keyword — BPT, portfolio optimization, SP/A theory, mental accounts

1. INTRODUCTION

Standard economic models assume that investors understand the nature of their investing dilemma, and know how to elucidate and solve this problem given appropriate information and figures, thus, rational financial market players. However, investors often face a myriad of factors (i.e., transaction costs, financial information, risk-return trade-offs, and personal attributes) that affect investment decisions, and at times complicate the selection of a portfolio at any given time.

Markowitz (1952) through its Mean-Variance Theory (MVT) stated that investors tend to select the security with the higher return given specific risks; or the one with the lower risk given a specific expected return. Stock portfolio selection has been studied heavily since then, and MVT has been the so-called rational way of investing. However, numerous studies have shown that investors do not behave rationally as initially expected; and assumptions on rationality and risk attitudes of investors towards their portfolio have long been challenged. For example, Amirshahi and Siahtiri (2010) stated that rational behavior is one of the most challenging assumptions in the economy. Typical investors sell their winning stocks, due to the fear of a price drop, or keep their losing stocks, because of the hope of a price increase in the future. The latter can also be related with the overestimation of one's investing skills and knowledge as explained by Barber and Odean (2001) through active trading, and that men lose more compared to woman traders because of too much assertion and overconfidence in the market.

Investors also have the tendency to follow the crowd or the "herding phenomenon" as studied by Hirshleifer et al. (2001); and added that traders react too emotionally in times of market stress, and in the absence of timely information. In consonance with this study, Glaser and Weber (2007) observed that irrational investors do not really know how their portfolio performs, while the rational investors are technically aware of what is happening with their positions. The authors also stated that irrational investors do not really learn from their past investment, and would normally still commit the same mistakes. Agnew and Szykman (2005) further explained that low-knowledge investors when making decisions to buy the default and known portfolio allocation more, compared to the objective analysis of high-knowledge investors. A more recent study of Amin et al. (2009) observed that the gambler's fallacy also

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affects the decision of some investors causing them to make biased decisions.

The abovementioned evidence of irrational behaviors challenges basic economic assumptions of positive utility, which result to abnormal movements of security prices. Thus, often violating the assumption of normally and independently distributed return rates. Clearly, the MVT model and the traditional utility function are not sufficient to describe investors' preferences in selecting their portfolio; and as Islam (2012) puts it, there is a lack of professional practice and objective mechanisms which are necessary to control and correct irrational behavior in portfolio selection.

Optimizing portfolio selection, which considers investors' behavior was first proposed by Shefrin and Statman (2000) through the behavioral portfolio theory (BPT). The theory suggests that investors build their portfolios based on their own belief, behavior, and perceptions of the market performance. BPT uses the foundation laid by Lopez's (1987) security-potential aspiration (SP/A) theory, and Thaler's (1985) mental accounting in determining the individual optimal portfolio of an investor. Through these foundations, investors build their portfolios using a multi-layered pyramid called mental accounts with the corresponding aspiration levels and risk attitudes for each layer. BPT emphasizes the role of behavioral preferences in portfolio selection and encompasses individual investors' portfolio choices. The portfolio return performance reflects the investors' characteristics such as aspirations, hope, fear and even the tendency to not see the investment context through "narrow framing".

This study applies BPT on stock portfolio optimization. Portfolio selection has 3 stages: (1) estimation of future returns; (2) assignment of probability weights on estimated returns; (3) selection of the optimal portfolio for each mental account. First, this study quantitatively models the investor' behaviour in generating return scenarios through regression models with the consideration of the market trend. This study utilizes the relative strength index (RSI) in the estimation of the future market trend. Considering different market trends, 4 indices (earnings-price ratio for representativeness; turnover-rate for over-confidence; short-term return proxy for over-reaction; long-term return proxy for under-reaction), that may probably reflect the biases of investors, are used to generate future return scenarios through regression models. Investors perception on future returns are then added into the allocation of weights to these return scenarios using SP/A theory (fear and hope levels). This study also proposes an aggregate portfolio selection model that simultaneously considers two opposing mental accounts (risk-seeking and safety-first).

1.1 Risk-return Trade-off

The utility function for loss aversion instead of risk aversion of Kahneman and Tversky's (1979) prospect theory explained that investors care more about the losses in relation to the risks involved. Likewise, some investors may hold onto stocks that are losing due to the belief that these stocks will eventually go up and experience returns, which Shefrin and Statman (2000) referred to as the disposition effect. Massa and Simonov (2002) add that behavioral investor decides how much to invest in risky assets mainly on the basis of prior gains and losses and selects the individual risky securities. Bighiu (2010) thinks that people have the tendency to compare their thinking to a group. In instances where an investor made decisions carrying less risk than a group, typically the individual will revise his decisions to an increased level of risk to follow group thinking. With regards to personality and gender effects, Olsen and Cox (2001) observed that women are more risk-averse than men, which was supported in the latter study of Durand et al. (2008).

1.2 Fear and Hope Levels

Investor behaviors show that there are different thought-patterns emanating from the expectation of the future status of the stock market. Thaler (1980) earlier stated that some investors may think that the market will likely to rise (bull market) and some may think that the market will likely to fall (bear market) just based on their own beliefs. Lopes (1987) on the other hand, introduced a theory of choice under uncertainty called SP/A theory, on the assumption that investors make their decisions based on their fear and hope levels. The study attested that investors will likely put more weight on the worst (best) outcomes when they are in fear (hope). The study of Hoffmann and Post (2013) found that individual investors update their confidence (or return expectations) and fear (or risk perceptions) in response to their individual return and risk experiences. This is also consistent with the earlier paper of Statman et al. (2006), which claimed that irrational investors tend to be more over-confident after having a positive return, thus increasing their investment activity; similarly, they tend to be less over-confident after having a negative return thus decreasing their investment activity. Furthermore, Shiller (2000), and Barber and Odean (2001) observed that investors that are over-confident with their trading skills, increase their trading activities, but generally have below-average returns.

1.3 Framing Mental Accounts

The earliest study to consider the framing of mental accounts was done by Roy (1952), which proposed a risk management tool for investors to select the optimal portfolio, called the safety-first model. The objective is to minimize the probability that the portfolio return will fail to reach the threshold level. Improvements were made by Telser (1955) and Kataoka (1963) by maximizing the threshold level (i.e., expected return) under a predetermined acceptable probability of the return failing to reach the threshold level. Thaler's (1985) mental accounting illustrates that investors tend to segregate different types of gambles; and for each gamble faced, decision-makers tend to have different risks attitudes and do not consider any interactions among other gambles.

In a more recent study, Norkin and Boyko (2010) developed a model reducing the safety-first portfolio selection problem to a linear mixed-Boolean programming in a finite number of yield scenarios. The studies of Shefrin and Statman (2000), and Das et al. (2010) about the new mental accounting (MA) framework, where the sub-portfolio within any given account is chosen by maximizing the accounts expected return, subject to a constraint that reflects the accounts motive. This constraint specifies the sub-portfolios threshold return and the maximum probability of failing to reach that threshold in the account. Baptista (2012) assumed that the investor faces background risk in all accounts but may face different levels of risk for different accounts. The study then provided models to find the optimal portfolios within mental accounts and the aggregate portfolio considering the background risks. In another study, Jiang et al. (2013) analyzed the international portfolio selection based on BPT using exchange rate risk. The study found that investors select the optimal BPT portfolio in each market, overlooking covariance between the markets, and allocating funds across markets to minimize losses.

2. METHODOLOGY

2.1 Estimating Future Returns

This study considers 4 irrational behaviors with their corresponding probable indexes to estimate the future stock returns. Following the work of Lee (2009), price-earnings ratio (EP) was used as an index for representativeness; turnover rate (TO) was used as an index for over-confidence; short-term return proxy was used as an index for over-reaction (OR); long-term return proxy was used as an index for under-reaction (UR). These indices may or may not fully reflect the irrational behavior of investors. Nonetheless, for testing purposes, these 4 indices are used for back-testing. Each week, the market status (MS) is determined and classified into 6 market conditions through the RSI value of the market. For testing purposes, the following assumptions were made: $MS=1$ when RSI value is $0<RSI<20$; $MS=2$ when RSI value is $20<RSI<40$; $MS=3$ when RSI value is $40<RSI<50$; $MS=4$ when RSI value is $50<RSI<60$; $MS=5$ when RSI value is $60<RSI<70$; and $MS=6$ when RSI value is $80<RSI<100$. $MS(1,2,3)$ and $MS(4,5,6)$ are respectively considered as bearish and bullish market scenarios. The top 150 stocks in the Taiwan Stock Exchange (TWSE) are tested for significant effects of irrational behaviors through the 4 indexes. From the Taiwan Economic Journal Co. Ltd (TEJ_t), the weekly closing stock price ($P_{S,t}$), turnover rate ($TO_{TEJ,t}$), price-earnings ratio (PE_t) of stocks, and Market Index (MI_t) were collected. The suffix t added to the index denotes that the corresponding index is obtained from time t. The irrational behavior indexes were obtained at time t as follows:

$$EP_t = \frac{1}{PE_t} \tag{1}$$

$$TO_t = \frac{TO_{TEJ,t}}{100} \tag{2}$$

$$OR_t = \frac{P_{S,t} - P_{S,t-4}}{P_{S,t-4}} \tag{3}$$

$$UR_t = \frac{P_{S,t} - P_{S,t-52}}{P_{S,t-52}} \tag{4}$$

and the market returns ($R_{M,t}$) and individual stock returns ($R_{S,t}$) at time t denoted by:

$$R_{S,t} = \frac{P_{S,t} - P_{S,t-1}}{P_{S,t-1}} \tag{5}$$

$$R_{M,t} = \frac{MI_t - MI_{t-1}}{MI_{t-1}} \tag{6}$$

Subsequently, regression analysis was used to determine which of the irrational behaviors have a significant effect on the returns. The significant indices, current market and stock returns, and the forecasted market return are then used to estimate individual stock return for the next period using the regression equation:

$$R_{S,t+1} = \beta_0 + \beta_1 EP_t + \beta_2 TO_t + \beta_3 OR_t + \beta_4 UR_t + \beta_5 R_{S,t} + \beta_6 R_{M,t} + \beta_7 R_{M,t\ 1} + \sigma_{ei} \tag{7}$$

To forecast $R_{M,t+1}$, the exponential weighted moving average (EWMA) and exponential weighted moving variance (EWMV) was used. For each week there are 6 sets of equations to generate scenarios, one of which is updated depending on the forecasted market status. Note that in order to deal with the multicollinearity problems, independent variables that exceeded $VIF = 5$ for each stock are excluded in the generation of scenarios. The respective return of each stock in a generated return scenario is correlated with respect to the market return ($R_{M,t+1}$).

2.2 Weighting Return Scenarios

After generating return scenarios using eq. (7) they should be ranked from worst to best in order to apply the SP/A weights the investor. The scenarios are ranked according to the future market return estimates ($R_{M,t+1}$). Suppose the estimate value for scenario $V_j = R_{M_j,t+1}$, then V_j is ranked such that $V_1 < V_2 < \ldots < V_m$. Considering equal likelihood for all m scenarios, let p_j be the probability of V_j to occur, then the decumulative probabilities D for scenario j is calculated as $D_j = p_j + p_{j+1} + p_{j+2} + \ldots + p_m$. Then, the probability on scenario j (p_j) is calculated as $p_j = D_{j-1} - D_j$.

According to SP/A theory, some investor may overweight the best (worst) scenarios relative to their hope (fear) level. To consider an investor perception to future performance of the market in re-assigning probabilities to scenarios, it is necessary to have their q_p (measures hope level) and q_s (measures fear level). To the authors' knowledge, there is currently no clear way on how to estimate q_s and q_p. Following Lopes (1987) that when people are in fear they tend to overweight the unfavorable outcomes with a transformed D of $h_s(D) = D^{1+q_s}$ and when people are hopeful they tend to overweight the favorable outcomes with a transformed D of $h_p(D) = 1 - (1-D)^{1+q_p}$. From a sample mapping of the individual prospect of q_s and q_p, it is assumed that there are 20 possible scenario outcomes. Outcome 1 is the worst one and outcome 20 is the best one. Investors were asked to weight these 20 outcomes when they are in fear and when they are hopeful, respectively. The range of their weights is from 1 to 10. A weight of 10 (1) means a greater (lesser) probability of occurrence. Using these weights, the least squares method can be used to estimate q_s in $h_s(D) = D^{1+q_s}$ as follows:

$$q_s = \frac{\sum(\ln D' \times \ln D - \ln D^2)}{\sum \ln D^2} \tag{8}$$

where D (D') is the original (fear influenced) decumulative probability. Similarly, for q_p in $h_p(D) = 1 - (1-D)^{1+q_p}$ is estimated as follows:

$$q_p = \frac{\sum\left(\ln D'' \times \ln(1-D) - \ln(1-D)^2\right)}{\sum(1-D)^2} \tag{9}$$

where D'' is the hope influenced decumulative probability. $h_s(D)$ & $h_p(D)$ makes D in to

$$h(D) = \delta h_s(D) + (1-\delta)h_p(D) \tag{10}$$

where δ should also be given by the investor. When $\delta = 0$, the investor is purely potential-minded; When $0 < \delta < 1$, the decision maker is cautiously hopeful where he is experiencing both fear and hope. For testing purposes, RSI index was used as $\delta = \dfrac{100 - RSI}{100}$ in order to reflect investors' perception on future market conditions. Using the estimated q_p and q_s, the decumulative probability can be written as follows:

$$h(D) = \frac{100 - RSI}{100} \times h_s(D) + (1 - \frac{100 - RSI}{100}) \times h_p(D) \tag{11}$$

Eq. (11) makes the decumulative probability of scenario j equal to $h(D_j)$. Let p_j^r be the rebalanced probability weights on scenario j based on the SP/A parameters of an investor. Then, p_j^r is now calculated as $p_j^r = h(D_{j-1}) - h(D_j)$ so

$$\begin{aligned} p_j^r = &\left(\frac{100 - RSI}{100} \times h_s(D_{j-1}) + (1 - \frac{100 - RSI}{100}) \times h_p(D_{j-1})\right) - \\ &\left(\frac{100 - RSI}{100} \times h_s(D_j) + (1 - \frac{100 - RSI}{100}) \times h_p(D_j)\right) \end{aligned} \tag{12}$$

For this study, for testing purposes, the q_s and $\mathrm{q_p}$ values are set at 3.71. This is an estimated value obtained from one of the investors surveyed for the test using eq. (8) and eq. (9). Eq. (12) is then used for the assignment of weights for each scenario j.

2.3 Determining Mental Account

Investors have different objectives (mental accounts) and corresponding risk attitudes. For this study, the focus is on two common risk-attitude of any investor, the risk-seeking (RS) attitude in investing for a shot at riches, and the safety-first (SF) attitude in investing for future security. For the RS account (MA-1), the investor wants to earn as much as possible no matter the risk involved. Money is viewed as a dispensable tool to obtain the desired goal. RS portfolio selection model was then used for this mental account. On the contrary, for the safety-first mental account (MA-2), the investor wants to ensure that the money invested will be available for future use. Usually, these investments are for retirement funds and educational plans. Thus, SF portfolio selection model was utilized for this mental account.

In this study, it is assumed that there are n assets and m generated scenarios. The variables used are denoted as follows: w_i denotes the percentage of wealth invested in asset i; x_i denotes the portfolio of risk-seeking mental account; y_i denotes the portfolio of safety-first mental account; s_i denotes the initial price of asset i; $r_{i,j}$ denotes the return of asset i in simulated scenario j; p_j denotes the probability of scenario j to occur; p_j^r denotes the probability of scenario j to occur considering SP/A parameters; $\overline{r}_i$ denotes the mean return of asset i; $R_p = \sum_{i=1}^{n} w_i r_{ij}$ denotes the return of the portfolio; $\overline{R}_p = \sum_{i=1}^{n} w_i \overline{r}_i$ denotes the expected return of the portfolio; R_L denotes the tolerance level of loss; R_H denotes the desired level of gain; z_j is a binary indicator such that $z_j = 1$ when R_H is reached at scenario j, otherwise $z_j = 0$; ω_j is a binary indicator such that $\omega_j = 1$ when R_L is reached at scenario j, otherwise $\omega_j = 0$; α denotes the acceptable probability of reaching R_L; M is a large number, $i = 1, 2, \cdots, n$ and $j = 1, 2, \cdots, m$.

2.3.1 Risk-Seeking Mental Account Model

The first mental account, MA-1 or the risk-seeking mental account uses the risk-seeking portfolio selection model, which is derived from safety-first portfolio selection model. Instead of minimizing the probability of losing a certain high amount of money, the focus for this mental account is to maximize the probability of obtaining a desired high profit. Accordingly, the objective function is presented as:

$$Max \quad \Pr\left(R_p \geq R_H\right) \tag{13}$$

Similar to Norkin & Boyko (2010), the model can be written as follows:

$$Max \quad \sum_{j=1}^{m} p_j z_j \tag{14}$$

$$s.t. \quad R_p - R_H \leq M z_j \tag{15}$$

$$M - \left(R_H - R_p\right) \geq M z_j \tag{16}$$

To be more practical, the portfolio should be presented in terms of the number of shares or units purchased on the assets. Let x_i be the numbers of units of asset i purchased in the portfolio, considering an initial capital B, the percentage of wealth on asset i is

$$w_i = \frac{x_i \times S_i}{B} \tag{17}$$

In this setting, let $C = R_H B$ be the desired wealth level. To consider the SP/A weights of the investor, p_j is to be replaced with p_j^r as the probability given to scenario j. After writing w_i in terms x_i, $R_p = \sum_{i=1}^{n} x_i S_i r_{ij}$, writing R_H in terms of C, and assigning weights using eq. (12), the first model, MA-1, is as follows:

$$Max \quad \sum_{j=1}^{m} p_j^r z_j \tag{18}$$

$$s.t. \quad \sum_{i=1}^{n} x_i S_i r_{ij} - C \leq M z_j \tag{19}$$

$$M - \left(C - \sum_{i=1}^{n} x_i S_i r_{ij}\right) \geq M z_j \tag{20}$$

$$x_i \geq 0, \text{integer}; \; z_j \in \left(1, 0\right); \; i = 1, 2, \cdots, n \;\text{ and }\; j = 1, 2, \cdots, m \tag{21}$$

Eq. (18) maximizes the total probability of having a high return considering the SP/A parameters of an investor over the return scenarios. Eq. (19) and eq. (20) identify the scenarios where the desired high return is reached. Eq. (19) makes $z_j = 1$ when $R_p > C$ and $z_j = 0$ or $z_j = 1$ (unwanted 1s) when $R_p \leq C$. Eq. (20) makes $z_j = 0$ when $R_p < C$ and $z_j = 0$ (unwanted 0s) or $z_j = 1$ when $R_p \geq C$. Collectively, eq. (19) forces those unwanted 0s from eq. (20) to be 1s; eq. (20) forces unwanted 1s from eq. (19) to be 0s. Together, Eq. (19) and eq. (20) ensures that only the scenarios when $R_p \geq C$ are considered.

2.3.2 Safety-First Mental Account Model

The second mental account, MA-2, uses Telser (1955) safety-first portfolio model which deals with the risk-aversion attitude. The safety-first portfolio selection model focuses on maximizing the expected returns at a given loss threshold. Safety-first investors are willing to have a small probability of losing a certain amount of money, in order to maximize their possible earnings. Accordingly, the MA-2 model is written as:

$$Max \quad \overline{R}_p \tag{22}$$

$$s.t. \quad \Pr\left(R_p \leq R_L\right) \leq \alpha \tag{23}$$

Similar with Norkin & Boyko (2010), the model is written as follows:

$$Max \quad \overline{R}_p \tag{24}$$

$$s.t. \quad R_L - R_p \le M\omega_j \tag{25}$$

$$\sum_{j=1}^{m} p_j \omega_j \le \alpha \tag{26}$$

To be more practical, similar with MA-1, after writing w_i in terms of y_i and writing R_L in terms of $E = R_L B$, $R_p = \sum_{i=1}^{n} y_i S_i r_{ij}$, and assigning scenario weights using eq. (12) the second model, MA-2, is as follows:

$$Max \quad \sum_{i=1}^{n} y_i S_i \overline{r}_i \tag{27}$$

$$s.t. \quad E - \sum_{i=1}^{n} y_i S_i r_{ij} \le M\omega_j \tag{28}$$

$$M - \left(\sum_{i=1}^{n} y_i S_i r_{ij} - E \right) \ge M\omega_j \tag{29}$$

$$\sum_{j=1}^{m} p_j^r \omega_j \le \alpha \tag{30}$$

$$y_i \ge 0, \text{integer}; \; w_j \in (1,0); \; i = 1,2,\cdots,n \quad and \quad j = 1,2,\cdots,m \tag{31}$$

Eq. (27) maximizes the average portfolio return. Eq. (28) and eq. (29) identify and count the number of scenarios where the portfolio return falls below the acceptable loss. Eq. (28) makes $\omega_j = 1$ when $R_p < E$ and $z_j = 0$ or $z_j = 1$ (unwanted 1s) when $R_p \ge E$. Eq. (29) makes $\omega_j = 0$ when $R_p > E$ and $z_j = 0$ (unwanted 0s) or $z_j = 1$ when $R_p \le E$. Collectively, eq. (28) forces those unwanted 0s from eq. (29) to be 1s; eq. (29) forces unwanted 1s from eq. (28) to be 0s. Together, Eq. (28) and eq. (29) ensures that only the scenarios when $R_p \le E$ are considered. Eq. (30) with the help of eq. (28) and eq. (29) ensures that the total probability of having a portfolio return far below the acceptable loss E is limited to a threshold probability α.

3.3.3 Aggregate Portfolio Selection Model (Maximize Expected Wealth)

Through (Lopez, 1987) SP/A theory, wherein people's choices are affected by their fear and hope levels, an aggregate portfolio selection model for both mental accounts is developed by maximizing the combined expected wealth and considering the mood factors β_s (fear ratio) and β_r (hope ratio). Nofsinger (2005) stated that the stock market itself can be used to measure the social mood. Thus, for this study, considering RSI, it is assumed that $\beta_s = \delta$ and $\beta_r = 1 - \delta$, where $\delta = \dfrac{100 - RSI}{100}$. The fear ratio indicates the preferred ratio of the investor to his SF account, while the hope ratio indicates the preferred ratio of the investors to his RS account. The expected returns considering the SP/A weights are $\left(\sum_{j=1}^{m} p_j^r z_j \right) C$ for RS or MA-1 and $(1-\alpha)\left(\sum_{i=1}^{n} \overline{r}_i y_i \right)$ for SF or MA-2. Thus, the aggregate objective function can be written as follows:

$$Max \quad \beta_r \left(\left(\sum_{j=1}^{m} p_j^r z_j \right) C \right) + \beta_s \left((1-\alpha) \left(\sum_{i=1}^{n} \overline{r}_i y_i \right) \right) \tag{32}$$

However, the main objective of an investor for any investment is to ultimately earn more profit. Therefore, for each period, the hope factor β_r is dynamically used as the weight for both SF and RS expected returns. The aggregate portfolio selection model with the new objective function is now written as follows:

$$Max \quad \beta_r\left(\left(\sum_{j=1}^{m} p_j^r z_j\right)C + (1-\alpha)\left(\sum_{i=1}^{n} \overline{r}_i y_i\right)\right) \tag{33}$$

$$s.t. \quad \sum_{i=1}^{n} x_i S_i r_{ij} - C \le M z_j \tag{34}$$

$$M - \left(C - \sum_{i=1}^{n} x_i S_i r_{ij}\right) \ge M z_j \tag{35}$$

$$E - \sum_{i=1}^{n} y_i S_i r_{ij} \le M\omega_j \tag{36}$$

$$M - \left(\sum_{i=1}^{n} y_i S_i r_{ij} - E\right) \ge M\omega_j \tag{37}$$

$$\sum_{j=1}^{m} p_j^r \omega_j \le \alpha \tag{38}$$

$$x_i, y_i \ge 0, integer; \; z_j \in (1,0); \; \omega_j \in (1,0) \tag{39}$$

Eq. (33) maximizes the total expected return for both mental accounts (MA-1 and MA-2) at the same time. Eq. (34) and eq. (35) identify the scenarios where the desired high return for MA-1 is reached. Eq. (34) makes $z_j = 1$ when $\sum_{i=1}^{n} x_i S_i r_{ij} > C$ and $z_j = 0$ or $z_j = 1$ (unwanted 1s) when $\sum_{i=1}^{n} x_i S_i r_{ij} \le C$. Eq. (35) makes $z_j = 0$ when $\sum_{i=1}^{n} x_i S_i r_{ij} < C$ and $z_j = 0$ (unwanted 0s) or $z_j = 1$ when $\sum_{i=1}^{n} x_i S_i r_{ij} \ge C$. Collectively, eq. (34) forces those unwanted 0s from eq. (35) to be 1s; eq. (35) forces unwanted 1s from eq. (34) to be 0s. Together, Eq. (34) and eq. (35) ensures that only the scenarios when $\sum_{i=1}^{n} x_i S_i r_{ij} \ge C$ are considered. $C = R_H B$ in eq. (34) and eq. (35) represents the expected high return. $\sum_{i=1}^{n} x_i S_i r_{ij}$ is the return on scenario j for MA-1. Eq. (36) and Eq. (37) counts the number of scenarios where the MA-2 return falls below the acceptable loss. Eq. (36) makes $\omega_j = 1$ when $\sum_{i=1}^{n} y_i S_i r_{ij} < E$ and $z_j = 0$ or $z_j = 1$ (unwanted 1s) when $\sum_{i=1}^{n} y_i S_i r_{ij} \ge E$. Eq. (37) makes $\omega_j = 0$ when $\sum_{i=1}^{n} y_i S_i r_{ij} > E$ and $z_j = 0$ (unwanted 0s) or $z_j = 1$ when $\sum_{i=1}^{n} y_i S_i r_{ij} \le E$. Collectively, eq. (36) forces those unwanted 0s from eq. (37) to be 1s; eq. (37) forces unwanted 1s from eq. (36) to be 0s. Together, Eq. (36) and eq. (37) ensures that only the scenarios when $\sum_{i=1}^{n} y_i S_i r_{ij} \le E$ are considered. Eq. (38) with the help of eq. (36) and eq. (37) ensures that the total probability of having a MA-2 return far below the acceptable loss $E = R_L B$ is limited to a threshold probability α. $\sum_{i=1}^{n} y_i S_i r_{ij}$ is the return on scenario j for MA-2.

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3. EMPIRICAL RESULTS

3.1 Data Description

Considering weekly investments in TWSE, stocks are bought at the beginning of the week and then sold at the end of the week. For each week, the market status is identified and classified into 6 categories using RSI . The top 150 stocks are tested for significant effects of irrational behaviors. The stock return for the next period is estimated using 6 sets of regression equations for each market status considering the significant indices of irrational behavior. The stock pool is all 150 stock securities, the back-test period is from 6-15-2012 to 5-9-2014 and the initial data is from 1-4-2002 to 6-15-2012. Using eq. (7) and the initial data, 5000 weekly scenarios of stock returns showing significant irrational behavior are generated. Overall, the back-test period is for 100 weeks. For comparison purposes, the aggregate model was run using the historical data (HD) where it is assumed that the scenarios are equally likely to occur.

For the risk-seeking (RS) portfolios, MA-1, the desired return rate, R_H , is set at 5%. For the safety-first (SF) portfolios, MA-2, the return rate tolerance level, R_L , is set at -5%, and the acceptable probability α that the return will be less than -5% is also set at 5%. The combination of MA-1 and MA-2 is the aggregate portfolio. To check the performances of MA-1, MA-2, and the Aggregate portfolio using the proposed aggregate selection model, the optimal portfolios are compared in terms of data used (generated and historical data). These aggregate portfolios are also compared to the portfolio obtained using the mean-variance (MV) portfolio selection model utilizing historical returns. Moreover, the performances of these aggregate portfolios and the embedded RS and SF portfolios within them are also compared with the market. Overall, there are 8 portfolios. A1 denotes the aggregate portfolio using the generated scenarios and scenario weights of the investor using the proposed aggregate portfolio selection model. RS1 (SF1) represents the embedded MA-1 (MA-2) portfolio in A1. A2 denotes the aggregate portfolio utilizing the proposed aggregate model but using the historical data as return scenarios with the assumption that these return scenarios have an equal likelihood to occur. Similarly, RS2 (SF2) represents the embedded MA-1 (MA-2) portfolio in A2. MV denotes the mean-variance portfolio and Market denotes the Market returns.

3.2 Portfolio Comparison

In MA-1 (MA-2) or risk-seeking (safety-first) mental account the focus of the investor is on the returns (losses). The usual benchmark of comparison is with the market. Accordingly, similar with Chang et al. (2013) the down-side cases for SF1 and Market and the up-side cases for RS1 and Market are compared as shown in Table 1.

Table 1. RS1 (SF1) Up-Side (Down-side) Case Comparison vs Market over 100 Week Test Period

Case	RS1	Market	Case	SF1	Market
(+,+)	27	16	(+,-)	16	0
(+,-)	14	0	(-,+)	0	19
(-,+)	0	18	(-,-)	18	5
	41	34		34	24

The return comparison are classified into 4 cases (+,+), (+,-), (-,+), & (-,-). The first (second) + or - symbol represent the portfolio (market) return. A + sign indicates a positive or up-side return, while a – sign indicates a negative or down-side return. In (+,+), the portfolio outperforms the market when it has higher returns; In (-,-), the portfolio outperforms the market when it has a lower loss; In both (+,-) and (-,+), the one with the + sign outperforms its counterpart. The main focus for risk-seeking (safety-first) portfolio is the upside (downside) returns. The upside cases include (+,+), (+,-), and (-,+) while the downside cases include (+,-), (-,+), & (-,-). The number of times the portfolio outperforms the market during upside (downside) cases for risk-seeking (safety-first) account are counted which show that the RS1 (SF1) outperform the Market with better returns for upside (downside) cases.

Table 2. Portfolio Return Distributions over 100 Week Test Period

Upside Comparison of RS Portfolios					Downside Comparison of SF Portfolios				
Return	RS1	RS2	MV	Market	Return	SF1	SF2	MV	Market
≥5%	2	2	2	0	<0%	42	47	48	39
≥4%	8	4	6	1	<-1%	16	31	28	21
≥3%	11	11	12	5	<-2%	6	18	20	7
≥2%	22	22	21	10	<-3%	4	0	8	4
≥1%	40	31	35	32	<-4%	3	2	1	0
≥0%	57	49	52	61	<-5%	0	0	0	0

Table 2 shows the upside and downside distribution of the portfolios. All RS portfolios except Market have equal chances (2%) of having the high return of 5%. Only the Market has more positive returns than RS1 but RS1 have more high positive returns than the Market. All SF portfolios have no returns falling below the loss threshold (-5%). Only the Market has fewer negative returns than SF1 but SF1 has a lower frequency for smaller loss range than all other SF portfolios. RS1 seems to be riskier (more likely to have high returns) and SF1 seems to be safer (smaller loss range).

Table 3. Portfolios and Market Return Statistics Over 100 week Test Period

	RS1	RS2	SF1	SF2	MV	Market
Weekly Average Return	0.0028	-0.0010	0.0064	0.0014	0.0021	0.0023
Standard Deviation	0.0266	0.0260	0.0224	0.0228	0.0229	0.0156
Sharpe Ratio	0.1025	-0.0380	0.2837	0.0583	0.0901	0.1447
Cumulative Return	0.2727	-0.1210	0.8460	0.1170	0.2020	0.2424

Table 3 shows the return statistics for the portfolios. The portfolio with the higher weekly average return (more profit), standard deviation (more chances of getting high return), Sharpe ratio (more profit relative to risk), and cumulative return (more profit) outperforms the portfolio with lower values, but, with the exception for SF portfolios where a lower standard deviation (less riskier investment) is desired. RS1 outperforms its counterparts in all aspect except for the Sharpe ratio where the Market is a little bit better. SF1 outperforms its counterparts in all aspect except for the standard deviation where the Market is a little bit better. These observations imply that RS1 and SF1 are more profitable than their counterparts. Moreover, over the 100-week period, RS1 (SF1) has a better cumulative return compared to RS2 (SF2), MV, and Market. (See Fig.1 and Fig. 2).

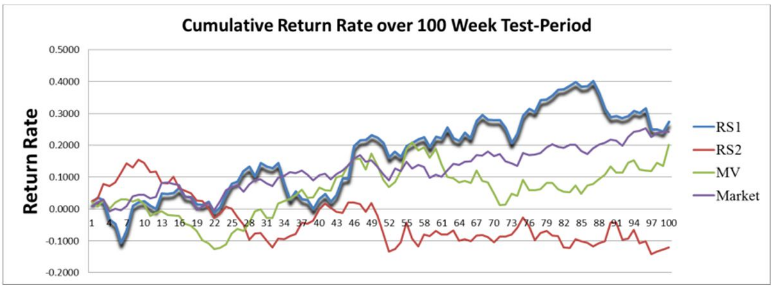

Figure 1 Cumulative Return Rates of RS1, RS2, MV, and Market over Test Period

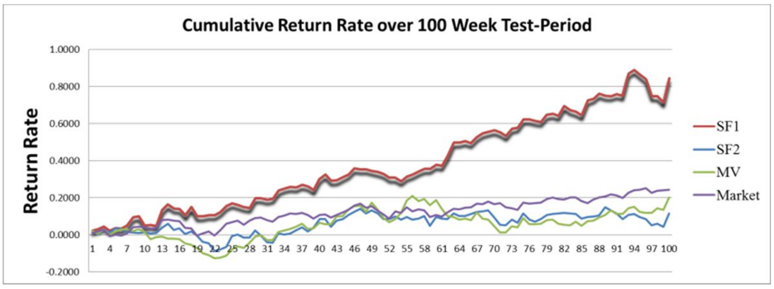

Figure 2 Cumulative Return Rates of SF1, SF2, MV, and Market over Test Period

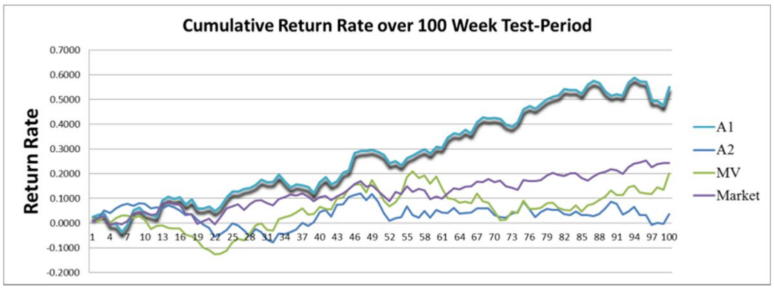

Figure 3 Cumulative Return Rates of A1, A2, MV, and Market over Test Period

Table 4. Aggregate Portfolios and Market Return Statistics Over 100 week Test Period

	A1	A2	MV	Market
Weekly Average Return	0.0046	0.0006	0.0021	0.0023
Standard Deviation	0.0205	0.0208	0.0229	0.0156
Sharpe Ratio	0.2224	0.0259	0.0901	0.1447
Cumulative Return	0.5486	0.0364	0.2020	0.2424

After showing the dominance of RS1 and SF1 for their respective mental account, the focus of comparison is shifted to the aggregate portfolio A1 against A2, MV, and Market. See Fig. 3, it shows that A1 have better cumulative returns over the 100-week test period. In Table 4, the descriptive statistics also reflect similar results where A1 outperforms A2, MV, and Market in all aspect, except for the standard deviation where the Market is slightly better. Although the comparison results show that A1 dominates its counterparts, these comparisons are still not enough to statistically conclude that A1 is the superior portfolio. Since the returns of the portfolios are in a time-series data and heterogeneous that depends on a particular period (week), in order to eliminate the period effects and variances of the test when comparing two portfolios, a pair-t test should be performed to have a more accurate result. The null (alternative) hypothesis for the test is that the average pair difference on the weekly return is less than or equal to (greater than) 0. The acceptance of the alternative hypothesis is justified with sufficient evidence. The rejection of the alternative hypothesis is justified when the null hypothesis is true or there is insufficient evidence to back up the alternative hypothesis. The result of the pair-tests between portfolios is shown in Table 5. The pair-return difference is the return of the row portfolio deducted by the return of the column portfolio.

Table 5. P-values of Pair-T tests

	RS2	SF2	A2	MV	Market
RS1	0.141			0.416	0.417
RS2				0.894	0.908
SF1		0.008***		0.045**	0.026**
SF2				0.662	0.818
A1			0.037**	0.158	0.069*
A2				0.828	0.837
MV					0.538
Market				0.462	

*,**, and *** respectively indicate significance at 0.1, 0.05, and 0.01 level

In Table 5, outperformance means that the row portfolios have a weekly average return that is greater than the column portfolio. It shows that there is strong evidence that SF1 outperforms SF2, A1 outperforms A2, SF1 outperforms MV and Market, and A1 outperforms Market. Only A1 portfolios were able to outperform the MV portfolio and Market. This support the initial comparison result that A1 portfolios and the embedded RS1 (MA-1) and SF1 (MA-2) can outperform other benchmark portfolios.

4. CONCLUSION

This study proposes an investment procedure utilizing the BPT framework. This study provides methods for each of the 3 stages of portfolio selection. The regression model, Eq. (7), was used to generate return scenarios considering the possible index of investors' biases. These return scenarios are then ranked and given weights (probabilities) based from the SP/A parameters (fear and hope levels) of an investor. Considering these return scenarios with the SP/A weights, portfolios are then selected through a proposed aggregate portfolio selection model that chooses the best portfolio for the risk-seeking (MA-1) and safety-first (MA-2) mental account at the same time. The resulting aggregate portfolio (A1) and the embedded MA-1 (RS1) and MA-2 (SF1) portfolios are then compared to A2 and the embedded RS2 and SF2 portfolios, mean-variance portfolio, and Market. Back-test results show that in some shape or form A1 and the embedded portfolios are superior portfolios than their benchmark counterparts. These results imply the following key observations on improving portfolio performance (better portfolio returns): (1) better estimation of returns and allocation of scenario weights; (2) integration of mental accounts into an aggregate portfolio selection model; (3) finding the best way for each stage of portfolio selection. This study could be improved upon in many different ways by adapting the best strategy for the estimation of returns, allocation of weights on return scenarios, and modelling the appropriate portfolio selection model for the mental accounts. Future back-tests should consider also the transaction costs in order to determine whether the proposed strategy actually works on real investment conditions.

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