

A Critical Survey on the Network Optimization Algorithms for Evacuation Planning Problems

Tanka Nath Dhamala^{1*}, Urmila Pyakurel¹, Stephan Dempe²

¹Central Department of Mathematics, Tribhuvan University,
P.O.Box 13143, Kathmandu, Nepal

²TU Bergakademie, Fakultät für Mathematik und Informatik,
09596 Freiberg, Germany

Received July 2018; Revised August 2018; Accepted August 2018

Abstract: In the last decades, research on emergency traffic management has received high attention from the operations research community and many pioneer researchers have established it as one of the most fertile research areas. We consider the computationally hard flows over time problems from wider perspective including flow/time dependent attributes (dynamic flows), a possibility of flows loss on paths while travelling (lossy network problems), arcs/path reversal capability (contraflow models) and possibilities of eliminating merging and crossing conflicts at intersections (abstract flows). The topics also include the networks for relief distribution, location-allocation of facilities, multi-criterion characteristics and transit based flow models in brief. The issues are highly motivated from the perspective of traffic control and emergency route choice and scheduling.

Despite of many directions such as differential equations for fluid flows, measure and function theory, cell transmission approach and optimal control theory, we have restricted to the perhaps most computationally acceptable research domain, the network flow optimization approach with macroscopic behavior. We compactly review the contributions, explore the featured results, present structured systematic analysis and state the weakness and strength of the models and solution strategies the authors carried out during several years. A large number of problems belong to the category of strongly \mathcal{NP} -hard problems and demand efficient computational techniques that at least yield acceptable approximate solutions. This comprehensive survey on evacuation problems complements the number of previous reviews by adding many recent results obtained in the field so far. Moreover, it highlights the main stream research and most promising challenges in modeling and solving more realistic real-life scenarios and explores some possible future research fields.

Keyword — Network optimization, dynamic flows, transportation problems, evacuation planning, algorithms, complexity

1. INTRODUCTION

The rapid increase of natural (for example, earthquakes - 1255, 1934 and April 2015 in Nepal, March 2011 in Japan, Haiti, Chichi, Bam, Kashmir and Chile; volcanic eruptions, landslides, floods, tsunamis - Japan and the Indian Ocean; hurricanes - Katrina, Rita and Sandy in USA; typhoons) and human crated (for example, chemical explosions and terrorist threats - September 11 attacks in USA) disasters forces the emergency planning researchers to build up an effective evacuation plan that saves the people and property, and also supports humanitarian relief with optimal use and equitable distribution of available resources. Authors in (Kruhl et al. 2018) address the issues in Nepal earthquake 2015 and the consequences. Unplanned saturation of metropolitan cities and heavy population migration into them overloads the existing traffic capacity, demand an effective and universally acceptable emergency plan that could address urgent emergency exit of the evacuees, emergency logistic supports to them and temporarily shelter location-allocation management, which is still in lacking.

There exist a flow of research ranging from wide spectrum of mathematical frameworks, like variational inequality, optimal control, differential equations of fluids, numerical simulations, like cell transmission models to the approaches of mathematical programming, like continuous as well as discrete network flow models. The existing models and methods seek to address the issues from different point of views, for example, single commodity or multi commodity

*Corresponding author's e-mail: amb.dhamala@daadindia.org

flows, single or multi-modal vehicles, quickest or largest flows, transit dependent or self vehicle movements, continuous or discrete time settings, queuing constant or variable attributes, macro or micro behaviors, single or multi objectives, and so on. Unfortunately, none of them beat the others as more accurate models seeking to capture the closer reality may not yield efficient practical implementations for the real-world large scale problems.

In general, the continuous models yield more accurate results over the discrete ones but are computationally more challenging. Likewise the models with flow/time dependent attributes imply heavy computational costs caused by the nonlinearities which suggests to relax the models with constant attributes for computationally possible approximations. The flow models with differential equations yield accurate results for small size problems but are rather complicated for large scale realistic problems. The macroscopic models are the aggregate results of group behaviors as a single object instead of computationally harder microscopic models which include individual characteristics. An appropriate choice of vehicle (transit or car) dependent models depends on their availability and users ability, but the most effective integrated one is almost unsolvable. Naturally, user optima and system optima objectives might have their particular interests conflicting each other. Most general problems that have to address the characteristics of diversified vehicles and commodities are rather complicated from the computational point of view.

One of the most effective approach for handling evacuation problems in different disasters is the dynamic network flow (flow over time) model as it performs better approximation results in comparison to the others in reasonable computational time. Discretization of the models is being a better option for good approximations to real-life solutions. Even nonlinearities caused by flow and/or time dependent attributes could be approximated by constant attributes which may be obtained by some queuing rules or travel experiences and continuous time settings replaced with discrete time steps at pre-specified time points. Here, the evacuation region is represented by a network where nodes represent the locations, where evacuees are gathered or waiting for transshipment and the arcs are the road segments or node connections through which the flow (evacuees) travel. The nodes and arcs should be equipped with capacities and travel times. An assignment or prediction of flows for varying demands over time on peak period traffic imposes a heavy pressure on capacity and transit times. More widely, modeling of flows over time ranges in varieties of networks, for example, financial and commodity flows, production systems, political and social networking, communication, information, public utilities and services, computers, air and road traffic control, and evacuation plans, see for instance (Aronson 1989, Dhamala 2015, Powell et al. 1995). From macroscopic analysis, the arcs on the network can also be thought as fluid transporting pipelines, where the length and width determine arc transit time and its capacity, respectively.

As mentioned, there exists a wide range of literature on network flow problems, we refer to some survey papers and the references therein for more information, for instance, Akter and Wamba (2017), Altay and Green III (2006), Anaya-Arenas et al. (2014), Aronson (1989), Cova and Johnson (2003), Dhamala and Adhikari (2018), Dhamala (2015), Hamacher and Tjandra (2002), Hamacher et al. (2013), Kotsireas et al. (2015), Moriarty et al. (2007), Pascoal et al. (2006), Powell et al. (1995), Yusoff et al. (2008).

Covering all aspects of these model varieties and solution strategies in a single survey is impossible and quite diverging because of which we selected the most prominent ones - the dynamic traffic assignment problem, the flow maximization and time minimization flow over time problems, the transshipment problems and the minimum cost flow problems mostly for single commodity flow in discrete time settings. A so-called natural transformation translates these solutions in case of certain continuous flows. The node-arc structure is also viewed with arc-path structure in this study, called the abstract flow problem. One of the main approaches of this work is also to cover the contraflow reconfiguration approach which is very effective from managerial implications and also a widely accepted model in use. Though most of literature and software considers them as heuristic solutions, we summarize a number of current results that explain the contraflow technique analytically. In fact, results show that contraflow increases the flow values and decreases the evacuation time significantly. Few applications related to our flow over time problems, such as location-allocation benefits, multi-objective applications, logistic supports, tollbooth problem, supply chains, coupling of car/transit based and pedestrian movements, are considered and highlighted here. It is tried to summarize the results together with their insights giving hints to most of the models illustrating the networks.

The paper is organized as follows. Section 2 defines all parameters used in the network flow theory, gives dynamic flow varieties and illustrates the ways how to represent them using convenient networks. Different solution strategies for flow maximization, time minimization, quickest transshipment, generalized flow over time (Subsection 3.2) and abstract flows (Subsection 3.3) are compactly explained in Section 3. Subsection 3.1 and Subsection 3.4, respectively, describe the flow models with constant and variable attributes (inflow, load and flow dependent transit times and time dependent attributes). The contraflow approach is surveyed in Section 4 with heuristic and analytical solutions in Subsection 4.1 and Subsection 4.2, respectively. The concluding remarks are given in Section 6 after few highlights on certain applications in Section 5.

2. PRELIMINARIES

We give some basic definitions and fix few notations that are used in the following sections. Notice that for static models all time requirements are irrelevant and they are removed from the dynamic flow models.

2.1 Basic Denotations

Flow rates. Besides the static flows that pass with a single wave and do not vary over time, defining a flow rate on an arc that is the number of flow units (e.g. cars, etc.) traveling the arc per unit time at a specific point of time is vague and rather complicated. Among many other possibilities of defining the flow rates, the inflow, outflow and intermediate flow rates are, respectively, measured at the entry, exit and intermediate points on an arc. Notice that none of them are satisfactory as homogeneous flow on an arc can rarely be observed in real-world applications. We denote the flow rate on arc e at time θ by $\Phi_e(\theta)$ and the dynamic flow function as $\Phi : A \times T \rightarrow \mathbb{R}^+$. The flows from node v at time θ to the same node with travel time one will be denoted by $\phi_v(\theta + 1) = \Phi(v_\theta, v_{\theta+1})$

Capacity. Arc upper capacity (bound) function $b^u : A \times T \rightarrow \mathbb{R}^+$ on network $N = (G, A)$ limits the flow rate passing along the arc for each point in time. Likewise, an arc lower capacity (bound) function $b^l : A \times T \rightarrow \mathbb{R}^+$ limits the flow from below. Although the lower bound restrictions usually are not taken into account in the literature, it is beneficial in introducing them to maintain certain load of the transportation network that is applicable in absence of feasible flow. Usually, for time independence, the upper capacity is denoted by b_e on arc e and the lower capacity is treated as zero if not mentioned otherwise. The capacities may depend upon many factors like the condition of roads, movement of vehicles, presence of traffic control resources, climatic conditions, and so on, Bozhenyuk et al. (2017). For example, the number of lanes on roads naturally limits the vehicle counts. A node may also be associated with a capacity function a_v for $v \in V$, that allows holdover of flows at nodes and can be set to zero if not used at the nodes other than sources and sinks. Similarly the nodes may be equipped with the initial occupancy, if any.

Transit times. A combination of density, speed and flow rate evolving along the arc taken into account realizes a complete realistic flow model with flow-dependent transit times (FDTT) on arcs. But such a model requires rather complicated analysis and computational challenges. A transit time function $\tau : A \times T \rightarrow \mathbb{R}^+$ measures the amount of time it takes for the flow to travel along an arc on network $N = (G, A)$. Since the flow values on arcs may change over time in general, FDTT function may be interpreted as in the definition. The inflow-dependent transit times (IFDTT) and load-dependent transit times (LDTT) on arcs basically relax the more general setting of (FDTT), where transit time of an arc depends on the behavior of the current flow on this arc. With (IFDTT), the transit time $\tau_e(\Phi_e(\theta))$ is a function of inflow rate $\Phi_e(\theta)$ on the arc e at given time θ , so that time flow units enter an arc with uniform speed and their speed remains uniform through traveling this arc. Whereas with (LDTT), the transit time $\tau_e(l_e(\theta))$ depends on the total amount of flow on an arc e at a given time θ , i.e., the load $l_e(\theta)$ on e . In this approach all units of flow on an arc enjoy the same speed and the transit time of an arc varies with each unit of flow entering or leaving the arc with the continuous changes of flow on this arc. There exist various ways of considering the transit time attribute, however, we represent them by the same denotation τ_e as for the constant transit times (CTT) on the arc $e \in A$ to be understood with any specific context chosen. An accurate estimation of link travel times is quite complicated as it is a nonlinear function of probable congestion. We recommend Smith and Cruz (2014) for an overview and various approaches of travel times estimation on arterial links, free and high ways. Note that non-negativity property of transit times can be dropped applicable to the residual networks.

Costs. On network $N = (G, A)$, the arcs may be associated with non-negative costs caused by many reasons at each point of time. The cost per unit flow on an arc $e \in A$ at time θ , if it exists, is denoted by $c_e(\theta)$. If the cost remains constant throughout the give time horizon, it is a function on the set of arcs only.

Example 1 Assume that Figure 1(a) represents a dynamic network in which each arc has constant capacity and constant transit time. Arc (s, x) has capacity 3 and transit time 1. Assume that the unit time is 5 minutes and unit flow is 100 evacuees. Then, it takes 5 minutes for evacuees to travel arc (s, x) and a maximum of 300 evacuees can simultaneously travel through this arc. During most of the evacuation processes, movement of evacuees towards the sources is not allowed as in Figure 1(b). However, these arcs may still be useful for other humanitarian logistic supports and facility locations. For static network, the transit time is considered as cost. The maximum static flow through paths $s - x - d$ and $s - y - d$ is 7 and the corresponding minimum cost 32. The dynamic flow through the same paths with $T = 5$ is 10.

Figure 2 depicts the time expanded network corresponding to Figure 1(b) with respect to the increment of arc travel times by one unit on each arcs (s, x) , (s, y) , (x, y) , (x, d) , (y, d) at time $\theta = 1, 2, 3, 4, 5$, respectively, and remaining all other arc travel times the same as shown in the figure. This illustrates a time expanded network of the time dependent dynamic network.

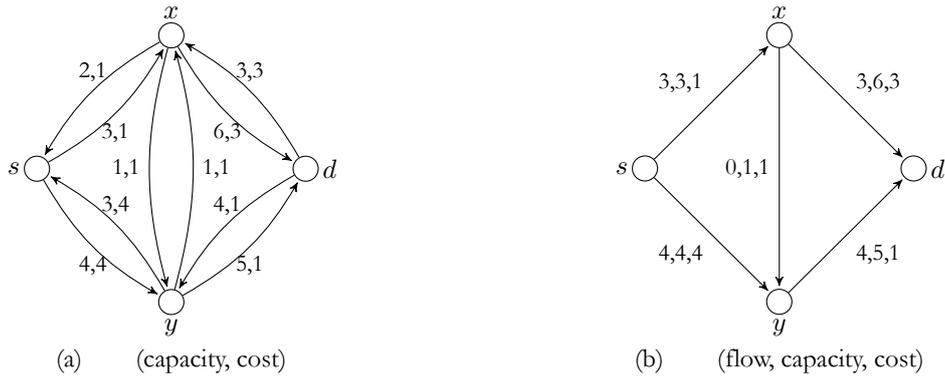


Figure 1: (a) Existing evacuation network (b) Maximum static flow solution with time as cost on arcs.

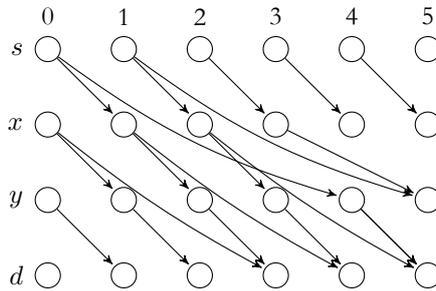


Figure 2: Time expanded network with cost as travel time on arcs.

Natural transformation. The continuous and discrete flow models are connected by the natural transformation (1) dealing with the same time complexity, Fleischer and Tardos (1998).

$$\Phi_e^c(\lambda) = \Phi_e(\theta), \text{ for all } \theta \text{ and } \lambda \text{ with } \theta \leq \lambda < \theta + 1 \tag{1}$$

if $\Phi_e(\theta)$ is the amount of discrete dynamic flow that enters arc e at time $\theta = 0, 1, \dots, T$ with constant capacities on the arcs. For static flow Ψ_e on arc e , the discrete dynamic flow with travel time τ_e is

$$\Phi_e(\theta) = \sum_{\sigma=0}^{\tau_e(\theta)-1} \Psi_e(\theta - \sigma), \text{ for all } \theta = 0, 1, \dots, T - 1 \tag{2}$$

The flow that enters e at time $\theta - \tau_e$ arrives at the head node at time θ in discrete time, but at time $[\theta + 1)$ in continuous time. Then the flow Φ^c is feasible and the amount of source-sink flow at any integer time interval $[\theta, \theta + k)$, for $\theta = 0, 1, \dots, T, k \in \mathbb{N}$, will be the same for both settings.

2.2 Dynamic Flow Model Varieties

Minimization problems. Let $\Phi_e(\theta)$ be the volume of flow on arc e at time θ , $b_e(\Phi_e(\theta))$ be the maximum amount of flow out from arc e , $d_e(\theta)$ be an amount of flow entering arc e , $f_u(\theta)$ be an external input in node u and $h_{e\theta}(\Phi_e(\theta))$ be the travel cost function associated to e . Then, Merchant and Nemhauser (1978) give the most general non-linear non-convex formulation of the dynamic traffic assignment problem (DTAP) as (3-6). The exogenous flows in (DTAP) allowed only at the beginning can be considered as initial occupancies in evacuation problems.

$$\text{minimize } \sum_{\theta=0}^T \sum_{e \in A} h_{e\theta}(\Phi_e(\theta)) \tag{3}$$

such that for all time periods $\theta = 0, 1, \dots, T - 1$

$$\Phi_e(\theta + 1) - \Phi_e(\theta) - d_e(\theta) + b_e(\Phi_e(\theta)) = 0, \forall e \in A \tag{4}$$

$$\sum_{uv \in A} d_{uv}(\theta) - f_u(\theta) - \sum_{wu \in A} b_{wu}(\Phi_{wu}(\theta)) = 0, \forall u \neq d \tag{5}$$

$$(\Phi_e(\theta), h_{e\theta}) \geq 0, \forall e \in A \tag{6}$$

This single destination deterministic discrete time model, in which flow rates, travel costs and route choices are independent, represents the congestion explicitly in the constraints. As a result, the analytical and numerical solutions are hard to realize for large size network problems. They proposed a piecewise linearization and a linear programming technique satisfying a certain ordered set property. They show that the simplex algorithm yields the optimal objective function value for the modified constrained linear problem.

In general, the cost function $h_{e\theta}(\cdot)$ has been assumed to be nonnegative, continuous, convex and nondecreasing, for example $h_{e\theta}(\Phi_e(\theta))$ is directly proportional to the flow volume $\Phi_e(\theta)$ on e . Also, the maximum (capacity) outflow $b_e(\Phi_e(\theta))$ associated with arc e can be taken as an upper bound to the actual outflow $a_e(\theta)$ in presence of the flow control $b_e(\Phi_e(\theta)) - a_e(\theta) \geq 0$. With this assumption, Carey (1987) converted the non-convex (DTA) model to system cost minimization convex problem as (7-11). In both models the inflows and outflows are non-negative.

$$\text{minimize } \sum_{\theta=0}^T \sum_{e \in A} h_{e\theta}(\Phi_e(\theta)) \quad (7)$$

such that for all time periods $\theta = 0, 1, \dots, T - 1$

$$\Phi_e(\theta + 1) - \Phi_e(\theta) - d_e(\theta) + a_e(\Phi_e(\theta)) = 0, \quad \forall e \in A \quad (8)$$

$$\sum_{uv \in A} d_{uv}(\theta) - f_u(\theta) - \sum_{wu \in A} a_{wu}(\Phi_{wu}(\theta)) = 0, \quad \forall u \neq d \quad (9)$$

$$b_e(\Phi_e(\theta)) \geq a_e(\Phi_e(\theta)) \geq 0, \quad \forall e \in A \quad (10)$$

$$(\Phi_e(\theta), h_{e\theta}) \geq 0, \quad \forall e \in A \quad (11)$$

Carey's model is a convex programming problem as the capacity outflow function $b_e(\Phi_e(\theta))$ is also convex in addition to the cost function $h_{e\theta}(\Phi_e(\theta))$ by which the latter model exploits rich convex optimality properties. Moreover, it benefits on the improvements of aggregate network costs by reducing the arc flow rates below their maximum level for new optimal pattern of the flow controls. Their main result is to establish sufficient conditions for optimal flow controls to be zero, i.e., $b_e(\Phi_e(\theta)) - a_e(\theta) = 0$ for realizing a practical flow. For approximate solutions, they extend the model with multiple destinations by adding a super destination and introducing either (i) fixed aggregate demands at each destination, or (ii) negative costs with the out-flows at each destination and in the objective function. Moreover, the flexible departure times and elastic demands are also addressed with this model.

Carey (1992) extensively discusses the complications of including nonconvex constraints in the dynamic assignment problem with multiple destinations or commodities. The model has been modified to convex linear approach for addressing the flow behavior with link travel times, mainly depending on inflow-outflow rates and some traffic control effects ahead on the links, Carey and Subrahmanian (2000). The first-in-first-out (FIFO) property which could be satisfied with restricted inflows not decreasing sharply after a sharp increase has been discussed extensively. Realizing the nonlinearity and nonconvexity relationships between travel times and flows on arcs, they use them to define sets of time-space links avoiding nonlinearly of these relationships in the mathematical programming. They also discuss different ways of handling the FIFO property, either ignore them if present in small size and not very critical links, or introduce additional nonconvex constraints and relax them to linear constraints with zero-one integer variables. Also the FIFO property can be maintained by starting a solution process with a small set of time-space links and make more iterations with additions or subtractions of such links in the process. Moreover, the conditions of holding back of flow on links, an extension of the model with multiple destinations and possible extensions to user equilibrium are discussed in Carey and Subrahmanian (2000).

Papageorgiou (1990) presented a macroscopic modeling for multi sink dynamic traffic freeway and/or road networks with time varying demands and illustrated various interesting instances. The other models for the (DTAP) deal with simulation-based, variational inequality and optimal control approaches, for example see Daganzo (1994), Harker and Pang (1990), Peeta and Ziliaskopoulos (2001). The cell-transmission approach may be of more practical interest since it incorporates the user-equilibrium constraints. In this model, the highway links are segmented into cells with a specific capacity that could be traversed in an unit time. A limitation of flow from one cell to another controls the congestion. The transportation cost has to be minimized over the set of arc-flows using the mathematically richer variational inequality models. The single and multi-destination problems with time-varying parameters depending on inflow-outflow and flow controls on entrance, arc-path links and exit are modeled with constrained optimal control (DTA) models. A number of system optimization and user equilibrium problems on transportation networks have been dealt with continuous as well as discrete time settings. However, all of these formulations differently suffer from a number of theoretical optimality conditions and practical dealings like inefficiencies in handling many behavioral and FIFO constraints.

The field of (DTAP) includes a rich class of research with density dependent travel times on a dynamic network. The nonlinear density dependent travel time function imposes the dynamic flow conservation constraints also to be

nonlinear. This happens since the density dependent travel times implies nonexistence of an arc belonging to a source-sink path caused by higher congestion, for example, to the preceding arc. The first among the two common objectives in (DTAP) take care of the system optimum minimizing the average travel time where the individuals should share the lateness. However, the second objective considers user optimum where the individuals do not care each other.

In order to deal with this nonlinearity, Kaufman et al. (1998) approximate the travel times by 0-1 decision variables. This leads to (DTAP) as a minimum cost capacitated dynamic network flow problem. The other iterative approaches fix the travel times temporarily according to the current flow and update them with respect to the new flow until the iterative travel time is convergent (Chen and Hsueh 1998, Janson 1991, Jayakrishnan et al. 1995 and Ran and Boyce 1996). In each iteration a nonlinear programming problem is solved as a convex optimization problem, where the travel time has to be temporally fixed firstly before finding an optimal flow under this constrained finally.

Note that the following maximization, universally maximization, quickest, lexicographic maximum and transshipment flow problems relax the (DTA) models by considering the arc travel times as constant with free-flow speed environment. For general time/flow dependent problems, special donations will be made whenever necessary.

Maximization problems. The maximum flow problems are more useful when there is no reliable information regarding the initial occupancies and source nodes are capable to house a significant number of evacuees before the evacuation process starts. With an inflow rate $\Phi_e(\theta)$ on arc e at discrete time θ that may change over the planning horizon T , the dynamic flow Φ satisfies the flow conservation and the capacity constraints (12-14). The flow may wait at intermediate nodes with inequality constraints, however, waiting is not permitted otherwise.

$$\sum_{\sigma=\tau_e}^T \sum_{e \in B(v)} \Phi_e(\sigma - \tau_e) - \sum_{\sigma=0}^T \sum_{e \in A(v)} \Phi_e(\sigma) = 0, \forall v \notin \{s, d\} \quad (12)$$

$$\sum_{\sigma=\tau_e}^{\theta} \sum_{e \in B(v)} \Phi_e(\sigma - \tau_e) - \sum_{\sigma=0}^{\theta} \sum_{e \in A(v)} \Phi_e(\sigma) \geq 0, \forall v \notin \{s, d\}, \theta \in \mathbf{T} \quad (13)$$

$$b_e(\theta) \geq \Phi_e(\theta) \geq 0, \forall e \in A, \theta \in \mathbf{T} \quad (14)$$

Note that the inequalities like (13) could be replaced with an equality by addressing the waiting flows $\phi(\cdot)$ at intermediate nodes and the initial occupancies at sources (see, Hamacher and Tjandra (2002), for details).

The earliest arrival flow problem (EAFP), also known as the universal maximum flow problem (UMFP), maximizes the objective function $val(\Phi, \theta)$ in (15) for all $\theta \in \mathbf{T}$ satisfying the constraints (12-14). However, for a given time T , the maximum dynamic flow problem (MDFP) maximizes the objective function $val(\Phi, T)$ in (15) for $\theta = T$ satisfying the constraints (12-14). The maximum flow value is denoted by $val_{\max}(\Phi, \theta)$.

$$val(\Phi, \theta) = \sum_{\sigma=0}^{\theta} \sum_{e \in A(s)} \Phi_e(\sigma) = \sum_{\sigma=\tau_e}^{\theta} \sum_{e \in B(d)} \Phi_e(\sigma - \tau_e) \quad (15)$$

For a given value Q_0 , the quickest flow problem (QFP) looks for the minimal time $\min T = T(Q_0)$ such that the flow value is at least Q_0 satisfying the constraints (12-14) with equality in (13). As the universal maximum flows yield optimal solutions from the start of an evacuation plan without an earlier estimation of the completion time, this model captures the essence of most realistic scenarios.

For a given time horizon T and an ordered set of multi-terminals, the lex-maximum dynamic flow problem (LMDFP) finds a feasible flow that lexicographically maximizes the amount leaving (entering) each terminal in the given priority order.

Let us introduce an additional parameter $\lambda_e \in \mathbb{R}^+$ on arc e , say a gain factor, in order to model a generalized dynamic flow when only λ_e units of flow leave from w at time $\theta + \tau_e$ by entering a unit of flow on $e = (v, w)$ at time θ . If the flow is only lost but never gained along all arcs, then $\lambda_e \leq 1$ for all arcs $e \in A$ holds, and we call the network as lossy. Then the generalized dynamic flow (GDF) $\Phi : A \times T \rightarrow \mathbb{R}^+$ for given time T satisfies the constraints (16-18).

$$\sum_{\sigma=\tau_e}^{\theta} \sum_{e \in B_v} \lambda_e \Phi_e(\sigma - \tau_e) \geq \sum_{\sigma=0}^{\theta} \sum_{e \in A_v} \Phi_e(\sigma), \forall v \notin \{s, d\}, \theta \in \mathbf{T} \quad (16)$$

$$\sum_{\sigma=\tau_e}^T \sum_{e \in B_v} \lambda_e \Phi_e(\sigma - \tau_e) = \sum_{\sigma=0}^T \sum_{e \in A_v} \Phi_e(\sigma), \forall v \notin \{s, d\} \quad (17)$$

$$b_e(\theta) \geq \lambda_e \Phi_e(\theta) \geq 0, \forall e \in A, \theta \in \mathbf{T} \quad (18)$$

The generalized maximum dynamic flow problem (GMDFP) is to find a GDF of maximum value $val_{\max}(\Phi, T)$ in (19). The total amount of flow that arrives at the sink in time steps $\{0, 1, \dots, \theta\}$ for all $\theta \in \mathbf{T}$ gives the earliest

arrival flow of a GDF. Note that no flow remains in the dynamic network at time T which is ensured by assuming that $\Phi_e(\theta) = 0$ for all $\theta \geq T - \tau_e$.

$$\text{val}(\Phi, T) = \sum_{e \in B_d} \sum_{\sigma = \tau_e}^T \lambda_e \Phi_e(\sigma - \tau_e) \quad (19)$$

Let $\Psi : A \rightarrow \mathbb{R}^+$ be a non-negative static flow function with uniform unit costs on arcs satisfying the objective function (20) and the constraints (21-22), respectively.

$$\text{val}(\Psi) = \sum_{e \in B_d} \Psi_e = \sum_{e \in A_s} \Psi_e \quad (20)$$

$$\sum_{e \in B_v} \Psi_e - \sum_{e \in A_v} \Psi_e = 0, \quad \forall v \in V \setminus \{s, d\} \quad (21)$$

$$b_e \geq \Psi_e \geq 0, \quad \forall e \in A \quad (22)$$

The static flow that maximizes the objective (20) turns into a zero circulation by adding an extra arc (d, s) with value $\text{val}(\Psi)$ through it and the flow conservation condition is also satisfied at terminals. If the objective function is multiplied by cost coefficients, then it is a minimum cost static flow (MCSF) problem. For the fixed flow value $\text{val}(\Psi)$, the (MCSF) problem seeks to shift this value with minimum cost $\sum_{e \in A} c_e \Psi_e$. A (MCSF) problem with zero circulation turns into a minimum cost circulation flow (MCCF) problem. Also the generalized maximum static flow problem (GMSFP) has analogous formulation by dropping out the time parameters in constraints (16-18) and the objective function (19), respectively.

Goldberg and Tarjan (1990) state a summary of existing results on the polynomial time algorithms for the minimum cost circulation problem relying on cost and capacity scaling algorithms that require a maximum flow and a shortest path computation as subroutines, respectively. Moreover, they presented various pseudo-polynomial, polynomial and strongly polynomial minimum cost circulation algorithms. Their approach solves the maximum flow problem with successive approximations based on cost scaling and conclude that the minimum cost circulation algorithm is not much harder than the maximum flow problem. Naturally, the pseudo-polynomial complexity depends on the maximum absolute values of arc cost and capacity, respectively.

Transshipment problems. Let $\mathcal{N} = (V, A, b, \tau, S, D, \mu(s), \mu(d))$ be a multi-terminal network with a source-supply and sink-demand vectors $\mu(s)$ and $\mu(d)$, respectively, such that $\mu(S \cup D) = \sum_{v \in S \cup D} \mu(v) = 0$. The supply and demand at sources and sinks are positive and negative, respectively, whereas these rates are balanced at intermediate nodes. The multi-terminal (EAFP) sends the total supply $\mu(S) = \sum_{s \in S} \mu(s)$ from S to the total demand $\mu(D) = \sum_{d \in D} \mu(d)$ in D with maximum value at each point of time $\theta > 0$. If all demands are fulfilled with supplies by shifting them within given time T , then the problem turns into the transshipment problem. The earliest arrival transshipment problem (EATP) maximizes $\text{val}(\Phi, \theta)$ in the objective function (23) satisfying the multi-terminal constraints (12-14) for all time points $\theta \in \{0, 1, \dots, T\}$.

$$\text{val}(\Phi, \theta) = \sum_{\sigma=0}^{\theta} \sum_{e \in A(s):s \in S} \Phi_e(\sigma) = \sum_{\sigma=\tau_e}^{\theta} \sum_{e \in B(d):d \in D} \Phi_e(\sigma - \tau_e) \quad (23)$$

If all supplies are transshipped from S to D satisfying its demands in minimum time $\min T = T(\mu(S \cup D))$, then the problem is called quickest transshipment problem (QTP).

Minimum cost flows. Let $c_e(\theta)$ be the cost per unit flow and $\Phi_e(\theta)$ be the flow rate on arc e at time θ , then the total cost of dynamic flow $\Phi : A \times T \rightarrow \mathbb{R}^+$ is defined by

$$c(\Phi) = \sum_{\theta=0}^T \sum_{e \in A} c_e(\theta) \Phi_e(\theta). \quad (24)$$

Variants of operational research models and algorithms on minimum cost flow problems are available in literature, for example, the multicommodity quickest minimum cost flow problem by Fleischer and Skutella (2002), the minimum cost flow over time problems (either find minimum cost flow for a given time horizon or find a quickest flow within a given total cost) by Klinz and Woeginger (1995, 2004), the time varying minimum cost flow problem by Cai et al. (2001)

and the infinite horizon minimum cost dynamic flow problem that maximizes throughput by Orlin (1984). However, most of the problems in this class are computationally strongly hard and no polynomial solutions exists unless $\mathcal{P} = \mathcal{NP}$ holds. Hardness occurs as neither the min-cost quickest flow nor the min-cost maximum flow solutions belong to the class of temporally repeated flows. Therefore, researchers seek for a best polynomial time approximation algorithms.

For the transportation of several distinct categorizes of flow through a single network, a number of attempts have been made to model multi-commodity flows over time, for example, the quickest minimum cost multi-commodity flow problem (Fleischer and Skutella 2007), the fractional multi-commodity flow problem over time (Hall et al. 2007a), the quickest multi-commodity flow problem with inflow-dependent transit times (Hall et al. 2007b, Hall et al. 2003). These problems are \mathcal{NP} -hard because of which algorithms are designed for acceptable approximate solutions including fully polynomial time approximation schemes (FPTAS). The static length bounded flows with bounded costs support for provably good multi-commodity flows over time and waiting at nodes may play significant roles in these solutions. Pattern-based mixed-integer programming models with the number of lanes represented by integer variables are discussed in Bretschneider (2013) which incorporates the two-commodities - vehicles of evacuees (e.g, cars) and buses.

2.3 Network Representations

Residual networks. Let $e' = (w, v)$ be the reverse of an arc $e = (v, w)$. The static residual network $\mathcal{N}(\Psi)$ w.r.t. a static flow Ψ is given by $(V, A(\Psi))$, where $A(\Psi) = A_F(\Psi) \cup A_B(\Psi)$ with $A_F(\Psi) = \{e \in A \mid \Psi_e < b_e\}$ and $A_B(\Psi) = \{e' \in A \mid \Psi_e > 0\}$. The arc length is given by τ_e for $e \in A_F(\Psi)$ and $-\tau_e$ for $e' \in A_B(\Psi)$. For nodes $v, w \in V$, let $P_{vw}(\Psi)$ be the shortest path from v to w in $\mathcal{N}(\Psi)$. The residual capacity $b(\Psi) : A(\Psi) \rightarrow \mathcal{R}$ is defined as $b_e(\Psi) = b_e - \Psi_e$ for $e \in A_F(\Psi)$ and $b_{e'}(\Psi) = \Psi_e$ for $e' \in A_B(\Psi)$. There may exist an augmenting path for increasing the source-sink flow.

Given a network $\mathcal{N} = (V, A, b, \tau, S, D, T)$, we define an admissible time a_e in (25) as additional attributes for finding the shortest augmenting path with static network.

$$A_e = \begin{cases} \{\theta : b_e(\theta) - \Psi_e(\theta) > 0\}, & \text{if } e \in A \\ \{\theta + \tau_{e'} : \Psi_{e'}(\theta) > 0\}, & \text{if } e' \in A. \end{cases} \quad (25)$$

Here in (25), first condition represents the set of times necessary to increase flow along e while the second represents the set of possible times to decrease flow along e' by sending back some flows along e . Then, for given feasible dynamic flow Φ , the dynamic residual network is defined as $\mathcal{N}(\Phi) = (V, A_F(\Phi) \cup A_B(\Phi))$ with $A_F(\Phi) = \{e : e \in A, A_e \neq 0\}$ and $A_B(\Phi) = \{e' : e' \in A, A_e \neq 0\}$. Moreover, the residual transit times are defined as

$$\tau_e = \begin{cases} \tau_e, & \text{if } e \in A_F \\ -\tau_{e'}, & \text{if } e \in A_B \end{cases} \quad (26)$$

Time-expanded network. The discrete dynamic flow problems are dealt with in Ford and Fulkerson (1962) introducing the static time-expanded network $\mathcal{N}(T) = (V_T, A_M \cup A_H)$, where $V_T = \{v_\theta \mid v \in V, \theta = 0, 1, \dots, T\}$, $A_M = \{(v_\theta, w_{\theta+\tau_e}) \mid e = (v, w) \in A, \theta = 0, 1, \dots, T - \tau_e\}$ and $A_H = \{(v_\theta, v_{\theta+1}) \mid v \in V, \theta = 0, 1, \dots, T - 1\}$. A moment arc $e \in A_M$ has the constant capacity b_e , whereas a holdover arc in A_H has infinite capacity that allows storage of flow at intermediate nodes. The latter are represented by the initial occupancies if they are known in advance. With these the network $\mathcal{N}(T)$ expands to the multi-terminal network and it requires to add the super-source and super sink nodes with specific costs and arc capacities. Links from the super source to the source nodes and from sink nodes to the super sink node are problem dependent. The dynamic flow in \mathcal{N} is equivalent to the static flow in $\mathcal{N}(T)$. To realize it, at a point of time θ , one can take a flow on $e_\theta \in A_M$ as the flow amount into $e \in A$, and reversely, the average flow on $e \in A$ of flow over time Φ in T as the flow value into $e_\theta \in A_M$. As $\mathcal{N}(T)$ has $O(nT)$ nodes and $O((n+m)T)$ arcs, any algorithm based on it has pseudo-polynomial running time. For example, the multi-terminal (MDFP) is solvable in pseudo-polynomial time on $\mathcal{N}(T)$. But it is still an open problem whether there is a polynomial solution for it in general.

Extended network. The construction of a two-terminal network \mathcal{N}^* , called the *extended network*, is generalized to the multi-terminal network \mathcal{N} by adding a super-terminal node (\star) and introducing arcs (\star, s_i) to each $s_i \in S$ with infinite capacity and zero transit time, and arcs (d_i, \star) to each $d_i \in D$ with infinite capacity and transit time $-(T + 1)$ for given time period T . For the network clearance problem, the capacities on (\star, s_i) and (d_i, \star) are replaced by the initial occupancies at s_i and the total demand at D , respectively (see, Hamacher and Tjandra (2002) for details).

Condensed network. By rescaling the time where each arc length (transit time) is a multiple of $\Delta > 0$ and capacity is multiplied by Δ , a condensed time expanded network $\mathcal{N}_\Delta(T)$ with time horizon $\lceil \frac{T}{\Delta} \rceil$ is constructed. In $\mathcal{N}_\Delta(T)$, node V is copied $\lceil \frac{T}{\Delta} \rceil$ times as $V_{\rho\Delta}$ for $\rho = 0, \dots, \lceil \frac{T}{\Delta} \rceil$ such that copy $V_{\rho\Delta}$ corresponds to flow through V in the

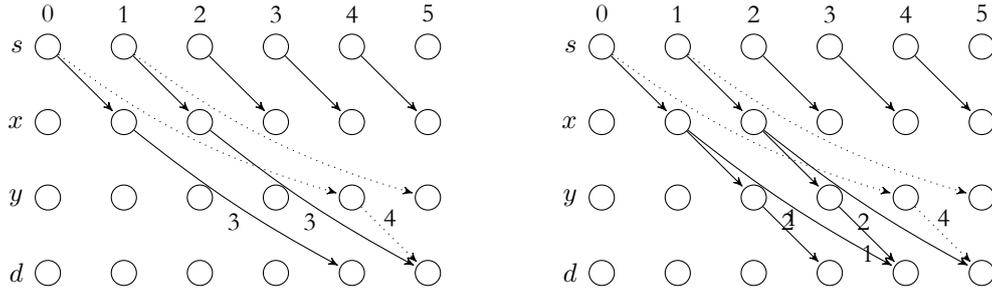


Figure 3: The maximum dynamic flow and earliest arrival flow solutions, respectively, in the time-expanded networks corresponding to Figure 1(b), with source node s , sink node d , and intermediate nodes x and y .

interval $[\rho\Delta, (\rho + 1)\Delta)$. If $\lceil \frac{T}{\Delta} \rceil$ is integer, then any flow over time in T and a static flow of equal cost in $\mathcal{N}_\Delta(T)$ are equivalent. However, for non-integer time $\lceil \frac{T}{\Delta} \rceil$, any flow in $\mathcal{N}_\Delta(T)$ corresponds to a flow over time of equal value that completes before time $T + \Delta$. However, as the transit times are rounded up in $\mathcal{N}_\Delta(T)$, its maximum flow approximates the maximum flow of an optimal solution in the original setting and it can be transformed into a flow over time with original arc lengths without too much loss in flow value Fleischer and Skutella (2007). Only two paths $s - x - d$ and $s - y - d$ carry the flows in condensed time-expanded network.

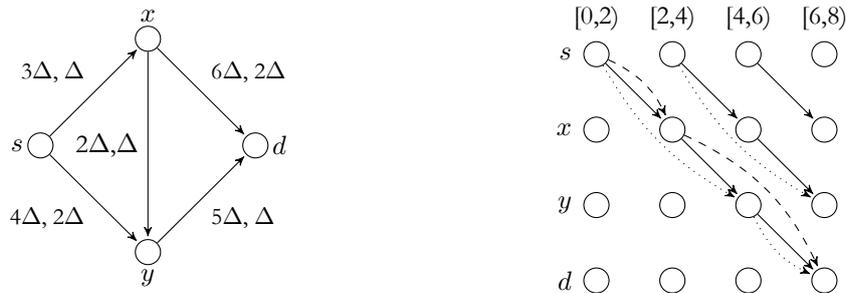


Figure 4: The condensed graph corresponding to Figure 1(b) and its time expansion with $\Delta = 2$

Standard chain decomposition. Most of the polynomial time dynamic flow algorithms are based on the temporally repeated static flow solutions introduced by Ford and Fulkerson (1962). By considering the constant arc travel times as costs, they decompose the obtained maximum flow minimum cost static flow solution $\Psi : A \rightarrow \mathbb{R}^+$ into p -chains (paths) γ_k , denoted by $\Gamma = \{\gamma_1, \gamma_2, \dots, \gamma_p\}$, each carrying the value $\text{val}(\gamma_k)$ to the sink such that $\Psi = \sum_{k=1}^p \gamma_k$. This standard chain decomposition uses its chain flows in the same direction as Ψ does. The decomposition Γ of a feasible static flow Ψ yields a feasible dynamic flow for given finite time horizon T by pushing the flow values of each chains from zero till the time permits, for example till time $T - \tau(\gamma)$ for the chain γ in Γ , whenever this chain has length at most T . The Γ -induced dynamic flow value denoted by Γ_T equals to $\text{val}(\Phi, T)$ for Φ , which depends only on Ψ and not on a particular choice of any standard chain decomposition, can be expressed as

$$\text{val}(\Phi, T) = \sum_{\gamma \in \Gamma} (T - \tau(\gamma) + 1) \text{val}(\gamma) = (T + 1) \text{val}(\Psi) - \sum_{e \in A} \tau_e \Psi_e \tag{27}$$

By calculating a minimum cost circulation to the network with additional arc $e = (d, s)$ having infinite capacity and cost $-(T+1)$, a maximum dynamic s - d solution has been obtained in polynomial time. Unfortunately, this efficient approach can not be applied for the exact solutions of many evacuation problems, like multi-terminal and two-terminal earliest arrival flows. However, there are many instances wherein this technique can be applied for calculating an approximate solution.

Non-standard chain decomposition. Hoppe and Tardos (1994), and Hoppe (1995) introduced a non-standard chain decomposition, wherein a chain flow might be canceled via another chain flow in a proper way (see, Figure 5). To verify dynamic feasibility, chains in this decomposition have to be checked very carefully. Consider an arc $e = (u, v)$ in the chains γ and γ' used in forward and backward directions. Then the portion of flow on γ from source to u that has to be canceled on e must be reached to u before γ' reaches there. Also, the flow from v to the sink through γ that

has to be canceled by γ' should not already left v . It also induces an optimal dynamic flow solution through, so-called generalized temporally repeated flows.

In this decomposition, consider the chains Γ_v and Γ'_v as the multi-sets of all chain flows in Γ whose extensions use edges $\star v$ and $v\star$, respectively. In Γ_v , flows start from v at time 0, but the flow from any chain $\gamma' \in \Gamma'_v$ takes $\tau(\gamma')$ time to reach terminal v . The time θ is taken sufficiently large, i.e., $\theta \geq \tau(\gamma')$ for each $\gamma' \in \Gamma'_v$ so that flow along each chain has reached the terminal v . Moreover, the arc set A on the left side of equation (28) contains only the arcs having positive capacities. Then the net value of flow Φ out of vertex v during the time θ is given by equation (28).

$$\text{val}(\Phi, T)_v = \sum_{\gamma \in \Gamma_v} \theta |\gamma| - \sum_{\gamma' \in \Gamma'_v} (\theta - \tau(\gamma')) = \sum_{e \in A} (\tau_e \sum_{\gamma \in \Gamma'_v} \gamma'_e) \quad (28)$$

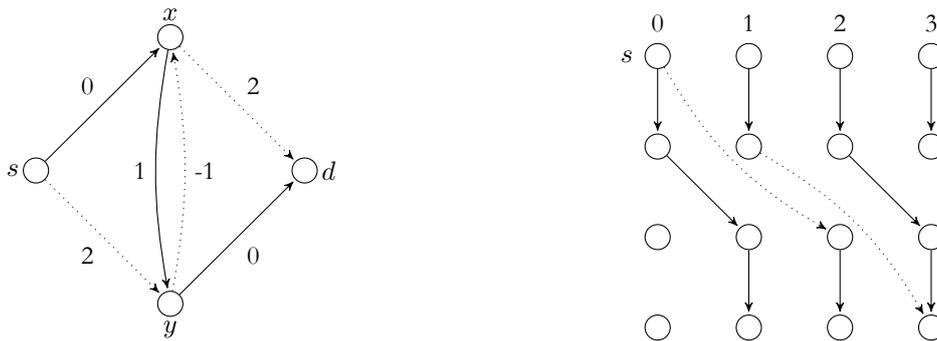


Figure 5: An illustration of backward flow cancellation in generalized temporally repeated flows

Fan and bow networks. Let us denote the non-decreasing left-continuous inflow dependent transit times step function by τ^s with only integer values. Then the graph $\mathcal{N}^F(T) = (V_T^F, A_M^F \cup A_H^F)$ represents a fan network, where $V_T^F = \{v_\theta \mid v \in V, \theta = 0, 1, \dots, T\}$ denotes the set of nodes, Köhler (2002). The set of holdover arcs $A_H^F = \{(v_\theta, v_{\theta+1}) \mid \theta = 0, 1, \dots, T-1\}$ with infinite capacity allows a possibility of holding a flow at node v . The set of fan arcs is denoted as $A_M^F = \{(v_\theta, w_{\theta+\tau_e^s(\theta)}) \mid e = (v, w) \in A, \theta = 0, 1, \dots, T-1, \tau_e^s \text{ is travel time on } e \text{ at time } \theta\}$. The capacitated horizontal fan arcs that control the flow distribution are continued with uncapacitated fan arcs pointing downwards that give the different possible transit times. For the flow rate y_e sent into the fan, transit time to the slowest portion of this flow is at least $\tau_e^s(y_e)$. Figure 6 represents an arc of time-expanded fan network with inflow rates 1 and 4, and $T = 6$. However, the fan network representation only realizes a relaxation of the real (IFDIT) settings. For example, as represented by Figure 6, every flow units should travel e with $\tau_e^s(z_2) = 4$. But only $z_2 - z_1$ units travel with time 4 as z_1 may already travel with time 1.

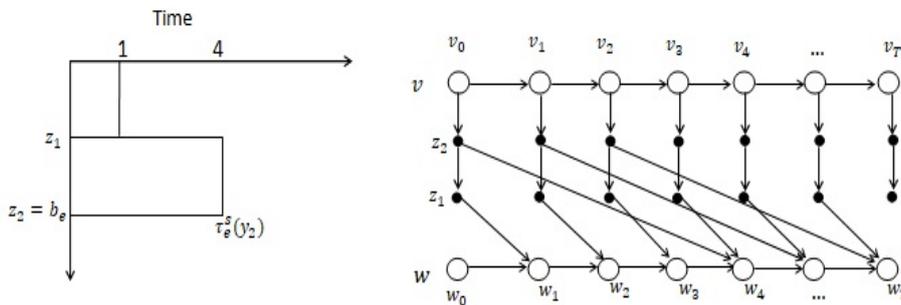


Figure 6: Expansion of a single arc $e = (v, w)$ in the fan network with travel times 1 and 4, for at most z_1 and z_2 flow units, respectively. All arcs except the regulating arc are of infinite capacity.

The size of fan network increases the size of time-expanded network by its step functions very largely. Therefore, it is quite hard to handle the flow over time with (IFDIT) even with approximate settings in the fan. In order to deal with a certain class of flows over time with (IFDIT), the bow network has been introduced, Köhler (2002).

Again define the transit times given by the step functions τ_e^s on arc e , and consider the breakpoints of flow rates $0 < z_1 < z_2 \dots z_k = b_e$ with the corresponding transit times $\tau^1 < \tau^2 < \dots < \tau^k$, respectively. Then the bow network is defined by $N^B = (V^B, E^B)$, where $V^B = V$ and $e \in E^B$ consists of two classes of arcs, namely, the

bow arcs b^i and the regulating arcs r_i , $i = 1, \dots, k$. The former are uncapacitated with transit times τ^i , whereas the latter have zero transit time and capacity z_i . The size of this expansion is linear in the number of arcs. Clearly, the fan-network is a time expansion of the bow network.

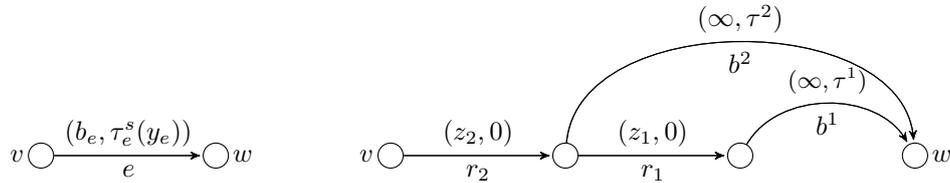


Figure 7: Expansion of $e = (v, w)$ in the bow network with travel times 1 and 4, for at most z_1 and z_2 flow units.

For given flow over time Φ in the original network N with (IFDTT) step functions τ_e^s and time horizon T given by the flow rates $\Phi_e(\theta) : A \times T \rightarrow \mathbb{R}^+$, the flow over time Φ^B on the bow expansion of e is considered as in Definition (29).

$$\Phi_a^B(\theta) = \begin{cases} \Phi_e(\theta), & \text{if } a = b^i \text{ or } a = r_j \text{ with } j \geq i \\ 0, & \text{otherwise} \end{cases} \quad (29)$$

With this, every flow over time with (IFDTT) in N can be considered as a flow over time with constant transit times in N^B , but not conversely. The problem on bow network is certainly a relaxation of the original (IFDTT) flow over time problem.

3. EXISTING APPROACHES

3.1 Evacuations With Constant Attributes

3.1.1 Maximum Evacuation Problems

The earliest arrival flow solutions of Miniéka (1973) and Wilkinson (1971) on s - d dynamic network could be viewed within the non-standard chain decomposition approaches as they apply the flow cancellation property in successive shortest augmenting path algorithms. Miniéka (1973) iterates the maximum flow algorithm of Ford and Fulkerson (1962, 1958) T -times and Wilkinson (1971) constructs a minimum cut. The non-standard chain decomposition here uses the minimum cost maximum flow static flow calculated by the shortest augmenting path algorithm of Ford and Fulkerson (1962, 1958) such that the shortest residual (s, v) and (v, d) lengths are nondecreasing in the process. The shortest directed path sends the incremental s - d flow in the residual network with remaining capacities. However, these algorithms are not polynomial but pseudo-polynomial in time as they are essentially represented by the time expanded networks with backward chain flows. The successive shortest path algorithm with scaling on bounded integer capacities yields a $(1 + \epsilon)$ -approximate earliest arrival flow for any fixed error factor $\epsilon > 0$ that is of polynomial time, Hoppe and Tardos (1994). Again a decomposition defined by the sequence of augmentations of the obtained flows induces a dynamic flow. For $O(\log U)$ scaling phases for capacity rounding with $O(\frac{m}{\epsilon})$, each of which computed with $O(m + n \log n)$, they proved Theorem 3.1.

Theorem 3.1 *Let $val_{\max}(\Phi_e, \theta)$ be the maximum value in time $\theta \leq T$ for any dynamic flow problem, then the capacity scaling successive shortest path algorithm computes dynamic flow $val_{\max}^{eaf}(\Phi_e, \theta)$ in time $O(\frac{m}{\epsilon}(m + n \log n) \log U)$ such that $val_{\max}(\Phi_e, \theta) \leq (1 + \epsilon)val_{\max}^{eaf}(\Phi_e, \theta)$.*

Earliest arrival solutions do not necessarily exist for the general S - D flows (Baumann and Skutella 2006, Fleischer 2001). Authors in (Richardson and Tardos 2002, Fleischer and Skutella 2007) show an existence using the lexicographically maximal flows in the time expanded network. In (Hajek and Ogier 1984, Fleischer 2001) a polynomial time solution is obtained for zero travel times on all arcs, however, an approximate solution is obtained for arbitrary arc travel times in (Fleischer and Skutella 2002). Authors in (Baumann and Skutella 2009, Baumann 2007) solve the S - D flow problem with a strongly polynomial time algorithm in the input plus output size. Authors recursively construct the piecewise linear function for the time-dependent maximum flow value followed by a submodular function minimization within the parametric search, Megiddo (1979). They make use of the quickest transshipment algorithm of Hoppe and Tardos (2000) to obtain the earliest arrival flows.

Consider any subset A of terminals with $\mu(A) = \sum_{w \in A} \mu(w)$ and the maximum flow $\max_{\theta}(A)$ possible to be sent from the sources in A to the sinks outside A in time θ without taking care of $\mu(w)$ for $w \in V$. Then there exists a feasible flow for time θ if and only if $\max_{\theta}(A) \geq \mu(A)$ for all $A \subseteq S \cup D$. Also it holds: $\max_{\theta}(A) = -\min\{\text{cost}_{\theta}(\Phi) \mid \Phi \text{ circulation in modified } \mathcal{N}'\}$, Ford and Fulkerson (1958). The function $\theta \mapsto \max_{\theta}(A)$ is a piecewise linear and

convex cost function of the parametric min-cost flow problem, and the function $\max_{\theta} : S \cup D \mapsto \mathbb{R}$ is submodular, Baumann and Skutella (2009). Define the earliest arrival flow pattern $h : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ that sends the amount of flow $h(\theta)$ into the sink at time θ without violating source constraints. It is piecewise linear but may be non-convex. The $h(\theta)$ units of flow have arrived at the sink by time θ for all $\theta \geq 0$ simultaneously in the earliest arrival flow. Let $\theta_1 = \max\{\theta \mid h(\theta) = \max_{\theta}(S)\}$ and $S_1 \subset S$ be such that $\max_{\theta_1}(S_1) = \max_{\theta_1}(S) - \mu(S - S_1)$. Then

$$h(\theta) = \begin{cases} \max_{\theta}(S) & \text{if } \theta < \theta_1, \\ h_1(\theta) + \mu(S - S_1) & \text{if } \theta \geq \theta_1, \end{cases}$$

where h_1 is the earliest arrival flow pattern for source S_1 . An earliest arrival flow pattern is reduced to the problem of computing an s - d earliest arrival flow pattern for S_1 . An earliest arrival flow pattern is recursively computed on the smaller set of sources with polynomial time on the input size plus the number of breakpoints. An earliest arrival flow can be calculated in a modified network with k additional nodes and arcs for given earliest arrival flow pattern with k breakpoints Baumann and Skutella (2009).

Let us consider the set of k terminals d_1, d_2, \dots, d_k in a multi-terminal network. Minięka (1973) and Megiddo (1974) obtain a static flow that lexicographically maximizes the flows leaving these terminals and lexicographically minimizes the flows entering the sinks in this given order. Minięka (1973) gives an existence proof that requires the max-flow min-cut theorem and bases on the independence departure-arrival flow patterns. The results are also valid if all sources are getting higher ranks than all sinks. Megiddo (1974) obtains a solution to this priority based problem imposing the source to be ranked first without pre-specified ordering of the sinks. Gallo et al. (1989) presents a more general parameterized maximum flow algorithm that has the complexity of a single maximum flow algorithm.

A dynamic lex-max flow solution has to perform this within a given time T . An easy extension of this static approach yields a higher complexity dynamic solution in time-expanded network $\mathcal{N}(T)$ with $d_i(0)$ for source d_i and $d_j(T)$ for sinks d_j . All of the time-expanded network size dependent lex-maximal flow algorithms are of exponential growth. Improving this complexity, Hoppe and Tardos (2000) present a chain-decomposable based polynomial time algorithm on the successive residual networks of the original network. Given zero flow at start, their lex-max dynamic flow algorithm calculates successive layers of minimum cost static flows in the residual network of previous layers and adds standard chains to the existing one. For given period T , the time complexity to compute this feasible and hence the lex-maximal flow takes k times the complexity of minimum cost flow computations.

Theorem 3.2 (Hoppe and Tardos 2000) *A k -terminal lexicographically maximum dynamic flow problem can be solved in polynomial time $O(k \cdot g(nm))$, where $O(g(nm))$ is required for one minimum cost flow computation.*

In a problem of building evacuation, the multiple zones can be prioritized based on emergency levels, like fire blocking and smoke, wherein priority based evacuation steps are quite important. Hamacher and Tufecki (1987) obtain a lexicographical minimum cost dynamic flow by augmenting the current flow in a lexicographically shortest augmenting path as long as it exists. To achieve this, higher costs on arcs of lower priority levels have to be assigned. Their intention is to prevent unnecessary movements within the building and complete an order based evacuation process as quickly as possible. The non-dominated evacuation routes are obtained using dynamic programming formulation of the multi-objective evacuation problems, where several attributes on arcs can be imposed using the dynamic severity matrix, Kostreva and Wiecek (1993).

3.1.2 Quickest Evacuation Problems

Quickest paths. As a variant of the shortest path problem, a solution to the single-source single-sink quickest path evacuation problem sends a predetermined number of evacuees in minimum possible time through a single path, Chen and Chin (1990). This approach has a relevance in an evacuation scenario when evacuees intend to choose a single path or tunnel from their initial location without having any other external evacuees from other positions, for example when the spectators are evacuated from a sports stadium. These shortest paths are of interest with respect to the travel times as well as that of the number of evacuees continuously passing over the time. As a sub-path of a quickest path may not necessarily be a quickest path, a careful analysis is required in to order to figure out the relations between quickest and shortest paths. Rosen et al. (1993) present an efficient algorithm of complexity $O(\kappa(m + n \log n))$, where κ denotes the number shortest paths of distinct capacity values. Additional polynomial algorithms and extensions to the quickest path evacuation problems with flow dependent transit times can be found in (Hamacher and Tjandra 2002, Hung and Chen 1993, Tjandra 2003). Theorem 3.3 states a nice property connecting quickest and shortest paths in single-source single network, Rosen et al. (1991).

Theorem 3.3 *Let $N_{\delta} = (V, A_{\delta}, b, q_o)$ be a sub-network of $N = (V, A, b, q_o)$ with capacity b and flow volume q_o , where $A_{\delta} = \{e \mid e \in A, b_e \geq \delta\}$ and P_k be a shortest s - d path in N_{δ_k} with distinct capacity values $k = 1, 2, \dots, \kappa$. If*

$$\tau(P_{\kappa_o}) + \frac{q_o}{b(P_{\kappa_o})} = \min_{k=1, \dots, \kappa} \left\{ \tau(P_k) + \frac{q_o}{b(P_k)} \right\},$$

then P_{κ_o} is the quickest s - d path in $N = (V, A, b, q_o)$, where $\tau(P_k)$ and $b(P_k)$ represent the length and capacity of the path P_k , respectively.

Quickest flows. The quickest (flow) evacuation problem, also known as the minimum time network clearing problem, extends the quickest path problem by allowing multiple paths for sending an initially located predetermined volume of flow Q_0 in minimum time. The multi-source single-sink (QFEP) for building evacuation in emergencies has been considered in (Berlin 1978, Chalmet et al. 1982), in which Chalmet et al. (1982) study the network flow theory to analyze the evacuation times. A solution to the single-source single-sink quickest evacuation problem has been obtained by iteratively solving the maximum dynamic evacuation problem, Burkard et al. (1993) (cf. Algorithm 1). They establish that the s - d quickest flow problem has strong relation to the corresponding maximum flow problem and to linear fractional programming problems.

Algorithm 1 *The s - d quickest evacuation algorithm*

1. Apply any method to estimate the time T (binary search, interpolation or Newton's method).
2. Solve the parametric cost linear program-minimum cost circulation for given parameter T .
3. Repeat Steps 1 and 2 until the minimum time $T(Q_0)$ with $\text{val}(\Phi, T) = Q_0$.

In fact the quickest flow problem can be considered as the inverse problem of the maximum dynamic flow problem. The dynamic flow Φ is quickest of value $\text{val}(\Phi, T)$ with minimum time T , if $\text{val}_{\max}(\Phi, T - 1) < \text{val}(\Phi, T)$ for an integer time $T \geq 0$. Algorithm 1 computes a quickest flow determining a MDF Φ for $T^*(\text{val}(\Phi, T))$ and decreasing its value to $\text{val}(\Phi, T)$ if $\text{val}_{\max}(T^*(\text{val}(\Phi, T))) > \text{val}(\Phi, T)$, where $T^*(\text{val}(\Phi, T))$ denotes the minimum time for the transshipment of value $\text{val}(\Phi, T)$.

Theorem 3.4 *The time depending maximum value function $\text{val}_{\max}(\Phi, T)$ is monotonically increasing and increases strictly if $T \geq \min\{T' \mid \text{val}_{\max}(\Phi, T') > 0\}$. The function $\Delta(T) = \text{val}_{\max}(\Phi, T) - \text{val}_{\max}(\Phi, T - 1)$ is monotonically increasing for all $T > 0$ in the range $\{0, 1, \dots, \text{val}_{\max}(\Psi)\}$, where Ψ represents an arbitrary MSF.*

Exploiting the properties of piecewise-linear and increasing function $\text{val}_{\max}(\Phi, T)$ as a continuous function, several efficient algorithms including also a strongly polynomial one are presented for the s - d quickest flow problem in Burkard et al. (1993). The obtained earliest arrival flow $\text{val}_{\max}(\Phi, T)$ simultaneously yields the quickest flow time $T(Q_0)$ with $\text{val}_{\max}(\Phi, T) = Q_0$, Jarvis and Ratliff (1982). However, with this approach the solution complexity is earliest arrival flow solution dependent, for example a pseudo-polynomial algorithm of Wilkinson (1971) can be considered for computing of a s - d EAF solution. For an integer T , a binary search procedure takes time $O(m \log n(m + n \log n) \min\{\log \text{val}(\Phi, T), \text{val}_{\max}(\Psi)\})$ and an interpolation and Newton's approximation based algorithm takes time $O(m \log n(m + n \log n) \log \Delta_{\max})$, where $\Delta_{\max} = \min\{\text{val}(\Phi, T), \text{val}_{\max}(\Psi)\}$ represents an upper bound for the slope $\Delta(T^*(\text{val}(\Phi, T)))$ of the line segment containing $T^*(\text{val}(\Phi, T))$. Here $O(m \log n(m + n \log n))$ costs for the minimum cost circulation, Orlin (1988). Although not very practical, authors in (Burkard et al. 1993) present a strongly polynomial time algorithm taking time $O(m^2 \log^3 n(m + n \log n))$ using the linear fractional programming problem in Megiddo (1979), where the total time period has been expressed as quotient of two linear functions of flow variables. Several computational tests and worse case comparisons are made for randomly generated instances.

Lin and Jaillet (2015) proposed an improved algorithm for the s - d quickest flow problem considered in (Burkard et al. 1993) by formulating it as a fractional programming problem (30-32).

$$T^* = \min \frac{Q_0 + \sum_{e \in A} \tau_e \cdot \Psi_e}{\text{val}(\Psi)} \quad (30)$$

$$\text{such that } \sum_{e \in A_v} \Psi_e - \sum_{e \in B_v} \Psi_e = \begin{cases} \text{val}(\Psi) & \text{if } v = s \\ -\text{val}(\Psi) & \text{if } v = d \\ 0 & \text{otherwise} \end{cases} \quad (31)$$

$$0 \leq \Psi_e \leq b_e, \quad \forall e \in A \quad (32)$$

A temporally repeated flow of any optimal static flow Ψ is an optimal quickest flow to transship the value Q_0 . To minimize the ratio $T = \frac{Q_0 + \sum_{e \in A} \tau_e \cdot \Psi_e}{\text{val}(\Psi)}$, the residual network $\mathcal{N}(\Psi)$, node potentials π (i.e., dual variables corresponding to the flow conservation constraints) are introduced, and the reduced cost $\pi(v) - \pi(u) + \tau_e$ is calculated for each arc $e = (u, v) \in \mathcal{N}(\Psi)$. The overall time complexity of their algorithm is $O(n^3 \log(nC))$, where C denotes the maximum travel time on arcs.

Improving all of the approaches given in (Burkard et al. 1993 and Lin and Jaillet 2015), a strongly polynomial time algorithm, making the use of a cancel-and-tighten algorithm, has been recently presented by Saho and Shigeno (2017) that solves the single-source single-sink quickest flow problem in time $O(nm^2(\log n)^2)$.

A polynomial time algorithm depending on $\log T$ has been presented for the general quickest flow problem in Hoppe and Tardos (1994, 2000) and Hoppe (1995). Their non-standard flow decomposition approach within time-expanded graph allows simultaneous flows on different but not necessarily disjoint source-sink paths. They study the multi-terminal quickest evacuation problem with predefined node-arc capacities and arc travel times.

For time period T and $\mu(A) = \sum_{v \in A} \mu(v)$, let $\max_T b_v(A)$ with $A \subseteq S \cup D$, be the maximum flow value that could be shifted from $v \in A \cap S$ to $v \in D - A$ without taking account the needs of other terminals. Let $\mathcal{N}_v = (V, A, b, \tau, \mu)$ be the extended network, where s (respectively, d) denotes the super source (respectively, sink) connected to $v \in A \cap S$ (respectively, from $v \in D - A$).

Algorithm 2 *Multi-terminal dynamic transshipment feasibility test*

1. Solve MDF in $\mathcal{N}_v = (V, A, b, \tau, \mu)$ for $\max_T b_v(A)$ with static minimum cost flow calculation.
2. Check the feasibility (i.e., $\max_T b_v(A) \geq \mu(A)$ for any $A \subseteq S \cup D$) of the proposed QFP.
3. Find a violated set by minimizing the submodular function $\max_T b_v(A) - \mu(A)$ based on Grötschel et al. (1988).

With $O(\alpha(mn))$ as the computational cost required for Step 1, a trivial exponential time $O(2^k \alpha(mn))$ algorithm finds a violating set in Step 3. But authors in Hoppe and Tardos (2000) propose a strongly polynomial time algorithm for Step 3 based on submodular function minimization oracle of Grötschel et al. (1988), and also present an algorithm of complexity $O(k^2 \alpha(mn) \log(nTB_o))$ based on Vaidya (1989), where k and B_o denote the number of terminals and the maximum arc capacity, respectively.

They also present a polynomial time algorithm by combining Algorithm 2 and binary search to find the minimum time T for the quickest transshipment problem. With nT as an upper time step for reaching the first flow to a sink, their binary search starts with the bound $nT + \mu(S)$. It requires $(k^2 \alpha(mn) \log^2(nB_o T_o \mu(S)))$ time, where the optimal time T is bounded by $nT_o + \mu(S)$. A trivial check requires $(2^k \alpha(mn) \log(nT_o \mu(S)))$ time. They also present a strongly polynomial time algorithm using the parametric search technique of Megiddo (1979) for the minimum transshipment time T .

The dynamic transshipment problem is reduced in Hoppe and Tardos (2000) to an equivalent lex-max dynamic flow problem in more complex graph using the already obtained time bound of the former. In this reconstructed network, the terminals are connected with extra terminals with adjusted transit times and capacity functions to get a parameterized dynamic transshipment problem. An algorithm for equivalence of solutions among these problems generate a chain of tight subsets (i.e., $\max_T b_v(A) = \mu(A)$) with modified terminal sets ordered by inclusion. After all, a binary search technique has to be applied to determine the adjusted transit times on artificial arcs. Finally, Algorithm 2 terminates with its final feasibility check of the single lex-max problem. As a result, the dynamic transshipment problem can be solved in polynomial time using binary search and also in strongly polynomial time using parametric search. The time required for checking the feasibility of a dynamic transshipment problem and finding a violated set is denoted by β .

Theorem 3.5 *The dynamic transshipment problem can be solved in strongly polynomial time and also in time $O(k\beta \log(nTB_o))$ time. Moreover, the quickest transshipment problem can be solved in $O(k\beta \log(nT_o B_o \mu(S)))$ time, and also in strongly polynomial time.*

There are nice optimality properties between the universal maximal flows and minimum average and total evacuation times. Theorem 3.6 gives a strong connection between the three optimization problems - the universal maximal flow, minimum weighted sum and quickest flow problems, Jarvis and Ratliff (1982). Second is the minimum total turnstile cost problem with increasing turnstile costs $c_d(v_\theta, \star) = \theta$, where $v \in D$ and $\star = w_{\theta+\tau(v,w)}$ is the super-sink in $\mathcal{N}(T)$, (Chalmet et al. 1982, Hamacher and Tjandra 2002, Hamacher and Tufecki 1987). Kisko and Francis (1985) solved the problem of minimizing the average evacuation time $\sum_{\theta=1}^T \sum_{v \in D} \theta \cdot \Phi_e(\theta)$ with $e = (v_\theta, \star)$ satisfying the flow conservation and capacity constraints.

Theorem 3.6 *Consider the triple optimization problems as follows:*

$$\max \sum_{\theta=0}^{\theta^*} \Phi_e(\theta), \forall \theta^* \leq T; \quad \min \sum_{\theta=0}^T c_d \Phi_e(\theta); \quad \min \{T \mid \Phi_e(T^*) = 0, \forall T^* > T\}.$$

Then, a feasible solution for either of the first two is also a feasible solution for the other two.

3.2 Generalized Flow

Onaga (1966, 1967) presented the successive shortest path algorithm that solves the generalized maximum static flow problem in pseudo-polynomial time. It starts with $\Psi = 0$ and the residual network $\bar{N}(\Psi)$. If there is no $s - d$ path in $\bar{N}(\Psi)$, the algorithm terminates. Otherwise, it augments flow along the $s - d$ path with highest gain. The process will be continued with the resulting Ψ flow and residual network $\bar{N}(\Psi)$. Moreover, if the network has no residual flow generating cycle, then only, the obtained flow is a generalized maximum static flow. Gondran and Minoux (1984) presented a generalized flow decomposition approach in which a generalized static $s - d$ flow Ψ can be decomposed into at most m generalized flows $\Psi_i, i = 1, \dots, k$ along $s - d$ paths and cycles.

Lemma 3.7 *Consider the decomposition of generalized flows Ψ_i with the value $\Psi = \sum_{i=1}^k \Psi_i$. Then each of these flows satisfies one of the following structures.*

1. A path from the source s to the sink d .
2. A flow-generating cycle connected to the sink d by a path.
3. A path from the source s to a flow-absorbing cycle.
4. A unit-gain cycle flow.
5. A flow-generating cycle connected to a flow-absorbing cycle by a path.

Fleischer and Wayne (2002) considered a two terminal lossy network without flow generating cycles in which the product of gain factors exceeds one. A lossy network cannot have flow generating cycles and networks with no flow generating cycles need not be lossy. However, the transformation of a network with no flow generating cycles into an equivalent lossy networks has been made in $O(mn)$ time. They presented the generalized max flow packing algorithm that computes an ϵ -approximate generalized maximum static flow on two-terminal lossy network without flow generating cycles for given $\epsilon > 0$ in $O(\epsilon^{-2}m(m + n \log m) \log m)$ time complexity.

By assuming $\epsilon = 1/2$, the generalized max flow packing algorithm has been run repeatedly in residual network that computes $1/2$ -approximate generalized maximum static flow Ψ Goldberg et al. (1991). As in each iteration, at least $1/2$ of the remaining flow is possible, the optimality gap decreases geometrically to zero. After $\log(1/\epsilon)$ iterations an ϵ -approximation will be obtained, Fleischer and Wayne (2002). If, without loss of generality ϵ is assumed to be sufficiently small, for example, say M^{-3m} where M denotes the biggest integer used to represent any of the costs, capacities or loss factors then, the ϵ -approximate flow can be efficiently rounded to an optimal flow Goldberg et al. (1991).

Gross and Skutella (2012) and Gross (2014) introduced a generalized maximum dynamic flow (GMDF) model where each arc contains both loss factors and transit times. They proved by reduction from PARTITION that there is neither a polynomial time nor a polynomial time approximation algorithm on both series-parallel networks with proportional losses and lossy networks with non- proportional losses and transit times, unless $\mathcal{P} = \mathcal{NP}$. However, on two terminal lossy network with the loss rate per time unit identical on all arcs, a pseudo-polynomial time algorithm has been presented that computes the (GMDF) using the (GMSF) algorithm of Onaga (1966, 1967) on generalized time expanded network. The generalized time expanded network is similar to the time expanded network, however, each arc has additional gain factor λ . As in the dynamic network flow, the relation as in Lemma 3.8 holds, (Gross and Skutella 2012, Gross 2014).

Lemma 3.8 *There is a one-to-one correspondence between a generalized dynamic flow in original network and a generalized static flow in generalized time expanded network.*

In the generalized time expanded network, the successive shortest path algorithm of Onaga (1966, 1967) solves the (GMDF) problem on the original network. Gross and Skutella (2012) and Gross (2014) considered the lossy network with the assumption that in each time unit the same percentage of the remaining flow value is lost and the lost value is equivalent to $\lambda \equiv 2^{c \cdot \tau}$ for some constant $c < 0$. This is motivated by the problems in which evacuees cannot be shifted totally due to unfortunate deaths. Their algorithm constructs the temporally repeated flow structure and applies on generalized time expanded networks.

Theorem 3.9 *The (GMDF) problem can be solved in $O(\text{max-flow} \cdot T)$, where a maximum flow algorithm with a running time $O(nm)$ has been obtained in (Orlin 2013).*

For the two terminal lossy network with unlimited supply and demand, the (GMDF) obtained by Gross and Skutella (2012) and Gross (2014) satisfies the earliest arrival property if the holdover at intermediate nodes is not allowed. Thus, the generalized earliest arrival flow problem is also solvable and it is solved with the same complexity as

the (GMDF) problem. But for special cases, the running time can be further improved. A (GMSF) problem has been solved in Krumke and Zeck (2013) in series parallel networks with a greedy-strategy that chooses always the highest gain path in original network but not in residual network. This is sufficient for finding an optimal solution.

Moreover, Gross and Skutella (2012) and Gross (2014) also presented an approximate (GMDF) solution on two terminal lossy network. They used an FPTAS which is obtained by terminating the (GMDF) algorithm after a polynomial number of iterations. It approximates the maximum flow value and produces a solution that is at least as good as an optimal exact solution. The error ϵ appears only logarithmical as $\log \epsilon^{-1}$ in the runtime of the FPTAS.

Theorem 3.10 *For given $\epsilon > 0$, $U = \max_{e \in E} b_E(e)$ and constant $c < 0$ on \mathcal{N} , the approximate (GMDF) problem can be solved in $O(\max\text{flow} \cdot (\log \epsilon^{-1} + \log U + \log T))$ time complexity.*

Stating other applications where the gain factors can be used to model physical changes such as leakage or theft, that should be equally applicable to loss factors in evacuation, Véghe (2018) presented a strongly polynomial time algorithm for solving the generalized flow maximization problem using a continuous scaling technique.

3.3 Problems In Abstract Networks

An abstract network consists of an abstract path system formed by a set of elements that may be nodes or arcs. Let the network $\mathcal{N} = (A, \mathcal{P}, b, \tau, c, S, D, T)$ be a dynamic abstract network with capacity $b : A \rightarrow \mathbb{R}^+$, weight $c : A \rightarrow \mathbb{R}^+$ and transit time $\tau : A \rightarrow \mathbb{Z}^+$ on A . A static abstract network is obtained by discarding the time factor from dynamic abstract network. Assume that $\mathcal{P} \subseteq 2^A$ and every path $P \in \mathcal{P}$ has a linear order $<_P$ of the elements. A path system in abstract network satisfies Property 1, known as the switching property.

Property 1 *The paths P and Q cross at element e if $e \in P \cap Q$ and if there exists a path R that only uses elements at the beginning of P and at the end of Q (and vice versa). Mathematically, both sets $\{R \subseteq \mathcal{P} \mid R \subseteq (P, e) \cup (e, Q)\}$ and $\{R \subseteq \mathcal{P} \mid R \subseteq (Q, e) \cup (e, P)\}$ are non-empty. The switching paths of P and Q are $P \times_e Q \in \{R \subseteq \mathcal{P} \mid R \subseteq (P, e) \cup (e, Q)\}$ and $Q \times_e P \in \{R \subseteq \mathcal{P} \mid R \subseteq (Q, e) \cup (e, P)\}$, where $(P, e) = \{p \in P : p <_P e\}$ and $(e, Q) = \{q \in Q : e <_Q q\}$.*

Based on the network flow model of Ford and Fulkerson (1956), Hoffman (1974) introduced the abstract flows generalizing the concept of paths. For the static case, he showed that the generalized abstract maximum weighted flow and the generalized abstract minimum weighted cut problems are primal and dual LP to each other. Moreover, weights are supermodular and the abstract path system satisfies the switching property so that the abstract maximum flow is totally dual integral with minimum cut. McCormick (1996) presented a polynomial algorithm to solve the abstract maximum static flow problem with unit weight on each element. For the weighted case, Martens and McCormick (2008) compute an abstract maximum weighted flow by using augmenting structure of decreasing total reward, where the shortest path has the largest reward. Their algorithm has been applied to augment flow without removing on the counting elements in the process of finding restricted abstract maximum flow.

Let $\Psi : \mathcal{P} \rightarrow \mathbb{R}^+$ be the static flow on an abstract network. The maximum static abstract flow problem maximizes the sum of flows on all paths satisfying the capacity and feasibility contractions, as in the formulation (33). In this formulation, it is assumed that each path has unit weight.

$$\max \sum_{P \in \mathcal{P}} \Psi(P) \tag{33}$$

$$\text{such that } \sum_{P \in \mathcal{P}: e \in P} \Psi(P) \leq b_e, \forall e \in A \tag{34}$$

$$\Psi(P) \geq 0, P \in \mathcal{P} \tag{35}$$

If \mathcal{C} is a cut set with an abstract minimum cut with value $\text{val}_{\min}(\mathcal{C})$ as a dual of the maximum abstract flow problem, the objective function (36) wants a minimum capacity weight z_e of every element $e \in A$ that covers all paths.

$$\min \sum_{e \in A} b_e \Omega_e \tag{36}$$

$$\text{such that } \sum_{e \in P} \Omega_e \geq 1, \forall P \in \mathcal{P} \tag{37}$$

$$\Omega_e \geq 0, \forall e \in A \tag{38}$$

The abstract flow over time problem is investigated with its mathematical formulation by Kappmeier et al. (2014). Let a nonnegative function $\Phi : \mathcal{P}_T \rightarrow \mathbb{R}_+$ be the dynamic abstract flow to all temporal paths $P_t \in \mathcal{P}_T$, where P_t and \mathcal{P}_T are, respectively,

$$P_t = \left\{ (e, \theta) \in A_T : e \in P, \theta = t + \sum_{p \in (P, e)} \tau_A(p) \right\},$$

$$\mathcal{P}_T = \left\{ P_t : P \in \mathcal{P}, t \in \mathcal{T}, t + \sum_{p \in P} \tau_A(p) \leq T \right\}.$$

Here, A_T is a time expanded element set defined as $A_T = A \times \mathcal{T}$ which is obtained by copying A on each time period. Moreover, the flow along path P_t enters element e at time $t + \sum_{p \in (P, e)} \tau_p$ and reaches the sink at time $t + \sum_{e \in P} \tau_e$. All the temporal paths should arrive to sink at latest at time T . Then, the abstract maximum dynamic flow maximizes the objective function in (39) satisfying the capacity and non-negativity constraints.

$$\max \sum_{P_t \in \mathcal{P}_T} \Phi(P_t) \quad (39)$$

$$\text{such that } \sum_{\substack{P_t \in \mathcal{P}_T \\ (e, \theta) \in P_t}} \Phi(P_t) \leq b_e, \forall e \in A, \theta \in \mathcal{T} \quad (40)$$

$$\Phi(P_t) \geq 0, \forall P_t \in \mathcal{P}_T \quad (41)$$

Moreover, the value of an abstract maximum dynamic flow equals the capacity of minimum dynamic abstract cut and vice versa as dynamic abstract cut bounds the maximum value of dynamic abstract flow as follows

$$\sum_{P_t \in \mathcal{P}_T} \Phi(P_t) \leq \sum_{(e, \theta) \in \mathcal{C}} b_e, \forall e \in A, \theta \in \mathcal{T}. \quad (42)$$

Here, $\mathcal{C} \subseteq \mathcal{A}_T$ represents an abstract dynamic cut set so that $P_t \cap \mathcal{C} \neq \emptyset$ for each $P_t \in \mathcal{P}_T$. As in network flow model, a solution of the abstract maximum dynamic flow problem is obtained using time expanded abstract network, Kappmeier et al. (2014). They also solved the abstract maximum dynamic flow problem with a polynomial algorithm by using the abstract time expanded network. The abstract time expanded network introduces the copies of each path as a whole. However, the time expansion of an abstract network may not be an abstract network because it may violate the switching property. To satisfy the switching property, the intermediate storage for all temporal paths is allowed that does not influence the solution, i.e., the temporally repeated solution is optimal even if intermediate storage is allowed in Matuschke (2014) and Kappmeier (2015). Moreover, the abstract lex-maximum flow and abstract earliest arrival flow problems are investigated by Kappmeier (2015) with efficient solution algorithms.

3.4 Evacuations With Variable Attributes

3.4.1 Load-Dependent Transit Times

In order to deal flows over time with the (LDTT), we pose Assumption 1. The independence of flows on different arcs weakens this model yielding only approximate solutions in reality.

Assumption 1 *At each moment of time, the entire flow on an arc travels with uniform speed that depends only on the current load (the amount of flow) of that arc.*

For the load l_e and flow rate Ψ_e on arc e , the relation $l_e = \Psi_e \tau_e(\Psi_e)$ holds for a static flow. With an assumption that the function $\Psi_e \rightarrow \Psi_e \tau_e(\Psi_e)$ is non-negative, strictly increasing and convex, we also denote the transit time as a function of the load $\hat{\tau}_e(l_e)$. Satisfying the former relation by the flow rate and the load, we note that $\tau_e(\Psi_e) = \hat{\tau}_e(l_e)$. Thus the (LDTT) model relies on the fact that the speed of flow along an arc e is proportional to the inverse of current transit time $\hat{\tau}_e(l_e(\theta))$ at any moment of time θ .

The flow over time problem with (LDTT) has been approximately solved in polynomial time by exploiting the property of temporarily repeated solution of the corresponding average static flow rate $\Psi_e = \frac{1}{T} \int_0^T \Phi_e(\theta) d\theta$. The authors in Köhler and Skutella (2005) proved Theorem 3.11. Moreover, they also proved that the problem of finding a quickest flow over time with (LDTT) is strongly \mathcal{NP} -hard and there does not exist an FPTAS unless $\mathcal{P} = \mathcal{NP}$ holds. The reductions are based on the \mathcal{NP} -complete PARTITION and SATISFIABILITY problems.

$$\max \sum_{e \in A_d^{in}} \Psi_e - \sum_{e \in A_d^{out}} \Psi_e \quad (43)$$

$$\text{such that } \sum_{e \in A_v^{in}} \Psi_e - \sum_{e \in A_v^{out}} \Psi_e = 0, \forall v \in V \setminus \{s, d\} \quad (44)$$

$$0 \leq \Psi_e \leq b_e, \forall e \in A \quad (45)$$

$$\sum_{e \in A} \Psi_e \tau_e(\Psi_e) \leq Q_0 \quad (46)$$

Theorem 3.11 *Suppose that there exists a flow over time with (LDTT) that sends Q_0 units of s - d flow within time period T . Then a flow over time satisfying the same demand within time horizon at least $2T$ exists in the class of temporally repeated flows. Also, this solution can be computed polynomially within time $(2 + \epsilon)T$ for any given $\epsilon > 0$.*

On the other hand, if one is seeking in improving the time factor of 2, it has to compromise in decreasing the amount of flow Q_0 . Notice that the use of temporarily repeated solution of the static flow problem (43-46) worsen the flow over time solution that could be obtained theoretically.

3.4.2 Inflow-Dependent Transit Times

Authors in Köhler et al. (2002) study the quickest flow over time with an additional restriction on the transit time which completely depends on the current rate of inflow into that arc at any point of time. With this, the arc wise entering flows impose the pace of every unit of flow and this remains fixed throughout. Although these restrictions violate the first-in-first-out (FIFO) property and do not capture the more realistic behavior of the flow over time with flow-dependent times, the authors are successful to give approximate results with polynomial time algorithms comparable to the results on quickest flows over time with (LDTT). In order to model the (IFDTT) flow over time problem, Assumption 2 has been made.

Assumption 2 *At any moment of time, the transit time function on an arc is given as a non-negative, piecewise constant, non-decreasing and left-continuous function of inflow rate.*

Note that the functions can be restricted to have only integral values as this can be easily relaxed to allow arbitrary rational values by scaling the time in a proper way.

Theorem 3.12 (Köhler et al. 2002) *Suppose that there exists a flow over time sending Q_0 flow units from s to d within time T for non-decreasing piecewise constant transit time functions. Then a temporarily repeated flow with (IFDTT) can be computed in strongly polynomial time that sends the same amount of flow from s to d within time horizon at most $2T$.*

Moreover, any general non-negative, non-decreasing and left-continuous function can be approximated by a step function within arbitrary precision, Köhler et al. (2002). This allows us to obtain a temporarily repeated quickest flow with left-continuous inflow-dependent transit time functions by a strongly polynomial time $(2 + \epsilon)$ -approximation algorithm. That means, the same amount of flow that a flow over time can send within a certain time period in the given network can also be obtained via temporarily repeated flow within a time horizon at most $(2 + \epsilon)$ of the this time. Moreover, the performance can be improved to $\frac{3}{2} + \epsilon$ if convexity is replaced by concavity in the transit time functions. Furthermore, given all transit time functions to be convex, the quickest flow problem can be converted into a minimum convex cost flow problem with transit times as the arc costs.

3.4.3 Flow Dependent Earliest Arrival Flows

Unlike Gale's result on the existence of earliest arrival flows for constant transit times on arcs, earliest arrival flows do not exist for (FDTT) flow over time problems even on very simply structured networks Baumann and Köhler (2007). This result is valid even (FFTT) are restricted to (LDTT) or (IFDTT) functions. Obtaining an earliest arrival flow with (FDTT) functions is computationally an \mathcal{NP} -hard problem as its relaxed version where obtaining a maximum flow at the end of time horizon only, i.e., the (MDFP), is already in this class. However, they introduced the closely related version of this problem, the so-called α -earliest arrival flow problem. They presented a time approximation algorithm to find this almost earliest arrival flow solution which required only α -times longer to send the maximum flow to the sink for each point in time. The authors pointed out that the value approximation algorithms should also be possible as in Hoppe and Tardos (1994) for the constant transit times on arcs. A proof of this theorem is based on the approximation algorithms for the quickest flow solutions with (LDTT) and (IFDTT) as in (Hall et al. 2003, Köhler et al. 2002, Köhler and Skutella 2005).

Theorem 3.13 *The α -approximation earliest arrival flow problem can be solved by Ac -algorithms, where $c = 2$ for the (IFDTT) and (LDTT) flows over time models. Moreover, $c = 1 + \epsilon$ for the (IFDTT) flow over time model for any $\epsilon > 0$.*

3.4.4 Time Dependent Attribute Solutions

The majority of the network flow research rely on a balanced configuration with small size discretization in predetermined discrete time steps of the continuous time structure at the computational cost. The time dependent continuous time flow models are compactly sketched here. Let the unit cost $c(\theta)$ and upper bound $b(\theta)$ on the rate of continuous flow $\Phi(\theta)$ be bounded measurable functions with m -components. Let a bounded measurable function $\nu : A \times T \rightarrow \mathbb{R}$ represent the rate of demand-supply vectors in each arc. If $\phi(\theta)$ is the level of storage at time θ , a general continuous network flow problem with cost minimization can be formulated as follows.

$$\min \zeta = \int_{\theta=0}^T c(\theta)\Phi(\theta)d\theta \quad (47)$$

such that for all points in time $\theta \in [0, T]$,

$$\int_{\sigma=0}^{\theta} \mathcal{M}\Phi(\sigma)d\sigma + \phi(\theta) + \int_{\sigma=0}^{\theta} \nu(\sigma)d\sigma = 0 \quad (48)$$

$$\Phi(\theta) \leq b(\theta) \quad (49)$$

$$\Phi(\theta), \phi(\theta) \geq 0 \quad (50)$$

where \mathcal{M} denotes the node-arc incidence matrix. Given piecewise analytic functions $\nu(\theta)$, $b(\theta)$ and $c(\theta)$ on the neighborhood of $[0, T]$, there exists a piecewise analytic solution $\Phi(\theta)$ of the problem (47-50) if its feasible region is bounded and nonempty, Anderson et al. (1983). The above model can be extended to continuous dynamic network flow problem (51-55) with travel times on arcs (for instance (Anderson 1989, Anderson and Philpott 1994, Philpott 1990, Pullan 1997).

$$\min \zeta' = \int_{\theta=0}^T \sum_{e \in A} c_e(\theta)\Phi_e(\theta)d\theta \quad (51)$$

such that for all points in time $\theta \in [0, T]$ and $e = (u, v) \in A$,

$$\int_{\sigma=0}^{\theta} [\nu_v(\sigma) + \sum_{e \in A} (\Phi_e(\sigma - \tau_e) - \Phi_{e-1}(\sigma))]d\sigma + \phi_v(0) = \phi_v(\theta) \quad (52)$$

$$\Phi_e(\theta) \leq b_e(\theta) \quad (53)$$

$$\phi_v(\theta) \leq a_v(\theta) \quad (54)$$

$$\Phi_e(\theta), \phi_v(\theta) \geq 0 \quad (55)$$

Philpott (1990) formulated three evacuation problems, analogous versions to the triple optimization results of discrete dynamic network, and proves their optimality relations. The max-flow min-cut result of (Anderson et al. 1982) is extended to the case where each arc has a traversal time. Anderson et al. (1982) extended the discrete version of maximizing the network flow problem with time-varying arc capacities and storage at the nodes to the continuous time problem and presented a continuous version of Ford-Fulkerson labelling algorithm to solve the latter problem. The corresponding duality theory and the convergence issues with computational challenges for this problem are discussed. Anderson and Philpott (1989) developed a continuous-time network simplex algorithm and solved the minimum cost network flow problem as an infinite dimensional linear program. Pullan (1993) and Philpott and Craddock (1995) look at the continuous time evacuation problems using discretization approach. We refer to Hamacher and Tjandra (2002), and Miller-Hooks and Patterson (2004) for the respective algorithms on time dependent (for storage and arc capacities, and costs) dynamic network models with continuous time and for additional classical references on them. Most of them deal with primal-dual relationships for continuous flow models.

Koch et al. (2011) consider two independent research directions, the discrete and continuous flow models, into integrated one modeling the flow on each arc as a Borel measure. The arc capacities are also given by Borel measures on the real line and flow storage at nodes is allowed. They extend the max-flow min-cut theorem for the Borel flows and open many directions for the extensions of this model, for example into time/inflow/load dependent attributes. The results they presented are of theoretical interest as the algorithmic procedures under the general assumptions are highly complex and even nonterminating. For possible practicable algorithms, various assumptions, like no node storage capability, have to be imposed.

Tjandra (2003) gives a pseudo-polynomial time algorithm in the time horizon and total supply at source nodes if the supplies and capacities in the network are time dependent. Philpott and Mess (1992) find a continuous time

least-cost path in a network with time dependent costs. The costs, travel times, stopping-starting penalties are all time-dependent for their general model having practical limitations of the algorithm with respect to the termination. They extend the result to finite-time algorithm, where cost functions are piecewise-linear and starting and stopping times are constant, Philpott and Mess (1993).

A pseudo-polynomial time algorithm is proposed for solving the integral time dependent quickest flow problem on time dependent dynamic network, where the node and arc capacities, arc travel times and supply at nodes vary over time. However, the arc travel time $\tau_e(\theta)$ entering an arc e remains constant during the time it travels through this arc and holdover capacity is unlimited independent of time with unit travel time at nodes. This may allow violation of the undesirable non FIFO property. It finds the paths for source-sink flows such that the completion time of the last unit is minimum. Their algorithm also determines the paths that yield the minimum total time taken for completing the shipment of all source-sink flows. This solution technique is also extended for the time dependent multi source and general multi terminal quickest transshipment problems by converting the multi terminal networks to two-terminal networks. The main idea of their algorithm is to use the successive shortest path algorithm for solving minimum cost static network flow problem by improving flows on time dependent static residual network. If holdover of flows at intermediate nodes is not allowed during certain intervals of time, then a more general version of the maximum dynamic flow problem with uniform travel times but time dependent capacities on arcs have been dealt which requires cycle flows for an optimal solution, Halpern (1979).

Klinz and Woeginger (1995) consider the minimum cost quickest flow problem where the objective is to find source-sink paths with minimum cost such that the last unit of flow is reached to the destination in least time. The problem is \mathcal{NP} -even (harder than the even-odd partition problem) in case of two-terminal series-parallel networks with unit capacities. Cai et al. (2001) provide a technique to solve the time varying (arc capacities, travel times and costs are all time dependent) minimum cost flow problem that sends given amount of source-sink flow in minimum shipment total cost within a given time horizon. A polynomial time algorithm is available for solving an infinite horizon minimum cost dynamic flow problem that maximizes throughput, Orlin (1994). However, it does not take care of specified demand and computing how a flow starts and stops.

Hamacher et al. (2006) generalized the classical shortest path problem to bicriteria shortest path problem with time dependent attributes. The latter problem is more interesting in evacuation modeling, where shortest paths represent evacuation routes while these routes might change over time with respect to the importance to its length or reliability. Chen and Miller-Hooks (2008) give a mixed integer programming formulation of building evacuation problem in time dependent network (arc transit times and capacities and node supply are time dependent) with shared information constraints (the evacuees's do receive online common instructions) and presented a real-world numerical result. Their objective is to determine the appropriate routes such that total evacuation time is the least.

4. CONTRAFLOW MODELS

During the evacuation process, evacuation planner discourage the movement of people towards the disastrous areas from the safer places because of which the corresponding road lanes are unoccupied. However, due to heavy traffic of emergency vehicles and evacuees on the streets, the lanes outwards from the sources become more congested. The contraflow configuration is the technique of optimal use of these empty lanes. It increases the outbound road capacities from the sources by reversing the direction of arcs satisfying the given constraints. Due to which the traffic will be systematic and smooth by removing the traffic jams caused by not only different large scale natural and man-made disasters but also busy office hours, special events and street demonstrations. Moreover the flow value will be increased. The average evacuation time will be decreased and some lanes with excess capacities can be saved for an use of emergency vehicles and logistic supports needed to move towards the sources. There are various operational research models, heuristics, optimization and simulation techniques to deal the contraflow configuration. However, an efficient and universally acceptable solution approach that meets the macroscopic and microscopic behavioral characteristics is still lacking. A trade-off between computational costs and solution quality should be a compromise (Arulselman 2009, Kim et al. 2008, Pyakurel 2016, Rebennack et al. 2010). Among the network flow problems, the contraflow reconfigurations based models are most essential. It has been established that significant flow increase and evacuation time savings promote the ideas greatly. The lane reversal strategy that supports not only to the evacuation planning but also for the distribution of emergency logistics are quite relevant to the issues of disaster management.

4.1 Heuristics Approaches

The heuristic approaches and applications of lane reversal techniques can be found in various literature. In many terrible disasters, authors from diverse fields of research have reported significant time saving possibilities and a need of effective adaptation of contraflow techniques. Different contraflow mathematical models, for example, a mesoscopic model based on the dynamic traffic assignment method, tabu search heuristic for very large spatial networks, microscopic

traffic simulation models, integer programming formulation, etc., are proposed in literature. We refer to Kim et al. (2008) and Dhamala (2015) for the references and detail overviews of contraflow in practice.

The first contraflow algorithms, known as all-links and fastest-links are developed to support a smart traffic evacuation management system established with the aim to make an efficient and rapid response to disasters by creating dynamic evacuation plans based on incident location, scope and current traffic conditions, Hamza-Lup et al. (2004). The all-links algorithm minimizes congestion by using all available streets visiting only once starting from the source. The fastest-links algorithm forces the traffic to fastest paths from the source to exit points constructed by an optimal multicast tree. However, these algorithms do not care about the overall capacity of road so that these are not effective if the number of evacuees, road capacity, specific sinks are fixed, or if evacuees are spread over many locations. From several years, contraflow has been widely applied to evacuate regions of the southeastern USA under threat from hurricanes. Litman (2006) not only identified the planning problems in hurricane Katrina and Rita but also criticized the unplanned contraflow ordered and failure to use contraflow lanes. A significant improvement has been achieved in flow and time immediately without the time or cost required to plan, design, and construct additional lanes, Wolshon (2008).

Using graph and flow theory, the flip high flow edge heuristic solves a minimum cost flow problem in time expanded network for time horizon T , Kim and Shekhar (2005). It records the flow history and flips the direction of each edge in favor of the direction of larger flow. A suboptimal contraflow solution is obtained by exploiting the minimum cost solution on the modified graph without iteration. Moreover, the simulated annealing heuristic yields a local minimum with evacuation time as the objective function, and random flipping based perturbations. The computational difficulties of the evacuation networks are categorized as a function of overload degree, i.e., the ratio of the number of traveling units to the bottleneck capacity without contraflow, Kim et al. (2008). For overload degree < 1 , contraflow configuration is not necessary but for > 1 , an optimal contraflow reconfiguration can be achieved by microscopic simulation, search based techniques or optimization methods obtaining an optimal solution. For the moderate sized problem with medium overload degree, non-iterative heuristics based on a greedy approach are applicable for best approximate solutions in reasonable time. In this approach, the highly congested arcs obtained by using any evacuation route planner are flipped as contraflow reconfiguration and the network is reevaluated by using the route planner in the greedy approach. Notice that an evacuation route planner is used to generate flow history and evacuation time of a given network. However, if the overload degree is large, there exists a bottleneck relief heuristic that starts by finding a maximum flow with min-cut in the given network, and runs at most m times, improving the maximum flow in each iteration by flipping the arcs across the min-cut. With these heuristics, the evacuation time can be improved by 40 percent or more in experiments and case studies. However, the general problem of minimizing the evacuation time is \mathcal{NP} -hard.

Wang et al. (2010) introduced a multi-model evacuation problem in which contraflow model and repair of road segments are simultaneously solved. The result shows that on the damaged transportation network, the evacuation time has been reduced with more than 50% by constructing one new road and with 20% by re-planning the resource. Considering the priority ordering of evacuee's flow, a relaxed contraflow model including setup time for contraflow operation has been investigated in Wang et al. (2013). Moreover, by ignoring the background traffic and performing complete contraflow reconfiguration, Lv et al. (2010) gave the root choice opportunity for evacuees in contraflow network model. It improves the evacuation efficiency and decrease evacuation time from 30 to 60%.

In real practice, the Monticello, Minnesota region was evacuated by using the lane based contraflow and crossing elimination strategies simultaneously. The experiment was conducted with fix number of terminals and full lane reversal of transportation network, Xie and Turnquist (2011). In the same region, a bi-level model was used to solve the Monticello nuclear plant evacuation problem with contraflow at road segments and crossing elimination at intersection jointly, Xie et al. (2010). The bi-level includes the lane-based network optimization and simulation models. A case study was done for a super typhoon on an evacuation network using the integrated contraflow approach, Hua et al. (2014). A multi-modal integrated contraflow model is solved for uncertain arrivals of evacuees in the region with low mobility population that has little access to personal vehicles, unable to drive due to age, sickness, or any other reasons. The integrated contraflow strategy has been based on the fact that the transit-based evacuees and the auto-based evacuees will be evacuated to different destinations. The transit-based models are initiated with vehicle routine problem whereas the integrated strategy contains non-contraflow to shorten the strategy setup time, full-lane contraflow to minimize the evacuation network capacity and bus contraflow to realize the transit cycle operation. Similarly, recently, Zhao et al. (2016) presented bi-level model integrating lane based reversal design and routing with intersection crossing conflict elimination for an efficient evacuation by minimizing the total evacuation time to leave the evacuation zone.

4.2 Analytical Solutions

The recent interest in contraflow configurations is to develop its analytical solution techniques. It has not long history. The contraflow plans were used depending on past evacuation experiences and there was a lack of satisfactory analytical result. The theoretical developments of contraflow solutions can be found in (Arulselvan 2009, Rebennack et al. 2010,

Pyakurel 2016, Pyakurel et al. 2017a).

Arulselvan (2009) and Rebennack et al. (2010) introduce the first analytical models and solution algorithms to the contraflow configuration. They allowed the reversal of arcs at time zero, i.e., if we choose to reverse an arc, it remains reversed for all time period. The capacities of two-way arcs are added to form new capacity but the transit time is similar to the transit time without contraflow. The processing cost of contraflow configuration is negated. However, the general contraflow evacuation problem by arc reversals is \mathcal{NP} -hard (Kim et al. 2008, Rebennack et al. 2010). Rebennack et al. (2010) adopt the maximum dynamic flow models of Ford and Fulkerson (1962) in the contraflow approach and introduced the maximum dynamic contraflow problem (MDCFP) in discrete time. But the (MDCFP) with arc reversals at time zero remains \mathcal{NP} -hard in the strong sense even with two sources and one sink or vice versa in multi-terminal network. When we choose arcs, we have to know if an arc has been reserved or not in every time. This memory and decision of reversing the arc now or at a later time makes the problem \mathcal{NP} -complete. The proofs follow by reductions from the problems 3-SAT and PARTITION.

On two terminal network, Arulselvan (2009) and Rebennack et al. (2010) presented a polynomial time algorithm to solve the (MDCFP) problem. The given network is converted into transformed network with arc reversals at time zero and run the temporally repeated flow algorithm. The obtained flow is decomposed into paths and removable cycles. Arc $e' \in A$ is reversed, if and only if the flow along arc e is greater than b_e or if there is a nonnegative flow along arc $e \notin A$.

Theorem 4.1 (Rebennack et al. 2010) *The s-d (MDCFP) is solved in time $O(h_2(n, m) + h_3(n, m))$, where $h_2(n, m) = O(n \cdot m)$ and $h_3(n, m) = O(n^2 \cdot m^3 \cdot \log n)$ are the time required for the flow decomposition and the maximum static flow computation, respectively.*

By computing the (MDCF) at every point in time from the very beginning, authors in (Dhamala and Pyakurel 2013, Pyakurel and Dhamala 2014, Pyakurel and Dhamala 2015) introduced the earliest arrival contraflow model and solved the earliest arrival contraflow problem (EACFP) in discrete time setting. This discrete solution has been transformed into continuous time and presented a strongly polynomial algorithm in a particular network in Pyakurel and Dhamala (2016). To the problem in two terminal general network, a pseudo-polynomial algorithm has been presented in Pyakurel and Dhamala (2017b) (see also Khadka and Bhandari 2016).

Theorem 4.2 (Dhamala and Pyakurel 2013, Pyakurel and Dhamala 2014) *The (EACFP) on s-d series parallel network is solved in $O(nm + m \log m)$ time with arc reversals at time zero.*

With natural transformation, same solution can be computed in same complexity to the (EACFP), Pyakurel and Dhamala (2016, 2017b). Moreover, in s-d general network, the (EACFP) is solved in pseudo-polynomial time with arc reversals at any time in both discrete and continuous time, Pyakurel and Dhamala (2015, 2017b). A polynomial approximation algorithm has been presented to the (EACFP) in s-d network that computes an (EACF) from s to d within a factor of $(1 + \epsilon)$, for $\epsilon > 0$ in time T if the direction of the arcs can be reversed for both discrete and continuous time, Pyakurel and Dhamala (2016). The complexity is $O(m\epsilon^{-1}(m + n \log n) \log U)$ time.

Based on the given priority orderings of terminals, finding a feasible maximum dynamic flow in given time horizon T with arc reversals capability is the lex-maximum dynamic contraflow problem (LMDCFP). Authors in Pyakurel and Dhamala (2015, 2017b) present polynomial algorithms to solve the (LMDCFP) on multi terminal network in which more dangerous areas can be evacuated first than others. The first one gives the discrete solution and the latter one gives continuous solution to the (LMDCFP).

Theorem 4.3 (Pyakurel and Dhamala (2015, 2017b)) *The (LMDCFP) is solved in $O(\delta \times MCF(m, n))$ time, where $MCF(m, n)$ represents the time complexity $O(m \log n)(m + n \log n)$ of the minimum cost flow problem in the residual network.*

For given equal supplies and demands at sources and sinks, respectively, the problem of shifting all the supplies from sources to sinks satisfying the demands, by sending maximum flow at each time point from the beginning with arc reversals capability, is the earliest arrival transshipment contraflow problem (EATCFP) (Pyakurel and Dhamala 2017a, Pyakurel et al. 2017b). The (EATCFP) is solved in multi-source or multi-sink networks with polynomial time algorithms for both discrete and continuous times. The complexity depends upon the input plus output size of the problem. In multi-sink network, the (EATCFP) is solved with zero transit time on each arc. However, in the multi-terminal network with arbitrary transit times and zero transit time on each arc, approximation algorithms are designed in (Pyakurel and Dhamala 2017a, Pyakurel et al. 2017a) that solves the problem in pseudo-polynomial and polynomial time complexity, respectively.

To transship the given amount of flow from sources to sinks in minimum time, the quickest contraflow problem (QCFP) is introduced in Arulselvan (2009) and Rebennack et al. (2010). They solved the s-d (QCFP) in strongly polynomial time. Computing $s - d$ paths, they first obtained an upper bound on the quickest time in time and applied a binary search repeatedly to compute MDCF along the path until all supplies at the source are sent to the sink. They

also proved that the multi-terminal (QCFP) is harder than 3-SAT and PARTITION. The both s - d (MDCFP) and s - d (QCFP) are solved in continuous time using natural transformation and with same complexity as in discrete time, Pyakurel and Dhamala (2016, 2017b). Moreover, authors in Pyakurel and Dhamala (2017b) introduce the quickest transshipment contraflow problem and solve in polynomial time. Recently, the first minimum cost flow algorithm to solve the s - d (QCFP) is presented in Pyakurel et al. (2017b) and verified its efficiency with experimental results. The algorithm is strongly polynomial time and solvable in both discrete and continuous time. They presented a polynomial time approximation algorithm to solve s - d (QCFP) with load-dependent transit time. The experiment they performed approve significant improvement if implemented a contraflow reconfiguration evacuation plan.

Moreover, the (MDCFP) and (EACFP) is generalized on s - d lossy network in (Pyakurel et al 2014, Pyakurel 2016) with an additional gain factor on each arc. They presented pseudo-polynomial time algorithm to solve both problems with arc reversals at time zero. Their solution is in discrete time setting.

5. OTHER APPLICATIONS

Research on network design and management for emergency mitigation, preparedness, response and recovery plays vital roles in today's very complex disastrous community. Altay and Green (2006) outlined many diversified fields of applications in disaster management operations.

Given a number of demands at risk nodes, Hanawa et al. (2017) interestingly formulated a variant of car and pedestrian mixed movement multi terminal quickest evacuation as a mixed evacuation problem and illustrated a case study of Japanese earthquake. When the number of terminals is bounded by $K_o \log_2 n$ for some constant K_o , then they make use of polynomial time algorithm presented by Hoppe and Tardos (2008) to solve this problem polynomially. They proved that the integer version of mixed evacuation problem is \mathcal{NP} -hard by reduction from disjoint paths with different costs (Li et al. 1992).

Focusing on the logistics aspects of the problem, Anaya-Arenas et al. (2014) presented a systematic and up-to-date survey of contributions on relief distribution networks in response to the disasters management. They draw attention of the researchers to develop more complex but realistic models from theoretical as well as practical implications for both network design and vehicle routine problems to be implemented in disaster preparedness, response and relief distributions. Akter and Wamba (2017) focused on complex disaster network and highlighted the importance of availability for big data for a better solution. Jin et al. (2015) proposed a mixed integer programming model from the prospective of logistics supports in emergency with capacity restrictions. They carried out a case study with an objective of maximizing the survival of patients and minimizing the total cycle time and illustrated evidences that the former objective is more effective. Colson et al. (2007) give a list of applications including congestion management, network design and management of hazardous materials in case of emergency. Kim and Lee (2015) investigate network design, capacity planning and vehicle routing in reverse logistics that determines the locations, capacities, and the number and routes of vehicles with an objective of minimizing the fixed opening costs for locations and transportation costs of the vehicles.

For the best allocation decision of the relief supplies, Yang et al. (2016) presented the model of reserve network by estimating the associate cost, risk and utility. Reserve relief supplies before disaster has many benefits to the emergency relief system such as offering supplies in the first time, reducing the inadequate risk of relief supplies and rescuing more victims. Selecting four warehouses, the optimal allocation strategies has been obtained under the principles of equity and effectiveness which should be obeyed in the network design for humanitarian assistance. Kalinowski et al. (2015) presented two mixed integer programming formulations for an incremental network design problem and made many numerical tests to compare them with several heuristics. The objective is to maximize the cumulative flow over the entire planning horizon, where the network demands to add an arc in each time period of the planning phase. Li et al. (2011) emphasize on an efficient plan and allocation of emergency response facilities that deliver effectively and timely relief to people in risk. They revisited different optimization models and techniques including heuristics, simulation and exact algorithms.

Hamacher et al. (2013) investigate the facility location-allocation models and solution techniques that are also very supportive in supporting facilities maintaining again as much flow as possible without destroying too many source-sink paths. Four exact algorithms are proposed on the top of best maximum flow algorithms to assign a single facility location and the running times are compared. They show that the multi-facility location-allocation problem is polynomial time reducible to 3-SAT, and thus proving that even no polynomial time approximation algorithm solves it unless $\mathcal{P} = \mathcal{NP}$ holds. Farahani (2018) investigated capacitated evacuation-location problem where the main objective is to determine the most safe destinations in order to maximize the number of evacuees. They introduced a mixed integer programming model to determine a single or multiple destinations and presented exact algorithms and heuristics in order to solve the proposed problem. The algorithms are compared for various network structures and tested for their performances with respect to the optimality gaps. Ng et al. (2010) combine both cooperative and noncooperative behavior of evacuees during an evacuation, namely optimal shelter assignment with free choice of evacuees to reaching their shelters. Xiang and Zhuan (2016) introduced a queueing network model to optimize the available medical resources with respect to

the deteriorated victims' health condition caused by large-scale disasters. Afterwards, they presented both analytical solutions and numerical results for this model. They also proposed two resource allocation models with an objective of minimizing the total expected death rate and total waiting time.

Stefanello et al. (2017) introduced the tollbooth problem that assign tolls to streets and roads with the objective of inducing drivers to take alternative routes. They presented mathematical formulations for the problem that use piecewise linear functions to approximate congestion cost. They also applied a biased random-key genetic algorithm on a set of real-world instances and analyzed solutions by computing shortest paths according to two different weight functions. With experimental results, they claimed that their models and algorithms give very high quality solutions. As, users naturally take the minimum cost path, the toll setting can be used to better distribute the flow in the network and consequently reduce traffic congestion. However the tollbooth problem, in general, is NP-hard.

Efficient and effective blood supply chain network in earthquakes is designed and analyzed solving multi-objective optimization techniques in their mixed-integer linear programming formulation, Khalilpourazari and Khamseh (2017). They used it to solve a real world problem with a case study. Gutjahr and Nolz (2016) examine different optimization criteria as well as multi-criteria decision making approaches applied in the network of humanitarian crises. These multi-criteria approaches seek to cover more than one objective functions which are very challenging but quite important techniques for emergency network designs and management. Saadatseresht et al. (2009) investigated the evacuation planning problems from the perspective of conflicting multi objectives where building blocks of greater population should have higher priority and the overload of safe destinations has to be mentioned. They used the multi-objective evolutionary algorithms and the geographical information system for solving them and also presented a case study for evacuating 118 building blocks of total population 22,000 to the 7 safe destinations having capacity of 20,000 persons.

If some paths are blocked by incidence during evacuation process, Lim et al. (2016) investigated a model for the alternative routes to clear the traffic. They presented a preprocessing algorithm to update the evacuation network where real time traffic information, for example, incident time, location, severity of congestion, incident end time, are known in advanced. Then, a multi-commodity network flow optimization model has been used to develop alternative paths and corresponding flow rates. However, the model is a mixed integer nonlinear programming formulation. This model has been relaxed to a linear formulation that gives the better computational results experimentally. Akbari and Salman (2017) introduced a new arc routing problem to clearance the blocked roads after a disaster and reconnect the post-disaster road network in a synchronized way considering necessary waiting times. They presented a mixed-integer programming formulation to solve the problem in small size and relaxation based heuristic for the larger size. Both approaches make use of randomly generated largest instances. Moreover, the optimality gap of the relaxation has been bounded by the number of vehicles.

Notice that obtaining an optimum solution satisfying multiple necessary requirements for large-scale evacuation planning and management in advanced society is a quite challenging optimization problem. We recommend the papers listed in Torre (2017) for recent models and techniques of multiple criteria optimization approaches. Vogiatzis et al. (2012) applied a clustering of nodes in an evacuation network to divide the problem into smaller and easier subproblems and demonstrate numerical tests efficiently. They note that lane reversals are demanding techniques for better utilization of a transportation network in emergency. Vogiatzis et al. (2013) proposed integer programming formulations of evacuation plan for livestock caused by Fukushima Daiichi nuclear power plant accident because of Great East Japan earthquake followed by a 23-foot tsunami. They presented two efficient algorithms based on augmented Lagrange multiplier heuristic approach to find sub-optimal solutions, where the first network flow algorithm seeks to maximize the transfer of livestock whereas the second vehicle routine algorithm tries to minimize the number of livestock carrying vehicles. The authors target to minimize the evacuation time as well.

A book on "dynamics of disasters - key concepts, models, algorithms and insights" edited by Kotsireas et al. (2015) includes varieties of natural and man-made disasters. The attractive insights are made on the multi-disciplinary theories, tools, techniques and methodologies for the disasters from mitigation and preparedness to response and recovery focusing also on infrastructure protection, resiliency, humanitarian logistic, supply chains, risk mitigation and uncertain risks. They emphasized on the interest of inter-disciplinary research from many fields such as social, managerial, government, business, medical, engineering and applied sciences. Duhamel et al. (2016) presented a non-linear simpler resilience model for post-disaster operations that deals with decisions about locating facilities and distributing supplies in order to improve the survival rate. They solved the problem by a decomposition approach using non-linear solver. The papers edited in Kruhl (2018) address earthquakes with a special focus on Nepal earthquake in April 2015. Dhamala et al. (2018) explain the usefulness of the operational research models and solution techniques for emergency planning caused by natural or man made disasters.

Pyakurel et al. (2016) studied the transit-dependent network approach and illustrated a case study for an evacuation plan of a core part of Kathmandu Valley, the capital of Nepal. These models do have particular importance when evacuees are unable to reach their destinations with private vehicles. Bish (2011) investigated the bus-based evacuation problem exploiting the matching properties of vehicle routine problem. In his approach a general mixed integer programming problem is formulated for general multi-terminal split delivery evacuation planning model along with

two general heuristics. With restrictions on split delivery constraint, Goerigk et al. (2013) used several schemes for a branch and bound algorithm for two-terminal problem. To realize the uncertain arrival rate of evacuees, Goerigk et al. (2014) presented a heuristic solution to robust bus evacuation model with delayed scenario information. Dhamala and Adhikari (2018) extensively study various evacuation problems from prospective of transit vehicles focusing on the need based people who are unable to use their own cars. Presenting varieties of models and solution strategies many research directions are opened with significant number of literature survey.

6. CONCLUDING REMARKS

This article presents a systematic review of the main literature that were originally developed for solving network flow problems but are now implementable in handling different evacuation planning problems. Among the large class of problems and their model varieties, we selected the clusters of maximum flow, lexicographically maximum flow, quickest flow, earliest arrival flow, generalized flow, abstract flow, dynamic traffic assignment and minimum cost flow problems. The problem of contraflow reconfiguration is specially focused in most of these clusters. The researchers focused on location and network design problems, and transportation and routing problems applicable to disaster preparedness and response phases. Within this boundary, it is our trust that we were able to sufficiently gather and present the major features of network optimization algorithms for evacuation planning problems.

The mathematical models and degrees of their difficulties significantly differ with respect to the dependencies on network parameters (flow/time dependent attributes yield more accurate results reflecting closer to the real scenario but paying too high computational costs). The studied single commodity (say cars) models are relatively easier than the multi commodity (for example cars and buses as two commodities) models. In general, the latter problems and the problem with multi terminal networks are almost \mathcal{NP} -hard. Likewise, the models with additional costs imposed on nodes/arcs/paths are also almost \mathcal{NP} -hard. All such problems requiring polynomial time approximation schemes for acceptable solutions in reasonable time. To deal the large scale problems where attributes are time/flow dependent, special time expanded networks (fan/bow network and condensed network) have been introduced in order to achieve approximate solutions. Certain quickest flow and earliest arrival flow problems are solved approximately with these techniques, but there is a great potential that other classes of problems can also be modeled for acceptable solutions within certain range of precision.

We also studied the abstract flow models where the same quality solution has to be obtained, provably reducing the merging and crossing conflicts at intersections. In this approach, the contraflow reconfiguration is applied to revert the whole path instead of the arcs piecewise in classical network flow models. The contraflow configuration is a technique which significantly increases the flow value and decreases the evacuation time by increasing outbound capacities towards the safe destinations to reduce traffic congestion in emergency period or rush hour traffic. Its heuristic solutions have been extensively applied in traffic management from the past. We give a recent status of the analytical solutions and also sketch the heuristic results. We also studied the dynamic flow model where flow conservation is not valid due to a possible flow loss on the way (called lossy network flows). These problems are also \mathcal{NP} -hard for which a number of approximate solution techniques have been proposed.

On the way of our research, we listed some major applications, for example, flow models for transit dependent population, issues on emergency logistic supports, multi-objective scenarios and facility location-allocation problems. We surveyed the macroscopic models instead of microscopic one but a coupling of their approaches do have a great benefit as a solution of one can be fed to the other for reducing the solution gaps. The multi-objective problems are more life reflecting as it is a reality and the conflicting interests do occur in real-world. The system optimization approaches (which we studied here) should also be incorporated with user optimum approaches for real implementation of evacuation plans. The \mathcal{NP} -hard transit dependent models, usually the mixed integer programming problems, have rarely been considered through most of the evacuation regions rely on transit vehicles. In fact, the car and transit based integrated mixed models would yield real scenarios but naturally yielding a higher computational complexity. We illustrated the location-allocation flow models and lane reversal strategies that are very relevant in saving the unused arcs for the placement of facilities with least lost on the flow value and least increase on evacuation times. The results will equally be important for logistic supports in emergencies. However, these integrated flow-location problems are harder than a single one itself.

A distribution in different classes of literature flow is illustrated in Table 1. This representative list covers most of the current and major important classical references in evacuation planning problems. We have sketched the main ideas of their results and analyzed the complexity and applicability they have presented therein. The insights into presented models and solution strategies explore different challenges for the operational research community in addressing still more complex models and possible solution approaches that would address more realistic problems. The research questions are of both theoretical and practical interests. Incorporating the evacuees' individual or group behavior in more accurate models, clearly understanding the traffic dynamics, estimating the unpredictable and fuzzy environment, and building up cooperative multi-objective or non-cooperative multi-level and integrated models is a big challenge in

Evacuation model	Reference citations
Maximum flow	[7] [44] [46] [48] [50] [65] [66] [115] [96] [127]
Lex-max flow	[44] [50] [69] [74] [75] [76] [113] [116]
Quickest flow	[16] [20] [25] [26] [27] [43] [44] [63] [64] [115] [75] [74] [76] [78] [80] [93] [97] [98] [106] [42] [112] [123] [125] [127] [147] [150] [155] [157]
Earliest arrival	[12] [13] [14] [15] [43] [42] [41] [44] [49] [61] [115] [75] [80] [113] [116] [146] [163]
Generalized flow	[45] [53] [55] [119] [120] [121]
Abstract flow	[37] [48] [73] [84] [85] [109] [110] [111]
Traffic assignment	[24] [22] [23] [28] [79] [81] [87] [114] [144]
Minimum cost	[5] [8] [6] [21] [25] [26] [54] [67] [68] [69] [87] [94] [95] [99] [122] [127] [129] [130]
Contraflow	[11] [36] [70] [77] [91] [92] [107] [108] [134] [135] [136] [137] [138] [139] [140] [141] [142] [145] [161] [162] [164] [166] [167] [170]
Reviews	[2] [3] [4] [10] [31] [34] [33] [68] [100] [117] [125] [131] [169]
Applications	[1] [3] [17] [38] [51] [52] [60] [66] [71] [82] [83] [89] [90] [100] [104] [105] [143] [149] [154] [159] [160] [165] [168]

Table 1: A distribution of literature flow in different classes of evacuation planning problems. The numbers refer to the item in the list of references.

this field. We aspect that this research opens multi directions for an advancement of the models and investigations of new solution strategies.

ACKNOWLEDGEMENTS

This research has been carried out during the research stays of the first author under the *AnH Research Group Linkage Program* and the second author as a research fellow with George Foster Fellowship for Post-doctoral Researchers at TU Bergakademie Freiberg. The authors acknowledge the supports of Alexander von Humboldt Foundation.

REFERENCES

1. Akbari, F.V. and Salman, S. (2017). Multi-vehicle synchronized arc routing problem to restore post-disaster network connectivity. *European Journal of Operational Research*, **257**, 625-640.
2. Akter, S. and Wamba, S.F. (2017). Big data and disaster management: a systematic review and agenda for future research. *Annals of Operation Research*, 1-21, doi:10.1007/s10479-017-2584-2.
3. Altay, N. and Green III, W.G. (2006). OR/MS research in disaster operations management. *European Journal of Operational Research*, **175**, 475–493.
4. Anaya-Arenas, A.M., Renaud, J. and Ruiz, A. (2014). Relief distribution networks: a systematic review. *Annals of Operation Research*, **223**, 53-79.
5. Anderson, E.J. (1989). Extreme-points for continuous network problems with arc delays, *Journal Information and Optimization sciences*, 10: 45-52.
6. Anderson, E.J., Nash, P and Perold, A.F. (1983). Some properties of a class of continuous linear programs, *SIAM Journal Control and Optimization*, 21(5): 758-765.
7. Anderson, E.J., Nash, P. and Philpott, A.B. (1982). A class of continuous network flow problems, *Mathematics of Operations Research*, 7: 501-514.
8. Anderson, E.J. and Philpott, A.B. (1994). Optimization of flows in networks over time, *Probability, Statistics and Optimization*, F.P. Kelly, ed. J. Wiley and Sons, 369-382.
9. Anderson, E.J. and Philpott, A.B. (1989). A continuous time network simplex algorithm, *Networks*, 19: 395-425.
10. Aronson, E.J. (1989). A survey of dynamic network flows. *Annals of Operations Research* **20**, 1-66.
11. Arulselman, A. (2009). *Network model for disaster management*. PhD Thesis, University of Florida, USA.
12. Baumann, N. (2007). *Evacuation by earliest arrival flows*, PhD thesis, Department of Mathematics, University of Dortmund, Germany.

13. Baumann, N. and Köhler, E. (2007). Approximating earliest arrival flows with flow-dependent transit times, *Discrete Applied Mathematics*, 155, 161-171.
14. Baumann, N. and Skutella, M. (2009). Earliest arrival flows with multiple sources. *Mathematics of Operations Research*, 34, 499-512.
15. Baumann, N. and Skutella, M. (2006). Solving evacuation problems efficiently, earliest arrival flows with multiple sources, *Foundations of Computer Science, FOCS '06. 47th Annual IEEE Symposium*, 399-410.
16. Berlin, G.N. (1978). The Use of Directed Routes for Assigning Escape Potential, *Fire Technology*, 14(2), 126–135.
17. Bish, D.R. (2011). Planning for a bus-based evacuation. *OR spectrum*, 33(3):629 - 654.
18. Bozhenyuk, A.V., Gerasimenko, E.M., Kacprzyk, J. and Rozenberg, I.N. (2017). *Flows in Networks Under Fuzzy Conditions*, Springer, (2017).
19. Bretschneider, S. (2013). *Mathematical Models for Evacuation Planning in Urban Areas*, Springer.
20. Burkard, R.E., Dlaska, K. and Klinz, B. (1993). The quickest flow problem. *ZOR-Methods and Models of Operations Research*, 37:31–58.
21. Cai, X., Sha, D. and Wong, C.K. (2001). Time-varying minimum cost flow problems, *European Journal of Operations Research*, 131: 352-374.
22. Carey, M. (1987). Optimal time-varying flows on congested networks. *Operations Research*, 35(1), 58-69.
23. Carey, M. (1992). Nonconvexity of dynamic traffic assignment problem. *Transportation Research B*, 26(2), 127-133.
24. Carey, M. and Subrahmanian, E. (2000). An approach to modeling time-varying flows on congested networks. *Transportation Research Part B*, 34, 157-183.
25. Chalmet, L.G., Francis, R.L. and Saunders, P.B. (1982). Network models for building evacuation, *Fire Technology*, 18, 90-113.
26. Chalmet, L.G., Francis, R.L. and Saunders, P.B. (1982). Network models for building evacuation, *Management Science* 28, 86-105.
27. Chen, Y.L. and Chin Y.H. (1990). The quickest path problem, *Computers and Operations Research*, 17, 153-161.
28. Chen, H.K. and Hsueh, C.F. (1998). A model and an algorithm for the user-optimal route choice problem, *Transportation research B*, 32(3): 219-234.
29. Chen, L. and Miller-Hooks, E. (2008). The building evacuation problem with shared information. *Naval Research Logistics*, 55, 363–376.
30. Colson, B., Marcotte, P. and Savard, G. (2007). An overview of bilevel optimization, *Annals of Operations Research*, 153:235-256.
31. Cova, T. and Johnson, J.P. (2003). A network flow model for lane-based evacuation routing. *Transportation Research Part A: Policy and Practice*, 37, 579–604.
32. Daganzo, F. (1994). The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B* 28(4), 269-287.
33. Dhamala, T.N. (2015). A survey on models and algorithms for discrete evacuation planning network problems. *Journal of Industrial and Management Optimization*, 11, 265-289.
34. Dhamala, T.N. and Adhikari, I. M. (2018). On Evacuation Planning Optimization Problems from Transit-based Perspective, *International Journal of Operations Research*, 15(1), 29–47.
35. Dhamala, T.N., Adhikari, I.M., Nath, H.M. and Pyakurel, U. (2018). Meaningfulness of OR models and solution strategies in emergency planning, In [101], 175-194.
36. Dhamala, T.N. and Pyakurel, U. (2013). Earliest arrival contraflow problem on series-parallel graphs. *International Journal of Operations Research*, 10, 1-13.

37. Dhungana, R.C., Pyakurel, U. and Dhamala, T. N. (2018). Abstract Contraflow Models and Solution Procedures for Evacuation Planning, *Journal of Mathematics Research*, 10(4), 89-100, doi:10.5539/jmr.v10n4p89.
38. Duhamel, C., Santos, A.C., Brasil, D., Châtelet, E. and Birregah, B. (2016). Connecting a population dynamic model with a multi-period location-allocation problem for post-disaster relief operations. *Annals of Operation Research*, 247, 693-713.
39. Ferrari, P. (1990). Capacity constraints in urban transport networks, *Transportation Res E*, 31(4): 291-301.
40. Farahani, M.M., Chaharsooghi, S.K., Woensel, T. V. and Veelenturf, L.P. (2018). Capacitated network-flow approach to the evacuation-location problem, *Computers and Industrial Engineering*, 115: 407-427.
41. Fleischer, L.K. (2001). Faster algorithms for quickest transshipment problem, *SIAM Journal on Optimization* 12, 18-35.
42. Fleischer, L.K. and Skutella, M. (2002). Quickest multicommodity flow problem, *Integer Programming and Combinatorial Optimization, Lecture Notes in Computer Science*, Springer, 2337, 36-53.
43. Fleischer, L. and Skutella, M. (2007). Quickest flows over time, *SIAM Journal of Computing* 36, 1600–1630.
44. Fleischer, L.K. and Tardos, E. (1998). Efficient continuous-time dynamic network flow algorithms. *Operations Research Letters*, 23, 71-80.
45. Fleischer, L.K. and Wayne, K.D. (2002). Fast and simple approximation schemes for generalized flow. *Mathematical programming*, 91:215 -238.
46. Ford, L.R. and Fulkerson, D.R. (1962). *Flows in Networks*, Princeton University Press, Princeton, New Jersey.
47. Ford, L.R. and Fulkerson, D.R. (1958). Constructing maximal dynamic flows from static flows, *Operations Research* 6, 419-433.
48. Ford, L.R. and Fulkerson, D.R. (1956). Maximal flow through a network. *Canadian Mathematical Society*, 8: 399-404.
49. Gale, D. (1959). Transient flows in networks. *Michigan Mathematical Journal*, 59-63.
50. Gallo, G.M., Grigoriadis, M.D. and Tarjan, R.E. (1989). A fast parametric maximum flow algorithm and applications, *SIAM Journal of Computing* 18, 30-55.
51. Goerigk, M. and Grün, B. (2014). A robust bus evacuation model with delayed scenario information. *OR Spectrum*, 36(4):923-948.
52. Goerigk, M., Grün, B. and Heßler, P. (2013). Branch and bound algorithms for the bus evacuation problem. *Computers and Operations Research*, 40(12):3010-3020.
53. Goldberg, A.V., Plotkin, S.A. and Tardos, E. (1991). Combinatorial algorithms for the generalized circulation problems. *Mathematics of Operations Research*, 16:351–381.
54. Goldberg, A.V. and Tarjan, R.E. (1990). Finding minimum-cost circulations by successive approximation, *Mathematics of Operations Research*, 15(3), 430-466.
55. Gondran, M. and Minoux, M. (1984). *Graphs and Algorithms*. Wiley.
56. Grötschel, M., Lovász, L. and Schrijver, A. (1988). *Geometric algorithms and combinatorial optimization*, Springer, doi: 10.1007/978-3-642-97881-4.
57. Groß, M. (2014). *Approximation algorithms for complex network flow over time problems*. PhD Thesis, Technical University, Berlin, Germany.
58. Groß, M., Kappmeier, J-P. W., Schmidt, D.R. and Schmidt, M. (2012). Approximating earliest arrival flows in arbitrary networks. *L. Epstein and P. Ferragina (Eds.): ESA 2012, LNCS, 7501*, 551-562.
59. Groß, M. and Skutella, M. (2012). Generalized maximum flows over time. *Proceedings of the 9th Workshop on Approximation and Online Algorithms, Lecture Notes in Computer Science*, 7164:247-260.
60. Gutjahr, W.J. and Nolz, P.C. (2016). Multicriteria optimization in humanitarian aid. *European Journal of Operational Research*, 252, 351-366.

61. Hajek, B. and Ogier, R.G. (1984). Optimal dynamic routing in communication networks with continuous traffic, *Networks* 14, 457-487.
62. Hall, A., Hippler, S. and Skutella, M. (2007a). Multicommodity flows over time: Efficient algorithms and complexity, *Theoretical Computer Science* 379:387-404.
63. Hall, A., Langkau, K. and Skutella, M. (2007b). An FPTAS for quickest multicommodity flows with inflow-dependent transit times, *Algorithmica*, 47: 299-321.
64. Hall, A., Langkau, K. and Skutella, M. (2003). An FPTAS for quickest multicommodity flows with inflow-dependent transit times, in: S. Arora, K. Jansen, J.D. Rolim, A. Sahai (Eds.), *Approximation, Randomization, and Combinatorial Optimization (APPROX'03), Lecture Notes in Computer Science*, vol. 2764, Springer, 71-82.
65. Halpern, J. (1979). A generalized dynamic flows problem, *Networks*, 39:133-167.
66. Hamacher, H.W., Heller, S. and Rupp, B. (2013). Flow location (FlowLoc) problems: dynamic network flows and location models for evacuation planning. *Annals of Operations Research*, **207**, 161–180.
67. Hamacher, H.W., Ruzika, S. and Tjandra, S.A. (2006). Algorithms for time-dependent bicriteria shortest path problems, *Discrete Optimization* 3, 238-254.
68. Hamacher, H.W. and S.A. Tjandra, S.A. (2002). Mathematical modeling of evacuation problems: A state of the art. In: *M. Schreckenberger and S.D. Sharma (Eds.), Pedestrian and Evacuation Dynamics*, Springer, 227-266.
69. Hamacher, H.W. and Tufefcki, S. (1987). On the use of lexicographic min-cost flows in evacuation modeling, *Naval Research Logistics* 34, 487-503.
70. Hamza-Lup, G., Hua, K.A., Le, M. and Peng, R. (2004). Enhancing intelligent transportation systems to improve and support homeland security. *Proceedings of the Seventh IEEE International Conference, Intelligent Transportation Systems (ITSC)*, 250-255.
71. Hanawa, Y., Higashikawa, Y., Kamiyama, N., Katoh, N. and Takizawa, A. (2017). The mixed evacuation problem, *Journal of Combinatorial Optimization*, <https://doi.org/10.1007/s10878-017-0237-7>.
72. Harker, P.T. and Pang, J.S. (1990). Finite-dimensional variational inequality and nonlinear complementary problems: A survey of theory algorithms and applications, *Mathematical Programming* 48: 161-220.
73. Hoffman, A. (1974). A generalization of max flow-min cut. *Mathematical Programming*, 6(1), 352 - 359.
74. Hoppe, B. (1995). *Efficient dynamic network flow algorithms*. PhD thesis, Cornell University.
75. Hoppe, B. and Tardos, E. (1994). *Polynomial time algorithms for some evacuation problems*, in: Fifth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA'94), 1994, 433 - 441.
76. Hoppe, B. and Tardos, E. (2000). The quickest transshipment problem. *Mathematics of Operations Research*, **25**, 36-62.
77. Hua, J., Ren, G., Cheng, Y. and Ran, B. (2014). An integrated contraflow strategy for multimodal evacuation, *Hindawi Publishing Corporation Mathematical Problems in Engineering* (Volume 2014).
78. Hung, Y.C. and Chen, G.H. (1993). On the quickest path problem, *Inform. Process. Lett.*, 46, 125-128, (1993).
79. Janson, B.N. (1991). Dynamic traffic assignment for urban road network, *Transportation Research B: Methodological*, 25: 143-161.
80. Jarvis, J. and Ratliff, H. (1982). Some equivalent objectives for dynamic network flow problems,, *Management Science*, 28, 106-108.
81. Jaykrishnan, R., Tsai, W.K. and Chen, A. (1995). A dynamic traffic assignment model with traffic-flow relationship, *Transportation Research C*, 3(1): 51-72.
82. Jin, S., Jeong, S., Kim, J. and Kim, K. (2015). A logistics model for the transport of disaster victims with various injuries and survival probabilities. *Annals of Operation Research*, 230, 17-33.
83. Kalinowski, T., Matsypura, D. and Savelsbergh, M.W. (2015). Incremental network design with maximum flows. *European Journal of Operational Research*, 242, 51-62.

84. Kappmeier, J.-P.W. (2015). *Generalizations of flows over time with application in evacuation optimization*. PhD Thesis, Technical University, Berlin, Germany.
85. Kappmeier, J.-P.W., Matuschke, J. and Peis, B. (2014). Abstract flows over time: A first step towards solving dynamic packing problems. *Theoretical Computer Sciences*, 544, 74-83.
86. Karger, D.R. and Stein, C. (1996). A new approach to the minimum cut problem. *Journal of the ACM*, 43(4), 601-640.
87. Kaufman, D.E., Nonis, J. and Smith, R.L. (1998). A mixed integer linear programming model for dynamic route guidance. *Transportation Research B* 32(6): 431-440.
88. Khadka, S.R. and Bhandari, P.P. (2017). Dynamic Network Contraflow Evacuation Planning Problem with Continuous Time Approach, *International Journal of Operations Research*, 14 (1) 27-34.
89. Khalilpourazari, S. and Khamseh, A.A. (2017). Bi-objective emergency blood supply chain network design in earthquake considering earthquake magnitude: a comprehensive study with real world application. *Annals of Operation Research*, 1-39, doi:10.1007/s10479-017-2588-y.
90. Kim, J.S. and Lee, D.H. (2015). An integrated approach for collection network design, capacity planning and vehicle routing in reverse logistics, *Journal of the Operational Research Society*, 66(1): 76-85.
91. Kim, S., Shekhar, S. and Min, M. (2008). Contraflow transportation network reconfiguration for evacuation route planning. *IEEE Transactions on Knowledge and Data Engineering*, 20, 1-15.
92. Kim, S. and Shekhar, S. (2005). Contraflow network reconfiguration for evacuation planning: a summary of results. *Proceedings of 13th ACM Symposium on Advances in Geographic Information Systems (GIS 05)*, 250–259.
93. Kisko, T.M. and Francis, R.L. (1985). EVACNET+: A computer program to determine optimal building evacuation plans, *Fire Safety Journal* 9, 211-220.
94. Klinz, B. and Woeginger, G.J. (2004). Minimum cost dynamic flows: the series-parallel case, *Networks*, 43:153-162.
95. Klinz, B. and Woeginger, G.J. (1995). Minimum cost dynamic flows, the series-parallel case, in *Proceedings of the 4th Conference on Integer Programming and Combinatorial Optimization*, 329-343.
96. Koch, R., Nasrabadi, E. and Skutella, M. (2011). Continuous and discrete flows over time - a general model based on measure theory, *Mathematical Methods of Operations Research*, 73: 301-337.
97. Köhler, E., Langkau, K., and Skutella, M. (2002). Time expanded graphs for flow depended transit times. R. *Moehring and R. Raman (Eds.): ESA 2002, LNCS 2461*, Springer-Verlag, 599-611.
98. Köhler, E. and Skutella, M. (2005). Flows over time with load-dependent transit times. *SIAM Journal of Optimization*, 15(4): 1185–1202.
99. Kostreva, M.M. and Wiecek, M.M. (1993). Time dependency in multiple objective dynamic programming, *Journal of mathematical analysis and application*, 173(1): 289-307.
100. Kotsireas, I.S., Nagurney, A. and Pardalos, P.M. (Eds.) (2015). *Dynamics of disasters - key concepts, models, algorithms, and insights*. Springer Proceedings in Mathematics and Statistics.
101. Kruhl, J.H., Adhikari, R. and Dorka, U.E. (Eds.) (2018), *Living Under the Threat of Earthquakes: Short and Long-term Management of Earthquake Risks and Damage Prevention in Nepal*, Springer Natural Hazards.
102. Krumke, S.O. and Zeck, C. (2013). Generalized max flow in series-parallel graphs. *Discrete Optimization*, 10:155-162.
103. Li, C.L., McCormick, S.T., and Simchi-Levi, D. (1992). Finding disjoint paths with different path-costs: complexity and algorithms, *Networks* 22(7):653-667.
104. Li, X., Zhao, Z., Zhu, X. and Wyatt, T. (2011). Covering models and optimization techniques for emergency response facility location and planning: a review, *Mathematical Methods of Operations Research*, 74(3), 281-310.
105. Lim, G.J., Baharnemati, M.R. and Kim, S.J. (2016). An optimization approach for real time evacuation reroute planning. *Annals of Operation Research*, 238, 375-388.

106. Lin, M. and Jaillet, P. (2015). On the quickest flow problem in dynamic networks— a parametric min-cost flow approach. *Proceedings of the Twenty-Sixth Annual ACM-SIAM Symposium on Discrete Algorithms*, 1343-1356.
107. Litman, T. (2006). Lessons from Katrina and Rita: what major disasters can teach transportation planners. *Journal of Transportation Engineering*, **132**(1), 11–18.
108. Lv, N., Yan, X., Xu, K. and Wu, C. (2010). Bi-level programming based contraflow optimization for evacuation events. *Kybernetes*, **39**(8), 1227-1234.
109. Martens, M. and McCormick, S.T. (2008). A polynomial algorithm for weighted abstract flow. Proceedings of the 13th Conference on Integer Programming and Combinatorial Optimization. Volume 5035 of Lecture Notes in Computer Science, Springer, Berlin, Heidelberg, 97-111, 2008.
110. Matuschke, J. (2014). *Network flows and network design in theory and practice*. PhD Thesis, Technical University, Berlin, Germany.
111. McCormick, S.T. (1996). A polynomial algorithm for abstract maximum flow. Proceedings of the 7th Annual ACM-SIAM Symposium on Discrete Algorithms, 490-497.
112. Megiddo, N. (1979). Combinatorial optimization with rational objective functions,, *Mathematics of Operations Research*, 4, 414-424.
113. Megiddo, N. (1974). Optimal flows in networks with multiple sources and sinks. *Mathematical Programming*, 7(1), 97–107.
114. Merchant, V. and Nemhauser, G. (1978). A model and an algorithm for the dynamic traffic assignment problems. *Transportation Science*, 12 (3), 183-199.
115. Miller-Hooks, E. and Patterson, S.S. (2004). On solving quickest time problems in time-dependent dynamic flows, *Journal of Mathematical Modelling and Algorithms*, 3: 39-71.
116. Minioka, E. (1973). Maximal, lexicographic, and dynamic network flows. *Operations Research*, 21, 517-527.
117. Moriarty, K.D, Ni, D. and Collura, J. (2007). Modeling traffic flow under emergency evacuation situations: Current practice and future directions. *86th Transportation Research Board Annual Meeting*, Washington, D.C. [http : //works.bepress.com/daiheng_ni/24](http://works.bepress.com/daiheng_ni/24).
118. Ng, M.W., Park, J. and Waller, S.T. (2010). A hybrid bilevel model for the optimal shelter assignment in emergency evacuations, *Computer-Aided Civil Infrastructure Engineering*, 25: 547-556.
119. Onaga, K. (1966). Dynamic programming of optimum flows in lossy communication nets. *IEEE Transactions on Circuit Theory*, 13:282-287.
120. Onaga, K. (1967). Optimal flows in general communication networks. *Journal of the Franklin Institute*, 283:308 - 327.
121. Orlin, J.B. (2013). Max flow in $O((nm)$ time, or better. *Proceedings of the 45th ACM Symposium on the Theory of Computing (STOC 13)*, 765–774.
122. Orlin, J.B. (1984). Minimum convex cost dynamic network flows. *Mathematics of Operations Research*, 9:190-207.
123. Orlin, J.B. (1988). A faster strongly polynomial minimum cost flow algorithm, Proc.20-th Annual Syrup. Theory of Computing, 377-387.
124. Papageorgiou, M. (1990). Dynamic modeling, assignment, and route guidance in traffic networks, *Transportation Research B*, 24(6): 471-495.
125. Pascoal, M.M.B, Captivo, M.E.V. and Climaco, J.C.N. (2006). A comprehensive survey on the quickest path problem. *Annals of Operations Research*, 147(1), 5-21.
126. Peeta, S. and Ziliaskopoulos, A. (2001). Foundations of dynamic traffic assignment: Past, present and the future. *Networks and Spatial Economics*, 1, 233-265.
127. Philpott, A.B. (1990). Continuous time flows in networks, *Mathematics of Operations Research*, 15(4): 640-661.

- 132 **Dhamala, Pyakurel, Dempe** : *A Critical Survey on the Network Optimization Algorithms for Evacuation Planning Problems* IJOR Vol. 15, No. 3, 101-133 (2018)
128. Philpott, A.B. and Craddock, M. (1995). An adaptive discretization algorithm for a class of continuous network programs, *Networks*, 26: 1-11.
129. Philpott, A.B. and Mees, A.I. (1993). Finite-time algorithm for shortest path problems with time-varying costs, *Applied mathematics Letters*, 6(2): 91-94.
130. Philpott, A.B. and Mees, A.I. (1992), Continuous-time shortest path problems with stopping and starting costs, *Applied mathematics Letters*, 5: 63-66.
131. Powell, W.B., Jaillet, P. and Odoni, A. (1995). Stochastic and dynamic networks and routing. In M. O. Ball, T. L. Magnanti, C. L. Monma, and G. L. Nemhauser, editors, *Network Routing*, volume 8 of *Handbooks in Operations Research and Management Science*, chapter 3, pages 141-295. North-Holland, Amsterdam, The Netherlands.
132. Pullan, M.C. (1997). A study of general dynamic network problems with arc time-delays, *SIAM Journal Optimization*, 7(4): 889-912.
133. Pullan, M.C. (1993). An algorithm for a class of continuous linear programs. *SIAM Journal on Control and Optimization*, 31(6): 1558-1577.
134. Pyakurel, U. (2016). *Evacuation planning problem with contraflow approach*. PhD Thesis, IOST, Tribhuvan University, Nepal.
135. Pyakurel, U. and Dhamala, T.N. (2014). Earliest arrival contraflow model for evacuation planning. *Neural, Parallel, and Scientific Computations [CNLS-2013]*, 22, 287-294.
136. Pyakurel, U. and Dhamala, T.N. (2015). Models and algorithms on contraflow evacuation planning network problems. *International Journal of Operations Research*, 12, 36-46.
137. Pyakurel, U. and Dhamala, T.N. (2017a). Evacuation planning by earliest arrival contraflow, *Journal of Industrial and Management Optimization* 13, 487-503.
138. Pyakurel, U. and Dhamala, T.N. (2016). Continuous time dynamic contraflow models and algorithms. *Advances in Operations Research - Hindawi*; Article ID 368587, 1-7.
139. Pyakurel, U. and Dhamala, T.N. (2017b). Continuous dynamic contraflow approach for evacuation planning *Annals of Operation Research*, 253, 573-598.
140. Pyakurel, U., Dhamala, T.N. and Dempe, S. (2017a). Efficient continuous contraflow algorithms for evacuation planning problems. *Annals of Operations Research (ANOR)*, 254, 335-364.
141. Pyakurel, U., Nath, H.N. and Dhamala, T.N. (2017b). Efficient contraflow algorithms for quickest evacuation planning. *Science China Mathematics*, DOI 10.1007/s11425-017-9264-3.
142. Pyakurel, U., Hamacher, H.W. and Dhamala, T.N. (2014). Generalized maximum dynamic contraflow on lossy network. *International Journal of Operations Research Nepal*, 3, 27-44.
143. Pyakurel, U., Goerigk, M., Dhamala, T.N. and Hamacher, H.W. (2016). Transit dependent evacuation planning for Kathmandu valley: A case study. *International Journal of Operations Research Nepal*, 5: 49-73.
144. Ran, B. and Boyse, D. (1996). *Modeling dynamic transportation network*, Springer Heidelberg.
145. Rebennack, S., Arulselvan, S. A., Elefteriadou, L. and Pardalos, P.M. (2010). Complexity analysis for maximum flow problems with arc reversals. *Journal of Combinatorial Optimization*, 19, 200-216.
146. Richardson, D. and Tardos, E. (2002b). Cited as personal communication in [43].
147. Rosen, J.B., Sun, S.Z. and Xue, G.L. (1991). Algorithms for the quickest path problems and the enumeration of quickest paths, *Computers and Operations Research*, 18, 579-584.
148. Ruzika, S., Sperber, H. and Steiner, M. (2011). Earliest arrival flows on series- parallel graphs. *Networks*, 10, 169-173.
149. Saadatseresht, M., Mansourian, A. and Taleai, M. (2009). Evacuation planning using multiobjective evolutionary optimization approach, *European Journal of Operational Research* 198: 305-314.

150. Saho, M. and Shigeno, M. (2017). Cancel-and-tighten algorithm for quickest flow problems. *Network*, 69(2), 179–188.
151. Schmidt, M. and Skutella, M. (2010). Earliest arrival flows in networks with multiple sinks. *Electronics Notes in Discrete Mathematics*, 36, 607-614.
152. Smith, J.M.G. and Cruz, E.R.B. (2014). $M/G/c/c$ state dependent travel time models and properties, *Physica A*, 395, 560-579.
153. Schroeder, J., Guedes, A. and Duarte, E.P. (2004). *Computing the minimum cut and maximum flow of undirected graphs*. Technical Report Federal University of Paraná.
154. Stefanello, F., Buriol, L.S., Hirsch, M.J., Pardalos, P.M., Querido, T., Resende, M.G.C. and Ritt, M. (2017). On the minimization of traffic congestion in road networks with tolls. *Annals of Operation Research*, 249, 119-139.
155. Tjandra, S.A. (2003). Dynamic network optimization with applications to evacuation planning, Ph.D. thesis, University of Kaiserslautern.
156. Torre, D. L. (2017). Multiple criteria optimization and goal programming in science, engineering, and social sciences, *Annals of Operations Research*, 251:1-5.
157. Vaidya, P.M. (1989). A new algorithm for minimizing convex functions over convex sets, *Proc. 30th Ann. IEEE Symposium Foundations Computer Science*.
158. Végh, L.A. (2018). A Strongly Polynomial Algorithm for Generalized Flow Maximization, *Mathematics of Operations Research*, 42(1): 179-211.
159. Vogiatzis, C., Yoshida, R., Aviles-Spadoni, I., Imamoto, S. and Pardalos, P.M. (2013). Livestock evacuation planning for natural and man-made emergencies. *International Journal of Mass Emergencies and Disasters*, 31(1), 25-37.
160. Vogiatzis, C., Walteros, J.L. and Pardalos, P.M. (2012). Evacuation through clustering techniques. In: *Models, Algorithms, and Technologies for Network Analysis*, Springer, Proceedings in Mathematics and Statistics, 32, 185-198.
161. Wang, J.W., Ip, W.H. and Zhang, W.J. (2010). An integrated road construction and resource planning approach to the evacuation of victims from single source to multiple destinations. *IEEE Transactions on Intelligent Transportation Systems*, 11(2), 277–289.
162. Wang, J.W., Wang, H.F., Zhang, W.J., Ip, W.H. and Furuta, K. (2013). Evacuation planning based on the contraflow technique with consideration of evacuation priorities and traffic setup time. *IEEE Transactions on Intelligent Transportation Systems*, 14(1), 480–485.
163. Wilkinson, W.L. (1971). An algorithm for universal maximal dynamic flows in a network. *Operations Research*, 19:1602-1612.
164. Wolshon, B. (2008). Contraflow for evacuation traffic management. *Encyclopedia of GIS*, 165–170, doi:10.1007/978-0-387-35973-1_210.
165. Xiang, Y. and Zhuang, J. (2016). A medical resource allocation model for serving emergency victims with deteriorating health conditions, *Annals of Operations Research* 236: 177-196.
166. Xie, C. and Turnquist, M.A. (2011). Lane-based evacuation network optimization: An integrated Lagrangian relaxation and tabu search approach. *Transportation Research Part C*, 19(1), 40-63.
167. Xie, C., Lin, D.Y. and Waller, S.T. (2010). A dynamic evacuation network optimization problem with lane reversal and crossing elimination strategies. *Transportation Research Part E*, 46(3), 295-316.
168. Yang, F., Yuan, Q., Du, S. and Liang, L. (2016). Reserving relief supplies for earthquake: a multi-attribute decision making of China Red Cross. *Annals of Operation Research*, 247, 759-785.
169. Yusoff, M., Ariffin, J. and Mohamed, A. (2008). Optimization approaches for macroscopic emergency evacuation planning: a survey. *Information Technology, ITSim, International Symposium, IEEE*, 3, 26–28, 1–7.
170. Zhao, X., Feng, Z-Y, Li, Y. and Bernard, A. (2016). Evacuation network optimization model with lane-based reversal and routing. *Mathematical Problems in Engineering*, Article ID 1273508, 1-12.