

# Establishment of EOQ Model with Quadratic Time-Sensitive Demand and Parabolic-Time Linked Holding Cost with Salvage Value

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*Received August 2018; Accepted August 2018*

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**Abstract:** In the present study, we analyze an economic order quantity (EOQ) model for deteriorating surroundings under the quadratic time-linked demand with parabolic time changeable holding cost related with salvage value. Mathematical formulation and solution procedure is developed for determining the most advantageous solution. The theoretical expressions are obtained for finest cycle time and order quantity. The result is verified with the help of numerical examples. Sensitivity investigation and graphical representations are used to study the outcome with a choice of parameters.

**Keyword** — Inventory, Deterioration, Parabolic holding cost, Quadratic demand rate, Salvage value

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## 1. INTRODUCTION

In most of the standard inventory models, demand rate has been assumed stable. However, in factual existence, market demand is forever rise and fall. Demand is the most impulsive of all market forces, as inventory manager has least control over it. It is frequently observed that demand for a particular article can be affected by numerous variables such as price, time, new inventions and availability of the product. Also, researchers felt the necessity to use the weakening factor in their models to get nearer to the real-world situations, as spoilage of the physical goods in store is a wide-ranging occurrence and cannot be overlooked. The rate of deterioration is very high in a few items like food grains, blood, fish etc., whereas some items like toys, glassware, clothes etc., decays at a slower speed. Moreover, it is also significant to diminish the inventory carrying cost. The economic order quantity model assumes that the holding cost is invariable. However, this assumption is not very common in practice and could be more sensible to consider that holding cost may fluctuate over the storage time.

The important literature is classified into three types of EOQ models as follows. The first type models are related to inconsistent demand. The second type is models categorized with corrosion. The third category is considered for the changeable selection holding cost.

### 1.1 Inventory Models with Variable Demand and Salvage Value

Attempting the phenomenon of time unstable demand pattern, as linear or exponential, in inventory model not yields much real time application. EOQ models in which demand rate assumed to be quadratic function of time are very much reasonable for definite commodities. Bhandari and Sharma (2000) projected a single-period inventory problem with quadratic time-induced demand allocation under the control of marketing policies. Kharna and Chaudhuri (2003) explored the EOQ model for a failing item with quadratic (accelerated growth/decline) time dependent demand patterns. Ghosh and Chaudhuri (2004) discussed a model for a deteriorating article having an direct supply, a quadratic time unstable demand in view of shortages. Begum et al. (2010) discussed an EOQ model with quadratic time varying demand rate. In this model, unit production cost is inversely comparative to time dependent demand rate. Khanra et al. (2011) widespread an EOQ model with time sensitive quadratic demand when delay in payments is acceptable. The deterioration rate is assumed to be unvarying. Sharma et al. (2012) studied Weibull deteriorating inventory supervision with quadratic time unsteady demand rate with variable holding cost. Amutha and Chandrasekaran (2013) proposed a model for deteriorating items with quadratic time dependent holding cost. Rangarajan and Karthikeyan (2015) analyzed EOQ models for deteriorating objects with a variety of demand rates such as stable, linear and quadratic function of time and time-induced holding cost to minimize the entire cost.

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A concept of salvage value is new in inventory management. It has been used by extremely few researchers. Salvage value, also called scrap value, is the predictable value that will realize upon its sale at the end of its practical life. Salvage values are important in business because they affect the size of a company's downgrading expense and thus they affect the net income. Mohan and Venkateshwarlu (2013) studied a model with quadratic time varying demand allowing for variable holding cost and salvage value. An inventory model is established by Venkateshwarlu and Mohan (2014) for invariable deteriorating foodstuffs with quadratic time varying demand rate. Venkateshwarlu and Reddy (2014) considered time-linked quadratic demand EOQ model for deteriorating items. The author assumed that deterioration rate is regular and the supplier offers his vendor the credit period to settle the account. Salvage value is also taken in the model to see its effect on the total cost.

### **1.2 Inventory Models with Deterioration**

In practice, deterioration can take place during the storage period of items of our daily requirements and for such products losses cannot be ignored. Hence a deteriorating inventory system has received much attention of several modelers. Ghare and Schrader (1963) were the two researchers who first studied an EOQ model for an exponentially decaying item with constant demand. To accommodate more practical features of the real inventory systems, Aggarwal and Jaggi (1995), Hwang and Shinn (1997), extended Goyal's (1985) model to consider the deterministic model with a steady corrosion rate. Chang et al. (2002) extended this issue to the unstable rate of deterioration. Chu et al. (1998) and Chung et al. (2001) also investigated the deteriorating objects under this condition and developed well-organized move towards to decide the finest cycle time. Moon et al. (2005) constructed a model to incorporate two extreme physical distinctiveness of stored items into inventory model ameliorating and fading. Skouri and Konstantaras (2009) deliberated an order echelon inventory model for deteriorating seasonable/designer products with permitted shortages. An inventory model for deteriorating items with recurring products was presented by Tayal et al. (2014). Recently, Singh et al. (2015) investigated an EOQ model for deteriorating products having stock-linked demand with trade credit phase and protection equipment.

### **1.3 Inventory Models with Variable Holding Cost**

Various models have been proposed with stable holding cost. Holding cost was assumed to be changeable over time within few inventory models. Vander Veen (1967) presented an inventory system by means of holding cost as a non-linear function of inventory. Muhlemann and Valtis-Spanopoulos (1980) investigated invariable rate EOQ model with changeable holding cost articulated as a fraction of the normal value of capital investigated in stock. Weiss (1982) deliberated classical EOQ model with unit holding cost as a non-linear function of length of time. Giri et al. (1996) offered a comprehensive EOQ model for fading items with shortages, in which both the demand rate and the holding cost were continuous function of time. Alfares (2007) presented the step formation of holding cost considering inventory policy for an article with a stock-level dependent demand rate and time-dependent holding cost. Kumar et al. (2009) presented an inventory model for power demand rate incremental holding cost under allowed delay in payments. Kumar et al. (2012) proposed an inventory model with Weibull distribution deteriorating item for control blueprint demand with shortage and time-sensitive holding cost. Kumar et al. (2013) developed a deterministic inventory model for price-linked demand with parabolic time varying holding cost and trade credit. Tripathi et al. (2018) analyzed an EOQ model for time-dependent holding cost under unlike trade credits.

The objective of this manuscript is to develop an EOQ model in which demand rate is a quadratic increasing function of time and unit holding cost depends on the storage time. The deterioration of items is assumed to be invariable and recover value is connected with the deteriorated units. A mathematical model is constructed based on these assumptions to minimize the total inventory cost.

The rest of the paper is planned as follows. Section 2 introduces the notations and assumptions used all over the paper. The formulation of the cost function for obtaining optimal order quantity, cycle time and minimum total cost is specified in section 3. Section 4 proposes a solution procedure to locate most favorable solution. To validate theoretical results of the problem, numerical results are presented in section 5. In section 6, sensitivity study with graphical interpretations is agreed out to examine the effect of changes in optimal solution with respect to change in one parameter at a time. Finally, Conclusion is drawn in section 7.

## **2. NOTATIONS AND ASSUMPTIONS**

### **2.1 Notations**

In order to formulate the mathematical model of the present model, we use the following notations:

$O$	Ordering cost/order
$C$	Unit purchasing price
$I(t)$	Inventory level at time $t$
$R(t)$	( $= a + bt + ct^2$ ); Annual demand rate at any time $t \geq 0$ , ( $a > 0, b \neq 0, c \neq 0$ ) where $a$ is the initial rate of demand and $b, c$ denotes the rate of change at which the demand rate itself increases.
$h(t)$	( $= h + \alpha t^2$ ), $h > 0, \alpha > 0$ the holding cost/unit
$\theta$	Deterioration rate(constant), $0 \leq \theta < 1$
$\delta$	Salvage Coefficient
$Q$	Order quantity
$T$	Length of replenishment cycle time
$K(T)$	Total inventory cost per unit time

## 2.2 Assumptions

Inventory system will be assumed to have the following properties:

- Model considers only one item in inventory.
- Replenishment rate is infinite, that is, lead time is negligible.
- Shortages are not acceptable.
- Holding cost has two components: a unvarying module  $h$  and a changeable component  $\alpha$  that increases linearly with the length of storage time.
- Salvage value  $\delta C$  ( $0 \leq \delta < 1$ ) is linked to deteriorated units throughout the cycle time.

## 3. MATHEMATICAL FORMULATION

The inventory depletes due to demand and weakening. Thus, the governing differential equation of the inventory level at time  $t$  is as follows:

$$\frac{dI(t)}{dt} + \theta I(t) = -R(t), \quad 0 \leq t \leq T \quad (1)$$

With boundary condition  $I(T) = 0$ , solution of (1) is:

$$I(t) = \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \left( \frac{e^{\theta(T-t)} - 1}{\theta} \right) + \left( b - \frac{2c}{\theta} + cT \right) \frac{T e^{\theta(T-t)}}{\theta} - \left( b - \frac{2c}{\theta} + ct \right) \frac{t}{\theta} \quad (2)$$

Thus, the order quantity leads to:

$$Q = I(0) = \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \left( \frac{e^{\theta T} - 1}{\theta} \right) + \left( b - \frac{2c}{\theta} + cT \right) \frac{T e^{\theta T}}{\theta} \quad (3)$$

Total annual variable cost consists of following cost components:

$$OC = \frac{O}{T} \quad (4)$$

Deterioration cost/time units:

$$\begin{aligned} DC &= \frac{C}{T} \left( Q - \int_0^T R(t) dt \right) \\ &= \frac{C}{T} \left\{ \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \left( \frac{e^{\theta T} - 1}{\theta} \right) + \left( b - \frac{2c}{\theta} + cT \right) \frac{T e^{\theta T}}{\theta} - \left( a - \frac{bT}{2} - \frac{cT^2}{3} \right) T \right\} \quad (5) \end{aligned}$$

Inventory holding cost/unit time is:

$$\begin{aligned}
 IHC &= \frac{1}{T} \int_0^T (h + \alpha t^2) I(t) dt \\
 &= \frac{h}{T} \left\{ \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} + bT - \frac{2cT}{\theta} + cT^2 \right) \left( \frac{e^{\theta T} - 1}{\theta^2} \right) + \left( \frac{2c}{\theta} - b \right) \frac{T^2}{2\theta} - \left( \frac{c}{\theta} \right) \frac{T^3}{3} - \right. \\
 &\quad \left. \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \frac{T}{\theta} \right\} + \frac{\alpha}{T} \left\{ \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} + bT - \frac{2cT}{\theta} + cT^2 \right) \frac{1}{\theta^2} \left( \frac{2e^{\theta T}}{\theta^2} - T^2 - \frac{2T}{\theta} - \frac{2}{\theta^2} \right) \right. \\
 &\quad \left. + \left( \frac{2c}{\theta} - b \right) \frac{T^4}{4\theta} - \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \frac{T^3}{3\theta} - \left( \frac{c}{\theta} \right) \frac{T^5}{5} \right\} \quad (6)
 \end{aligned}$$

Salvage Value of deteriorated units is:

$$SV = \frac{\delta C}{T} \left\{ \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \left( \frac{e^{\theta T} - 1}{\theta} \right) + \left( b - \frac{2c}{\theta} + cT \right) \frac{T e^{\theta T}}{\theta} - \left( a + \frac{bT}{2} + \frac{cT^2}{3} \right) T \right\} \quad (7)$$

Thus, optimum total cost  $K(T)$  of an inventory system per unit time is:

$$K(T) = \frac{1}{T} [OC + DC + IHC - SV] \quad (8)$$

$$\begin{aligned}
 K(T) &= \frac{O}{T} + \frac{C}{T} \left\{ \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \left( \frac{e^{\theta T} - 1}{\theta} \right) + \left( bT - \frac{2cT}{\theta^2} + \frac{cT^2}{\theta} \right) \frac{e^{\theta T}}{\theta} - \left( a + \frac{bT}{2} + \frac{cT^2}{3} \right) T \right\} \\
 &\quad + \frac{h}{T} \left\{ \left( a + bT + cT^2 - \frac{b}{\theta} + \frac{2c}{\theta^2} - \frac{2cT}{\theta} \right) \left( \frac{e^{\theta T} - 1}{\theta^2} \right) + \left( \frac{2c}{\theta} - b \right) \frac{T^2}{2\theta} - \left( \frac{c}{\theta} \right) \frac{T^3}{3} \right. \\
 &\quad \left. - \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \frac{T}{\theta} \right\} + \frac{\alpha}{T} \left\{ \left( a + bT + cT^2 - \frac{b}{\theta} + \frac{2c}{\theta^2} - \frac{2cT}{\theta} \right) \frac{1}{\theta^2} \left( \frac{2e^{\theta T}}{\theta^2} - T^2 - \frac{2T}{\theta} - \frac{2}{\theta^2} \right) \right. \\
 &\quad \left. + \left( \frac{2c}{\theta} - b \right) \frac{T^4}{4\theta} - \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \frac{T^3}{3\theta} - \left( \frac{c}{\theta} \right) \frac{T^5}{5} \right\} \\
 &\quad - \frac{\delta C}{T} \left\{ \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \left( \frac{e^{\theta T} - 1}{\theta} \right) + \left( bT - \frac{2cT}{\theta} + cT^2 \right) \frac{e^{\theta T}}{\theta} - \left( a + \frac{bT}{2} + \frac{cT^2}{3} \right) T \right\} \quad (9)
 \end{aligned}$$

In order to find minimum variable cost/ unit time, necessary and sufficient conditions to minimize  $K(T)$  for a given value  $T$  are respectively  $\frac{dK(T)}{dT} = 0$  and  $\frac{d^2K(T)}{dT^2} > 0$ .

Now,  $\frac{dK(T)}{dT} = 0$  gives the following equation:

$$\begin{aligned}
 & - \frac{O}{T^2} + (1 - \delta) C \left\{ \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \frac{1}{\theta T} \left( \theta e^{\theta T} - \frac{e^{\theta T}}{T} + \frac{1}{T} \right) + \left( b - \frac{2c}{\theta} \right) e^{\theta T} + \frac{c e^{\theta T}}{\theta} (\theta T + 1) \right. \\
 & \quad \left. - \left( \frac{b}{2} + \frac{2cT}{3} \right) \right\} + h \left\{ \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \left( e^{\theta T} - \frac{e^{\theta T}}{\theta T} + \frac{1}{\theta T} \right) \frac{1}{T\theta} + \left( b - \frac{c}{\theta} \right) \frac{e^{\theta T}}{\theta} + \frac{c}{\theta^2} (\theta T e^{\theta T} - 1) \right. \\
 & \quad \left. + \frac{1}{2\theta} \left( \frac{2c}{\theta} - b \right) - \frac{2cT}{3\theta} \right\} + \alpha \left\{ \left( a - \frac{b}{\theta} + \frac{2c}{\theta^2} \right) \frac{1}{\theta} \left( \frac{2e^{\theta T}}{\theta^2 T} - \frac{2}{\theta^3 T^2} (e^{\theta T} - 1) - \frac{1}{\theta} - \frac{2T}{3} \right) \right. \\
 & \quad \left. + \left( b - \frac{2c}{\theta} \right) \left( \frac{2e^{\theta T}}{\theta} - 2T - \frac{2}{\theta} \right) \frac{1}{\theta^2} + \left( \frac{2c}{\theta} - b \right) \frac{3T^2}{4\theta} \right. \\
 & \quad \left. + \frac{2c}{\theta^2} \left( \frac{T}{\theta} (e^{\theta T} - 2) + \frac{(e^{\theta T} - 1)}{\theta^2} - \frac{3T^2}{2} \right) - \frac{4cT^3}{5\theta} \right\} = 0 \quad (10)
 \end{aligned}$$

The exponential approximation i.e.  $e^{\theta T} = 1 + \theta T + \frac{\theta^2 T^2}{2}$  is used to find the solution of equations. By solving (10), we compute the value of  $T$ . For such  $T$ , total cost is minimum only if

$$\frac{d^2K(T)}{dT^2} > 0 \quad (11)$$

#### 4. SOLUTION PROCEDURE

The optimal solution of the anticipated inventory system can be obtained from following algorithm.

- Step 1.** Initialize the parameters  $O, a, b, c, C, h, \theta, \delta, \alpha$  in equation (10).  
**Step 2.** Obtain the value of  $T$  using equation (10).  
**Step 3.** Check the optimality of  $T$  by using equation (11). If satisfied then go to step 4 otherwise repeat procedure from step 1 to step 3 with new values of  $T$ .  
**Step 4.** Using the optimal value of  $T$  in (3) and (9), we find optimal order size  $Q$  & minimum total cost  $K(T)$ .  
**Step 5.** Stop

#### 5. NUMERICAL EXAMPLES

To demonstrate above solution procedure, we consider following examples:

**Example 1** Consider parametric values  $[O, a, b, c, C, h, \theta, \delta, \alpha] = [150, 125, 35, 0.2, 35, 5, 0.02, 0.1, 0.2]$  in appropriated units. Using the solution algorithm in section 4, we get optimal results as follows:

$$T = 0.0218, Q = 2.7413, K(T) = 13736.67 \quad \text{and} \quad \frac{d^2 K(T)}{dT^2} = 29989223.6383 > 0.$$

**Example 2** Consider parametric values  $[O, a, b, c, C, h, \theta, \delta, \alpha] = [300, 175, 75, 0.9, 50, 9, 0.09, 0.7, 1]$  in appropriate units. Applying solution procedure in section 4, we get the optimal results as follows:

$$T = 0.0725, Q = 13.0378, K(T) = 8243.70 \quad \text{and} \quad \frac{d^2 K(T)}{dT^2} = 15668620.67 > 0.$$

#### 6. SENSITIVITY ANALYSIS

For the sensitivity study, we now learn various effects of parameters  $O, a, b, c, C, h, \theta, \alpha$  and  $\delta$  on the optimal solutions of the system.

##### 6.1 Sensitivity by Values Variation

The following tables designate change in inventory policy by shifting value of one parameter at a time and maintaining the remaining parameters fixed. We fix the variables as  $O = 200, a = 100, b = 50, c = 0.1, C = 30, h = 4, \theta = 0.05, \delta = 0.1, \alpha = 0.2$ . The results are shown in Table 1-9.

Table 1: Variation in  $T, Q$  &  $K(T)$  with ordering Cost ( $O$ ) $c$

$O$	$T$	$Q$	$K(T)$
100	0.0245	2.4803	8154.35
200	0.0346	3.5246	11529.71
300	0.0424	4.3326	14118.82
400	0.0490	5.0184	16300.92
500	0.0548	5.6259	18222.93
600	0.0600	6.1780	19960.19
700	0.0648	6.6880	21557.44
800	0.0693	7.1647	23043.87

Table 2: Variation in  $T, Q$  &  $K(T)$  with initial demand (a)

$a$	$T$	$Q$	$K(T)$
50	0.03367	1.7380	11871.98
100	0.0346	3.5246	11529.71
150	0.0357	5.4255	11176.93
200	0.0369	7.4579	10812.61
250	0.0382	9.6436	10435.53
300	0.0397	12.0096	10044.24
350	0.0414	14.5909	9636.99
400	0.0433	17.4330	9211.66

Table 3: Variation in  $T, Q$  &  $K(T)$  with coefficient (b)

$b$	$T$	$Q$	$K(T)$
10	0.2153	21.9085	1804.71
20	0.0667	6.7594	5977.79
30	0.0484	4.9063	8254.37
40	0.0398	4.0465	10026.66
50	0.0346	3.5246	11529.71
60	0.0310	3.1650	12858.29
70	0.0284	2.8980	14061.93
80	0.0263	2.6896	15170.39

Table 4: Variation in  $T, Q$  &  $K(T)$  with coefficient (c)

$c$	$T$	$Q$	$K(T)$
0.05	0.0338	3.4416	11807.39
0.10	0.0346	3.5246	11529.71
0.15	0.0355	3.6139	11245.19
0.20	0.0364	3.7105	10953.27
0.25	0.0374	3.8152	10653.37
0.30	0.0386	3.9293	10344.77
0.35	0.0398	4.0544	10026.69
0.40	0.0412	4.1923	9698.18

Table 5: Variation in  $T, Q$  &  $K(T)$  with holding cost (h)

$h$	$T$	$Q$	$K(T)$
2	0.0347	3.5346	11497.72
4	0.0346	3.5246	11529.71
6	0.0345	3.5147	11561.61
8	0.0344	3.5050	11593.42
10	0.0343	3.4953	11625.14
12	0.0342	3.4856	11656.78
14	0.0342	3.4761	11688.33
16	0.0341	3.4666	11719.79

Table 6: Variation in  $T$ ,  $Q$  &  $K(T)$  with holding coefficient ( $\alpha$ )

$\alpha$	$T$	$Q$	$K(T)$
0.5	0.0346	3.5246	11529.71
0.7	0.0293	2.9775	13615.47
0.9	0.0259	2.6246	15421.81
1.1	0.0234	2.3730	17037.79
1.3	0.0215	2.1820	18513.32
1.5	0.0201	2.0307	19879.67
1.7	0.0188	1.9069	21158.00
1.9	0.0178	1.8033	22363.41

Table 7: Variation in  $T$ ,  $Q$  &  $K(T)$  with deterioration rate ( $\theta$ )

$\theta$	$T$	$Q$	$K(T)$
0.05	0.0346	3.5246	11529.71
0.10	0.0986	10.3459	4036.01
0.15	0.1793	19.5175	2203.90
0.20	0.263	29.7312	1486.26
0.25	0.3372	39.3706	1147.95
0.30	0.3954	47.3184	973.32
0.35	0.4377	53.3053	877.87
0.40	0.4673	57.6100	823.16

Table 8: Variation in  $T$ ,  $Q$  &  $K(T)$  with salvage coefficient ( $\delta$ )

$\delta$	$T$	$Q$	$K(T)$
0.05	0.0346	3.5225	11533.30
0.10	0.0346	3.5232	11534.26
0.15	0.0346	3.5242	11530.91
0.20	0.0346	3.5246	11529.71
0.25	0.0346	3.5250	11528.51
0.30	0.0346	3.5254	11527.32
0.35	0.0346	3.5257	11526.12
0.40	0.0346	3.5261	11524.92

Table 9: Variation in  $T$ ,  $Q$  &  $K(T)$  with purchasing cost ( $C$ )

$C$	$T$	$Q$	$K(T)$
10	0.0347	3.5286	11516.93
20	0.0346	3.5266	11523.32
30	0.0346	3.5246	11529.71
40	0.0346	3.5226	11536.1
50	0.0346	3.5207	11542.48
60	0.0346	3.5187	11548.86
70	0.0345	3.5167	11555.24
80	0.0345	3.5147	11561.61

## 6.2 Sensitivity by Percentage Variation

Graphical representation in Figures 1 and 2 exhibit the percentage deviation in total cost with percentage variation in model parameters. We draw results by changing values of parameters in relative steps of 20% (+40%, +20%, -20%, -40%). We fix the variables as  $O = 200$ ,  $a = 100$ ,  $b = 50$ ,  $c = 0.1$ ,  $C = 30$ ,  $h = 4$ ,  $\theta = 0.05$ ,  $\delta = 0.1$ ,  $\alpha = 0.2$ .

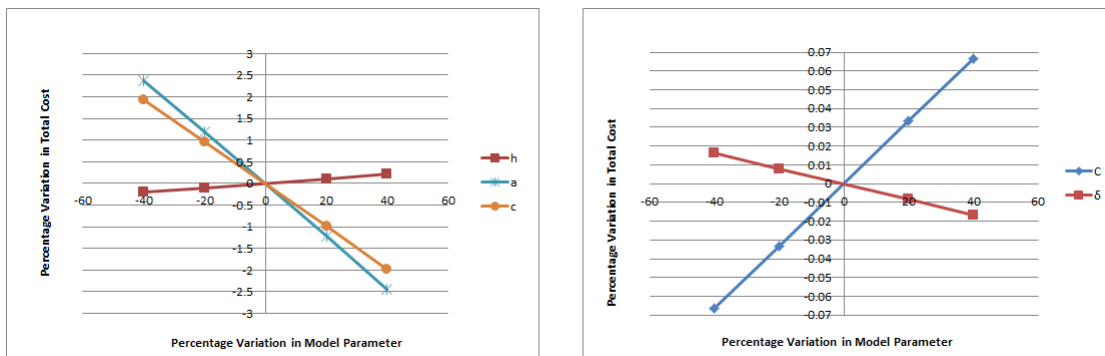


Figure 1: Percentage Variation in Model Parameters with Percentage Variation in Total Cost

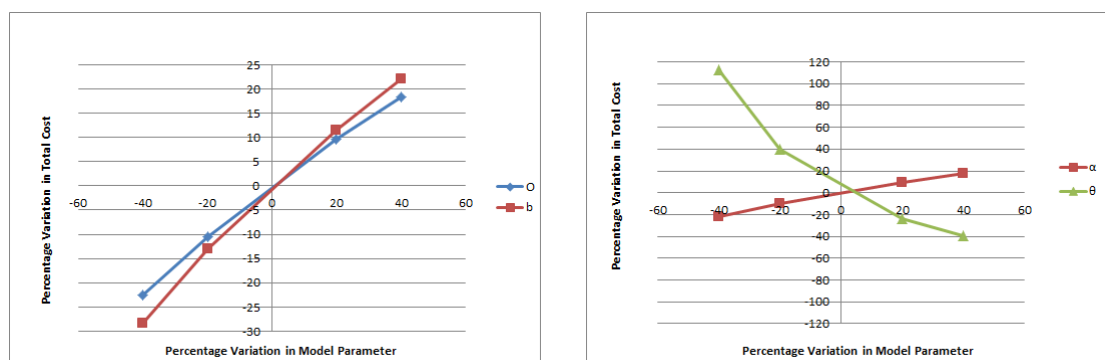


Figure 2: Percentage Variation in Model Parameters with Percentage Variation in Total Cost

### 6.3 Effect of Parameters on the Cost Function

(a) *Effect of Ordering Cost:* Table 1 represents that amplified ordering cost outcomes in higher increment in the optimal cycle length  $T$ , most favorable quantity size  $Q$  and in total inventory cost  $K(T)$ . But after reaching at a certain value the optimum values increases at a gradual rate than before. From Fig. 2, it has also been observed that with the variation in ordering cost in the range of -40% to +40%, total cost varies from -22.52% to +18.30%.

(b) *Effect of Initial Demand ('a'):* An increased demand rate compels the supplier to keep the sufficient stock in advance which increases the replenishment cycle length and it results in the decrease of the total cost. Hence, Table 2 shows that any enlarge in initial demand results to an increase in optimal cycle length  $T$  and optimal order quantity  $Q$  but it decreases total cost  $K(T)$ . Fig. 1 also displays that with the variation with initial demand in the given range, total cost varies from +2.38% to -2.44%.

(c) *Effect of Coefficients ('b' and 'c'):* 'b' and 'c' are the coefficients of the demand at time  $t$ . Table 3 and Table 4 shows that an increase value of 'b' results to a decrease in optimal replenishment period  $T$  and optimum order quantity  $Q$  but there is an increment in total cost whereas any increase in 'c' results to an increase in total cycle length  $T$  and order size  $Q$  but it decreases total cost  $K(T)$ . Total cost varies from -28.40% to +21.96% and +1.93% to -1.96% respectively as b and c varies in the range of -40% to +40%. Results are shown in Fig. 1 and Fig. 2.

(d) *Effect of Holding Cost and Coefficient ( $\alpha$ ):* In real-world situation, if the holding cost increases, it will increase total cost and it is true for the proposed model too. From table 5 and Table 6, it can be easily seen that as we increase the value of holding cost with time it results in the increment of total cost  $K(T)$  but decreases the cycle length  $T$  and order size  $Q$ . It has been investigated that with the variation in holding coefficient, the total cost  $K(T)$  varies from -22.18% to +18.09% whereas it varies a lesser amount from -0.19% to +0.22% with the percentage change in holding cost.

(e) *Effect of Deterioration:* Table 7 and Fig. 2 depicts that a small increment in deterioration rate results in higher increments in replenishment cycle time  $T$  and order quantity  $Q$ . On the other hand, it decreases the total cost  $K(T)$  as we increase the deterioration rate  $\theta$ . Also, in real time, as the deterioration becomes higher, the supplier replenishes the stock sooner and the supplier has to place the order earlier to avoid the condition of shortages. Total cost fluctuates from +112.45% to -39.79% with the change in deterioration in the assumed range. It can be realized that deterioration has extreme impact on optimal quantity size  $Q$ .

(f) *Effect of Salvage Coefficient:* An increase in ' $\delta$ ', decreases the optimal cycle length  $T$  and the total cost  $K(T)$  whereas it increases the optimal order quantity  $Q$ . It can be shown in Table 8. From Fig. 1, it is noted that total cost diverges from +0.01% to -0.01% with the variation in  $\delta$ . The effect of salvage coefficient is insignificant on the optimum values.



(g) *Effect of Purchasing Cost:* If the supplier increases the unit purchase cost of an item then the demand goes down and it results in the decrement of the quantity ordered. If the order size is small then the replenishment period is also very short. This result is drawn in Table 9, that on increasing the purchase cost, there will be increment in the total cost  $K(T)$  but it decreases the total cycle length  $T$  and optimal order quantity  $Q$ . With the variation in purchasing cost in the specified range, the total cost fluctuates from -0.06% to +0.06%. The effect of  $C$  is not so significant.

## 7. CONCLUSION AND FUTURE WORK

This study deals with a deterministic EOQ model for fading products assuming demand rate as quadratic function of time. Deterioration rate is considered steady with parabolic holding cost. Salvage value is also measured in calculating total cost. Shortages are not allowed. Economic order quantity, optimal cycle length and total cost were determined using solution algorithm of the model. Sensitivity investigation with respect to parameters has been carried out with the help of tables and graphs. Summarizing, we can say that the enlarged holding cost results in the augmentation of total cost and it matches real-world situations. Total cost is incredibly sensitive to deterioration rate and holding coefficient. Whereas, salvage coefficient and purchasing cost has least effect on the total cost.

The research can be further extended in different ways. For instance, model can be extended for non-instantaneous deteriorated stuffs, stable demand and tolerable shortages. This model can also be extended for fuzzy environment.

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