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Maximum FlowLoc Problems with Network Reconfiguration

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Abstract: The large scale calamities caused by natural or human-created disasters are the challenging issues to protect the life and their surroundings. It is a complex task to develop the significant and universally accepted solution strategy for handling these issues. Appropriate facility locations and transportation facilities are essential components for the efficient evacuation planning flow models. Contraflow, the lane reversal strategy, is one of the widely accepted solution approach for evacuation planning as it maximizes the outbound capacities of roads by reversing the required road directions and makes the traffic smooth. This significantly increases the flow value and decreases the evacuation time. In this work, the network facility location and the contraflow approach are incorporated to the flow models and some efficient algorithms are presented to locate the facility with an objective of minimum flow loss on the evacuation network. Our facility location contraflow solutions obtain the optimal plans with respect to the given and arbitrary locations.
Keyword — Evacuation plan, dynamic network, flow model, facility location, contraflow, ContraFlowLoc

1. INTRODUCTION

There has been a growing number of natural as well as man-made disasters worldwide. An essential aspect of the awareness to such events is the choice of shelter locations such that everyone who has to be evacuated can be accommodated. Nevertheless, one must take into consideration the traffic that will result from evacuation network as it has a big influence on the duration of the evacuation process. Due to the importance of the best possible choice of shelter locations with minimum flow loss or minimum increase in the network clearance time, the efficient evacuation planning has drawn the interest for current researchers.

An evacuation network represents the intersections of roads (i.e., rooms in a building or intersection of streets in a region) as nodes and road segments between nodes (i.e., doors between rooms, or streets in region) as arcs. The sources represent initial places where evacuees are located and start to move and the sinks represent safe places where they are supposed to arrive. Most traffic delays occur in roads due to different facilitity locations around the roads. Shifting people with efficient routes to the sinks with proper facility locations is an innermost challenge to manage a regional evacuation plan. In order to cope such real-world problems, the decision maker has to target more than one objectives or consider different factors or measures, then the problems should be changed into multi-criteria decision making problems.

After the development of maximal static and maximum dynamic flow models and the respective algorithms by Ford and Fulkerson (1958), various network flow problems are studied that are applicable for evacuation planning. For example, the maximum dynamic flow problem to shift maximal amount in a given time, the earliest arrival flow problem to maximize the number of evacuees in every possible time, the quickest flow problem for allocating the evacuees to a safer zone in minimum time and the lexicographically maximum dynamic flow problem to send maximum number of evacuees in given priority within the given time period. We refer to Dhamala, Pyakurel, and Dempe (2018) for the summery of results and the extensive references. Philpott (1982) firstly introduced network flow model for continuous time setting. The efficient continuous-time dynamic network flow algorithms have been developed by Fleischer and Tardos (1998) using the natural transformation.

The location theory was introduced by Weber (1909) with applications for industries. Different discrete network location models and algorithms have been investigated by Daskin (1995). For details, we refer to Drezner and Hamacher (2002). The influence of facilities on the walking speed, the walking behaviour of pedestrians, the necessity of placing security personnels to guide the pedestrians to the locations, different positions of facilities and their influence on the

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behaviour and walking speed of pedestrians have been investigated. However, Hamacher, Heller, and Rupp (2013) uses location theory to improve the existing evacuation models where two different models, network flow and location theory have been integrated to introduce FlowLoc theory in evacuation modeling.

Contraflow approach is another emerging and widely accepted model for evacuation planning that increases the outbound road capacities by reversing the direction of roads towards the safe destinations. For the large scale evacuations, the evacuation time has been improved by at least 40 percent with at most 30 percent of the total arc reversals, Kim, Shekhar, and Min (2008). Different contraflow models with their efficient solution algorithms and a greedy heuristic have been developed after the first integer programming formulation (Kim et al., 2008). They showed that the general problem of minimizing the evacuation time is $\mathcal{NP}$-hard. Pyakurel and Dhamala (2016) introduced the first contraflow model for continuous time setting. They have presented efficient algorithms to solve the maximum continuous dynamic contraflow, quickest continuous contraflow and earliest arrival contraflow problems.

A polynomial time algorithm to solve the abstract maximum dynamic contraflow with path reversal capability in continuous time setting has been presented by Pyakurel, Dhamala, and Dempe (2017). Dhungana, Pyakurel, and Dhamala (2018) introduced the abstract contraflow problems and their solution procedures for discrete time setting with path reversals. The quickest contraflow problems with constant and load dependent transit times, their algorithms and computational experiments have been discussed by Pyakurel, Nath, and Dhamala (2018a). Various network flow models and solutions with partial contraflow based on path reversals have been presented by Pyakurel, Nath, and Dhamala (2018b). For the important property of the contraflow reconfiguration and the analytical solutions for various contraflow problem, we refer to Dhamala et al. (2018) and the references therein. The contraflow problem can be solved with the same complexity as without contraflow but flow value may be up to double and the evacuation time the half.

Xie and Turnquist (2011) have introduced and solved the lane based contraflow and crossing elimination strategies at intersections simultaneously. The evacuation network model with lane-based reversal and flow routing has been considered by Zhao, Feng, Li, and Bernard (2016). On the evacuation planning with contraflow and crossing elimination jointly, a bi-level lane-based network optimization and simulation model have been formulated by Xie, Lin, and Waller (2010). The multi-model integrated contraflow for uncertain arrivals of evacuees in an evacuation region with low mobility population is presented by Hua, Ren, Cheng, and Ran (2014). Transit-based models are initiated with vehicle routine problem whereas the integrated strategy contains non-contraflow to shorten the strategy setup time, full-lane contraflow to minimize the evacuation network capacity and bus contraflow to realize the transit cycle operation. The transit priority during multi-modal evacuation can be provided by network aggregation method and an integrated contraflow strategy (Hua et al., 2014).

The plan of the paper is given as follows. The notations and prior works in network flow, FlowLoc and contraflow models are given in Section 2. The set of locations can be fixed in advance or it can be an arbitrary. The former approach has its advantage in allocating facilities at predefined locations that a policy maker requires, however, the latter one gives us a flexibility of allocating a facility among the arbitrary locations. Based on given possible locations, we introduce the two-terminal FlowLoc problem in continuous time dynamic network and propose an efficient algorithm for its solution in Section 3. The corresponding results for two-terminal FlowLoc static and dynamic contraflow problems are presented in Section 4. The results with arbitrary locations are extended in Section 5. Section 6 concludes the paper.

2. DENOTATIONS AND PRIOR WORKS

An evacuation network $N = (V, A, u_e, \theta_e, S, D)$ is a directed graph where V, $A \subseteq V \times V$, S, D, T represent the set of vertices, arcs, sources, sinks and the time horizon, respectively. The capacity and travel time vectors of $e \in A$ are denoted by u_e and θ_e, respectively. The evacuation time is represented by $T_d = \{0, 1, \cdots, T\}$ in discrete model, whereas $T_c = \{[0,1), \cdots, [T, T+1)\}$ in continuous model. Let $f^c_{\text{dyna}} : A \times T_c \longrightarrow R^+$ and $f^d_{\text{dyna}} : A \times T_d \longrightarrow R^+$ be the dynamic flow function for continuous and discrete time setting, respectively. The incoming and outgoing arcs of the vertex v are denoted and defined by $A^-_v = \{(k, v) \in A \mid v, k \in V\}$ and $A^+_v = \{(v, w) \in A \mid v, w \in V\}$, respectively. It is assumed that the flow is possible only at positive time. The first mathematical formulation for continuous time network flow with $S = \{s\}$ and $D = \{t\}$ is considered by Philpott (1982) which satisfies the following constraints.

$$\begin{aligned}
\int_0^T \sum_{e\in A_v^-} f^c_{\text{dyna}}(e,\tau-\theta_e)d\tau - \int_0^T \sum_{e\in A_v^+} f^c_{\text{dyna}}(e,\tau)d\tau &= 0,\ \forall v\notin\{s,t\} &(1)\\
\int_0^\tau \sum_{e\in A_v^-} f^c_{\text{dyna}}(e,\tau-\theta_e)d\tau - \int_0^\tau \sum_{e\in A_v^+} f^c_{\text{dyna}}(e,\tau)d\tau &\geq 0,\ \forall v\notin\{s,t\},\ \tau\in[0,T] &(2)\\
\int_0^T \sum_{e\in A_s^-} f^c_{\text{dyna}}(e,\tau-\theta_e)d\tau - \int_0^T \sum_{e\in A_t^+} f^c_{\text{dyna}}(e,\tau)d\tau &= 0 &(3)\\
0\leq f^c_{\text{dyna}}(e,\tau)\leq u(e,\tau) \qquad & \forall e\in A,\ \tau\in[0,T]. &(4)
\end{aligned}$$

The objective of the maximum dynamic flow problem is to maximize the value of flow for given time T satisfying the Constraints (1-4). The maximum flow value at given time T is defined by

$$\text{val}(f^c_{\text{dyna}},T) = \int_0^T \sum_{e\in A_s^-} f^c_{\text{dyna}}(e,\tau-\theta_e)d\tau = \int_0^T \sum_{e\in A_t^+} f^c_{\text{dyna}}(e,\tau)d\tau \tag{5}$$

If $f : A \longrightarrow R^+$ be the static flow and if $f^d_{\text{dyna}}(\tau)$ be the dynamic flow that enters arc e at time $\tau = 0, 1, \cdots, T-1$, then the following relation holds true.

$$f^d_{dyna}(\tau) = \sum_{\sigma}^{\theta_e-1} f(\tau-\sigma),\ \text{for all}, \tau = 0,1,\cdots,T-1 \tag{6}$$

The flow that enters e at $\tau-\theta_e$ arrives at the head node at time τ in discrete time, but at time $[\tau+1)$ in continuous time. Then the continuous dynamic flow f^c_{dyna} is feasible and the amount of source-sink flow at any integer time interval $[\tau,\tau+\gamma)$, for $\tau = 0,1,\cdots,T-1$, $\gamma\in N$ will be the same for both settings. The dynamic flow models for discrete and continuous time setting have been connected by natural transformation (7) introduced by Fleischer and Tardos (1998).

$$f^c_{dyna}(\alpha) = f^d_{dyna}(\beta) \text{ for all } \beta \text{ and } \alpha \text{ with } \beta\leq\alpha<\beta+1 \tag{7}$$

Efficient continuous time dynamic network flow algorithms can be found by Fleischer and Tardos (1998), whereas the same solution can be obtained by solving the discrete problem, Equation (7).

The static network flow model can be obtained from the above dynamic flow model by omitting the time factor. The maximum static flow model maximize the value val(f) satisfying Constraints (8 - 9).

$$\sum_{e\in A_v^+} f(e) - \sum_{e\in A_v^-} f(e) = \begin{cases} \text{val}(f) & \text{if } v=s\\ 0 & \text{if } v\neq s,t\\ -\text{val}(f) & \text{if } v=t\end{cases} \tag{8}$$

$$0\leq f(e)\leq u(e),\quad \forall\, e\in A \tag{9}$$

2.1 FlowLoc

Let $L\subseteq A$ be the set of all feasible locations, P the set of all facilities, $r : P\to N$ the size of the facilities and nol : $L\to N$ the number of facilities that can be placed on the possible locations. The FlowLoc problem asks for an allocation loc : $P\to L$ of the facilities to the arcs, such that the $s-t$ flow value in the network $N^{\text{loc}}_{\text{dyna}} = (V, A, u'_e, \theta_e, s, t)$ is maximized where $u'_e = u_e - \max\{r_p : \text{loc}(p) = e\}$. If more than one facility are placed on location l only the size of the largest facility determines the reduction of the capacity on the arc. Other modelling alternatives for placing more than one facility on an edge has been discussed by Hamacher et al. (2013). The multi-facility FlowLoc problems (q-FlowLoc) find locations for the q facilities $p\in P$ with size r_p such that the reduction of the maximum flow value is as small as possible and not more than nol(l) facilities are placed on each arc $l\in L$. In particular, the Single-FlowLoc problem assigns one facility for $q=1$ among the given set of facilities.

2.2 Contraflow

In contraflow approach, the auxiliary network of given network will be constructed by adding the capacities of two way arcs and allowing the directions in both ways with symmetric capacities and transit times. The auxiliary network $\tilde{N} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \theta_{\tilde{e}}, S, D)$ is constructed from given evacuation network N as $\overrightarrow{e} = (i, j) \in \tilde{A}$, if $\overrightarrow{e} \in A$ or $\overleftarrow{e} = (j, i) \in A$. The arc capacity function $\tilde{u}$ is given by $\tilde{u}_{\tilde{e}} = u(\overrightarrow{e}) + u(\overleftarrow{e})$ for all arcs $\overrightarrow{e} \in \tilde{A}$. The transit time is defined as follows

$$\tilde{\theta}(\tilde{e}) = \begin{cases} \theta(\overrightarrow{e}) & \text{for } \overrightarrow{e} \in A, \\ \theta(\overleftarrow{e}) & \text{else,} \end{cases} \quad \forall\, \tilde{e} \in \tilde{A}.$$

3. FLOWLOC PROBLEMS

The location of emergency units or other supports are most affecting factors in the evacuation network. Placing any facilities on arcs can affect the size of maximum flow value and quickest evacuation time. The multi terminal q-FlowLoc problem (q-MT-FlowLoc) is $\mathcal{NP}$-complete, Heller and Hamacher (2011). They provide some heuristic solutions for it. Here, we consider the Single-FlowLoc problem with $q = 1$ and present polynomial time algorithms to solve it.

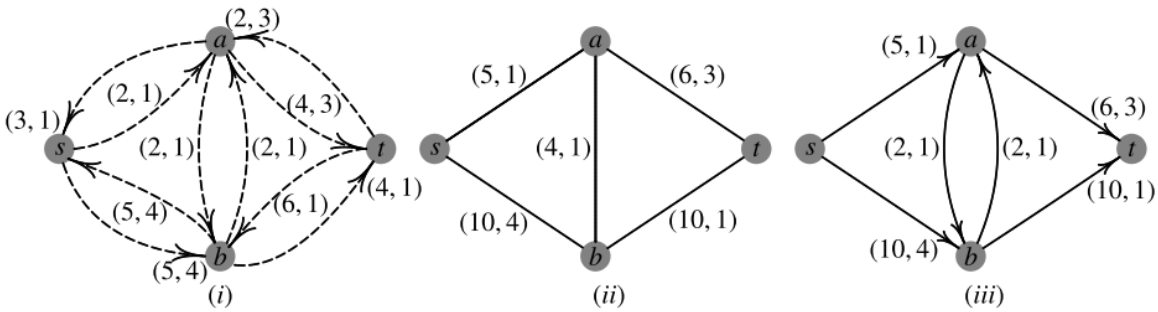

Figure 1: Evacuation, auxiliary and contraflow reconfiguration networks, respectively.

3.1 Maximum Static FlowLoc

The maximum flow can be solved by using labelling algorithm in pseudo-polynomial time $O(mnU)$, where m, n and U represent number of arcs, number of vertices and maximum capacity of arcs, respectively. The shortest augmenting path algorithm finds the shortest augmenting path in polynomial time $O(m^2n)$ from the residual network considering the number of arcs as cost. Several algorithms can be found to improve the complexity of the maximum flow algorithm. We consider the maximum flow algorithm of time complexity $O(nm)$, Orlin (2013). The maximum FlowLoc problem on static network and its solution procedures have been presented by Hamacher et al. (2013). They solved the problem in time $O(|L|n^3)$. But the same problem can be solved using Algorithm 1 in time $O(|L|nm)$. The maximum FlowLoc problem and its efficient solution for the two-terminal static network will be studied in this section.

Problem 1. The maximum FlowLoc problem on static network $N_{stat} = (V, A, u_e, s, t)$ asks to locate the facility in possible locations of the network such that the resulting static flow is maximum in the updated network $N^{\text{loc}}_{stat} = (V, A, u'_e, s, t)$.

Theorem 1. The maximum FlowLoc problem can be solved optimally in time $O(|L|nm)$.

Proof. Algorithm 1 iterates through all possible locations $l \in L$, determines the maximum flow from source to sink if location l hosts facility p and thus finds the optimal location for facility p by comparing all those maximum flow values. For every possible location, maximum flow has to be determined. Thus, Algorithm 1 has the complexity $O(|L|nm)$, where $O(nm)$ is the complexity of a maximum flow algorithm. Hence, Algorithm 1 solves Problem 1 optimally in polynomial time. □

Example 1. Consider the static network given by Figure 1(i) ignoring all arc transit times. Suppose $L = \{(s, a), (s, b), (a, b), (a, t), (b, t)\}$, nol $= 1$ and $r_p = 2$. Before locating facility on the network, 7 units of flow can be sent to the sink through the paths $s - a - t$, $s - b - a - t$ and $s - b - t$ with path flows 2, 2 and 3, respectively. If we pick location (a, b) for the facility it will not reduce any flow from the given network. Thus, optimal flow after providing location remains the same. But it is not necessary that the optimal flow should remain the same in all networks after providing a facility on given network, for example the placement of facility on any other arcs of L reduces the flow value. Thus, the location (a, b) and flow value 7 are the optimal FlowLoc solutions.

Algorithm 1: Maximum Static FloLoc

Input : A directed static network $N_{stat} = (V, A, u_e, s, t)$, locations L, size r_p of facility p
Output: Maximum flow value max_flow, location loc(p) of facility p in the network $N_{stat} = (V, A, u_e, s, t)$

Set: max_flow := -1
  for all $l \in L$ **do**
    if $u_l \geq r_p$ **then**
      $u_l = u_l - r_p$
      max_flow_temp = v(max_flow(N_{stat}))
      $u_l = u_l + r_p$
      if max_flow <max_ flow_temp **then**
        max_flow = max_flow_temp
        loc$(p) = l$
      end if
    end if
  end for
  return max_flow, loc(p)

3.2 Maximum Dynamic FlowLoc

In this section, we introduce FlowLoc problem on dynamic network for continuous time setting that locate given facility in given possible location and maximize the flow on updated dynamic network. We also proposed an efficient algorithm for the problem. Either temporally repeated solution, minimum cost circulation solution considering time as cost or static solution with time expanded network can be used to find the maximum dynamic solution. The complexity of static solution with time expanded network depends in given time horizon so that it is not polynomial. Thus, we have considered the most efficient temporally repeated solution approach which solves the maximum dynamic flow problem in time $O((m \log n)(m + n \log n))$.

Problem 2. The maximum dynamic FlowLoc problem is to locate the facility in a possible location such that the dynamic flow is maximum in $N_{\text{dyna}}^{\text{loc}} = (V, A, u'_e, \theta_e, T, s, t)$.

Algorithm 2: Maximum Dynamic FlowLoc

Input : A dynamic network $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$, locations L, size r_p of the facility p
Output: Maximum flow value max_dyna_flow, location loc(p) of facility p in $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$

Set: max_dyna_flow := -1
for $l \in L$ **do**
  if $u_l \geq r_p$ **then**
    $u_l = u_l - r_p$
    max_dyna_ flow_ temp = v(max_dyna_flow(N_{dyna}))
    $u_l = u_l + r_p$
    if *max_dyna_flow* $<$ *max_dyna_flow_ temp* **then**
      max_dyna_flow = max_dyna_flow_temp
      loc$(p) = l$
    end
  end
end
return max_dyna_flow, loc(p)

Theorem 2. The maximum dynamic FlowLoc problem can be solved optimally.

Proof. Algorithm 2 iterates through all possible locations $l \in L$, gives the maximum dynamic flow value in updated $N^{\text{loc}}_{\text{dyna}} = (V, A, u'_e, \theta_e, T, s, t)$ if location l hosts facility p in given network. The optimal location for facility p is obtained by comparing all those maximum dynamic flow values. Hence, Algorithm 2 solves Problem 2 optimally. □

Corollary 1. Algorithm 2 solves the maximum dynamic FlowLoc problem in polynomial time.

Proof. The complexity of the Algorithm 2 is $O(|L|(m \log n)(m + n \log n))$, where $O((m \log n)(m + n \log n))$ represents the complexity of maximum dynamic flow. □

Example 2. Consider the network of Figure 1(i) with $L = \{(s, a), (s, b), (a, b), (a, t), (b, t)\}$, nol $= 1$, $r_p = 2$ and $T = 8$. The solution can be obtained by applying Algorithm 2. The optimal flow is 27 with optimal location (a, b), shown in Table 1.

Table 1: Maximum dynamic flow corresponding to location (a, b).

Time Horizon	Paths	Length	Flow	Total Dynamic Flow
4	$s-a-t$	4	2	2
5	$s-a-t$	4	2	
	$s-b-t$	5	4	8
6	$s-a-t$	4	2	
	$s-b-t$	5	4	14
7	$s-a-t$	4	2	
	$s-b-t$	5	4	20
8	$s-a-t$	4	2	
	$s-b-t$	5	4	
	$s-b-a-t$	8	1	27

The maximum dynamic flow problem maximizes the solution at given time horizon, where as the earliest arrival flow problem maximizes the solution at every possible time from the beginning. This implies that, every earliest arrival solution is maximum dynamic solution but the converse may not be true. The earliest arrival flow problem has been solved by using the successive shortest augmenting path algorithm (Wilkinson, 1971). The standard chain decomposition could not be adopted to solve the earliest arrival problem as we can not repeat the same path for each time. The non-standard chain decomposition uses the minimum cost maximum flow approach assuming backward chain flows. We can not fix the direction of any arc to get the earliest arrival flow. Thus, the FlowLoc problem could be extended to different dynamic flow problems in various networks, but it can not be extended to the earliest arrival flow models. For the justification we provide following example.

Example 3. Consider the network of Figure 1(iii) with $L = \{(s, a), (s, b), (a, b), (a, t), (b, t)\}$, nol $= 1$ and $r_p = 2$. The earliest arrival flow without any facilities on the network is 65, as shown in Table 2. Applying successive shortest path algorithm on the network, different solutions with respect to different locations have been shown in Table 3 and Table 4. From the table, the facility on the location (a, b) reduces the minimum flow but it is not earliest arrival flow, since 2 units of flow can be sent in $[0, 3]$. If we look for earliest arrival flow we loss the maximality in some interval of time. Again, facility on locations $\{(s, a), (s, b), (a, t), (b, t)\}$ will send 2 units of flow in $[0, 3]$ but could not send as much flow as we send by considering the location (a, b) from some points of time. For example, if we consider location (a, t) we can send 33 units of flow in $[0, 6]$ but by considering location (a, b), 35 units of flow can be sent in the same time.

4. CONTRAFLOWLOC PROBLEMS

In this section, we define both static and dynamic maximum ContraFlowLoc problems and present efficient algorithms to solve them. The proposed maximum ContraFlowLoc algorithms solve the problems in the same time complexity as the maximum FlowLoc problems but the value of flow can be double after contraflow reconfiguration.

4.1 Maximum Static ContraFlowLoc

The maximum static FlowLoc problem locates the facility on possible locations and maximize the flow on updated network whereas the contraflow allows arc reversals to improve the solution. The maximum FlowLoc problem is introduced by Hamacher et al. (2013) and the maximum contraflow problem by Rebennack, Arulselvan, Elefteriadou,

Table 2: Earliest arrival flow on network given in Figure 1 (iii).

Time Horizon	Paths	Length	Flow	Earliest Arrival Flow
3	$s-a-b-t$	3	2	2
4	$s-a-b-t$	3	2	
	$s-a-t$	4	3	7
5	$s-a-t$	4	3	
	$s-b-t$	5	10	20
6	$s-a-t$	4	5	
	$s-b-t$	5	10	35
7	$s-a-t$	4	5	
	$s-b-t$	5	10	50
8	$s-a-t$	4	5	
	$s-b-t$	5	10	65

and Pardolas (2010). We define the maximum ContraFlowLoc problem in static network. We also propose an efficient algorithm to solve the Problem 3.

Problem 3. Given network $N_{stat} = (V, A, u_e, s, t)$, locations L and size r_p of facility p, the maximum static ContraFlowLoc problem finds the maximum flow in $N_{stat}^{\text{loc}} = (V, A, u'_e, s, t)$ providing efficient location for the facility with arc reversals capability.

Algorithm 3: Maximum Static ContraFlowLoc

Input : A static network $N_{stat} = (V, A, u_e, s, t)$, locations L, size r_p of facility p
Output: Maximum contraflow value max_cont, location loc(p) of facility p in $N_{stat} = (V, A, u_e, s, t)$

1 Construct auxiliary $\tilde{N}_{stat} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, s, t)$ with new capacity $\tilde{u}(\tilde{e}) = u(\overrightarrow{e}) + u(\overleftarrow{e})$
2 Apply Algorithm 1 in $\tilde{N}_{stat} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, s, t)$ considering locations L
3 Decompose the maximum flow resulting from Step 2 into chain and cycle flows then remove the cycle flow
4 A arc $\overleftarrow{e} \in A$ is reversed if and only if the flow along $\overrightarrow{e} \in A$ is greater than $u'(\overrightarrow{e})$ or there is a non-negative flow along the arc $\overrightarrow{e} \notin A$

Theorem 3. Algorithm 3 solves the maximum ContraFlowLoc problem optimally.

Proof. The Steps 1 and 2 are feasible by definition. If $r_p > u_{\overrightarrow{e}}$ and $loc(p) = \overrightarrow{e}$ on the auxiliary network then the capacity of $\overleftarrow{e}$ is defined by $u'(\overleftarrow{e}) = u(\overleftarrow{e}) + u(\overrightarrow{e}) - r_p$, since the facility will occupy all capacity of $\overrightarrow{e}$ and remaining from the capacity of $\overleftarrow{e}$. Thus, Step 4 is well defined, i.e. not both arcs $\overrightarrow{e}$ and $\overleftarrow{e}$ have to be reversed at a time which is ensured by using Step 3. There is flow along $\overrightarrow{e}$ or $\overleftarrow{e}$ but not in both directions at the same time. Hence, the resulting flow from Step 4 is a feasible flow with arc reversals in $N_{stat} = (V, A, u_e, s, t)$.

Algorithm 1 has been used in Step 3 for the solution on $\tilde{N}_{stat} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, s, t)$ with new capacity $\tilde{u}(\tilde{e}) = u(\overrightarrow{e}) + u(\overleftarrow{e})$. In fact, any optimal solution to the FlowlLoc maximum flow problem with arc reversals on $N_{stat} = (V, A, u_e, s, t)$ is also a feasible solution to the maximum FlowLoc problem on $\tilde{N}_{stat} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, s, t)$ with new capacity $\tilde{u}(\tilde{e}) = u(\overrightarrow{e}) + u(\overleftarrow{e})$. Moreover, the amount of flow sent from s to t in Steps 2 is not changed in Step 4, the resulting solution is an optimal for the Problem 3. □

Corollary 2. Algorithm 3 finds optimal solution for the maximum ContraFlowLoc problem in polynomial time.

Proof. The direction of paths can be reversed using Step 3 in linear time thus the construction of auxiliary network takes linear time. Thus, the complexity of Algorithm 3 depends on the complexity of Step 2. Hence, Algorithm 3 finds optimal solution for the Problem 3 in polynomial time as designed by Theorem 3. □

Example 4. As in Example 1, let $L = \{(s, a), (s, b), (a, b), (a, t), (b, t)\}$ be the locations, nol $= 1$ and $r_p = 2$. Using contraflow approach on the same network, 15 units of flow can be send to the sink through the paths $s-a-t$ and $s-b-t$ with path flows 5 and 10, respectively. Thus, the optimal flow value is 15 with optimal location (a, b). Note that the optimal locations could be changed after using contraflow technique. However, the optimal location remains the same in this example.

Table 3: Earliest arrival FlowLoc on the network shown in Figure 1 (iii).

Locations	Time Horizon	Paths	Length	Flow	Total Earliest Arrival Flow	Remark
(s,a)	3	$s-a-b-t$	3	2	2	
	4	$s-a-b-t$ $s-a-t$	3 4	2 1	5	
	5	$s-a-t$ $s-b-t$	4 5	1 10	16	
	6	$s-a-t$ $s-b-t$	4 5	3 10	29	
	7	$s-a-t$ $s-b-t$	4 5	3 10	42	
	8	$s-a-t$ $s-b-t$	4 5	3 10	55	
(s,b)	3	$s-a-b-t$	3	2	2	
	4	$s-a-b-t$ $s-a-t$	3 4	2 3	7	
	5	$s-a-b-t$ $s-a-t$ $s-b-t$	3 4 5	2 3 8	20	
	6	$s-a-b-t$ $s-a-t$ $s-b-t$	3 4 5	2 3 8	33	
	7	$s-a-b-t$ $s-a-t$ $s-b-t$	3 4 5	2 3 8	46	
	8	$s-a-b-t$ $s-a-t$ $s-b-t$	3 4 5	2 3 8	59	
(a,t)	3	$s-a-b-t$	3	2	2	
	4	$s-a-b-t$ $s-a-t$	3 4	2 3	7	
	5	$s-a-t$ $s-b-t$	4 5	3 10	20	
	6	$s-a-t$ $s-b-t$	4 5	3 10	33	
	7	$s-a-t$ $s-b-t$	4 5	4 10	47	
	8	$s-a-t$ $s-b-t$	4 5	4 10	61	
(a,b)	4	$s-a-t$	4	5	5	
	5	$s-a-t$ $s-b-t$	4 5	5 10	20	
	6	$s-a-t$ $s-b-t$	4 5	5 10	35	
	7	$s-a-t$ $s-b-t$	4 5	5 10	50	
	8	$s-a-t$ $s-b-t$	4 5	5 10	65	Maximum but not earliest arrival

Table 4: Remaining part of Table 2.

Locations	Time Horizon	Paths	Length	Flow	Total Earliest Arrival Flow	Remark
(b,t)	3	$s-a-b-t$	3	2	2	
	4	$s-a-b-t$	3	2		
		$s-a-t$	4	3	7	
	5	$s-a-t$	4	3		
		$s-b-t$	5	8	18	
	6	$s-a-t$	4	5		
		$s-b-t$	5	8	31	
	7	$s-a-t$	4	5		
		$s-b-t$	5	8	44	
	8	$s-a-t$	4	3		
		$s-b-t$	5	10		
		$s-b-a-t$	8	2	59	

4.2 Maximum Dynamic ContraFlowLoc

The maximum dynamic ContraFlowLoc problem is introduced to locate the facility on given network such that the loss in maximum dynamic contraflow is minimum on given network, i.e., the dynamic flow on updated network is maximum. We also present an efficient algorithm for solving Problem 4.

Problem 4. Given a network $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$, locations L and size r_p of facility p, the maximum dynamic ContraFlowLoc problem on $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$ is to find the maximum dynamic flow on updated network $N^{\text{loc}}_{\text{dyna}} = (V, A, u'_e, \theta_e, T, s, t)$ with arc reversals allowed initially.

Algorithm 4: Maximum Dynamic ContraFlowLoc

Input : A dynamic network $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$, locations L, size r_p of facility p

Output: Maximum dynamic contraflow value max_dyna_cont, location $\text{loc}(p)$ of facility p in $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$

1 Construct auxiliary $\tilde{N}_{\text{dyna}} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \tilde{\theta}_{\tilde{e}}, T, s, t)$ with new capacity $\tilde{u}(\tilde{e}) = u(\overrightarrow{e}) + u(\overleftarrow{e})$ and symmetric transit time

2 Apply Algorithm 2 in $\tilde{N}_{\text{dyna}} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \tilde{\theta}_{\tilde{e}}, T, s, t)$ considering locations L

3 Decompose the maximum dynamic flow resulting from Step 3 into chain and cycle flows then remove the cycle flow

4 A arc $\overleftarrow{e} \in A$ is reversed if and only if the flow along $\overrightarrow{e} \in A$ is greater than $u'(\overrightarrow{e})$ or there is a non-negative flow along the arc $\overrightarrow{e} \notin A$

Theorem 4. The maximum dynamic ContraFlowLoc problem can be solved optimally in time $O(|L|(m \log n)(m + n \log n))$.

Proof. As in Theorem 3, Steps 1, 2 and 4 are feasible and well defined. The solution on $\tilde{N}_{\text{dyna}} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \tilde{\theta}_{\tilde{e}}, T, s, t)$ has been obtained using Algorithm 2 in Step 2. Indeed, any optimal solution to the problem with arc reversals on $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$ is also a feasible solution to the problem on $\tilde{N}_{\text{dyna}} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \tilde{\theta}_{\tilde{e}}, T, s, t)$. Every feasible flow of the maximum dynamic FlowLoc problem in $\tilde{N}_{\text{dyna}} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \tilde{\theta}_{\tilde{e}}, T, s, t)$ is feasible to the maximum dynamic ContraFlowLoc problem in $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$. The maximum dynamic contraflow with facility in $N_{\text{dyna}} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \tilde{\theta}_{\tilde{e}}, T, s, t)$ is not more than the maximum dynamic flow in $\tilde{N}_{\text{dyna}} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \tilde{\theta}_{\tilde{e}}, T, s, t)$ with facility. The maximum dynamic flow can be obtained by temporally repeating a path flow of a static graph, Ford and Fulkerson (1958). Since the amount of flow sent from s to d in Steps 3 is not changed in Step 4, the resulting FlowLoc solution is optimal for the Problem 3. As described in Corollary 2, the running time of the Algorithm 4 depends on the running time of Step 3, indeed in the running time of the Algorithm 2. Hence, the Algorithm 4 solves the Problem 4 in polynomial time, Corollary 1. □

Example 5. Apply the Algorithm 4 on the network of Figure 1(i) for $T = 8$, $r_p = 2$ and $L = \{(s,a), (s,b), (a,b), (a,t), (b,t)\}$. The optimal flow is 65 which can be send through the paths $s-a-t$and $s-b-t$ in different time intervals providing optimal location (a, b) (see Table 2 corresponding to location (a, b)). Here, the optimal flow value after contraflow is more than the double flow value before contraflow configuration.

The earliest arrival flow model has been integrated with contraflow to introduce earliest arrival contraflow by Dhungana, Pyakurel, Khadka, and Dhamala (2015). They have presented different algorithms to get exact and approximate solutions. According to the idea of FlowLoc presented in Section 3.2 and Example 3 we are not combining the earliest arrival contraflow model with location analysis to define the FlowLoc earliest arrival contraflow problem.

5. CONTRAFLOWLOC PROBLEMS FOR EXTENDED LOCATIONS

The FlowLoc problems with given possible facility locations are studied in Section 4. In this section we consider a FlowLoc problem, where a location will be feasible if its size is at least the respective arc capacity. The problem is modeled with and without the lane reversal permissibility. In both cases, efficient algorithms are presented. In this approach, the contraflow configuration increases the number of facilities with modified arc capacities.

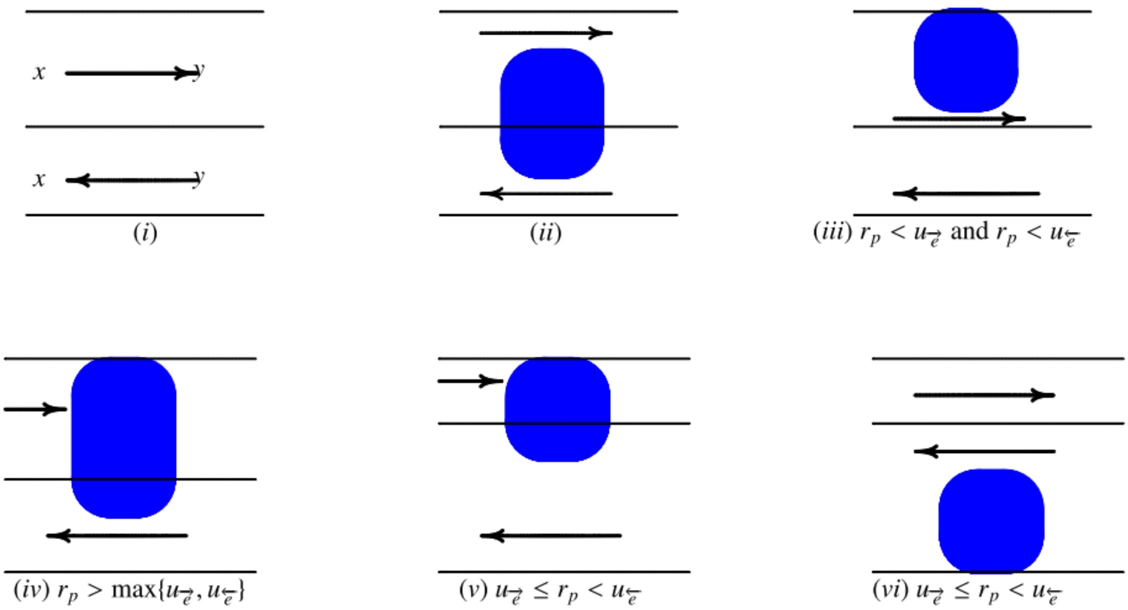

Figure 2: Variations of facility locations on auxiliary network with respect to the modified capacities of arcs and facility locations determined by a given facility size.

Consider Figure 2(i), which represents two way arcs (x, y) and (y, x). The maximum ContraFlowLoc solution locates a facility on the auxiliary network and finds an optimal flow in the network with reduced capacity by r_p. We exclude of locating a facility on the middle of a road as shown in Figure 2(iii). We consider the following cases.

1. Suppose the facility is located on (x, y) as shown in Figure 2(ii). If a maximum contraflow is in the direction of (x, y) and it is greater than $u'(x, y)$, then the direction of arc (y, x) has to be reversed.
2. Suppose the capacity of facility is greater than the capacities of both arcs (x, y) and (y, x), then the arc reversal problem reduces to the single arc problem. In this situation, the facility can be located on any side, for example as shown in Figure 2(iv).
3. Suppose the capacity of only one arc is sufficient to allocate facility but not its backward arc, then the problem becomes as shown in Figures 2(v) and 2(vi). The former is as in Figure 2(iv) and the latter is as in Figure 2(ii).

Example 6. Let $r_p = 3$, $u(a, b) = 2$ and $u(b, a) = 4$ on a single arc of a network. Suppose that we locate facility on (a, b) in the auxiliary network and a maximum-flow solution in the auxiliary network yields a flow 2 along the direction of (a, b). Then, the arc (b, a) should be reversed. But if we would have located the facility on (b, a) in the auxiliary network then there was no need to reverse the above arc to pass this flow value. it matches to the case of Figure 2 (v-vi) . Indicating that a wrong decision may lead to unnecessary arc reversals.

5.1 Maximum Static ContraFlowLoc

The maximum static ContraFlowLoc problem and its solution procedure has been presented in this section.

Problem 5. Given network $N_{stat} = (V, A, u_e, s, t)$, locations $L = \{e \in A \mid u_e \geq r_p\}$, where r_p denotes the size of facility p, the maximum static ContraFlowLoc problem is to find the maximum flow in $N_{stat}^{\text{loc}} = (V, A, u'_e, s, t)$ with arc reversal capability.

Algorithm 5: Maximum Static ContraFlowLoc with Extended Locations

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Input : A static network $N_{stat} = (V, A, u_e, s, t)$, locations $L = \{e \in A \mid u_e \geq r_p\}$ where r_p denotes the size of facility p

Output: Maximum contraflow value max_cont, location loc(p) of facility p in $N_{stat} = (V, A, u_e, s, t)$

1 Construct auxiliary network $\tilde{N}_{stat} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, s, t)$ with new capacity $\tilde{u}(\tilde{e}) = u(\overrightarrow{e}) + u(\overleftarrow{e})$ and locations $\tilde{L} = \{\tilde{e} \in \tilde{A} \mid \tilde{u}_{\tilde{e}} \geq r_p\}$

2 Apply Algorithm 1 in $\tilde{N}_{stat} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, s, t)$ considering modified set of locations $\tilde{L} = \{\tilde{e} \in \tilde{A} \mid \tilde{u}_{\tilde{e}} \geq r_p\}$

3 Decompose the maximum flow resulting from Step 2 into chain and cycle flows then remove the cycle flow

4 An arc $\overleftarrow{e} \in A$ is reversed if and only if the flow along $\overrightarrow{e} \in A$ is greater than $u(\overrightarrow{e})$ or there is a non-negative flow along the arc $\overrightarrow{e} \notin A$

5 If facility is located on $\tilde{e} \in \tilde{A}$ and the flow is positive along the direction of the arc $\overrightarrow{e} \in A$ then allocate the facility along the direction of $\overleftarrow{e} \in A$

Theorem 5. The maximum ContraFlowLoc problem with extended locations can be solved optimally in time $O(|\tilde{L}|nm)$.

Proof. The construction of possible location set $L = \{e \in A \mid u_e \geq r_p\}$ is feasible as the facility will host in any location if it has sufficient capacity. Step 1 constructs the auxiliary network with new location set $\tilde{L} = \{\tilde{e} \in \tilde{A} \mid \tilde{u}_{\tilde{e}} \geq r_p\}$ and capacity $\tilde{u}$ that are feasible. Step 2 is feasible as it calculates FlowLoc solution in the auxiliary network considering location $\tilde{L}$. Step 4 is well defined, i.e. not both arcs $\overrightarrow{e}$ and $\overleftarrow{e}$ have to be reversed at a time, this is ensured by the solution in auxiliary network. The resulting flow is decomposed into path and cycle flows and the cycle flows are removed in Step 3. So that there is flow along $\overrightarrow{e}$ or $\overleftarrow{e}$ but not in both directions at the same time. Hence, the resulting flow from Step 4 is a feasible flow with arc reversals in $N_{stat} = (V, A, u_e, s, t)$.

Optimal locations for the facility will be obtained in Step 2 for auxiliary network. If there are two way arcs on given network and the auxiliary arcs host the facility on auxiliary network then Step 5 decides the location of the facility in the given network according to the direction of flow provided from Step 3. Thus, Step 5 is feasible. Further proof of this theorem can be completed as in Theorem 3. ☐

Example 7. Let nol $= 1$ and $r_p = 4$ with ignored travel times in Figure 1 (i). Then, $L = \{(a, t), (s, b), (b, s), (b, t), (t, b)\}$ be the possible locations. Applying Algorithm 1, we can send 7 units of flow before contraflow. In this case, both locations (b, s) or $(t, b)\}$ are the optimal location as the resulting flow remains the same. The arc (b, a) or (a, b) was not able to host the facility due to low capacities.

But using contraflow approach, both arcs have been combined to increase the capacity. As a result, all arcs become feasible to host the facility with capacity 4. After locating facility on the arc, 15 units of flow can be send to the sink through the paths $s - a - t$ and $s - b - t$ with path flows 5 and 10, respectively. Thus, the optimal flow value is 15 with optimal location (a, b). Here, the facility on the network did not affect the optimality of the flow.

5.2 Maximum Dynamic ContraFlowLoc

The maximum dynamic ContraFlowLoc problem with capacity constrained locations and its procedure has been introduced in this section.

Problem 6. Given a network $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$, locations $L = \{e \in A \mid u_e \geq r_p\}$, where r_p denotes the size of facility p, the maximum dynamic ContraFlowLoc problem on $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$ is to find the maximum dynamic flow on updated network $N_{\text{dyna}}^{\text{loc}} = (V, A, u'_e, \theta_e, T, s, t)$ with arc reversals allowed initially.

Theorem 6. Algorithm 6 solves Problem 4 optimally in time $O(|\tilde{L}|(m \log n)(m + n \log n))$.

Proof. The feasibility of Algorithm 6 can be proved as in Theorem 5. The solutions on $\tilde{N}_{\text{dyna}} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \tilde{\theta}_{\tilde{e}}, T, s, t)$ with locations $\tilde{L} = \{\tilde{e} \in \tilde{A} \mid \tilde{u}_{\tilde{e}} \geq r_p\}$ have been calculated using Algorithm 2 in Step 2. The maximum dynamic flow obtained from Step 2 has been decomposed into temporally repeated path flows in Step 3 which is optimal flow for the Problem 6. A proof can be shown as in Theorem 4. ☐

Example 8. Consider the network of Figure 3 with values from Table 5 and $T = 9$. Without providing facility on network, maximum 61 and 137 units of flows can be send before and after contraflow, respectively. Our objective is to locate a facility on arc with minimum flow loss in the given network. Suppose nol $= 1$, $r_p = 10$ and $T = 9$. Before contraflow, possible locations are $L = \{(s,a),(s,d),(c,t)\}$. Construct the auxiliary network (shown in Figure 4) with values of Table 6. Then, possible locations set becomes $\tilde{L} = \{(s,a),(s,d),(c,t),(d,a),(d,e),(b,t)\}$. Algorithm 6 locates facility on given evacuation network of Figure 3 with minimum flow loss. Different maximum dynamic flow solutions after providing facility on different locations with different time horizons are shown in Table 7. Before contraflow, the arc (d,a) or (a,d) was not feasible to host the facility due to low capacities. But using contraflow approach, arc capacity has been increased by reversing the direction. As a result, the arc become feasible to host the facility and it is optimal location for the problem. If we consider different time horizon less than the transit time of longest path (longest in the sense of transit time) optimal location can be different for different time horizons, details is shown in Table 7.

Algorithm 6: Maximum Dynamic ContraFlowLoc with Extended Locations

Input : A dynamic network $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$, locations $L = \{e \in A \mid u_e \geq r_p\}$, size r_p of facility p

Output: Maximum dynamic contraflow value max_dyna_cont, location loc(p) of facility p in $N_{\text{dyna}} = (V, A, u_e, \theta_e, T, s, t)$

1. Construct auxiliary $\tilde{N}^{\text{loc}}_{\text{dyna}} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \tilde{\theta}_{\tilde{e}}, T, s, t)$ with new locations $\tilde{L} = \{\tilde{e} \in \tilde{A} \mid \tilde{u}_{\tilde{e}} \geq r_p\}$, capacity $\tilde{u}(\tilde{e}) = u(\overrightarrow{e}) + u(\overleftarrow{e})$ and symmetric transit time $\tilde{\theta}$
2. Apply Algorithm 2 in $\tilde{N}_{\text{dyna}} = (V, \tilde{A}, \tilde{u}_{\tilde{e}}, \tilde{\theta}_{\tilde{e}}, T, s, t)$ for locations $\tilde{L} = \{\tilde{e} \in \tilde{A} \mid \tilde{u}_{\tilde{e}} \geq r_p\}$
3. Construct temporally repeated flow resulting from Step 2
4. An arc $\overleftarrow{e} \in A$ is reversed if and only if the flow along $\overrightarrow{e} \in A$ is greater than $u(\overrightarrow{e})$ or there is a non-negative flow along the arc $\overrightarrow{e} \notin A$
5. If facility is located on $\tilde{e} \in \tilde{A}$ and the flow is positive along the direction of the arc $\overrightarrow{e} \in A$ then allocate the facility along the direction of $\overleftarrow{e} \in A$

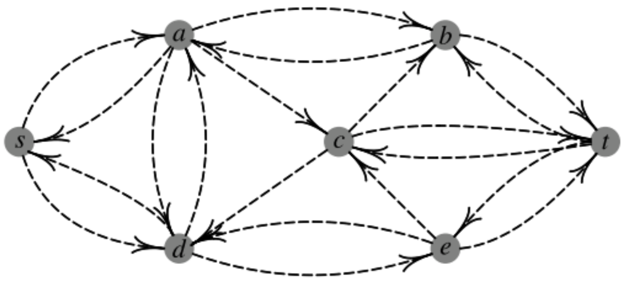

Figure 3: Evacuation network.

Table 5: Capacities and transit times of arcs corresponding to Figure 3.

Arcs	(s,a)	(a, s)	(s, d)	(d, s)	(d, a)	(a,d)	(a, c)	(c, d)	(a, b)	(b, a)
Capacities	10	5	10	7	7	4	3	5	5	3
Transit times	1	1	2	2	1	1	2	2	1	1
Arcs	(c, b)	(e, c)	(d, e)	(e, d)	(c, t)	(t, c)	(b, t)	(t, b)	(e, t)	(t, e)
Capacities	6	5	8	3	10	5	2	9	4	4
Transit times	1	1	2	2	1	1	1	1	3	3

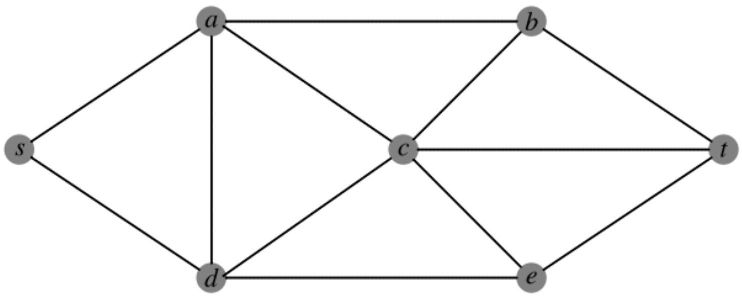

Figure 4: Auxiliary network corresponding to Figure 3.

Table 6: Capacities and transit times of arcs corresponding to Figure 4.

Auxiliary Arcs	(s,a)	(s, d)	(d, a)	(a, c)	(c, b)	(d, c)	(a, b)	(e, c)	(d, e)	(t, c)	(b, t)	(e, t)
Capacities	15	17	11	3	6	5	8	5	11	15	11	8
Transit times	1	2	1	2	1	2	1	1	2	1	1	3

Table 7: Variants of FlowLoc solutions for Network 4 considering the parameters of Table 5.

T	Before contraflow				After contraflow							Difference of Two Optimal Flows
	MDF after placing facility on			Location decisions	Maximum dynamic flow(MDF) after placing facility on						Location decisions	
	(s, a)	(s, d)	(c, t)		(s, a)	(s, d)	(c, t)	(d, a)	(d, e)	(b, t)		
3	0	2	2	(s, d) or (c, t)	5	8	8	8	8	1	(s, d) or (c, t) (d, a) or (d, e)	6
4	0	7	4	(s, d)	10	19	19	19	19	5	(s, d) or (c, t) (d, a) or (d, e)	11
5	2	12	6	(s, d)	23	35	32	31	35	14	(s, d) or (d, e)	23
6	12	21	8	(s, d)	44	56	48	52	53	28	(s, d)	35
7	22	30	14	(s, d)	66	78	72	79	70	48	(d, a)	49
8	32	39	20	(s, d)	88	100	96	106	87	68	(d, a)	67
9	42	48	26	(s, d)	110	122	120	133	104	88	(d, a)	85

6. CONCLUSIONS

In this paper, we studied the maximum FlowLoc and contraflow models on both static and dynamic networks. The maximum FlowLoc model locates facility in the possible locations and maximize the flow on the updated network whereas the contraflow approach reverses the direction of arcs to increase the flow value.

We introduced the ContraFlowLoc model that locates facility on given network and maximize the flow on updated network where arcs can be revered if the optimal solution can be improved from the given network. The maximum FlowLoc over continuous time, maximum static ContraFlowLoc and maximum dynamic ContraFlowLoc problems are studied in this paper. We also proposed efficient algorithms for the maximum dynamic FlowLoc, maximum ContraFlowLoc and maximum dynamic ContraFlowLoc in two-terminal networks for given possible locations as well as extended locations.

The extension of ContraFlowLoc problem to achieve different objectives such as lexicographically maximum ContraFlowLoc, lexicographically maximum dynamic ContraFlowLoc, earliest arrival transshipment ContraFlowLoc and quickest ContraFlowLoc for different networks with both given and arbitrary locations could be the further research interest.

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