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# Decision Making of Pricing Policies in Growing Market for Deteriorating Items

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**Abstract:** This paper studies the optimal pricing policy and optimal lot-sizing decision in growing market for deteriorating products. Pricing policy for growing market is entirely different than the decline market. Price reducing is the best strategy in decline market whereas price increasing may be the best-strategy in growing market. When a product is introduced in the market the demand of product increases slowly according to the performance of the product and depends on market conditions. In maturity stage, the demand is gradually increasing and thus it needs to obtain a pricing policy in growing market. This enables in deriving a new optimal lot-sizing and optimal price setting that maximizes the profit of the system. A numerical example presented to illustrate the proposed model and for practical use. The sensitivity analysis studied for key parameters and recommendations are made to be based on it.
**Keyword —** Inventory, Price sensitive demand, Dynamic pricing, Deterioration, Growing market, Optimization

## 1. INTRODUCTION

In a business regimentation the selling price of a product has a significant role to attract the clients. Nowadays many factors are affecting the demand of product as well as their price. Therefore, the dynamic price strategy is more suitable than static pricing policy. Thus, it has been increasingly adopted in many industries. Because of its potential application, the dynamic pricing control has also received considerable attention in the research community. In declining market, the price reducing is beneficial to the manufacturer and as well as for retailers, it also help to sustain the product in declining markets. In growing market the increasing price strategy is beneficial to the manufacturer and retailers both, thus the pricing strategy in growing market has seen interest for researchers. The pricing strategy is useful for every products and obviously, it needs to find out the best strategy depends on market conditions. Whitin (1955) comprehended the idea of price theory in inventory model, and thus research identifies the important concept for pricing policy. Lau and Lau (1988) incorporated the pricing strategy and compare it with constant pricing strategy. While the running of this path Abad (1996) developed an inventory model in which demand depends on pricing and lot-size also with partial backordering.

Baker and Urban (1988) established the economic order quantity model with a new idea of considering the demand which depends on price, time and inventory level. Arcelus and Srinivasan (1987) introduced a concept of discounts in price. Datta and Paul (2001) developed an economic order quantity model in which the demand rate is price and stock level dependent.

In real life problem demand is less price sensitive, if the price increases slowly, then there is no effect on demand, but if price increases suddenly, then the demand will decrease certainly. Datta and Paul (2001) analyzed a multi period economic order quantity model which is useful in the retail business. They proved how the selling price could be marked under a stock-dependent demand situation. Ray, Gerchak, and Jewkes (2005) considered selling price as a decision variable and established an analogous model, when price increases over working cost per unit. They suggested that in growing market price increasing is better than reducing price. Wen and Chen (2005) developed dynamic price model where time and stock is available to sell over a limited time on the internet. They had shown, that if the objective of a seller is to find a dynamic pricing policy, then they maximized the total expected revenue. Yeh, Chen, and Wang (2005) proposed an inspection model with discount factor for products having Weibull lifetime, in which they investigated the

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case where a field product can be detected only through inspections. By price changing strategy, You and Hsieh (2007) estimated the sum of all its component costs and optimize the profit function.

Moreover, by using the demand function of the stock level and the selling price Roy and Chaudhuri (2012) designed a model in which the production rate also depends on selling price per unit. They have considered deterioration as a constant function, extended the proposed demand function to time-price or quadratic-price dependent demand or stochastically fluctuating demand pattern. The two stage supply chain consisting one vendor and one buyer. Joglekar, Lee, and Farahani (2008) designed an inventory model for e-tales in which he proved that increasing price strategy is better than constant price strategy. The model is applicable for products that are more price sensitive. The model is illustrated with a numerical example and comparison of price and time. However, deterioration factor is not considered in this model. Sajadieh, Akbari, and Jokar (2009) proposed a model to find the relevant profit maximizing decision variable values. This model based on the joint total profit of both the vendor and the buyer. If buyer and vendor cooperate with each other and demand is more price sensitive than the model is more beneficial.

We observed that commodities have a seasonal track when the rate of demand depends upon price and time both. Banerjee and Sharma (2010) developed the model for seasonal product when the item deliberate has general the seasonal demand rate depends on time and price both. They have taken the price as a decision variable and shown the profit is concave function of selling price and time. Tripathy and Mishra (2010) presented an economic quantity model in which they have taken the demand rate as a function of selling price and holding cost which is time dependent. This model is deterministic inventory model for deterioration items. In this model they have used two cases, one with shortage and the other without shortage. They found that the optimum average profit in without shortage is more than that with shortage.

Sana (2010) designed an EOQ by assuming the demand function as price dependent and assumed that deterioration rate of defective item is time proportional. They developed this model over an infinite time horizon for perishable products. Sana (2011) developed an inventory model in which they have taken the demand function as quadratic function and the selling price increases in each cycle, but demand decreases quadratically with selling price. They studied a lot of changes in the rate of demand, if we take the demand function as the negative power function of price. So it is not possible in practical situations. Shah, Patel, and Lou (2011) extended the model of Soni and Shah (2008, 2009) model by assuming the selling price to be a decision variable and ending inventory to be positive or zero. They also assumed limited floor space, maximum profit and kept constant deteriorating, with this assumption they developed an algorithm to find out the optimal decision policy. Yang, Wee, Chung, and Huang (2013) designed a piecewise production inventory model for deteriorating products of price sensitive demand. They proved that the multiple production cycle is better than single production cycle. It was a good opportunity to raise product prices if demand parameters increase.

Pricing strategy play an essential role on deteriorating items and it is very important to look for appropriate optimal pricing for deteriorating items. In this direction Khedlekar and Shukla (2013) presented an economic production quantity model for deteriorating items, in this model he suggested subject to condition of disruption occurring at input level. At this time preservation technology is a very important for perishable type inventory. This idea provides the more effectiveness of business regimentation. Use of this item preservation concept Khedlekar, Shukla, and Namdeo (2016) developed an EOQ, in this model the demand of items is price sensitive and linearly decreases. He had shown that the profit is concave function of optimal selling price, they also find out the optimal selling price, the length of the replenishment cycle and the optimal preservation concept investment simultaneously. Khedlekar, Namdeo, and Chandel (2017) introduced an inventory model for deteriorating items, in this model they explored the preservation technology and exponentially declining demand. Khedlekar, Nigwal, and Tiwari (2017) addressed an inventory model on optimal pricing policy for manufacturer and retailer using item preservation technology for deteriorating items. They presented a continuous supply chain inventory model by optimizing the selling price of seasonal items.

Mishra (2016) proposed a single-manufacturer single-retailer inventory model by incorporating preservation technology cost for deteriorating items and determined optimal retail price, replenishment cycle and the cost of preservation technology. Duong, Wood, Wang, and Wang (2017) studied an supply chain model, in this model they identified how different variables influence backlogs during busy periods and service performance. Sekar and Uthayakumar (2018) presented an optimal inventory model for exponentially increasing demand with preservation technology. In this model they introduced four stages of production inventory model for deterioration items, they also considered three different stages of production and decline stage.

As in above literature, there is lack of contribution for price sensitive, time dependent, and stock dependent demand. Also there is lack of suggestions especially for growing market. In growing market price could be increased accordingly with increasing demand. So a motivation is derived to apply increasing price scenario, especially in a growing market. The seasonal products and daily uses products deteriorates, so we also incorporated the deterioration rate in the proposed model.

## 2. ASSUMPTIONS AND NOTATIONS

### Assumptions:

Following assumptions are made to propose the model

- The model is designed for finite time horizon,
- This model is designed for single item ,
- In this model the deterioration rate of defective item is considered,
- Shortages are not allowed,
- In each distinct cycle author take the selling price as decision variables,
- In each cycle the demand function is quadratic with selling price and depends on time.

### Notations:

$D(p_j, t)$ – Demand function varying quadratically with $p_j$ and dependent on time $t$,

$I_j(t)$ – On-hand inventory $t$ in $j^{th}$ cycle at time t,

$p_j$ – Selling price at $j$ period and it is decision variable,

$j$ – Index of period, where $j$ refers [($j$-1)$T$,$jT$],

$n$ – Number of cycle of different price,

$T$ – Length of each cycle, $T = \frac{L}{n}$,

$Q$ – Preliminary lot-size,

$L$ – Time compass,

$q_j$ – Inventory level at the starting of cycle $j$,

$C_h$ – Inventory holding cost unit per unit time,

$C_p$ – Purchasing cost per unit of item,

$\theta$ – Deterioration rate $(0 < \theta < 1)$,

$K$ – Price setting cost. This cost includes the resetting of the price label,

$a$ – Time scale parameter,

$\beta$ & $\gamma$ – Price sensitive parameters,

$\alpha$ – Initial constant demand,

$H_j(p_j, T)$ – Inventory holding cost at $j^{th}$ cycle,

$H(p_j, T)$ – Total inventory holding cost for n periods,

$R(p_j, T)$ – Sales revenue,

$\Pi(p_j, n)$ – Total profit.

## 3. THE MATHEMATICAL MODEL

Now we will develop an economic order quantity model for price sensitive seasonal items. Suppose an entrepreneur purchases/manufactured quantity Q of the seasonal product. The time horizon L is divided equal part as $T = \frac{L}{n}$. It is assumed that in growing market the demand of products increasing accordingly with price $p_j$, where $(j = 1, 2, ....n)$. So the entrepreneur decides to increase the selling price in different sub cycle accordingly with market conditions and demand. The demand of seasonal and household product follows a quadratic demand. It is assumed that $\alpha$ is the initial demand of the product, while $\beta$ and $\gamma$ are positive price sensitive parameters. Then the demand will be

$$D(p_j, t) = \alpha e^{at} - \beta p_j - \gamma {p_j}^2, \quad \alpha > 0, \quad \beta > 0, \quad \gamma > 0, \qquad (j = 1, 2, 3..., n) \tag{1}$$

The inventory depicted due to sum of the demand of products and deterioration. The governing differential equation in $j^{th}$ cycle is

$$\frac{d}{dt}I_j(t) = -\theta I_j(t) - D(p_j, t) \tag{2}$$

with initial conditions $I_j(0) = q_j$, $I_{j-1}(T) = I_j(0)$
Here, $q_j \quad (j = 1, 2..., n)$ is the stock level at the begining of cycle $j$.
from Eq. (1), we have

$$\frac{d}{dt}I_j(t) = -\Big(\alpha e^{at} - \beta p_j - \gamma {p_j}^2 + \theta I_j(t)\Big) \tag{3}$$

then the solution of above differential equation is

$$I_j(t) = q_j e^{-\theta t} + \frac{\alpha}{a+\theta}(e^{-\theta t} - e^{at}) + \frac{1}{\theta}(\beta p_j + \gamma {p_j}^2)(1 - e^{-\theta t}), \qquad j = 1, 2..., n \tag{4}$$

$$I_{j-1}(t) = q_{j-1} e^{-\theta t} + \frac{\alpha}{a+\theta}(e^{-\theta t} - e^{at}) + \frac{1}{\theta}(\beta p_{j-1} + \gamma {p_{j-1}}^2)(1 - e^{-\theta t}), \qquad j = 1, 2..., n \tag{5}$$

Now $I_{j-1}(T) = I_j(0)$

$$q_j = q_{j-1} e^{-\theta T} + \frac{\alpha}{a+\theta}(e^{-\theta T} - e^{aT}) + \frac{1}{\theta}(\beta p_{j-1} + \gamma {p_{j-1}}^2)(1 - e^{-\theta T}) \tag{6}$$

$$q_2 = q_1 e^{-\theta T} + \frac{\alpha}{a+\theta}(e^{-\theta T} - e^{aT}) + \frac{1}{\theta}(\beta p_1 + \gamma {p_1}^2)(1 - e^{-\theta T}) \tag{7}$$

if initial lot-size $q_1 = Q$
and let $e^{\theta T} = x$, then Eq. (7), leads to

$$q_2 = Qx^{-1} + \frac{\alpha}{a+\theta}(x^{-1} - e^{aT}) + \frac{1}{\theta}(\beta p_1 + \gamma {p_1}^2)(1 - x^{-1})$$

$$q_3 = q_2 x^{-1} + \frac{\alpha}{a+\theta}(x^{-1} - e^{aT}) + \frac{1}{\theta}(\beta p_2 + \gamma {p_2}^2)(1 - x^{-1})$$

$$q_3 = Qx^{-2} + \frac{\alpha}{a+\theta}(x^{-1} - e^{aT})(1 + x^{-1}) + \frac{\beta}{\theta}((1 - x^{-1})(x^{-1}p_1 + p_2) + \frac{\gamma}{\theta}(1 - x^{-1})(x^{-1}{p_1}^2 + {p_2}^2)$$

by mathematical induction

$$\begin{aligned} q_j = Qx^{j-1} &+ \frac{\alpha}{a+\theta}(x^{-1} - e^{aT})\sum_{i=2}^{j} x^{-(i-1)} + \frac{\beta}{\theta}(1 - x^{-1})\sum_{i=2}^{j} x^{-(j-i)} p_{i-1} \\ &+ \frac{\gamma}{\theta}(1 - x^{-1})\sum_{i=2}^{j} x^{-(j-i)} p_{i-1}^2, j \geq 2 \end{aligned} \tag{8}$$

$$q_n x^{-1} + \frac{\alpha}{a+\theta}(x^{-1} - e^{aT}) + \frac{1}{\theta}(\beta p_j + \gamma {p_j}^2)(1 - x^{-1}) = 0$$

$$q_n = \frac{\alpha}{a+\theta}(xe^{aT} - 1) - \frac{1}{\theta}(\beta p_n + \gamma {p_n}^2)(x - 1) \tag{9}$$

Eq. (8) and Eq. (9), lead to

$$\begin{aligned} Qx^{n-1} &+ \frac{\alpha}{a+\theta}(x^{-1} - e^{aT})\sum_{i=2}^{n} x^{-(i-1)} + \frac{\beta}{\theta}(1 - x^{-1})\sum_{i=2}^{n} x^{-(n-i)} p_{i-1} \\ &+ \frac{\gamma}{\theta}(1 - x^{-1})\sum_{i=2}^{n} x^{-(n-i)} p_{i-1}^2 = \frac{\alpha}{a+\theta}(xe^{aT} - 1) - \frac{1}{\theta}(\beta p_n + \gamma {p_n}^2)(x - 1) \end{aligned}$$

$$Q = \frac{\alpha x^{n-1}}{a+\theta}[(e^{aT} - x^{-1})\sum_{i=1}^{n-1} x^{-(i-1)} + xe^{aT} - 1] - \frac{\beta}{\theta}(1 - x^{-1})\sum_{i=1}^{n} x^i p_i - \frac{\gamma}{\theta}(1 - x^{-1})\sum_{i=1}^{n} x^i p_i^2 \tag{10}$$

The inventory carrying cost at $j^{th}$ cycle is

$$H_j(p_j, T) = C_h \int_0^T I_{j(t)} dt$$

$$H_j(p_j,T) = C_h \int_0^T \{q_j e^{-\theta t} + \frac{\alpha}{a+\theta}(e^{-\theta t} - e^{at}) + \frac{1}{\theta}(\beta p_j + \gamma {p_j}^2)(1-e^{-\theta t})\}dt$$

$$\begin{aligned} H_j(p_j,T) =& C_h \Bigg[ \frac{q_j}{\theta}(1-x^{-1}) + \frac{\alpha}{\theta(a+\theta)}(1-x^{-1}) + \frac{\alpha}{a(a+\theta)}(1-e^{aT}) \\ & + \frac{1}{\theta^2}(\beta p_j + \gamma {p_j}^2)(\theta T + x^{-1} - 1) \Bigg] \end{aligned} \tag{11}$$

Eq. (8) and Eq. (11), lead to

$$\begin{aligned} H_j(p_j,T) =& C_h \Bigg[ \left( \frac{(1-x^{-1})}{\theta} \right) \{Qx^{j-1} + \frac{\alpha}{a+\theta}(x^{-1} - e^{aT}) \sum_{i=2}^{j} x^{-(i-1)} \\ & + \frac{\beta}{\theta}(1-x^{-1}) \sum_{i=2}^{j} x^{-(j-i)} p_{i-1} + \frac{\gamma}{\theta}(1-x^{-1}) \sum_{i=2}^{j} x^{-(j-i)} p_{i-1}^2 \} \\ & + \frac{\alpha}{\theta(a+\theta)}(1-x^{-1}) + \frac{\alpha}{a(a+\theta)}(1-e^{aT}) + \frac{1}{\theta^2}(\beta p_j + \gamma {p_j}^2)(\theta T + x^{-1} - 1) \Bigg] \end{aligned} \tag{12}$$

The total inventory carrying cost for $n$ periods is

$$\begin{aligned} H(p_j,T) =& \sum_{j=1}^{n} C_h \Bigg[ \left( \frac{(1-x^{-1})}{\theta} \right) \{Qx^{j-1} + \frac{\alpha}{a+\theta}(x^{-1} - e^{aT}) \sum_{i=2}^{j} x^{-(i-1)} \\ & + \frac{\beta}{\theta}(1-x^{-1}) \sum_{i=2}^{j} x^{-(j-i)} p_{i-1} + \frac{\gamma}{\theta}(1-x^{-1}) \sum_{i=2}^{j} x^{-(j-i)} p_{i-1}^2 \} \\ & + \frac{\alpha}{\theta(a+\theta)}(1-x^{-1}) + \frac{\alpha}{a(a+\theta)}(1-e^{aT}) + \frac{1}{\theta^2}(\beta p_j + \gamma {p_j}^2)(\theta T + x^{-1} - 1) \Bigg] \end{aligned}$$

Putting the value of $Q$ then, we get

$$\begin{aligned} H(p_j,T) =& \frac{C_h}{\theta^2} \Bigg[ \theta(1-x^{-n}) \left( \frac{\alpha x^{n-1}}{a+\theta} [(e^{aT} - x^{-1}) \sum_{i=1}^{n-1} x^{-(i-1)} + x e^{aT} - 1 \right) \\ & - \beta(1-x^{-1}) \sum_{i=1}^{n} x^i p_i - \gamma(1-x^{-1}) \sum_{i=1}^{n} x^i p_i^2 + \frac{\alpha\theta}{a+\theta}(1-x^{-1})(x^{-1} - e^{aT}) \sum_{j=1}^{n} \sum_{i=2}^{j} x^{-(i-2)} \\ & + \beta(1-x^{-1})^2 \sum_{j=1}^{n} \sum_{i=2}^{j} x^{-(j-i)} p_{i-1} + \gamma(1-x^{-1})^2 \sum_{j=1}^{n} \sum_{i=2}^{j} x^{-(j-i)} p_{i-1}^2 \\ & + \frac{n\alpha\theta^2}{a+\theta} \left( \frac{(1-x^{-1})}{\theta} + \frac{(1-e^{aT})}{a} \right) + (\theta T + x^{-1} - 1) \sum_{j=1}^{n} (\beta p_j + \gamma {p_j}^2) \Bigg] \end{aligned}$$

After solving the above equation

$$
\begin{aligned}
H(p_j,T) = & \frac{C_h}{\theta^2}\Bigg[\frac{\alpha\theta}{a+\theta}(x^{n-1}-x^{-1})\left((e^{aT}-x^{-1})\sum_{j=1}^{n-1}x^{-(j-1)}+xe^{aT}-1\right)\\
& -\beta(1-x^{-1})\left\{\sum_{j=1}^{n}(x^j+1)p_j-2\sum_{j=1}^{n}x^{-(n-j)}p_j\right\}\\
& -\gamma(1-x^{-1})\left\{\sum_{j=1}^{n}(x^j+1)p_j^2-2\sum_{j=1}^{n}x^{-(n-j)}{p_j}^2\right\}\\
& +\frac{\alpha\theta}{a+\theta}(x^{-1}-e^{aT})\sum_{j=1}^{n}(1-x^{-(j-1)})\\
& +\frac{n\alpha\theta^2}{a+\theta}\left(\frac{(1-x^{-1})}{\theta}+\frac{(1-e^{aT})}{a}\right)+(\theta T+x^{-1}-1)\left\{\beta\sum_{j=1}^{n}p_j+\gamma\sum_{j=1}^{n}{p_j}^2\right\}\Bigg]
\end{aligned}
\tag{13}
$$

The sale revenue of $n$ cycles

$$
\begin{aligned}
R(p_j,T) &= \sum_{j=1}^{n}\left(\int_{(j-1)T}^{jT}D(p_j)(t)dt\right)p_j\\
&= \sum_{j=1}^{n}\left(\int_{(j-1)T}^{jT}(\alpha e^{at}-\beta p_j-\gamma {p_j}^2)dt\right)p_j\\
&= \sum_{j=1}^{n}\left[\frac{\alpha}{a}\left(e^{ajT}-e^{a(j-1)T}\right)p_j-(\beta {p_j}^2+\gamma {p_j}^3)T\right]
\end{aligned}
\tag{14}
$$

$$
R(p_j,T) = \frac{\alpha}{a}\sum_{j=1}^{n}p_j\left(e^{ajT}-e^{a(j-1)T}\right)-\left\{\beta\sum_{j=1}^{n}{p_j}^2+\gamma\sum_{j=1}^{n}{p_j}^3\right\}T
$$

Then the total net profit

$$
\Pi(p_j,n)=R(p_j,T)-H(p_j,T)-C_pQ-nK
$$

$$
\begin{aligned}
\Pi(p_j,n) = & \frac{\alpha}{a}\sum_{j=1}^{n}p_j\left(e^{ajT}-e^{a(j-1)T}\right)-\left\{\beta\sum_{j=1}^{n}{p_j}^2+\gamma\sum_{j=1}^{n}{p_j}^3\right\}T\\
& -\frac{C_h}{\theta^2}\Bigg[\frac{\alpha\theta}{a+\theta}(x^{n-1}-x^{-1})\left((e^{aT}-x^{-1})\sum_{j=1}^{n-1}x^{-(j-1)}+xe^{aT}-1\right)\\
& -\beta(1-x^{-1})\left\{\sum_{j=1}^{n}(x^j+1)p_j-2\sum_{j=1}^{n}x^{-(n-j)}p_j\right\}\\
& -\gamma(1-x^{-1})\left\{\sum_{j=1}^{n}(x^j+1)p_j^2-2\sum_{j=1}^{n}x^{-(n-j)}{p_j}^2\right\}+\frac{\alpha\theta}{a+\theta}(x^{-1}-e^{aT})\sum_{j=1}^{n}(1-x^{-(j-1)})\\
& +\frac{n\alpha\theta^2}{a+\theta}\left(\frac{(1-x^{-1})}{\theta}+\frac{(1-e^{aT})}{a}\right)+(\theta T+x^{-1}-1)\left\{\beta\sum_{j=1}^{n}p_j+\gamma\sum_{j=1}^{n}{p_j}^2\right\}\Bigg]\\
& -C_p\Bigg[\frac{\alpha x^{n-1}}{a+\theta}\left((e^{aT}-x^{-1})\sum_{i=1}^{n-1}x^{-(i-1)}+xe^{aT}-1\right)\\
& -\frac{\beta}{\theta}(1-x^{-1})\sum_{i=1}^{n}x^ip_i-\frac{\gamma}{\theta}(1-x^{-1})\sum_{i=1}^{n}x^ip_i^2\Bigg]-nK
\end{aligned}
\tag{15}
$$

**Theorem** A solution $p^*$ of equation $p^2 + \eta_1 p + \eta_2 = 0$, in the interval $(C_p, \infty)$ satisfying $\{-2\beta - 6\gamma p\} - \frac{C_h}{\theta^2}\left[2\gamma(\theta T + x^{-1} - 1) - (1 - x^{-1})\Big\{2\gamma(x^j + 1) - 4x^{-(n-j)}\Big\}\right] + \frac{2C_p}{\theta}\gamma x^j(1 - x^{-1}) < 0$, then $\Pi(p_j^*, n)$ has maximum value at p*, for fixed value of n.

**Proof** Differentiate partialy with respect to $p_j$ to Eq. (15), we have,

$$\begin{aligned}\frac{\partial \Pi(p_j, n)}{\partial p_j} = &\left[\frac{\alpha}{a}(e^{ajT} - e^{a(j-1)T}) - (2\beta p_j + 3\gamma {p_j}^2)T\right] - \frac{C_h}{\theta^2}\Bigg[-(1 - x^{-1})\beta\{(x^j + 1) - 2x^{-(n-j)}\} \\ &- (1 - x^{-1})\gamma\{2(x^j + 1)p_j - 4x^{-(n-j)}p_j\} + (\theta T + x^{-1} - 1)\{\beta + 2\gamma p_j\}\Bigg] \\ &+ C_p\left[\frac{\beta}{\theta}(1 - x^{-1})x^j + \frac{2\gamma}{\theta}(1 - x^{-1})x^j p_j\right]\end{aligned} \tag{16}$$

$$\frac{\partial^2 \Pi(p_j, n)}{\partial p_i \partial p_j} = 0, \quad \text{for} \quad i \neq j \tag{17}$$

$$\begin{aligned}\frac{\partial^2 \Pi(p_j, n)}{\partial {p_j}^2} = &- 2\beta - 6\gamma p_j - \frac{C_h}{\theta^2}\left[2\gamma(\theta T + x^{-1} - 1) - (1 - x^{-1})\Big\{2\gamma(x^j + 1) - 4x^{-(n-j)}\Big\}\right] \\ &+ \frac{2C_p}{\theta}\gamma x^j(1 - x^{-1})\end{aligned} \tag{18}$$

$$\begin{aligned}&Now, \\ &\frac{\partial \Pi(p_j, n)}{\partial p_j} = 0, \quad \text{imply} \quad {p_j}^2 + \eta_1 p_j + \eta_2 = 0\end{aligned} \tag{19}$$

$$\begin{aligned}&where \\ \eta_1 = &\frac{1}{3\gamma}\Bigg[2\beta T + \frac{C_h}{\theta^2}\Big\{-2(1 - x^{-1})\gamma(x^j + 1 - 2x^{-(n-j)}) \\ &+ 2\gamma(\theta T + x^{-1} - 1)\Big\} - 2\frac{C_p}{\theta}\gamma(1 - x^{-1})x^j\Bigg] \\ \eta_2 = &- \frac{\alpha}{3a\gamma}(e^{ajT} - e^{a(j-1)T}) - \frac{C_h}{3\gamma\theta^2}(1 - x^{-1})\beta(x^j + 1 - 2x^{-(n-j)}) \\ &- \beta\frac{C_h}{3\gamma\theta^2}(1 - x^{-1}) - \frac{C_p}{3\gamma\theta}\beta(1 - x^{-1})x^j + \beta\frac{C_h}{3\gamma\theta}T < 0\end{aligned}$$

Since $\eta_2 < 0$,
i.e ${\eta_1}^2 - 4\eta_2 > 0$, then Eq.(19) produce real roots.
If $\frac{\partial^2 \Pi(p_j, n)}{\partial {p_j}^2} < 0$
or $\{-2\beta - 6\gamma p\} - \frac{C_h}{\theta^2}\left[2\gamma(\theta T + x^{-1} - 1) - (1 - x^{-1})\Big\{2\gamma(x^j + 1) - 4x^{-(n-j)}\Big\}\right] + \frac{2C_p}{\theta}\gamma x^j(1 - x^{-1}) < 0$,
Then profit $\Pi(p_j, n)$ is maximum at $p^*$ for fixed *n*.

## 4. NUMERICAL EXAMPLES & SENSITIVITY ANALYSIS

To illustrate the proposed model we have presented two numerical examples

### 4.1 Numerical Example for Proposed Model

**Example 1.**To illustrate the proposed model we are considering a data set, $a = 0.0001$, $\alpha = 100$, $\beta = 5$, $\gamma = 0.005$, $\theta = 0.01$, $L = 120$, $C_h = .005$, $C_p = 4$, $K = 1000$, and the demand function $D(p_j, t) = \alpha e^{at} - \beta p_j - \gamma {p_j}^2$, on applying the output of proposed model and obtaining the solution procedure we get,

Put $n = 1, j = 1$ in Eq. (19), get, $\eta_1 = 6661.20$ & $\eta_2 = -94403.1$, then ${p_j}^2 + (6661.20)p_j - 94403.1 = 0$, get positive value $p_1 = 13.98$, in interval $[0, 120]$ by theorem,
$\frac{\partial \Pi(p_j,n)}{\partial p_j} = -0.5789 < 0$, and
$Q_1 = 6921$, by Eq. (10),
$R_1 = 4985.50$, by Eq. (14),
$\Pi(p_1, 1) = 19492.70$, by Eq. (15).
Put $n = 2, j = 1$ in Eq. (19), get, $\eta_1 = 6662.66$ & $\eta_2 = -86884.3$, then ${p_j}^2 + (6662.66)p_j - 86884.3 = 0$, get positive value $p_1 = 12.86$, in interval $[0, 60)$,
$\frac{\partial \Pi(p_j,n)}{\partial p_j} = -6.782 < 0$,
and $n = 2, j = 2$ in Eq. (19), get, $\eta_1 = 6659.51$ & $\eta_2 = -103053$, then ${p_j}^2 + (6659.51)p_j - 103053 = 0$, get positive value $p_2 = 15.27$, in interval $[60, 120]$ by theorem,
$\frac{\partial \Pi(p_j,n)}{\partial p_j} = -4.0171 < 0$,
$Q_2 = 6309$, by Eq. (10),
$R_2 = 48558.54$, by Eq. (14),
$\Pi(p_2, 2) = 21256.26$, by Eq. (15).
Put $n = 3, j = 1$ in Eq. (19), get, $\eta_1 = 6663$ & $\eta_2 = -85089.8$, then ${p_j}^2 + (6663)p_j - 85089.8 = 0$, get positive value $p_1 = 12.59$, in interval $[0, 40)$,
$\frac{\partial \Pi(p_j,n)}{\partial p_j} = -8.1830 < 0$,
and $n = 3, j = 2$ in Eq. (19), get, $\eta_1 = 6661.32$ & $\eta_2 = -9384.3$, then ${p_j}^2 + (6661.32)p_j - 9384.3 = 0$, get positive value $p_2 = 13.89$, in interval $[40, 80)$
$\frac{\partial \Pi(p_j,n)}{\partial p_j} = -7.206 < 0$,
and $n = 3, j = 3$ in Eq. (19), get, $\eta_1 = 6658.79$ & $\eta_2 = -106717$, then ${p_j}^2 + (6658.79)p_j - 106717 = 0$, get positive value $p_3 = 15.81$, in interval $[80, 120]$ by theorem,
$\frac{\partial \Pi(p_j,n)}{\partial p_j} = -5.748 < 0$,
$Q_3 = 6151$, by Eq. (10),
$R_3 = 48190.53$, by Eq. (14),
$\Pi(p_3, 3) = 17516.52$, by Eq. (15).

because the profit for $n^* = 2$, ($\Pi(p_j, n^*) = 21256.26$) is higher(Table 1) than for $n = 1$ $(19492.70)$ and for $n = 3$ $(17516.52)$, therefore the optimal selling price in first cycle [0, 60] is ${p_1}^* = 13.98$, and optimal selling price in second cycle (60, 120] (fig.1) is ${p_2}^* = 15.27$, then optimal profit ($\Pi(p_j, n) = 21256.26$) for $n = 2$ (fig. 3), is higher than profit for $n = 1$ and $n = 3$. Therefore in this case optimal number of price settings is 2.

**Table 1: Optimal solution of the numerical example**

| $n$ | $p_1$ | $p_2$ | $p_3$ | $Q$ | $R$ | $\Pi(n,p)$ |
|---|---|---|---|---|---|---|
| 1 | 13.98 | - | - | 6921 | 49857.50 | 19492.70 |
| 2* | **12.86** | **15.27** | **-** | **6309** | **48555.54** | **21256.26** |
| 3 | 12.59 | 13.89 | 15.81 | 6151 | 48190.53 | 17516.52 |

### 4.2 Numerical Example for *a* = 0

**Example 2.** If we put $a = 0$, in demand function of proposed model then the demand function will be $D(p_j, t) = \alpha - \beta p_j - \gamma {p_j}^2$, which does not depend on time t. Putting $\alpha = 150$, $\beta = 1.6$, $\gamma = 0.25$, $\theta = 0.01$, $L = 150$, $C_h = 0.0005$, $C_p = 4$, and $K = 600$, on applying the output of proposed model for $a = 0$ and obtain the solution procedure according to numerical Ex.4.1, Then the optimal solution is (Table 2) $n^* = 2$ and selling price are $p_1 = 14.27$, in interval $[0, 75)$ $p_2 = 16.17$, in interval $[75, 150]$ $\Pi({p_j}^*, n^*) = 60617$, $Q = 15099$, $R(p_j, T) = 122415$, $H(p_j, T) = 199.27$.

**Table 2: Optimal solution of the numerical Example 2**

| $n$ | $p_1$ | $p_2$ | $p_3$ | $Q$ | $R$ | $\Pi(p_j,n)$ |
|---|---|---|---|---|---|---|
| 1 | 15.19 | - | - | 15791 | 124018.6 | 59873.07 |
| 2* | 14.17 | 16.15 | - | 15099 | 122415.3 | 60617.88 |
| 3 | 14.05 | 15.03 | 16.58 | 14942 | 122041.8 | 60346.39 |

### 4.3 Sensitivity Analysis

To simulate the proposed model we use the original data as in Ex. 4.1, To examine the effect of various parameters on the output, we vary only one parameter and taking other parameter same.

**Table 3: Sensitivity Analysis for the numerical example 1**

| **Sensitivity analysis for parameter $\beta$&$\gamma$** | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\beta$ | n | $p_1$ | $p_2$ | $p_3$ | Q | $R$ | $\Pi(p_j, n)$ |
| 5 | 2 | 12.86 | 15.27 | - | 6309 | 48558.54 | 21256.26 |
| 6 | 2 | 11.25 | 13.66 | - | 5250 | 36435.99 | 13704.34 |
| 7 | 2 | 10.10 | 12.50 | - | 4194 | 27078.53 | 8901.05 |
| 8 | 2 | 09.22 | 11.62 | - | 3141 | 19469.00 | 02323.57 |
| $\gamma$ | n | $p_1$ | $p_2$ | $p_3$ | $Q$ | $R$ | $\Pi(p_j, n)$ |
| .0005 | 2 | 13.01 | 15.43 | - | 6339 | 19294.68 | 21840.04 |
| 0.005 | 2 | 12.86 | 15.27 | - | 6309 | 48558.54 | 21256.26 |
| 00.05 | 2 | 11.68 | 14.01 | - | 5943 | 7407879 | 16699.61 |
| 000.5 | 2 | 8.08 | 10.39 | - | 1378 | 9970.43 | 679.81 |

From table 3, we studied the variation in model outputs with respect to price sensitive parameter $\beta$. On increasing parameter $\beta$, the sales revenue decreases ( fig. 2) so that the product which is more price sensitive permits to change the price frequently. In order to this, the optimal net profit is highly sensitive on the parameter $\beta$. i.e the less price sensitive product gets high profit in decreasing $\beta$.

Moreover from table 3, we study the variation in model outputs with respect to the price sensitive parameter $\gamma$. On increasing price sensitive parameter $\gamma$, the optimal profit decreases. Similarly the profit decreases on increases the parameter $\beta$ (fig. 1). that is optimal numbers of price setting increases and profit decreases. It reveals that the optimal net profit is high for low price sensitive demand.

### 4.4 Numerical Example for higher value of *n*

**For n=3**

**Example 3.** The value of parameter in appropriate units are considered as follows $a = 0.0009$, $\alpha = 150$, $\beta = 8$, $\gamma = 0.0001$, $\theta = 0.009$, $L = 100$, $C_h = 0.009$, $C_p = 1$, $K = 1000$, and the demand function $D(p_j, t) = \alpha e^{at} - \beta p_j - \gamma {p_j}^2$. Then the required optimal solution is (see table 2) $n^* = 3, p_1^* = 8.77, p_2^* = 9.39, p_3^* = 10.17$, $Q^* = 9808$, $R^* = 57905.05$, $\Pi^*(p_j, n) = 44825.08$. i.e the optimal profit $\Pi^*(p_j, n) = 44825.08$, for $n = 3$, is higher than profit for $n = 1, 2, 4$ and 5.

**Table 4: Optimal solution of the numerical example 3**

| $n$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $Q$ | $R$ | $\Pi(n, p)$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 9.21 | - | - | - | - | 11704 | 58586.93 | 40533.33 |
| 2 | 8.85 | 9.90 | - | - | - | 10248 | 58108.86 | 44141.37 |
| 3* | **8.77** | **9.39** | **10.17** | - | - | **9808** | **57905.05** | **44825.08** |
| 4 | 8.73 | 9.17 | 9.69 | 10.32 | - | 9597 | 57732.31 | 44705.54 |
| 5 | 8.71 | 9.06 | 9.44 | 9.89 | 10.41 | 18125 | 86232.78 | 4202.81 |

**For n=4**

**Example 4.** The value of parameter in appropriate units are considered as follows $a = 0.001$, $\alpha = 150$, $\beta = 3$, $\gamma = 0.005$, $\theta = 0.01$, $L = 100$, $C_h = 0.001$, $C_p = 4$, $K = 1000$, and the demand function $D(p_j, t) = \alpha e^{at} - \beta p_j - \gamma {p_j}^2$. Then the required optimal solution is (see table 2) $n^* = 4, p_1^* = 27.35, p_2^* = 28.23, p_3^* = 30.23$, $p_3^* = 31.96$ $Q^* = 10535$, $R^* = 202685$, $\Pi^*(p_j, n) = 188156.16$. i.e the optimal profit $\Pi^*(p_j, n) = 188156.16$, for $n = 4$, is higher than profit for $n = 1, 2, 3$ and 5.

**Table 5: Optimal solution of the numerical example**

| $n$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $Q$ | $R$ | $\Pi(n, p)$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 29.58 | - | - | - | - | 12005 | 202854.9 | 153316.19 |
| 2 | 28.10 | 31.08 | - | - | - | 10962 | 202733 | 156697.60 |
| 3 | 27.67 | 29.42 | 31.66 | - | - | 10670 | 202699 | 156957.42 |
| 4* | **27.43** | **28.75** | **30.23** | **31.96** | - | **10535** | **202685** | **188156.16** |
| 5 | 27.35 | 28.34 | 29.46 | 30.15 | 32.15 | 18367 | 276235 | 35134.66 |

**Table 6: Sensitive analysis for the numerical example 4**

| Parameters change (in %) | | $n$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $Q$ | $\Pi(n,p)$ |
|---|---|---|---|---|---|---|---|---|
| $c_h$ | -50 | 4 | 27.41 | 28.62 | 30.03 | 31.69 | 10596 | 186041.22 |
| | -25 | 4 | 27.42 | 28.62 | 30.05 | 31.71 | 10588 | 186017.16 |
| | +25 | 4 | 27.45 | 28.67 | 30.08 | 31.74 | 10572 | 185969.47 |
| | +50 | 4 | 27.46 | 28.68 | 30.10 | 31.76 | 10563 | 185945.84 |
| $c_p$ | -50 | 2 | 26.10 | 27.67 | - | - | 12383 | 189595.32 |
| | -25 | 2 | 26.31 | 27.20 | - | - | 11539 | 187911.47 |
| | +25 | 4 | 28.01 | 29.38 | 31.01 | 32.92 | 10100 | 184313.69 |
| | +50 | 4 | 28.57 | 30.11 | 31.93 | 34.12 | 9620 | 182153.21 |
| $\theta$ | -50 | 4 | 27.31 | 28.16 | 29.05 | 30.01 | 8432 | 190076.56 |
| | -25 | 4 | 27.37 | 28.39 | 29.52 | 30.77 | 9454 | 188247.88 |
| | +25 | 4 | 27.51 | 28.94 | 30.70 | 32.91 | 11788 | 183175.58 |
| | +50 | 4 | 27.58 | 29.26 | 31.46 | 34.40 | 13029 | 179603.99 |
| $L$ | -50 | 4 | 27.13 | 27.70 | 28.30 | 28.95 | 4288 | 88365.56 |
| | -25 | 4 | 27.29 | 28.16 | 29.14 | 30.23 | 1757 | 136631.87 |
| | +25 | 4 | 27.59 | 29.17 | 31.01 | 33.47 | 14567 | 236079.85 |
| | +50 | 4 | 27.75 | 29.72 | 32.24 | 35.52 | 19044 | 286327.13 |

When purchasing cost $c_p$ increases, then optimal selling price $p_j$ $(j = 1, 2, 3..., n)$ also increase and $Q$ decreases. In this case, more inventory and lower purchasing cost cause more profit (see table 6).
Optimal selling price $p_j$ $(j = 1, 2, 3..., n)$ increase with increases to $\theta$ and to compensate more deteriorated units. In this case, stock level $Q$ and profit are lower due to higher deterioration rate (see table 6).

When time horizon increases then optimal selling price $p_j$ $(j = 1, 2, 3..., n)$ are increased. In this case, prices and stock level are adjusted so that the profit is maximized (see table 6). This model is also validated for higher value of price setting $n$ ($n = 3$ and $n = 4$) (see fig. 4 and 5).


**Fig 1: Total profit versus parameter $\gamma$**

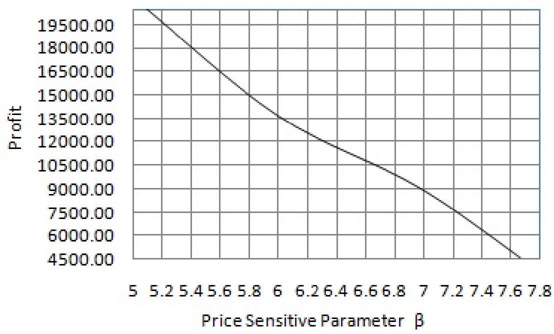


**Fig 2: Total profit versus parameter $\beta$**

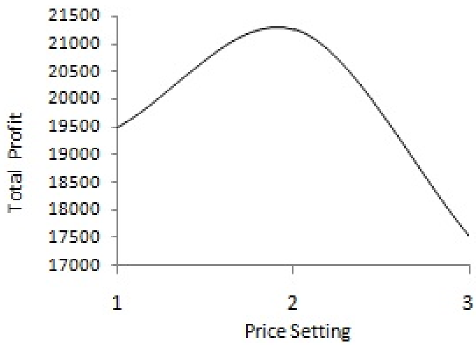


**Fig 3: Total profit versus price setting $n$**

 

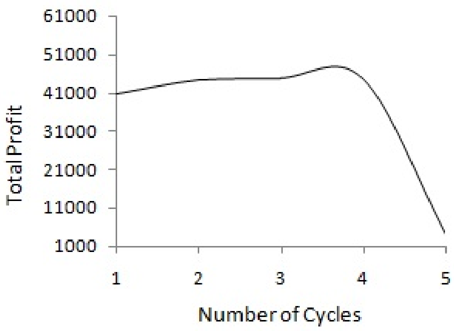


**Fig 4: Total profit versus price setting *n***

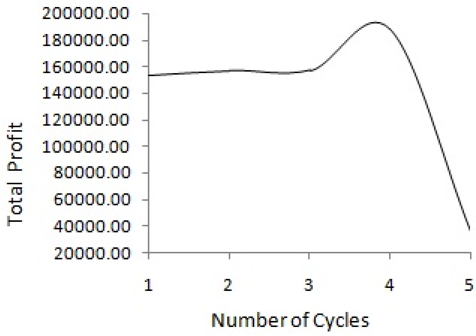


**Fig 5: Total profit versus price setting *n***

## 5. CONCLUSION

A dynamic pricing policy is developed for price sensitive product especially for a growing market. The sensitivity analysis reveals that high price sensitive product permits more optimal number of price settings. The pricing strategy in growing market is entirely different than the decline market, the increment of price in different subsequent interval are permitted to earn more and helps popularity of price sensitive product. We have considered the quadratic price sensitive demand which is more realistic and suitable for seasonal products. Numerical and computational study provides a better strategy for vendor, manufacturer and retailers. Proposed model is also validates for higher price settings.

We have considered constant deterioration rate so that one can extend the model by incorporating variable deterioration, probabilistic demand, variable holding and variable purchasing cost. Also one can formulate the proposed model in fuzzy environment.

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