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Cost Benefit and Mean Time to Failure Comparison between Network Flow Systems

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Abstract: The present paper deals with the mean time to failure and cost benefit comparison between dissimilar network flow systems. Five probabilistic models are discussed. It is assumed that failure and repair time of all units are assumed to be exponentially distributed. Explicit expressions for the mean time to failure are derived, examined and compared. The configurations are ranked based on their mean time to failure (MTTF). Numerical results for the mean time to failure and cost/benefit measure have been obtained for all configurations, where the benefit is mean time to failure.

Keyword — Mean time to failure, Network flow systems, Cost benefit

1. INTRODUCTION

Industrial and manufacturing systems are systems consisting of large components, units or subsystems arranged in either parallel, series, parallel-series or series-parallel systems. Example of these systems are feeding, crushing, refining, steam generation, evaporation, piston manufacturing, crystallisation unit in sugar plants, fertilizer plants, etc. These systems are usually studied with intention to the increase their reliability measures in terms of mean time to failure (MTTF), busy period of repairman, availability, generated revenue as well as profit. MTTF play a significant role in the overall performance of system in ensuring quality of products. Achieving a high or required level of MTTF is often an essential requisite. Because of its importance in power plants, manufacturing systems, and industrial systems, several results on mean time to system failure and cost analysis have widely been analysed in the literature. Study of MTSF and Cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection was performed in Bhardwaj and Malik (2010). Cost benefit analysis series systems with mixed was analysed by Wang and Kuo (2000) in which the optimal configuration in the study depend only on particular parameter using cost/MTTF and depend on the other parameter using cost/availability. Wang and Pearn (2003) studied cost benefit analysis of series systems with warm standby component where the optimality among the configuration depend also on a particular parameter using both cost/MTTF and cost/availability. Wang, Liu, and Pearn (2005) studied cost benefit of series system with warm standby components and general repairs. Wang, Hsieh, and Liou (2006) presented cost benefit of series systems with cold standby components and a repairable service station in which the optimal configuration depend on some parameters using cost/MTTF and cost/availability. Comparison of MTSF of 2-out-of-5 warm standby repairable system with replacement and without replacement at common cause failure was captured in Yusuf and Gimba (2013). Yusuf, Yusuf, and Lawan (2014) presented mean time to system failure modelling of a system connected supporting device for operation and a repairable service station. Some literatures on reliability and cost benefit analysis above have shown that the optimality among the systems depend on a particular parameter not on the entire parameters. Reliability and cost benefit models should be developed to show that optimality among the systems/configurations is uniform across all the parameters. The problem considered in this paper is different from the work discussed authors above.

The goal of this paper is threefold. The first goal is, to develop explicit expressions describing system mean time to failure. The second is to compare the five configurations in terms of their mean time to failure. The third is to capture the effect of system parameters on the cost benefit function.

The rest of the paper is organized as follows. Section 2 gives a description of the system. Section 3 deals with derivation of the models. The results of comparison and numerical simulations are presented and discussed in Section 4. The paper is concluded in Section 5.

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2. DESCRIPTION OF THE SYSTEMS EQUATIONS

In the present paper, five dissimilar network flow systems are considered as follows:

Configuration I consists of two subsystems A and B arranged in parallel with two unit each. From Configuration I, signal from the source is received by units in subsystem A. When one of the primary units fails, the standby unit in subsystem B is switched on to assume the role of the failed primary unit. The system failed when more than two units have failed.

Configuration II consists of three subsystems A, B and C. With subsystems A and B in series and parallel to subsystem C. Subsystem A has one unit while subsystems B and C having two units each. From Configuration II, signal is received from the source by unit in subsystem A which is distributed to one of the primary units in subsystem B. When one of the primary units fails, the standby unit in subsystem B is switched to operation to assume the role of the failed primary unit. At the failure of unit in subsystem A, subsystem B ceased to work and one of the primary unit in subsystem C is switched to operation to assume the role of subsystem A. The system failed when both units A and C have failed.

Configuration III consists of three subsystems A, B and C with subsystem A and B in parallel and series to subsystem C. Each subsystem has two units. from Configuration III, signal is received by primary units in subsystem A and is distributed any of the primary unit in subsystem C. Whenever of the primary unit in subsystem C fails, the standby unit in subsystem C is switched on to assume the role of failed primary unit. At the failure of a unit in subsystem A, units in subsystem B are switched on to assume the role of units in subsystem A. The system fails when both units in subsystems A and B and units in subsystem C have failed.

Configuration IV is parallel series system with subsystem A having units A1 and A2 in series and parallel to subsystems B and C. Subsystems B and C are in series and have two units each. In Configuration IV, Signal is received initially by unit by the two units in subsystem A. At the failure of subsystem A, both subsystems B and C are switched on, and the signal is then received by one of the primary unit in subsystem B and is then distributed to one of the primary unit in subsystem C. At the failure of either primary unit in subsystem B and C, the standby unit is switched to assume the role of failed primary unit. The system failed when subsystem A, and any of subsystem B and C have failed.

Configuration V consists of the three subsystems A, B and C in series. Subsystem A has two units A1 and A2 in cold standby, subsystem B consists of 2-out-of -3 units while subsystem C consists of one unit. In configuration V, signal is received from the source by any unit in subsystem A which is then distributed to two of the primary units in subsystem B. When one of the primary units fails, the standby unit is switched on to assume the role of the failed primary unit. System failure occurs when any of the three subsystems failed. It is assume that switching from standby to operation is perfect. It is also assume that the switchover time to operation is instantaneous.

Each of the primary units in any configuration fails independently of the state of the others and has an exponential failure time with parameter α_0 and is replace with cold standby unit if available while the failed unit is immediately sent to repair and the time to repair is exponential with parameter β_0. All failures are assumed to be repairable.

3. MEAN TIME TO FAILURE MODELS FORMULATION

3.1 Model formulation for Configuration 1

Let $p_i(t)$ be the probability that the system is in state $i$at time$t$. Let $P(t) = [p_0(t), p_1(t), p_2(t), ..., p_{12}(t)]$ be the probability row vector. Using the approach adopted in Wang and Kuo (2000) and Wang et al. (2006), the corresponding set of differential-difference equations for configuration I can be expressed as follows:

$$\dot{P} = Q_1 P, \tag{1}$$

with initial conditions

$$P_k(0) = \begin{cases} 1, & k = 0 \\ 0, & k = 1, 2, 3, ..., 12, \end{cases} \tag{2}$$

where $Q_1 = (a_{ij})$, $i, j = 1, 2, 3, ..., 12$, and $a_{11} = -2\alpha_0$, $a_{22} = a_{33} = a_{55} = a_{77} = -(2\alpha_0 + \beta_0)$, $a_{44} = a_{66} = -(\alpha_0 + \beta_0)$, $a_{88} = a_{99} = a_{10,10} = a_{12,12} = -\beta_0$, $a_{11,11} = -2\beta_0$, $a_{12} = a_{13} = a_{24} = a_{25} = a_{36} = a_{37} = a_{48} = a_{59} = a_{5,10} = a_{6,11} = a_{7,11} = a_{7,12} = -\beta_0$, $a_{21} = a_{31} = a_{42} = a_{52} = a_{63} = a_{73} = a_{84} = a_{95} = a_{10,5} = a_{11,6} = a_{11,7} = a_{12,7} = \alpha_0$, the rest equal to zero.

The time dependent analytic solution is difficult to obtain. So that we calculate the MTTF by taking the transpose matrix of Q_1 and delete the rows and columns for the absorbing state and designation the new matrix by M_1 following Wang and Kuo (2000), Wang and Pearn (2003), Wang et al. (2006). The explicit expression for the MTTF for

Configuration 1

$$MTTF_1 = \left[T_{P(0)\to P(absorbing)}\right] = P_1(0)(-M_1{}^{-1})[1,1,1,1,1,1,1]^T = \frac{13\alpha_0^3 + 15\alpha_0^2\beta_0 + 5\alpha_0\beta_0^2 + \beta_0^3}{2\alpha_0^3\left(4\alpha_0 + 3\beta_0\right)}, \tag{3}$$

where $P_1(0) = [1,0,0,0,0,0,0]$ and

$$M_1 = \begin{pmatrix} -2\alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ \beta_0 & -(2\alpha_0+\beta_0) & 0 & \alpha_0 & \alpha_0 & 0 & 0 \\ \beta_0 & 0 & -(2\alpha_0+\beta_0) & 0 & 0 & \alpha_0 & \alpha_0 \\ 0 & \beta_0 & 0 & -(\alpha_0+\beta_0) & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & -(2\alpha_0+\beta_0) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(\alpha_0+\beta_0) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(2\alpha_0+\beta_0) \end{pmatrix}.$$

3.2 Model formulation for Configuration 2

For the analysis of configuration II, the initial conditions are

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \cdots, P_{10}(0)\right] = [1,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P} = Q_2 P, \tag{4}$$

where $Q_2 = (b_{ij})$, $i,j = 1,2,3,...,11$ and $b_{11} = -2\alpha_0$, $b_{22} = b_{44} = b_{66} = b_{77} = b_{88} = b_{99} = -(\alpha_0+\beta_0)$, $b_{33} = -(2\alpha_0+\beta_0)$, $b_{55} = b_{10,10} = b_{11,11} = -\beta_0$, $b_{12} = b_{13} = b_{24} = b_{36} = b_{37} = b_{45} = b_{68} = b_{79} = b_{8,10} = b_{9,11} = \beta_0$, $b_{21} = b_{31} = b_{42} = b_{54} = b_{63} = b_{73} = b_{86} = b_{97} = b_{10,8} = b_{11,9} = \alpha_0$, the rest equal to zero.

The explicit expression for the MTTF for Configuration 2 is obtained as

$$MTTF_2 = \left[T_{P(0)\to P(absorbing)}\right] = P_2(0)(-M_2{}^{-1})[1,1,1,1,1,1,1,1]^T = \frac{11\alpha_0^5 + 14\alpha_0^4\beta_0 + 15\alpha_0^3\beta_0^2 + 10\alpha_0^2\beta_0^3 + 4\alpha_0\beta_0^4 + \beta_0^5}{\alpha_0^3\left(4\alpha_0^3 + 3\alpha_0^2\beta_0 + 3\alpha_0\beta_0^2 + \beta_0^3\right)}, \tag{5}$$

where $P_2(0) = [1,0,0,0,0,0,0,0]$ and

$$M_2 = \begin{pmatrix} -2\alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -(\alpha_0+\beta_0) & 0 & \alpha_0 & 0 & 0 & 0 & 0 \\ \beta_0 & 0 & -(2\alpha_0+\beta_0) & 0 & \alpha_0 & \alpha_0 & 0 & 0 \\ 0 & \beta_0 & 0 & -(\alpha_0+\beta_0) & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & 0 & -(\alpha_0+\beta_0) & 0 & \alpha_0 & 0 \\ 0 & 0 & \beta_0 & 0 & 0 & -(\alpha_0+\beta_0) & 0 & \alpha_0 \\ 0 & 0 & 0 & 0 & \beta_0 & 0 & -(\alpha_0+\beta_0) & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -(\alpha_0+\beta_0) \end{pmatrix}.$$

3.3 Model formulation for Configuration 3

For the analysis of configuration 3, the initial conditions are

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), \cdots, P_{10}(0)\right]$$

The differential equations are expressed in the form

$$\dot{P} = Q_3 P, \tag{6}$$

where $Q_3 = (n_{ij})$, $i,j = 1,2,3,...,11$ and $n_{11} = -2\alpha_0$, $n_{22} = n_{33} = n_{44} = n_{88} = -(2\alpha_0+\beta_0)$, $n_{55} = n_{66} = n_{77} = n_{99} = n_{10,10} = n_{11,11} = -\beta_0$, $n_{12} = n_{13} = n_{24} = n_{25} = n_{37} = n_{38} = n_{39} = n_{46} = n_{8,10} = n_{8,11} = \beta_0$, $n_{21} = n_{31} = n_{42} = n_{52} = n_{64} = n_{74} = n_{83} = n_{93} = n_{10,8} = n_{11,8} = \alpha_0$, the rest equal to zero.

The explicit expression for the MTTF for Configuration 3 is obtained as

$$MTTF_3 = \left[T_{P(0)\to P(absorbing)}\right] = P_3(0)(-M_3{}^{-1})[1,1,1,1,1,1,1,1]^T = \frac{9\alpha_0^2 + 5\alpha_0\beta_0 + \beta_0^2}{\alpha_0^2\left(4\alpha_0 + \beta_0\right)}, \tag{7}$$

where $P_3(0)=[1,0,0,0,0]$ and

$$M_3=\begin{pmatrix} -2\alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 \\ \beta_0 & -(2\alpha_0+\beta_0) & 0 & \alpha_0 & 0 \\ \beta_0 & 0 & -(2\alpha_0+\beta_0) & 0 & \alpha_0 \\ 0 & \beta_0 & 0 & -(2\alpha_0+\beta_0) & 0 \\ 0 & 0 & \beta_0 & 0 & -(2\alpha_0+\beta_0) \end{pmatrix}.$$

3.4 Model formulation for Configuration 4

For the analysis of configuration 4, the initial conditions are

$$P(0)=[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),\cdots,P_{11}(0)]=[1,0,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P}=Q_4P, \tag{8}$$

where $Q_4=(m_{ij}),\ i,j=1,2,3,...,12$ and $m_{11}=-\alpha_0$, $m_{ii}=-(2\alpha_0+\beta_0),\ i=2,3,4,5,6$, $m_{ii}=-\beta_0$, $i=7,8,9,10,11,12$, $m_{12}=m_{23}=m_{24}=m_{35}=m_{37}=m_{46}=m_{48}=m_{59}=m_{5,10}=m_{6,11}=m_{6,12}=\beta_0$, $m_{21}=m_{32}=m_{42}=m_{53}=m_{64}=m_{73}=m_{84}=m_{95}=m_{10,5}=m_{11,6}=m_{12,6}=\alpha_0$, the rest equal to zero.

The explicit expression for the MTTF for Configuration 4 is obtained as

$$\begin{aligned} MTTF_4=\left[T_{P(0)\to P(absorbing)}\right] &=P_4(0)(-M_4{}^{-1})[1,1,1,1,1,1]^T \\ &=\frac{72\alpha_0^5+101\alpha_0^4\beta_0+66\alpha_0^3\beta_0^2+26\alpha_0^2\beta_0^3+7\alpha_0\beta_0^4+\beta_0^5}{\alpha_0^3(32\alpha_0^3+32\alpha_0^2\beta_0+12\alpha_0\beta_0^2+\beta_0^3)}, \end{aligned} \tag{9}$$

where $P_4(0)=[1,0,0,0,0,0]$ and

$$M_4=\begin{pmatrix} -\alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ \beta_0 & -(2\alpha_0+\beta_0) & \alpha_0 & \alpha_0 & 0 & 0 \\ 0 & \beta_0 & -(2\alpha_0+\beta_0) & 0 & \alpha_0 & 0 \\ 0 & \beta_0 & 0 & -(2\alpha_0+\beta_0) & 0 & \alpha_0 \\ 0 & 0 & \beta_0 & 0 & -(2\alpha_0+\beta_0) & 0 \\ 0 & 0 & \beta_0 & \beta_0 & 0 & -(2\alpha_0+\beta_0) \end{pmatrix}.$$

3.5 Model formulation for Configuration 5

For the analysis of Configuration 5, the initial conditions are

$$P(0)=[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),\cdots,P_{11}(0)]=[1,0,0,0,0,0,0,0,0,0,0,0]$$

The differential equations are expressed in the form

$$\dot{P}=Q_5P, \tag{10}$$

where $Q_5=(t_{ij})$ and $t_{11}=-3\alpha_0$, $t_{22}=t_{33}=-(3\alpha_0+\beta_0)$, $t_{ii}=-\beta_0$, $i=4,5,6,8,9,10,11,12$, $t_{77}=-(3\alpha_0+2\beta_0)$, $t_{12}=t_{13}=t_{14}=t_{25}=t_{26}=t_{27}=t_{37}=t_{38}=t_{39}=t_{7,11}=t_{7,12}=\beta_0$, $t_{21}=t_{31}=t_{21}=t_{31}=t_{41}=t_{52}=t_{62}=t_{72}=t_{73}=t_{83}=t_{93}=t_{10,7}=t_{11,7}=t_{12,7}=\alpha_0$, the rest equal to zero.

The explicit expression for the MTTF for Configuration 5 is obtained as

$$\begin{aligned} MTTF_5=\left[T_{P(0)\to P(absorbing)}\right] &=P_5(0)(-M_5{}^{-1})[1,1,1,1]^T \\ &=\frac{17\alpha_0^2+11\alpha_0\beta_0+2\beta_0^2}{\alpha_0(27\alpha_0^2+15\alpha_0\beta_0+2\beta_0^2)}, \end{aligned} \tag{11}$$

where $P_5(0)=[1,0,0,0]$ and

$$M_5=\begin{pmatrix} -3\alpha_0 & \alpha_0 & \alpha_0 & 0 \\ \beta_0 & -(3\alpha_0+\beta_0) & 0 & \alpha_0 \\ \beta_0 & 0 & -(3\alpha_0+\beta_0) & \alpha_0 \\ 0 & \beta_0 & \beta_0 & -(3\alpha_0+2\beta_0) \end{pmatrix}.$$

All figures should be positioned at the top of the page when possible. All figures should be numbered consecutively and captioned; the caption should be centered under the figure as shown in Figure 1. All text within the figure should be no smaller than 9pt. There should be a minimum of two line spaces between figures and text.

4. COMPARISON BETWEEN THE CONFIGURATIONS

4.1 Analytical Comparison

In this section, the configurations are compared analytically in terms of their availability and mean time to failure to determine the optimal configuration by taking the difference between the configurations $\forall\alpha_0,\ \beta_0 > 0$ using MAPLE software package.

$$MTTF_2 - MTTF_1 = \frac{32\alpha_0^6 + 84\alpha_0^5\beta_0 + 111\alpha_0^4\beta_0^2 + 98\alpha_0^3\beta_0^3 + 59\alpha_0^2\beta_0^4 + 24\alpha_0\beta_0^5 + 5\beta_0^6}{2\alpha_0^3\,(4\alpha_0^3 + 3\alpha_0^2\beta_0 + 3\alpha_0\beta_0^2 + \beta_0^3)\,(4\alpha_0 + 3\beta_0)}$$

$$MTTF_2 - MTTF_3 = \frac{48\alpha_0^6 + 92\alpha_0^5\beta_0 + 109\alpha_0^4\beta_0^2 + 84\alpha_0^3\beta_0^3 + 44\alpha_0^2\beta_0^4 + 15\alpha_0\beta_0^5 + 2\beta_0^6}{2\alpha_0^3\,(4\alpha_0^3 + 3\alpha_0^2\beta_0 + 3\alpha_0\beta_0^2 + \beta_0^3)\,(4\alpha_0 + \beta_0)}$$

$$MTTF_2 - MTTF_4 = \frac{64\alpha_0^7 + 212\alpha_0^6\beta_0 + 341\alpha_0^5\beta_0 + 346\alpha_0^4\beta_0^3 + 250\alpha_0^3\beta_0^4 + 127\alpha_0^2\beta_0^5 + 4\alpha_0\beta_0^6 + 7\beta_0^7}{\alpha_0^3\,(4\alpha_0^3 + 3\alpha_0^2\beta_0 + 3\alpha_0\beta_0^2 + \beta_0^3)\,(32\alpha_0^3 + 32\alpha_0^2\beta_0 + 12\alpha_0\beta_0^2 + \beta_0^3)}$$

$$MTTF_2 - MTTF_5 = \frac{229\alpha_0^7 + 475\alpha_0^6\beta_0 + 587\alpha_0^5\beta_0 + 484\alpha_0^4\beta_0^3 + 273\alpha_0^3\beta_0^4 + 105\alpha_0^2\beta_0^5 + 23\alpha_0\beta_0^6 + 2\beta_0^7}{\alpha_0^3\,(4\alpha_0^3 + 3\alpha_0^2\beta_0 + 3\alpha_0\beta_0^2 + \beta_0^3)\,(27\alpha_0^2 + 15\alpha_0\beta_0 + 2\beta_0^2)}$$

$$MTTF_4 - MTTF_1 = \frac{128\alpha_0^6 + 312\alpha_0^5\beta_0 + 326\alpha_0^4\beta_0^2 + 218\alpha_0^3\beta_0^3 + 105\alpha_0^2\beta_0^4 + 33\alpha_0\beta_0^5 + 5\beta_0^6}{2\alpha_0^3\,(32\alpha_0^3 + 32\alpha_0^2\beta_0 + 12\alpha_0\beta_0^2 + \beta_0^3)\,(4\alpha_0 + 3\beta_0)}$$

$$MTTF_4 - MTTF_3 = \frac{256\alpha_0^6 + 472\alpha_0^5\beta_0 + 4186\alpha_0^4\beta_0^2 + 238\alpha_0^3\beta_0^3 + 91\alpha_0^2\beta_0^4 + 21\alpha_0\beta_0^5 + 2\beta_0^6}{2\alpha_0^3\,(32\alpha_0^3 + 32\alpha_0^2\beta_0 + 12\alpha_0\beta_0^2 + \beta_0^3)\,(4\alpha_0 + \beta_0)}$$

$$MTTF_4 - MTTF_5 = \frac{1400\alpha_0^7 + 2911\alpha_0^6\beta_0 + 2821\alpha_0^5\beta_0 + 1681\alpha_0^4\beta_0^3 + 676\alpha_0^3\beta_0^4 + 182\alpha_0^2\beta_0^5 + 29\alpha_0\beta_0^6 + 2\beta_0^7}{\alpha_0^3\,(4\alpha_0^3 + 3\alpha_0^2\beta_0 + 3\alpha_0\beta_0^2 + \beta_0^3)\,(27\alpha_0^2 + 15\alpha_0\beta_0 + 2\beta_0^2)}$$

$$MTTF_1 - MTTF_3 = \frac{16\alpha_0^4 + 24\alpha_0^3\beta_0 + 16\alpha_0^2\beta_0^2 + 6\alpha_0\beta_0^3 + \beta_0^4}{2\alpha_0^3\,(4\alpha_0 + 3\beta_0)\,(4\alpha_0 + \beta_0)}$$

$$MTTF_1 - MTTF_5 = \frac{242\alpha_0^5 + 425\alpha_0^4\beta_0 + 306\alpha_0^3\beta_0^2 + 120\alpha_0^2\beta_0^3 + 25\alpha_0\beta_0^4 + 2\beta_0^5}{2\alpha_0^3\,(27\alpha_0^2 + 15\alpha_0\beta_0 + 2\beta_0^2)\,(4\alpha_0 + 3\beta_0)}$$

$$MTTF_3 - MTTF_5 = \frac{134\alpha_0^4 + 163\alpha_0^3\beta_0 + 84\alpha_0^2\beta_0^2 + 21\alpha_0\beta_0^3 + 2\beta_0^4}{2\alpha_0^2\,(4\alpha_0 + \beta_0)\,(27\alpha_0^2 + 15\alpha_0\beta_0 + 2\beta_0^2)}$$

Thus,

$$MTTF_2 > MTTF_4 > MTTF_1 > MTTF_3 > MTTF_5\,,\ \forall\alpha_0\,,\ \beta_0 > 0.$$

4.2 Comparison between Configurations based on Ranking

The purpose of this section is to rank the configurations for their mean time to failure using MATLAB software package. The results are summarised in the figures below.

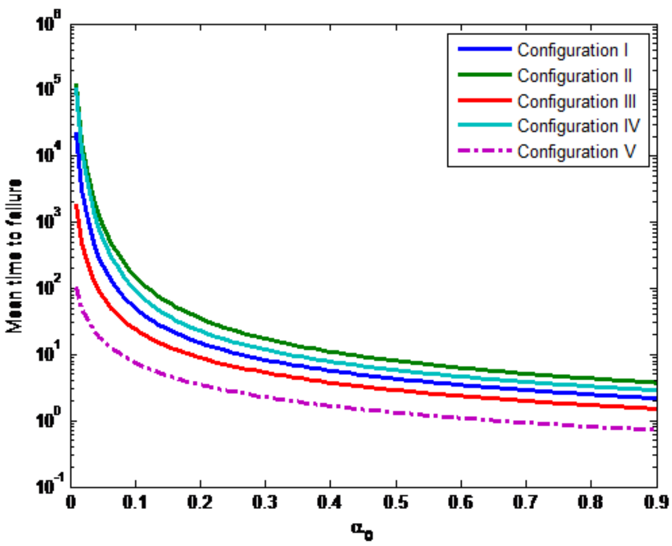

Figure 1: Mean time to failure of Configurations against α_0 for $\beta_0 = 0.3$

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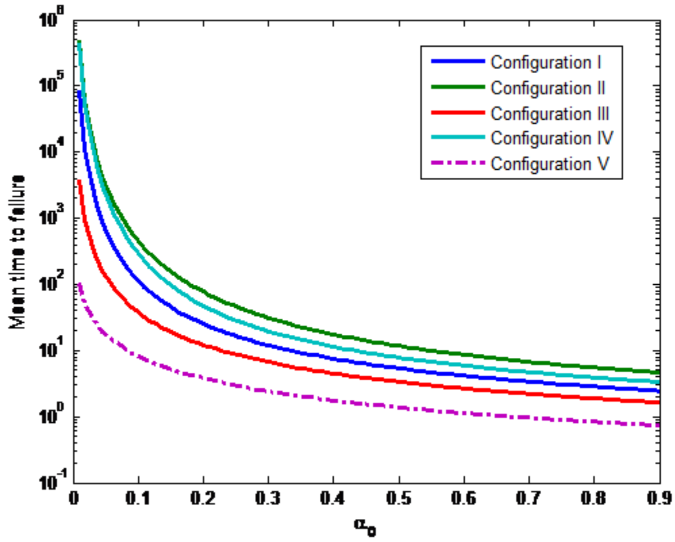

Figure 2: Mean time to failure of Configurations against α_0 for $\beta_0 = 0.6$

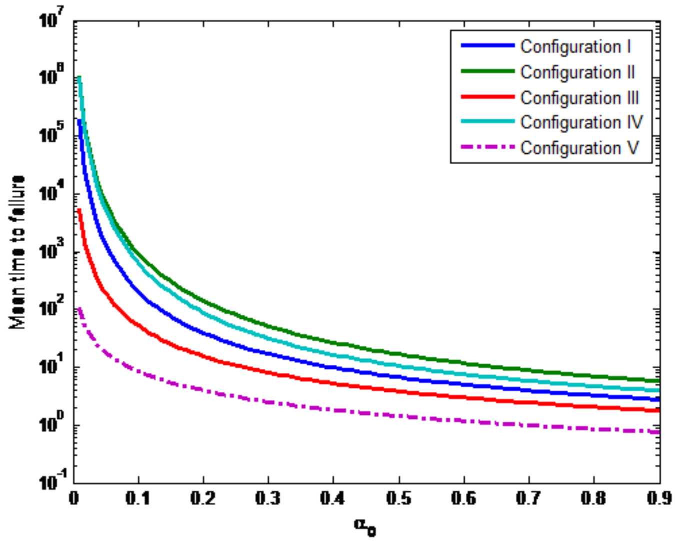

Figure 3: Mean time to failure of Configurations against α_0 for $\beta_0 = 0.9$

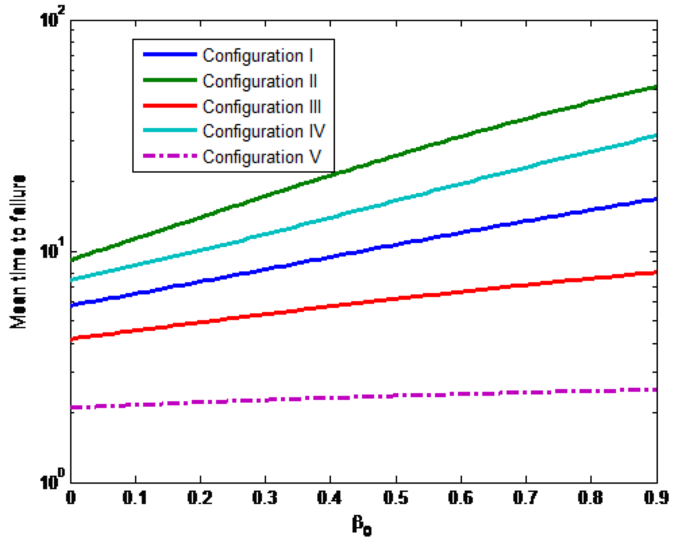

Figure 4: Mean time to failure of Configurations against β_0 for $\alpha_0 = 0.3$

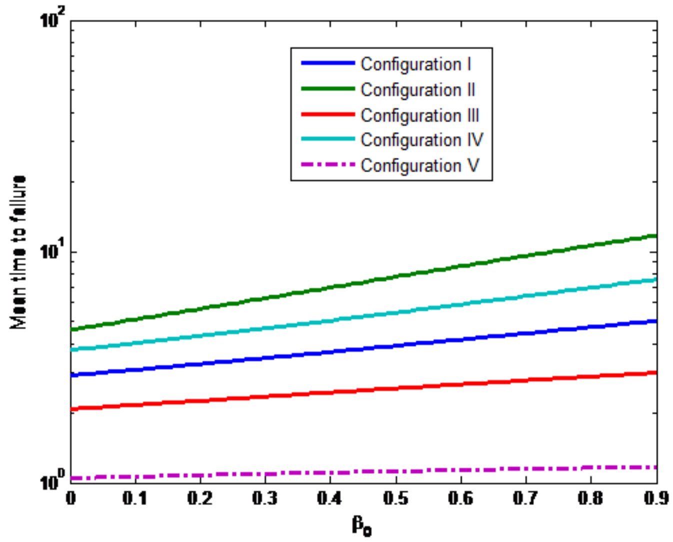

Figure 5: Mean time to failure of Configurations against β_0 for $\alpha_0 = 0.6$

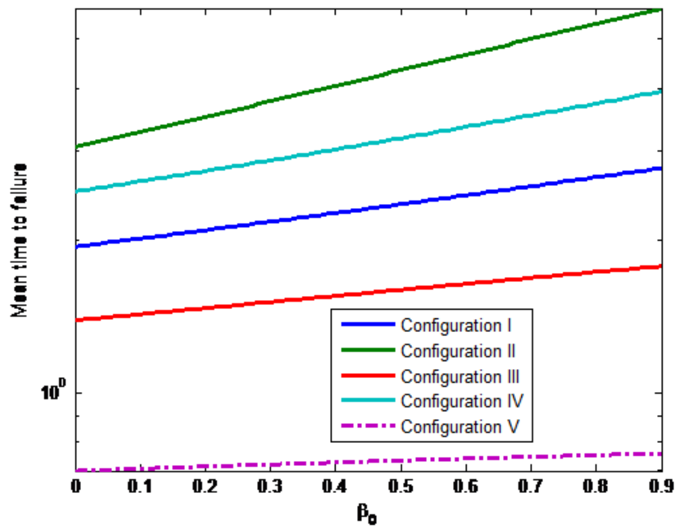

Figure 6: Mean time to failure of Configurations against β_0 for $\alpha_0 = 0.9$

The results which compare the MTTF with respect to α_0 for different values of β_0 and with respect to β_0 for different values of α_0 for all the three configurations considered depicted in Figure 1-3 and 4-6. Figure 1-3 and Figure 4-6 show that MTTF decrease as α_0 increases and increase as β_0 increases for any configuration respectively. Furthermore, Configuration I seems to be most effective and reliable configuration among all the three developed configurations. It is shown that Configuration I produces has more MTTF than the other configurations. It is evident from Figures 1–6 that Configuration I is more reliable. Thus, Configuration I is the optimal configuration in this study.

4.3 Comparison between Configurations based on Cost/Benefit

In this section, the configurations are compared based on their $B_k = C_k/MTTF_k$ using MATLAB software. The following set of parameters values are fixed for consistency:
$C_1 = 25,000,000$, $C_2 = 19,000,000$, $C_3 = 26,000,000$, $C_4 = 26,500,000$, $C_5 = 23,000,000$, $\beta_0 = 0.1$ are fixed and vary α_0 from 0 to 1 in Figures 1 and fixed $C_1 = 25,000,000$, $C_2 = 19,000,000$, $C_3 = 26,000,000$, $C_4 = 26,500,000$, $C_5 = 23,000,000$, $\alpha_0 = 0.1$ are fixed and vary β_0 from 0 to 1 in Figures 2 and obtained the following results.

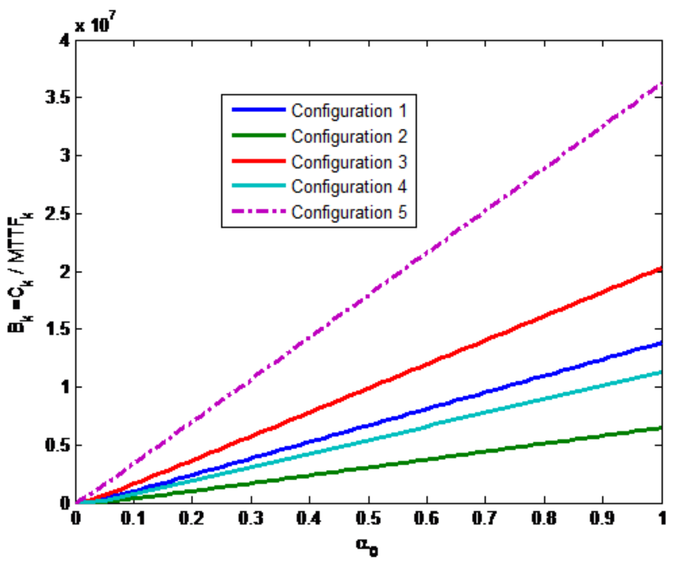

Figure 7: $C_k/MTTF_k$ against α_0

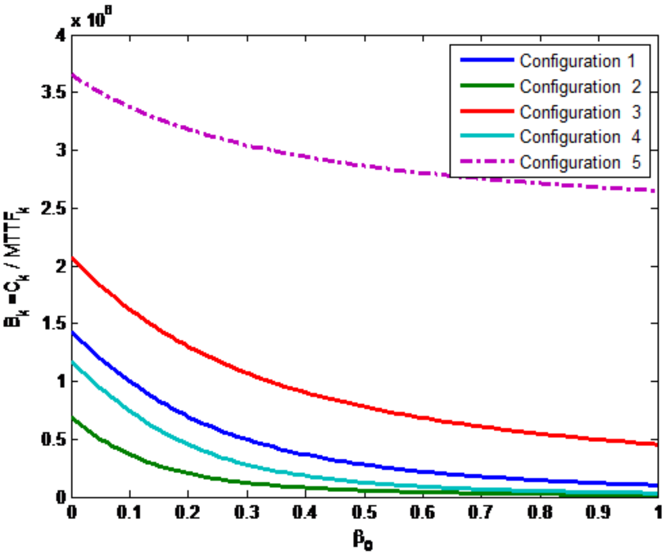

Figure 8: $C_k/MTTF_k$ against β_0

Figure 7 depicts the result of $B_k = C_k/MTTF_k$ for each configuration $i\,(i = 1, 2, 3, 4, 5)$ with respect to α_0. From the Figures, it is evident that $B_k = C_k/MTTF_k$ increase as α_0 increases for each configuration. It is clear from these Figures that the optimal configuration using $C_k/MTTF_k$ is Configuration II. On the other hand, Figure 8 depicts the result of $B_k = C_k/MTTF_k$ for each configuration $i\,(i = 1, 2, 3, 4, 5)$ with respect to β_0. From the Figure, it is evident that $B_k = C_k/MTTF_k$ decrease as β_0 increases for each configuration. It is clear from these Figures that the optimal configuration using $C_k/MTTF_k$ is Configuration II. It is evident from Figures 1 and 2 that the optimal configuration using is Configuration II. Thus,

$$C_2/MTTF_2 > C_1/MTTF_4 > C_3/MTTF_1 > C_3/MTTF_4 > C_5/MTTF_5.$$

5. CONCLUSION

In this paper, five dissimilar network configurations are studied. Explicit expressions for mean time to failure are derived. Comparisons are performed analytically and are presented numerically in Figures 1 to 6. Analysis of the effect of system parameters on $C_k/MTTF_k$, $k = 1, 2, 3, 4, 5$ are presented in Figures 7 and 8. On the basis of comparison analytical and numerical results presented in Figures 1 to 6, it is evident that the optimal configuration is configuration II for $\forall\,\alpha_0,\ \beta_0 > 0$. Similarly, the results of cost benefit $C_k/MTTF_k$ presented in Figures7-8 have shown that the optimal configuration is configuration II.

REFERENCES

Bhardwaj, R. K., & Malik, S. C. (2010). Mtsf and cost-effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection. *International Journal of Engineering Science and Technology*, *2*(1), 5882-5889.

Wang, K.-H., Hsieh, C.-H., & Liou, C.-H. (2006). Cost benefit analysis of series systems with cold standby components and a repairable service station. *Quality Technology & Quantitative Management*, *3*(1), 77-92.

Wang, K.-H., & Kuo, C.-C. (2000). Cost and probabilistic analysis of series systems with mixed standby components. *Applied Mathematical Modelling*, *24*(12), 957-967.

Wang, K.-H., Liu, Y.-C., & Pearn, W.-L. (2005). Cost benefit analysis of series systems with warm standby components and general repair time. *Mathematical Methods of Operations Research*, *61*(2), 329-343.

Wang, K.-H., & Pearn, W.-L. (2003). Cost benefit analysis of series systems with warm standby components. *Mathematical Methods of Operations Research*, *58*(2), 247-258.

Yusuf, I., & Gimba, B. (2013). Modeling and evaluation of mtsf of 2-out-of-5 warm standby repairable system with replacement at common cause failure. *Applied Mathematical Sciences*, *7*(129), 6403-6415.

Yusuf, I., Yusuf, B., & Lawan, M. A. (2014). Mathematical modeling approach to analyzing mean time to system failure of two unit cold standby system with a supporting device and repairable service station. *Open Journal of Mathematical Modeling*, *2*(2), 7-15. doi: 10.12966/ojmmo.05.01.2014