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Optimal Inventory Management for Perishable Goods when The Demand Depends on Product Price, Inventory Age, and Displayed Inventory Level

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Abstract: While the demand for perishable goods continuously increases, factors such as price, age, and displayed inventory may simultaneously affect the demand for such goods and make the inventory management of perishable goods even more difficult. In this study, an economic order quantity (EOQ) model with demand as a multivariate function of product price, inventory age, and displayed inventory level was developed. Zero ending inventory assumption was also relaxed to better formulate the strategies that companies may use to boost the sale and profit. Three major decision variables (i.e., product price, ending inventory level, and inventory cycle time) were considered in order to maximize the total profit. Analytical results of this study demonstrated that the total profit is strictly pseudo-concave in the three decision variables, and thus reduce the search for solutions to a unique local maximum. Numerical examples to illustrate the theoretical results and to highlight managerial insights are also presented.
Keyword — economic order quantity (EOQ); perishable goods; price; inventory age; displayed inventory level

1. INTRODUCTION

Nowadays, consumers are becoming more health-conscious than they used to be as their standard of living continues to increase, and the demand for fresh goods (e.g. vegetables, fruits, baked goods, bread, milk, meat, and seafood) has dramatically increased in recent years. However, the age of a fresh good has a negative impact on the demand due to the loss of the product freshness. In addition, it is also a well-known fact that increasing displayed inventory level may encourage consumers to purchase more. These effect time-dependent and stock-dependent demand makes inventory management for perishable products more complicated. Thus, to achieve optimal profit, a retailer must consider its pricing and inventory policy simultaneously.

According to classic marketing and economic theory, price is a major factor affecting the demand of a product. An early publication to incorporate the concepts of inventory and economic price theory can be traced back to Whitin (1955). Since then, many researchers (see, for example, Gallego & van Ryzin, 1994; Ladany & Sternlieb, 1974; Lee & Rosenblatt, 1986; Urban, 1992) have investigated inventory models that take the interaction between pricing policies and economic order quantity into consideration. Avinadav, Herbon, and Spiegel (2014) summarized six common price-dependent demand functions which are linear, iso-elastic, exponential, logit, logarithmic, and polynomial. Among those six functions, linear function has been most commonly used by other researchers (see, for example, Banerjee & Meitei, 2010; Mishra & Raghunathan, 2004).

In addition to the price of the product, the demand can also be affected by displayed inventory level. Wolfe (1968) presented empirical evidences of the existence of this type of phenomenon with style merchandise such as women's dresses or sports clothes. He found that the higher the displayed inventory level, the higher the demand. Not only does this phenomenon happen in style merchandising, but it also happens in consumer goods. Levin, McLaughin, Lamone, and Kotta (1972) observed that large piles of displayed consumer goods in a supermarket can lead consumers to buy more, partially owing to its visibility, popularity and also variety. To formulate such phenomenon, Baker and Urban (1988) developed an economic order quantity (EOQ) inventory model by specifying the stock dependent demand as a power function of displayed inventory level. This formulation was then adopted by other researchers (Feng, Chan, & Cardernas-Barron, 2017; Wu, Chang, Cheng, Teng, & Al-khateeb, 2016). Furthermore, since the higher the inventory the higher the demand, it is obvious that keeping higher levels of displayed inventory level may result in higher sales and profits. As a result, Urban (1992) extended the classical EOQ inventory model from zero ending inventory to non-zero ending inventory, which was later adopted by Urban and Baker (1997), Wu et al. (2016), and Feng et al. (2017).

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Not only is the demand affected by product price and displayed inventory level, but the demand is also affected by the age or freshness of a product. Sarker, Mukherjee, and Balan (1997) argued that this effect occurs because consumers tend to feel less confident in purchasing perishable items whose expiration dates are approaching. Therefore, in general, the age of inventory is likely to negatively affect the demand for perishable goods. To capture such phenomenon, Wu et al. (2016) and Feng et al. (2017) assumed that the age-dependent demand is linearly decreasing from 1 at the beginning to 0 at the maximum shelf life. However, in reality, it seems that different products may have different quality deterioration in aging. Some of their demand may only slightly decrease in the early period and then accelerate when the age of the product almost reaches its expiration time, some of their demand may linearly decrease from the beginning of the product age until its expiration time, and some of their demand may already significantly decrease in the early of product age. Avinadav, Herbon, and Spiegel (2013) thus argued that the polynomial function can better describe the demand change over time for perishable goods.

Recently Feng et al. (2017) formulated EOQ models that incorporate price, displayed inventory level, and inventory age on the demand rate to determine pricing and lot sizing policies. However, their model may not be general enough to accommodate different possibilities of product inventory age effect toward consumers demand discussed in Avinadav et al. (2013). The goal of this research is thus to deepen our understanding of the properties of the inventory and price decisions with respect to more plausible demand function. In particular, this research employs a polynomial decreasing demand function toward inventory age in its analytic model, and derive several theorems to give conditions for the existence of the optimal solution.

The rest of this paper is arranged as follows. In section 2, we discuss the notations and assumptions which will be used throughout the rest of the paper. In section 3, we formulate the mathematical model by incorporating the demand as a multivariate function of unit price, inventory age, and displayed inventory level. In section 4, we provide theoretical results and optimal solution. In section 5, we present numerical examples with sensitivity analysis and give some managerial insights. Finally, we conclude the paper with a discussion and suggest directions for future research in section 6.

2. NOTATIONS, ASSUMPTIONS, AND SYSTEM FORMULATION

The following notations and assumptions are used in the entire paper.

2.1 Notations

For simplicity, the notations are categorized into three groups: parameters, decisions variables, and functions.

Parameters:

- c = Purchasing cost
- h = Holding cost
- E = Maximum lifetime
- o = Ordering cost per order
- s = Salvage price
- W = Shelf space size
- β_0 = Potential market
- β_1 = Price elasticity
- n = Freshness (inventory age) elasticity
- γ = Displayed inventory level elasticity
- t_1 = Time at which the inventory level reaches W

Decision Variables:

- p = Product price
- Z = Ending inventory level
- T = Inventory cycle time

Functions:

- $D(p, t, i)$ = Demand rate, a multivariate function of price, inventory age, and inventory level.
- $I(t)$ = Inventory level at time t.
- $\prod(p, Z, T)$ = Total profit, which is a multivariate function of p, Z, and T

2.2 Assumptions

Considering an inventory system of a single perishable product that is replenished periodically over an infinite planning horizon with an ordering cost. Assumptions of the model are summarized below.

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1. Replenishment rate is infinite with a fixed lead time.
2. Backlogging is not possible.
3. All items in an order have identical shelf-life duration that is known by the consumer.
4. There is no demand for items that have passed their expiration time.
5. The demand rate function is deterministic and is a multiplication of three factors: product price, inventory age, and displayed inventory level.

In particular, for the effect of price on demand, the common negative linear relation was considered (see, for example, Avinadav et al., 2014; Banerjee & Meitei, 2010). We then modified inventory age function proposed by Avinadav et al. (2013) in our mathematical model to accommodate specific shelf space size in various freshness elasticity. In term of the effect of displayed inventory level, we adopted the function proposed by Urban (1992).

Similar to the function used in Avinadav et al. (2013), our inventory age function is general enough to support a variety of decreasing shapes as shown in Figure 1: no decrease for $n \longrightarrow 0$; concave decrease for $0 < n < 1$; linear decrease for $n = 1$; and convex decrease for $n > 1$.

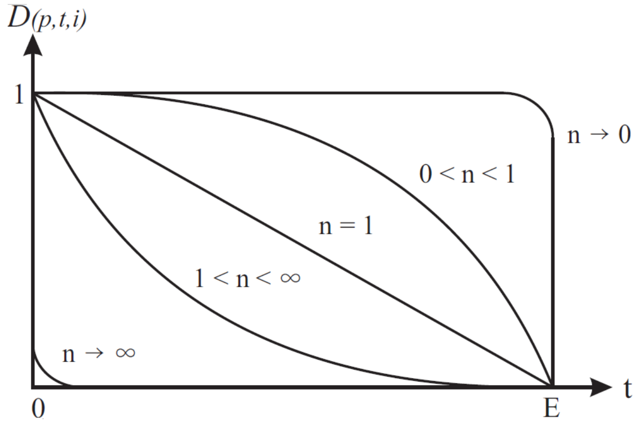

Figure 1: Demand function $D(p,t,i)$ for a given p and γ with various fresh elasticity n

To fully utilize the shelf space, it is assumed that Q is larger than or equal to W (i.e, $Q \geq W$). Otherwise, the retailer will just reduce the size of the shelf space to Q units to save the shelf space cost. Since $Q \geq W$, it is obvious that $t_1 \geq 0$. To put it more precisely, at time 0, the retailer will buy and receive Q units and displays W units on the shelf with the rest of the products (i.e. $Q - W$ units) stored in the backroom. When sales are made, inventory in the backroom is moved to shelf until there is no more inventory in the backroom (at time t_1). Thus, during period $[0, t_1]$, the shelf space will be full and the demand will only change in product age. However during $[t_1, T]$, as the displayed inventory level gradually decrease, the demand will change with respect to both the increase in product age and the decrease in displayed inventory level. The demand during $[0, t_1]$, the demand during $[t_1, T]$, and the graphical representation of the system can be seen in (1), (2), and Figure 2 respectively.

$$D(p,t,i) = (\beta_0 - \beta_1 p)(1 - t/E)^n W^\gamma \tag{1}$$
$$D(p,t,i) = (\beta_0 - \beta_1 p)(1 - t/E)^n I(t)^\gamma \tag{2}$$

3. MATHEMATICAL MODEL

From assumptions given in Section 2, the inventory level $I(t)$ at time t during the time interval $[0, t_1]$ is governed by the following differential equation:

$$\frac{dI\,(t)}{dt} = -\left(\beta_0 - \beta_1 p\right)\left(1 - \frac{t}{E}\right)^n W^\gamma. \tag{3}$$

Rearranging (3) and performing integration on both sides, with boundary conditions $I(t_0) = Q$ and $I(t_1) = W$, the inventory level is

$$I\,(t) = Q - \frac{(\beta_0 - \beta_1 p)\, W^\gamma E}{n+1}\left[1 - \left(1 - \frac{t}{E}\right)^{n+1}\right]. \tag{4}$$

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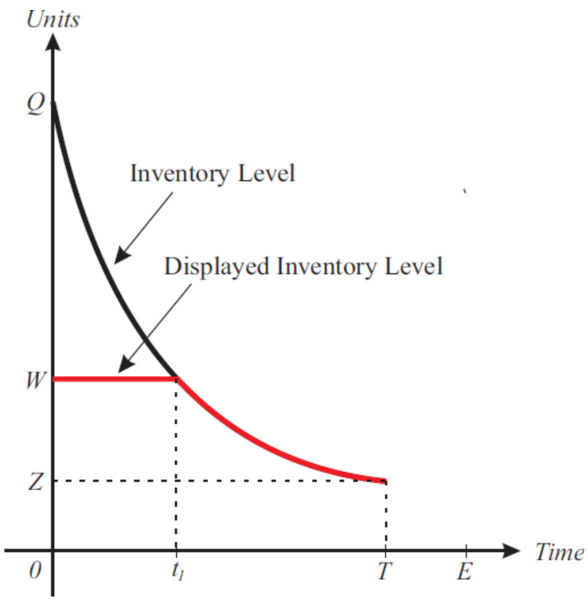

Figure 2: Graphical Representation of the System

Setting $t = t_1$ and $I(t) = W$ in (4), the time at which the inventory level is W (i.e., backroom inventory is empty) is given as

$$t_1 = E\left\{1 - \left[1 - \frac{(Q-W)(n+1)}{(\beta_0 - \beta_1 p) W^{\gamma} E}\right]^{\frac{1}{n+1}}\right\}. \tag{5}$$

Substituting (5) into (4), eventually the order quantity is

$$Q = W + \frac{(\beta_0 - \beta_1 p) W^{\gamma} E}{n+1}\left[1 - \left(1 - \frac{t_1}{E}\right)^{n+1}\right]. \tag{6}$$

Similarly, the inventory level $I(t)$ at time $t \in [t_1, T]$ can be described by the following differential equation

$$\frac{dI(t)}{d(t)} = -(\beta_0 - \beta_1 p)\left(1 - \frac{t}{E}\right)^{n} I(t)^{\gamma}. \tag{7}$$

Rearranging (7) and performing integration on both sides, with boundary condition $I(T) = Z$, the inventory level at time t is

$$I(t) = \left\{\frac{E(\beta_0 - \beta_1 p)(1-\gamma)}{n_1 + 1}\left[\left(1 - \frac{t}{E}\right)^{n_1+1} - \left(1 - \frac{T}{E}\right)^{n+1}\right] + Z^{1-\gamma}\right\}^{\frac{1}{1-\gamma}}. \tag{8}$$

Substituting t_1 into (8) and using the facts $I(t_1) = W$ and $t_1 \le T \le E$, the time at which the backroom is running out of inventory is

$$t_1 = E\left\{1 - \left[\frac{\left(W^{1-\gamma_1} - Z^{1-\gamma}\right)(n+1)}{E(\beta_0 - \beta_1 p)(1-\gamma)} + \left(1 - \frac{T}{E}\right)^{n+1}\right]^{\frac{1}{n+1}}\right\} \tag{9}$$

Substituting (9) into (6), and simplifying terms, the order quantity is

$$Q = W + \frac{(\beta_0 - \beta_1 p) W^{\gamma} E}{n+1}\left[1 - \frac{\left(W^{1-\gamma} - Z^{1-\gamma}\right)(n+1)}{E(\beta_0 - \beta_1 p)(1-\gamma)} - \left(1 - \frac{T}{E}\right)^{n+1}\right]. \tag{10}$$

By performing integration on (4), the holding cost during time $[0, t_1]$ is

$$H_1 = h\left\{Qt_1 + \frac{(\beta_0 - \beta_1 p) E^2\left[1 - \left(1 - \frac{t_1}{E}\right)^{n+2}\right]}{W^{-\gamma}(n+1)(n+2)} - \frac{(\beta_0 - \beta_1 p) W^{\gamma} E t_1}{n+1}\right\} \tag{11}$$

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Since it is intractable to derive an explicit analytic solution for the holding cost during $[t_1, T]$, for simplicity, a simple approximation equation is used to calculate it instead. The average inventory level during the time interval $[t_1, T]$ approximately equals $(W + Z)/2$. Therefore the holding cost during the time $[t_1, T]$ approximately equals to

$$\widehat{H_2} = \frac{h}{2}(W + Z)(T - t_1) \tag{12}$$

This approximation is also considered in Feng et al. (2017), Baker and Urban (1988), Urban (1992), Urban and Baker (1997). (Please be noted that based on our numerical results, the difference between optimal profits obtained by using (11) and (12) is less than 0.1%.) Maximizing total profit is considered as an objective of the optimization problem. Otherwise, if the objective is to minimize total cost, then both the inventory level and demand rate decrease, thus resulting in lower profits.

$$\begin{aligned}\text{Total Profit } =&\text{Total Selling Revenue} + \text{The Salvaging Revenue}\\ &- \text{The Purchasing Cost} - \text{The Ordering Cost} - \text{The Holding Cost}\end{aligned}$$

Therefore, the problem is to determine the optimal unit price p, ending inventory level Z, and inventory cycle time T simultaneously in order to maximize the total profit. The EOQ inventory model for perishable products with price-freshness-and-stock dependent demand to be solved is:

$$\max \quad \prod(p, Z, T) \;= \frac{1}{T}\left[p\,(Q - Z) + sZ - cQ - o - H_1 - \widehat{H_2}\right] \tag{13}$$

s.t.

$$\begin{aligned}
&Q = W + \frac{(\beta_0 - \beta_1 p)\, W^{\gamma} E}{n_1 + 1}\left[1 - \frac{\left(W^{1-\gamma} - Z^{1-\gamma}\right)(n+1)}{E\,(\beta_0 - \beta_1 p)\,(1-\gamma)} - \left(1 - \frac{T}{E}\right)^{n+1}\right] \geq W,\\
&t_1 = E\left\{1 - \left[\frac{\left(W^{1-\gamma_1} - Z^{1-\gamma}\right)(n+1)}{E\,(\beta_0 - \beta_1 p)\,(1-\gamma)} + \left(1 - \frac{T}{E}\right)^{n+1}\right]^{\frac{1}{n+1}}\right\} \geq 0,\\
&0 \leq Z \leq W\\
&0 \leq t_1 \leq T \leq E.
\end{aligned}$$

4. THEORETICAL RESULTS AND OPTIMAL SOLUTION

According to Cambini and Martein (2009), the real-value function

$$q(x) = \frac{y(x)}{z(x)} \tag{14}$$

is (strictly) pseudo-concave, if $y(x)$ is nonnegative, differentiable and (strictly) concave, and $z(x)$ is positive, differentiable and convex. For simplifying the representation used in this article, to solve the problem in (13), the following equation is defined

$$\begin{aligned}
J = -\frac{n}{E}\,(\beta_0 - \beta_1 p)\, W^{\gamma} E\left(1 - \frac{T_1}{E}\right)^{n-1}(p - c - ht_1) - \frac{h\left(1 - \frac{T}{E}\right)^{2n}}{\left(1 - \frac{t_1}{E}\right)^{n}} -\\
\left\{(\beta_0 - \beta_1 p)\, W^{\gamma} E + \frac{(W - Z)\, n}{2E}\left[\left(1 - \frac{t_1}{E}\right)^{-(n+1)} - \left(1 - \frac{T}{E}\right)^{-(n+1)}\right]\right\}
\end{aligned} \tag{15}$$

It seems impossible to prove that J is negative mathematically. However, the price (p) is generally higher than the sum of the purchase cost (c) and the holding cost (ht_1). So, there is just one positive term in J, and it is related to holding cost during $[t_1, T]$, which is significantly smaller than $p - c - ht_1$. Thus, given fixed p and Z, using results in (15) it can be proved that the retailer's total profit $\prod(p, Z, T)$ in (13) is strictly pseudo-concave in T (if $J < 0$). Consequently, there exists a unique optimal solution T^*.

Theorem 1. For any given product price (p) and ending inventory level (Z), if $J < 0$, then the objective function $\prod(p, Z, T)$ in (13) is a strictly pseudo concave function in T, and hence exists a unique maximum solution in T^*.

Proof. See Appendix A. □

Please be noted that the proof of Theorem 1 is similar to the one in Feng et al. (2017). Given the ending-inventory level E and the unit price p, taking the first order derivative of $\prod(p, Z, T)$ in (13) with respect to T, setting the result to zero and simplifying terms, we can obtain from Theorem 1 that the condition for the optimal replenishment cycle time T^* is as follows:

$$\left\{(\beta_0-\beta_1 p)W^\gamma E\Big(1-\frac{T}{E}\Big)^n(p-c-ht_1)-\frac{h}{2}\left[\frac{(W-Z)(1-\frac{T}{E})^n}{(1-\frac{t_1}{E})^n}+W+Z\right]\right\}T-$$
$$\left\{p(Q-Z)+sZ-cQ-o-h\left[Qt_1+\frac{(\beta_0-\beta_1 p)W^\gamma E^2\left[1-(1-\frac{t_1}{E})^{n+2}\right]}{(n+1)(n+2)}\right.\right.$$
$$\left.\left.-\frac{(\beta_0-\beta_1 p)W^\gamma E t_1}{n+1}\right]-\frac{h}{2}(W+Z)(T-t_1)\right\}=0. \tag{16}$$

For the detailed derivation, please see Appendix A and B. By using (9) and (10) the following results are obtained.

Corollary 1. Given the product price p and the ending-inventory level Z, both t_1 and Q are increasing and concave downward in T_1.

Proof. See (A3), (A4), (A6), and (A7). □

Next, to prove $\prod(p, Z, T)$ in (13) is strictly concave function in both p and Z for any given T, we define K, L, M as follows.

$$K=\left(\frac{W}{Z}\right)^\gamma-1+h\left[\frac{\left(W^{1-\gamma}-Z^{1-\gamma}\right)\beta_1}{(1-\gamma)\left(\beta_0-\beta_1 p\right)^2(1-\frac{t_1}{E})^n}\right]-\left[\left(\frac{W}{Z}\right)^\gamma-\frac{1}{2}\right]$$
$$-\frac{h\beta_1\left(W-Z\right)}{2(\beta_0-\beta_1 p)^2 Z^\gamma\left(1-\frac{t_1}{E}\right)^n}\left[1-\frac{n\left(W^{1-\gamma}-Z^{1-\gamma}\right)(1-\gamma)^{-1}}{E\left(\beta_0-\beta_1 p\right)\left(1-\frac{t_1}{E}\right)^{n+1}}\right] \tag{17}$$

$$L=-\left(p-c-ht_1\right)\frac{\gamma W^\gamma}{Z^{\gamma+1}}-\frac{h}{\left(\beta_0-\beta_1 p\right)\left(1-\frac{t_1}{E}\right)^n Z^\gamma}$$
$$-\left[\left(\frac{W}{Z}\right)^\gamma-1+\frac{\left(W-Z\right)n}{2E\left(\beta_0-\beta_1 p\right)\left(1-\frac{t_1}{E}\right)^{n+1}Z^\gamma}-\frac{\gamma\left(W-Z\right)}{2Z}\right] \tag{18}$$

$$M=-\left[\frac{\beta_1\left(Q-W\right)}{\left(\beta_0-\beta_1 p\right)}+\frac{\beta_1 W^\gamma\left(W^{1-\gamma}-Z^{1-\gamma}\right)}{\left(\beta_0-\beta_1 p\right)(1-\gamma)}\right]\left[2+\frac{h\left(W^{1-\gamma}-Z^{1-\gamma}\right)\beta_1}{(1-\gamma)\left(\beta_0-\beta_1 p\right)^2(1-\frac{t_1}{E})^n}\right]$$
$$-\frac{h\left(W-Z\right)\left(W^{1-\gamma}-Z^{1-\gamma}\right){\beta_1}^2}{(1-\gamma)\left(\beta_0-\beta_1 p\right)^3(1-\frac{t_1}{E})^n}\left[\frac{n\left(W^{1-\gamma}-Z^{1-\gamma}\right)}{2E\left(1-\gamma\right)\left(\beta_0-\beta_1 p\right)\left(1-\frac{t_1}{E}\right)^{n+1}}-1\right]$$
$$-\frac{h\beta_1 W^\gamma E}{(n+1)}\left[\left(1-\frac{t_1}{E}\right)^{n+1}-1\right]\left[\frac{\left(W^{1-\gamma}-Z^{1-\gamma}\right)\beta_1}{(1-\gamma)\left(\beta_0-\beta_1 p\right)^2(1-\frac{t_1}{E})^n}\right] \tag{19}$$

In general, the price (p) is higher than the sum of the purchase cost (c) and the holding cost (ht_1) in (18), and the positive terms come from holding cost during $[t_1, T]$, which is significantly smaller than $p-c-ht_1$. Hence we may assume that $L<0$. For M, it seems impossible to estimate whether $M<0$ or not therefore future calculation is needed.

Theorem 2. Given the replenishment cycle time T, if $L<0$, $M<0$, and $LM-K^2>0$, then in (13) is a strictly concave function in both p and Z, hence there exist a unique maximum solution p^* and Z^*.

Proof. See Appendix C. □

For any given replenishment cycle time T, taking the first order partial derivative of $\prod(p, Z, T)$ in (13) with respect to p and Z, setting the result to zero, and simplifying terms, we can obtain from Theorem 2 that the condition for the optimal ending inventory level is

$$(p-c-ht_1)\left(\frac{W}{Z}\right)^\gamma-p+s-\frac{h}{2}\left[\frac{\left(W-Z\right)\left(1-\frac{t_1}{E}\right)^{-n}}{\left(\beta_0-\beta_1 p\right)Z^\gamma}+T-t_1\right]=0 \tag{20}$$

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and the optimal condition for the optimal price per product is

$$Q - Z - (p - c - ht_1)\frac{\beta_1}{(\beta_0 - \beta_1 p)}\left[Q - W + \frac{W^{\gamma}\left(W^{1-\gamma} - Z^{1-\gamma}\right)}{(1-\gamma)}\right] + \frac{h(W-Z)\left(W^{1-\gamma} - Z^{1-\gamma}\right)\beta_1}{2(1-\gamma)(\beta_0 - \beta_1 p)^2\left(1 - \frac{t_1}{E}\right)^n} + \frac{h\beta_1 p W^{\gamma} E}{n+1}\left\{\frac{E\left[1 - \left(1 - \frac{t_1}{E}\right)^{n+2}\right]}{n+2} - t_1\right\} = 0 \tag{21}$$

For the detailed derivation, please see Appendix C and D. In addition, the following results are obtained based on (9) and (10).

Corollary 2. Given the replenishment cycle time T,

1. t_1 is increasing in Z, while decreasing in p_1, and
2. Q is increasing and concave downward in Z, but is decreasing linearly in p.

Proof. See (C1), (C2), (C7), (C8), (C10), and (C11). □

Based on the theorems, corollaries, and equations derived in this section, an algorithm similar to the one in Feng et al. (2017) is proposed to find the optimal solution of decision variables:

1. Input the parameter's value
2. Set i = 1, and choose a trial value of $T(E/2)$.
3. Substitute T_i into Equations (20) and (21) to get Z_i and p_i.
4. Substitute E_i and p_i into Equation (16) to obtain $T_{(i+1)}$.
5. If $-10^{-4} < T_{(i+1)} - T_{(i)} < 10^{-4}$, then set the optimal solution Z^*=Z_i, p^*=p_i, and T^*=T_i, and then stop, otherwise, set $i = i$ + 1, and go to step 3.

5. NUMERICAL EXAMPLES AND DISCUSSION

This section illustrates the theoretical results and managerial insights obtained by using two numerical examples run with MATHEMATICA 11. The details of numerical examples are as follows.

5.1 Example 1: Base Scenario

Assuming that the maximum number of potential consumer β_0=2000, price elasticity β_1=50, displayed inventory level elasticity γ=0.5, inventory age elasticity n=1, maximum product lifetime E=0.04 years, purchasing cost c=\$20/unit, holding cost h=\$5/unit/year, ordering cost o=\$20/order, shelf-space w=20, and salvage price s=\$10/unit, by using software MATHEMATICA 11 and Microsoft Excel, the local optimal solution to maximize $\prod(p, Z, T)$ is obtained as follows.
p^*=\$29.2096, Z^*=4.5265 units, T^*=0.0204 years, t_1=0.0071 years, Q^*=35.6485 units, $\prod(p, Z, T)^*$=\$10785.59/year.

We calculate $J = -2.15 \times 10^4 < 0$ at the optimal p^*, Z^*, and T^*. Thus, given p^*, and Z^*, $\prod(p, Z, T)$ in (13) is a pseudo concave function in T as shown in Figure 3. Similarly, given T^*, it is obtained that K = 1.0970, L = - 1.0038, M = - 15.7429, $LM - K^2$=14.5996 at the optimal solution. Hence, $\prod(p, Z, T)$ in (13) is a concave function both in p and Z as shown in Figure 4.

5.2 Example 2: Sensitivity Analysis

Using the same setting as those in Example 1, the sensitivity analysis of the optimal solution with respect to each parameter is obtained. The computational results are shown in Table 1.

In the first case, the value of β_0 (i.e., potential market) is permitted to vary while keeping other parameters constant. As shown in Table 1, the selling price, the order quantity, the ending inventory level, and the total profit all gradually increase as the value of β_0 increases. Furthermore, although the amount of product that sold by the retailer increases, the inventory cycle time gradually decreases as the value of β_0 increases. Conversely, in the second case, the selling price, the order quantity, the ending inventory level, and the total profit all gradually decrease as the value of β_1 (i.e., price elasticity) increases. In addition, although the amount of product that sold by the retailer decrease, the inventory cycle time gradually increases as the value of β_1 increases. The effects of changes in displayed inventory level elasticity γ on the selling price, the order quantity, the ending inventory level, the inventory cycle time, and the total profit are

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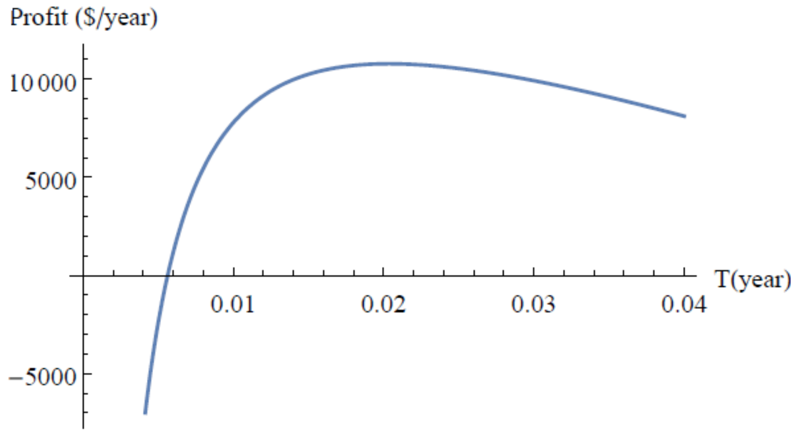

Figure 3: $\prod(p, Z, T)$ with respect to T for given p=$29.2096 and Z=4.5265

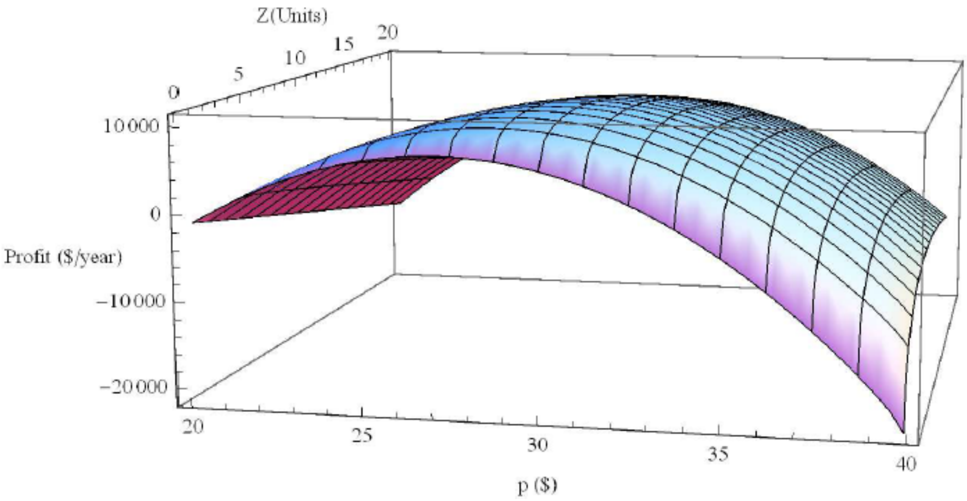

Figure 4: $\prod(p, Z, T)$ with respect to p and Z given T=0.0204

similar to the effects of changes in potential market. The directions of the effects are fundamentally the same while the magnitude of the effects of changes in displayed inventory elasticity is relatively smaller.

In the fourth case, all four decisions variables, as well as the total profit gradually decrease as the value of n (i.e., freshness or inventory age elasticity) increases. It means that if the customer is very sensitive to freshness of a perishable product, the retailer should compromise its ordering cost to achieve optimal profit. To achieve the optimal profit retailer will stock low amount of inventory and then replenish it with the new ones more frequently.

In the fifth case, all four decisions variables, as well as the total profit gradually increase as the value of E (i.e., maximum shelf lifetime) increases. Unlike previous cases, although the amount of sold product increases as E increases, the inventory cycle time also increases. This happens because the product demand over time does not significantly decrease when the shelf-life duration is increases. Consequently, the retailer may invest in preservation technology to prolong the product maximum lifetime to increase its total profit.

In the sixth case, the selling price and the inventory cycle time increase but the order quantity, the ending inventory level, and the total profit decrease when the purchasing cost c increases. In the seventh case, the increase of holding cost does not significantly impact all four decisions variable and the total profit. In the eighth case, all four decisions variables gradually increase while the total profit decreases when the value of ordering cost per order o increases.

In the ninth case, the order quantity, ending inventory level, the inventory cycle time, as well as the total profit gradually increase while the selling price decreases when the shelf space size W increases. Thus widening shelf space size such as applying vertical shelf layout is mandatory to optimize the retailer profit.

Finally, in the tenth case, the selling price, the order quantity, the ending inventory and the total profit increase while the cycle time decreases when the salvage price s increases. This indicates that if an earlier closeout sale increases the salvage price, then the retailer should try it to increase the total profit as well as the order quantity.

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Table 1: Sensitivity Analysis Computational Results.

Case	Parameter	Change in Parameter (%)	Deviation from Base Value (%) p	Q	Z	T	Profit
1	β_0	20	14.20	23.23	43.10	-23.25	145.50
		-10	-7.23	-16.00	-25.35	18.29	-48.78
		-20	-14.65	-36.74	-53.01	45.67	-81.97
		-20	-14.65	-36.74	-53.01	45.67	-81.97
2	β_1	20	-11.64	-17.66	-41.71	23.66	-60.56
		10	-6.34	-8.23	-22.12	11.03	-35.29
		-10	7.73	7.05	24.55	-9.87	48.99
		-20	17.37	12.91	51.44	-18.90	117.20
3	γ	20	0.69	28.89	30.52	-9.67	49.05
		10	0.37	13.98	15.72	-4.77	22.18
		-10	-0.42	-13.08	-16.50	4.51	-18.15
		-20	-0.87	-25.25	-33.47	8.55	-32.85
4	n	20	-0.28	-7.96	-0.78	-5.04	-7.90
		10	-0.14	-4.21	-0.39	-2.68	-4.06
		-10	0.14	4.77	0.39	3.09	4.32
		-20	0.29	10.24	0.79	6.68	8.95
5	E	20	0.35	12.33	0.91	9.58	9.09
		10	0.19	6.32	0.50	4.90	4.85
		-10	-0.23	-6.67	-0.63	-5.17	-5.64
		-20	-0.51	-13.76	-1.45	-10.65	-12.29
6	c	20	5.58	-20.41	-53.28	23.84	-52.81
		10	2.78	-9.72	-30.29	11.59	-29.30
		-10	-2.76	8.50	39.28	-11.43	36.01
		-20	-5.53	15.30	89.65	-23.19	80.10
7	h	20	0.00	-0.13	-0.29	-0.06	-0.16
		10	0.00	-0.07	-0.15	-0.03	-0.08
		-10	0.00	0.07	0.15	0.03	0.08
		-20	0.00	0.14	0.29	0.06	0.16
8	o	20	0.03	1.09	0.09	1.74	-1.81
		10	0.02	0.55	0.04	0.87	-0.91
		-10	-0.02	-0.56	-0.05	-0.88	0.91
		-20	-0.04	-1.13	-0.09	-1.77	1.84
9	W	20	-0.23	11.95	19.10	3.06	5.89
		10	-0.12	6.07	9.57	1.56	3.10
		-10	0.12	-6.28	-9.63	-1.63	-3.45
		-20	0.26	-12.78	-19.32	-3.32	-7.32
10	s	20	0.53	2.22	26.50	-4.36	4.74
		10	0.26	1.07	12.17	-2.02	2.21
		-10	-0.24	-0.99	-10.39	1.76	-1.93
		-20	-0.47	-1.91	-19.33	3.30	-3.64

In short, the profit is very sensitive to four demand parameters potential market (β_0), price elasticity (β_1), displayed inventory level elasticity (γ), and inventory age elasticity (n). Changes in these parameters significantly impact the values of all four decision variables, thus the retailer should pay more attention to these parameters than others. The profit is also moderately sensitive to maximum lifetime (E), purchasing cost (c), shelf-space size (W), and salvage price (s). The model thus enable us to evaluate whether the investment in managerial intervention to change some of these parameters is economically justified. In addition, a managerial insight can also be obtained when displayed inventory level elasticity (γ) is close to 0. Under this condition, the retailer will set the ending inventory level at zero unit (i.e., $Z = 0$) in optimal condition.

6. CONCLUSIONS

This paper proposes a novel and plausible inventory age polynomial function and develops an EOQ inventory model with demand depends on product price, inventory age, and displayed inventory level. Moreover, in order to better formulate the strategies that can be considered to boost the profit, this paper also relaxes zero ending inventory assumption to non-zero ending inventory.

Based on the calculation, the following instances are captured: (1) retailer's ordering cost should be compromised (i.e., more frequent orders with small amount of product quantity) if the consumer's demand is very sensitive to the freshness of a perishable product, (2) prolonging product's maximum lifetime by using preservation technology can influence the demand over time and thus increase the retailer's profit, (3) increasing the size of shelf space can significantly induce the retailer's profit higher, thus applying vertical shelf layout is mandatory to increase the retailer profit, (4) order quantity is significantly affected by displayed inventory level elasticity. In particular, when the displayed inventory level elasticity is zero (i.e., when the consumer's demand is not induced by displayed inventory level), the retailer ending inventory level is zero.

This research has given rise to some directions in need of further investigation. First, future work can relax the demand function's assumption from deterministic to stochastic. Second, the assumption of identical product age and inventory cycle time may not be realistic enough, future work can relax our assumption related to this issue. Last, the single player local optimal solution could be extended to an integrated cooperative solution for multiple players in a supply chain.

7. PROOF OF THEOREM 1.

Given p and Z, taking the first and second order derivatives of (9) and (10) with respect to T, the following results are obtained:

$$\frac{\partial t_1}{\partial T} = \left(1-\frac{T}{E}\right)^n \left[\frac{\left(W^{1-\gamma}-Z^{1-\gamma}\right)(n+1)}{E\left(\beta_0-\beta_1 p\right)(1-\gamma)} + \left(1-\frac{T}{E}\right)^{n+1}\right]^{-\frac{n}{n+1}} > 0 \tag{22}$$

$$\begin{aligned}\frac{\partial^2 t_1}{\partial T^2} = \frac{n}{E}\Bigg\{&\left(1-\frac{T}{E}\right)^{2n} \left[\frac{\left(W^{1-\gamma}-Z^{1-\gamma}\right)(n+1)}{E\left(\beta_0-\beta_1 p\right)(1-\gamma)} + \left(1-\frac{T}{E}\right)^{n+1}\right]^{-\frac{(2n+1)}{n+1}} \\ &-\left(1-\frac{T}{E}\right)^{n-1} \left[\frac{\left(W^{1-\gamma}-Z^{1-\gamma}\right)(n+1)}{E\left(\beta_0-\beta_1 p\right)(1-\gamma)} + \left(1-\frac{T}{E}\right)^{n+1}\right]^{-\frac{n}{n+1}}\Bigg\}\end{aligned} \tag{23}$$

$$\frac{\partial Q}{\partial T} = \left(\beta_0-\beta_1 p\right) W^\gamma \left(1-\frac{T}{E}\right)^n > 0 \tag{24}$$

$$\frac{\partial^2 Q}{\partial T^2} = -\frac{n}{E}\left(\beta_0-\beta_1 p\right) W^\gamma \left(1-\frac{T}{E}\right)^{n-1} < 0 \tag{25}$$

Rearranging (9), it is obtained that

$$\left(1-\frac{t_1}{E}\right)^{n+1} = \left(1-\frac{T}{E}\right)^{n+1} + \frac{\left(W^{1-\gamma}-Z^{1-\gamma}\right)(n+1)}{E\left(\beta_0-\beta_1 p\right)(1-\gamma)} \tag{26}$$

Substituting (A5) into (A1) and (A2), yields

$$\frac{\partial t_1}{\partial T} = \left(1-\frac{T}{E}\right)^n \left(1-\frac{t_1}{E}\right)^{-n} > 0 \tag{27}$$

$$\frac{\partial^2 t_1}{\partial T^2} = \frac{n}{E}\left[\left(1-\frac{T}{E}\right)^{2n}\left(1-\frac{t_1}{E}\right)^{-(2n+1)} - \left(1-\frac{T}{E}\right)^{n-1}\left(1-\frac{t_1}{E}\right)^{-n}\right] < 0 \tag{28}$$

From (14), set

$$\begin{aligned}y(T) = \Bigg\{&p(Q-Z)+sZ-cQ-o-h\Bigg[Qt_1+\frac{\left(\beta_0-\beta_1 p\right)W^\gamma E^2\left[1-\left(1-\frac{t_1}{E}\right)^{n+2}\right]}{(n+1)(n+2)} \\ &-\frac{\left(\beta_0-\beta_1 p\right)W^\gamma E t_1}{n+1}\Bigg]-\frac{h}{2}(W+Z)(T-t_1)\Bigg\}\end{aligned} \tag{29}$$

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and

$$x\left(T\right)=T>0. \tag{30}$$

Consequently,

$$q\left(T\right)=\frac{y(T)}{x(T)}=\prod\left(p,Z,T\right) \tag{31}$$

For any given p and Z, taking the first order derivative of $y(T)$, thus

$$y'(T)=\left(p-c-ht_1\right)\frac{\partial Q}{\partial T}-h\left[Q+\left(1-\frac{t_1}{E}\right)^{n+1}\left(\frac{\left(\beta_0-\beta_1p\right)W^{\gamma}E}{n+1}\right)-\frac{\left(\beta_0-\beta_1p\right)W^{\gamma}E}{n+1}\right.$$
$$\left.-\frac{W+Z}{2}\right]\frac{\partial t_1}{\partial T}-\frac{h}{2}\left(W+Z\right) \tag{32}$$

Rearranging (4) we get

$$W=Q-\frac{\left(\beta_0-\beta_1p\right)W^{\gamma}E}{n+1}\left[1-\left(1-\frac{t_1}{E}\right)^{n+1}\right] \tag{33}$$

Substituting (A12) into (A11), the following is obtained

$$y'(T)=\left(p-c-ht_1\right)\frac{\partial Q}{\partial T}-\frac{h}{2}\left(W-Z\right)\frac{\partial t_1}{\partial T}-\frac{h}{2}\left(W+Z\right) \tag{34}$$

Taking the derivative of (A13) with respect to T,

$$y^{''}\left(T\right)=\left(p-c-ht_1\right)\frac{\partial^2 Q}{\partial T^2}-h\left(\frac{\partial t_1}{\partial T}.\frac{\partial Q}{\partial T}\right)-\frac{h}{2}\left(W-Z\right)\frac{\partial^2 t_1}{\partial T^2} \tag{35}$$

Substituting (A3), (A4), and (A7) into (A14), and simplifying terms, the following result is obtained:

$$y^{''}\left(T\right)=J=-\frac{n}{E}(\beta_0-\beta_1p)W^{\gamma}E\left(1-\frac{T}{E}\right)^{n-1}\left(p-c-ht_1\right)-\frac{h(1-\frac{T}{E})^{2n}}{(1-\frac{t_1}{E})^{n}}$$
$$\left\{(\beta_0-\beta_1p)W^{\gamma}E+\frac{(W+Z)n}{2E}\left[\left(1-\frac{t_1}{E}\right)^{-(n+1)}-\left(1-\frac{T}{E}\right)^{-(n+1)}\right]\right\} \tag{36}$$

As a result, if $J<0$, then $y(T)$ is nonnegative differentiable and strictly concave. Thus, if $J<0$ then $\prod(p,Z,T)$ as in (13) is strictly pseudo-concave function in T, and there exists a unique optimal solution.

8. THE OPTIMAL REPLENISHMENT CYCLE TIME T*

From (A8), (A9), and (A10), it is obvious that $\prod(p,Z,T)=y(T)/T$. Thus, given E and P, taking the first order derivative of $\prod$(p,Z,T) with respect to T, and setting the result to zero, the necessary and sufficient condition to find T^* is given as follows:

$$\frac{d}{dT}\prod\left(\mathrm{p,Z,T}\right)\ =\frac{y'(T)}{T}-\frac{y\left(T\right)}{T^2}=0\ \Longrightarrow\ y'\left(T\right)T-y(T)=\ 0 \tag{37}$$

$$\left\{\left(\beta_0-\beta_1p\right)W^{\gamma}E\left(1-\frac{T}{E}\right)^{n}\left(p-c-ht_1\right)-\frac{h}{2}\left[\left(W-Z\right)\frac{\left(1-\frac{T}{E}\right)^{n}}{\left(1-\frac{t_1}{E}\right)^{n}}+W+Z\right]\right\}T$$
$$-\left[p\left(Q-Z\right)+sZ-cQ-o-h\left\{Qt_1+\frac{\left(\beta_0-\beta_1p\right)W^{\gamma}E^2\left[1-\left(1-\frac{t_1}{E}\right)^{n+2}\right]}{\left(n+1\right)\left(n+2\right)}\right.\right.$$
$$\left.\left.-\frac{\left(\beta_0-\beta_1p\right)W^{\gamma}Et_1}{n+1}\right\}-\frac{h}{2}\left(W+Z\right)\left(T-t_1\right)\right]=0 \tag{38}$$

9. PROOF OF THEOREM 2

For any given T, taking the first and second order partial derivatives of (9) with respect to Z and p, and applying (A5), the following results are obtained.

$$\frac{\partial t_1}{\partial Z} = \frac{1}{(\beta_0 - \beta_1 p)\left(1 - \frac{t_1}{E}\right)^n Z^\gamma} > 0 \tag{39}$$

$$\frac{\partial t_1}{\partial p} = -\frac{\left(W^{1-\gamma} - Z^{1-\gamma}\right)\beta_1}{(1-\gamma)(\beta_0 - \beta_1 p)^2\left(1 - \frac{t_1}{E}\right)^n} < 0 \tag{40}$$

$$\frac{\partial^2 t_1}{\partial Z \partial p} = \frac{\beta_1}{(\beta_0 - \beta_1 p)^2 Z^\gamma \left(1 - \frac{t_1}{E}\right)^n}\left[1 - \frac{n\left(W^{1-\gamma} - Z^{1-\gamma}\right)}{E(\beta_0 - \beta_1 p)(1-\gamma)\left(1 - \frac{t_1}{E}\right)^{n+1}}\right] \tag{41}$$

$$\frac{\partial^2 t_1}{\partial Z^2} = \frac{n}{E(\beta_0 - \beta_1 p)^2 Z^{2\gamma}\left(1 - \frac{t_1}{E}\right)^{2n+1}} - \frac{\gamma}{(\beta_0 - \beta_1 p)\left(1 - \frac{t_1}{E}\right)^n Z^{1+\gamma}} \tag{42}$$

$$\frac{\partial^2 t_1}{\partial p^2} = \frac{2\left(W^{1-\gamma} - Z^{1-\gamma}\right){\beta_1}^2}{(1-\gamma)(\beta_0 - \beta_1 p)^3\left(1 - \frac{t_1}{E}\right)^n}\left[\frac{n\left(W^{1-\gamma} - Z^{1-\gamma}\right)}{2E(1-\gamma)(\beta_0 - \beta_1 p)\left(1 - \frac{t_1}{E}\right)^{n+1}} - 1\right] \tag{43}$$

From (5) and (9), it is obtained that

$$\frac{(Q-W)(n+1)}{(\beta_0 - \beta_1 p) W^\gamma E} = 1 - \left(1 - \frac{T}{E}\right)^{n+1} - \frac{\left(W^{1-\gamma} - Z^{1-\gamma}\right)(n+1)}{E(\beta_0 - \beta_1 p)(1-\gamma)} \tag{44}$$

For any given T, taking the first and second order partial derivative of (10) with respect to Z and p, and applying (C6), the followings are obtained.

$$\frac{\partial Q}{\partial Z} = \left(\frac{W}{Z}\right)^\gamma > 0 \tag{45}$$

$$\frac{\partial Q}{\partial p} = -\frac{\beta_1 (Q-W)}{(\beta_0 - \beta_1 p)} - \frac{\beta_1 W^\gamma\left(W^{1-\gamma} - Z^{1-\gamma}\right)}{(\beta_0 - \beta_1 p)(1-\gamma)} < 0 \tag{46}$$

$$\frac{\partial^2 Q}{\partial Z \partial p} = 0 \tag{47}$$

$$\frac{\partial^2 Q}{\partial Z^2} = -\frac{\gamma W^\gamma}{Z^{\gamma+1}} < 0 \tag{48}$$

$$\frac{\partial^2 Q}{\partial p^2} = 0 \tag{49}$$

From (13), let

$$\begin{aligned} z(p,Z) = \Bigg[& p(Q-Z) + sZ - cQ - o - h\Bigg\{Qt_1 + \frac{(\beta_0 - \beta_1 p) W^\gamma E^2\left[1 - \left(1 - \frac{t_1}{E}\right)^{n+2}\right]}{(n+1)(n+2)} \\ & - \frac{(\beta_0 - \beta_1 p) W^\gamma E t_1}{n+1}\Bigg\} - \frac{h}{2}(W+Z)(T-t_1)\Bigg] \end{aligned} \tag{50}$$

Consequently, for any given T, the total profit is

$$\prod (p, \mathrm{Z}, \mathrm{T}) = \frac{1}{T} z(p,Z) \tag{51}$$

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Taking the first and second order partial derivatives of $z(p, Z)$ with respect to p and Z, applying (A12), (C1)-(C5), (C7)-(C11), the followings are obtained.

$$\frac{\partial z(p,Z)}{\partial Z} = (p-c-ht_1)\frac{\partial Q}{\partial Z} - p + s - h\left[(W-Z)\frac{\partial t_1}{\partial Z} + T - t_1\right] \tag{52}$$

$$\begin{aligned}\frac{\partial z(p,Z)}{\partial p} &= Q - Z + (p-c-ht_1)\frac{\partial Q}{\partial p} - \frac{h}{2}(W-Z)\frac{\partial t_1}{\partial p} \\ &\quad + \frac{h\beta_1 p W^\gamma E}{n+1}\left\{\frac{E\left[1-\left(1-\frac{t_1}{E}\right)^{n+2}\right]}{n+2} - t_1\right\}\end{aligned} \tag{53}$$

$$\begin{aligned}\frac{\partial^2 z(p,Z)}{\partial Z \partial p} &= \left(\frac{W}{Z}\right)^\gamma - 1 + h\left[\frac{\left(W^{1-\gamma}-Z^{1-\gamma}\right)\beta_1}{(1-\gamma)\left(\beta_0-\beta_1 p\right)^2\left(1-\frac{t_1}{E}\right)^n}\right]\left[\left(\frac{W}{Z}\right)^\gamma - \frac{1}{2}\right] - \\ &\quad \frac{h\beta_1(W-Z)}{2(\beta_0-\beta_1 p)^2 Z^\gamma\left(1-\frac{t_1}{E}\right)^n}\left[1-\frac{n\left(W^{1-\gamma}-Z^{1-\gamma}\right)}{E\left(\beta_0-\beta_1 p\right)(1-\gamma)\left(1-\frac{t_1}{E}\right)^{n+1}}\right] = K\end{aligned} \tag{54}$$

$$\begin{aligned}\frac{\partial^2 z(p,Z)}{\partial Z^2} &= -\left(p-c-ht_1\right)\frac{\gamma W^\gamma}{Z^{\gamma+1}} - \frac{h_1}{\left(\beta_0-\beta_1 p\right)\left(1-\frac{t_1}{E}\right)^n Z^\gamma}\left[\left(\frac{W}{Z}\right)^\gamma - 1 + \right. \\ &\quad \left.\frac{(W-Z)\,n}{2E\left(\beta_0-\beta_1 p\right)\left(1-\frac{t_1}{E}\right)^{n+1} Z^\gamma} - \frac{\gamma(W-Z)}{2Z}\right] = L\end{aligned} \tag{55}$$

$$\begin{aligned}\frac{\partial^2 z(p,Z)}{\partial p^2} &= -\left[\frac{\beta_1(Q-W)}{(\beta_0-\beta_1 p)} + \frac{\beta_1 W^\gamma\left(W^{1-\gamma}-Z^{1-\gamma}\right)}{(\beta_0-\beta_1 p)(1-\gamma)}\right]\left[2+\frac{h\left(W^{1-\gamma}-Z^{1-\gamma}\right)\beta_1}{(1-\gamma)\left(\beta_0-\beta_1 p\right)^2\left(1-\frac{t_1}{E}\right)^n}\right] \\ &\quad - \frac{h(W-Z)\left(W^{1-\gamma}-Z^{1-\gamma}\right){\beta_1}^2}{(1-\gamma)\left(\beta_0-\beta_1 p\right)^3\left(1-\frac{t_1}{E}\right)^n} - \left[\frac{n\left(W^{1-\gamma}-Z^{1-\gamma}\right)}{2E(1-\gamma)\left(\beta_0-\beta_1 p\right)\left(1-\frac{t_1}{E}\right)^{n+1}} - 1\right] \\ &\quad - \frac{h\beta_1 W^\gamma E}{(n+1)}\left[\left(1-\frac{t_1}{E}\right)^{n+1} - 1\right]\left[\frac{\left(W^{1-\gamma}-Z^{1-\gamma}\right)\beta_1}{(1-\gamma)\left(\beta_0-\beta_1 p\right)^2\left(1-\frac{t_1}{E}\right)^n}\right] = M\end{aligned} \tag{56}$$

If $L < 0$, $M < 0$, and $LM - K^2 > 0$, then the Hessian matrix associated with $z(p, Z)$ is negative definite. Thus, for any given T, if $L < 0$, $M < 0$, and $LM - K^2 > 0$, then $\prod(p, Z, T)$ in (13) is a strictly concave function in p and Z. Hence, there exists a unique optimal solution.

10. OPTIMAL ENDING INVENTORY LEVEL AND SHELF SPACE SIZE

For any given T, substituting (C1) and (C7) into (C14), and setting the result to zero, the necessary and sufficient condition for Z^* is given as follows.

$$(p-c-ht_1)\left(\frac{W}{Z}\right)^\gamma - p + s - \frac{h}{2}\left[\frac{(W-Z)\left(1-\frac{t_1}{E}\right)^{-n}}{\left(\beta_0-\beta_1 p\right)Z^\gamma} + T - t_1\right] = 0 \tag{57}$$

Similarly, substituting (C2) and (C8) into (C15), and setting the result to zero, the necessary and sufficient condition for p^* is:

$$\begin{aligned}&Q - Z - (p-c-ht_1)\frac{\beta_1}{(\beta_0-\beta_1 p)}\left[Q - W + \frac{W^\gamma\left(W^{1-\gamma}-Z^{1-\gamma}\right)}{(1-\gamma)}\right] \\ &+ \frac{h(W-Z)\left(W^{1-\gamma}-Z^{1-\gamma}\right)\beta_1}{2(1-\gamma)\left(\beta_0-\beta_1 p\right)^2\left(1-\frac{t_1}{E}\right)^n} + \frac{h\beta_1 p W^\gamma E}{n+1}\left\{\frac{E\left[1-\left(1-\frac{t_1}{E}\right)^{n+2}\right]}{n_1+2} - t_1\right\} = 0\end{aligned} \tag{58}$$

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