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Supply chain-transport supernetwork equilibrium under uncertain product demands

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Abstract: This paper proposes a supply chain-transport supernetwork equilibrium (SC-T-SNE) model, which takes into account inventory costs incurred due to the uncertainty of product demands. The model is also a modified version of supply chain network equilibrium models, dealing with a supernetwork explicitly integrating supply chain networks (SCNs) with a transport network. The inventory costs are estimated based on the probability distributions for representing the demand fluctuations, where wholesalers' and/or retailers' preference for inventory can be represented by setting the threshold. Decentralized decision-making processes and interactive behaviors among such entities as manufacturers, wholesalers, retailers, consumers (demand markets), freight carriers, and transport network users are incorporated within the model. These are formulated as variational inequality problems, and the equilibrium conditions governing the supernetwork and the solution procedures to the variational inequality problems are represented. Results of the model, as applied to a hypothesized supernetwork, reveal that the increased variability of consumers' demands would decrease the amount of products transacted between the entities and the efficiency of SCNs. Numerical tests using the model also show that the information sharing between wholesalers and retailers about such variable nature of the demands could enhance the efficiency of SCNs with the distribution channels on them being changed. Furthermore, it is indicated that the magnitudes of the effects attained from the information sharing might be comparable to those obtained by the improvement of capacity in congested links on the road network.
Keyword — Uncertainty, Inventory, Supply chain, Transport network

1. INTRODUCTION

A supply chain is a network of linkages between various economic entities for the passage of products from production to consumption, including manufacturers, wholesalers, retailers, consumers, and freight carriers. Supply chain management (SCM) has recently become a crucial long-term strategy for businesses(Mentzer, De Witt, Keebler, Min, & Nix, 2003), since it was firstly coined by Oliver and Webber (1982). The fundamental objective of SCM is to develop effective networks among companies and/or organizations including inventory control. That is, to create efficient supply chain networks (SCNs), even though the definition of SCM and its concept can somewhat vary (Christopher, 1992; Cooper & Ellram, 1993; Ellram, 1991; Kopczak, 1997; Lee & Billington, 1992; Towill, Naim, & Wikner, 1992). Decisions relating to goods distribution and freight transport are typically made looking over the entire SCNs, as SCM is positively implemented by companies for remaining competitive. Consequently, the comprehension of what occurs on the SCNs, namely, to describe the behavior of economic entities in the SCNs and the resulting flow of products, allows administrators and planners to understand the mechanism of the generation of products as well as to explore the effects of transport- and logistics-related measures. These can also encourage companies to recognize the necessity and effectiveness of such measures.

Supply chain network equilibrium (SCNE) models can be used as a tool for describing what happens on multi-tiered SCNs, incorporating decentralized decision-making by multiple agents on the SCNs and their behavioral interaction. The SCNE model, developed first by Nagurney, Dong, and Zhang (2002), can provide several important outputs, such as the amount of the products produced by manufacturers, that transacted and distributed between the entities involved in an oligopolistic three-tiered SCN (i.e., manufacturers, retailers, and consumers), and the prices of the products. The SCNE model has then been expanded to consider uncertain demands (Dong, Zhang, & Nagurney, 2010; Liu & Nagurney, 2013; Qiang, Ke, Anderson, & Dong, 2013; Zhou, Chan, & Wong, 2018), electronic commerce (Nagurney, Cruz,

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Dong, & Zhang, 2005; Zhao & Nagurney, 2008), reverse SCNs (Hammond & Beullens, 2007; Nagurney & Toyasaki, 2005; Toyasaki, Daniele, & Wakolbinger, 2014; Yamada, Russ, Castro, & Taniguchi, 2009), corporate social responsibility (Cruz, 2008; Cruz & Matsypura, 2009), production capacity constraints (Hamdouch, 2011; H. Y. C. R. Meng Q., 2009), the behaviour of raw material suppliers (Cruz & Liu, 2011; Yang, Wang, & Li, 2009), corporate financial risks (Liu & Cruz, 2012; Liu & Wang, 2018), competition between supply chains (Nagurney, 2010; Nagurney, Saberi, & Shukla, 2015; Nagurney & Shukla, 2017; Nagurney & Yu, 2012; Rezapour & Farahani, 2010), humanitarian relief (Nagurney, Flores, & Soylu, 2016), and has currently developed into a dynamic approach (Daniele, 2010)Cruz and Liu (2011); Hamdouch (2011); Liu and Nagurney (2012); Saberi, Cruz, Sarkis, and Nagurney (2018); Zhou et al. (2018). These existing SCNE studies, however, have not dealt with the endogenous decision-making process for determining transport costs and fares (i.e., carriage); therefore, its direct effects on the traffic conditions in a transport network has not properly been identified. The existing SCNE models do not focus on the reciprocal influences of the behavioral changes in the SCNs and the traffic conditions in the transport network. Accordingly, these models would not always be suitable for estimating the impact of freight transport measures to be implemented on the behavior of each entity on the SCNs.

Having further developed the SCNE model by integrating SCNs with a transport network,Yamada and Febri (2015); Yamada, Imai, Nakamura, and Taniguchi (2011) propose a supply chain–transport supernetwork equilibrium (SC-T-SNE) model. The SC-T-SNE model describes the behavior of six entities within a supernetwork as shown in Fig. 1, including manufacturers, wholesalers, retailers, freight carriers, consumers (i.e., demand markets), and transport network users. The model allows for endogenously determining transport costs based on freight carriers' decision-making, since the behavior of transport network users including freight vehicles is incorporated within it. The mutual effects between the behavioral changes in the SCNs and the transport network can also be identified, especially the effects of the traffic conditions in the road network on the behavior of the entities on each SCN and vice versa. To enhance the applicability of the model, the behavior of wholesalers and the facility costs for manufacturers, wholesalers, retailers, and freight carriers are also embedded within the model, which are not taken into consideration in the existing SCNE models. The SC-T-SNE is more oriented to SCNs as compared to the existing transport network models (Crainic, Florian, & Leal, 1990; Fernandez, Joaquin de Cea, & Alexandra, 2003; Friesz, Tobin, & Harker, 1983; Guelat, Florian, & Crainic, 1990; Harker & Friesz, 1986; Yamada et al., 2009), as the relationship between traffic flow and goods movement can be elucidated, looking over the entire SCNs. Although the existing SCNE studies (Hamdouch, 2011; Liu & Nagurney, 2012; Nagurney, 2006; Nagurney, Liu, Cojocaru, & Daniele, 2007) state that the SCNE can be reformulated and solved as transport network equilibrium problems, this indicates that the SCNE can be treated as mathematically equivalent to the transport network equilibrium. However, unlike the SC-T-SNE model, such SCNE studies disregard the integration of SCNs with a transport network and provide the equilibrium conditions of an SCN instead of showing those of a supply chain-transport supernetwork.

Inventory control is one of the most crucial functions for SCM, and therefore, there has been a lot of research undertaken on inventory management (Sherbrooke, 2004; Tiexin, Jingbo, & Tao, 2008), since Harris (1913) introduced the economic order quantity model. Most of them, however, are optimization models for a company or a group of companies to determine the optimal amount of products for inventory and ordering. The paper focuses on an inventory model based on the uncertain demands for products (i.e., risk of customer demand fluctuations), which is termed the stochastic inventory model, where the Poisson distribution or the normal distribution is generally utilized as a probability distribution for representing the demand variation (Chiang & Benton, 1994).Browne and Zipkin (1991); Hala and EI-Saadani (2006); Hariga and Ben-Daya (1999) provide a comprehensive overview of the stochastic inventory models. Existing research on SCNE models with inventory costs being generated by the uncertainty of demands (Dong et al., 2010; Liu & Nagurney, 2013; Qiang et al., 2013; Zhou et al., 2018) are inappropriate to deal with a variety of probability distributions. Multiperiod SCNE models (Cruz & Liu, 2011; Daniele, 2010; Hamdouch, 2011; Liu & Nagurney, 2012; Saberi et al., 2018) also take into account inventory costs with dynamically variable demand functions, which are however, not involving uncertainty in each period.

The highlight of the paper is to present the SC-T-SNE model taking into account the inventory costs being induced by the uncertain demands, even though the model is not a multiperiod one. As compared to the SC-T-SNE model proposed by Yamada and Febri (2015); Yamada et al. (2011), the model presented in the paper explicitly incorporates inventory costs, and consequently, is more realistic, practical, and useful for companies, administrators and planners. Characterized by the availability of a wider variety of probability distributions for representing the demand fluctuations, this model can facilitate the broader application than the existing SCNE models with inventory costs (Dong et al., 2010; Liu & Nagurney, 2013; Qiang et al., 2013; Zhou et al., 2018). The mutual effects between the behavioral changes in SCNs and a transport network, namely, the effects of the traffic conditions in the transport network on the activities of each agent on the SCNs and vice versa, can also be investigated in this model, taking into account the endogenous decision-making process for determining transport fares (i.e., carriage) and costs. These are allowed by the model handling the SC-T-SNE.

The rest of the paper is organized as follows. In the following section, the formulation of the SC-T-SNE model is given

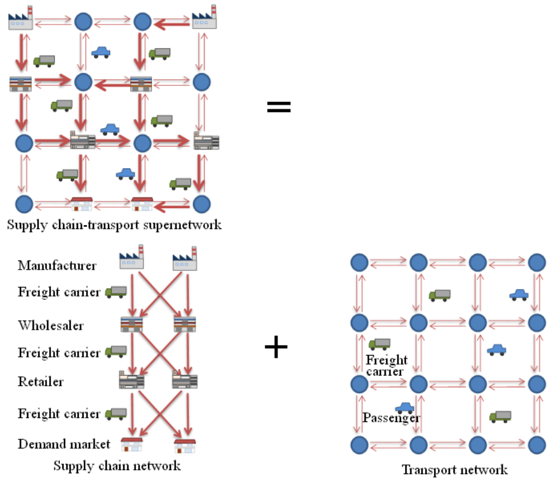

Figure 1: Supply chain-transport supernetwork

with the inventory costs being produced by the uncertainty of product demands, deriving the optimality conditions for the decision-makers. The governing equilibrium conditions are then presented, and the endogenous price variables are also discussed in this section. In Section 3, the qualitative properties of the solutions are provided. The solution procedures to the variational inequality problems formulated in the previous section are also outlined. The model is then tested and applied to a hypothetical supernetwork in Section 4, where several case studies are carried out for investigating the influence of the uncertain demands on goods movement and the efficiency of SCNs in comparison with the effects of the improvement of the road network. Finally, in Section 5, the methodologies, results, and analyses in the paper are summarised.

2. THE SUPPLY CHAIN–TRANSPORT SUPERNETWORK EQUILIBRIUM MODEL WITH UNCERTAIN PRODUCT DEMANDS

In this section, the SC-T-SNE model with uncertain demands for products is formulated for a supernetwork, where SCNs for various different products are involved in a transport network, which is assumed to be a road network in this paper. Here, Y kinds of oligopolistic four-tier SCNs lie on the transport network, each providing product $y(y = 1, ..., Y)$. As depicted in Fig. 2, the SCN for product y consists of my manufacturers, with a typical manufacturer denoted by i^y; n^y wholesalers, with a typical wholesaler denoted by j^y; o^y retailers, with a typical retailer denoted by k^y; u^y freight carriers, with a typical freight carrier denoted by h^y, and consumers associated with r demand markets, with a typical demand market denoted by l. Manufacturer $i^y(i^y = 1, \cdots, m^y)$ on the SCN for product y is involved in its production, which can then be purchased by wholesaler $j^y(j^y = 1, \ldots, n^y)$, who, in turn, sell the product to retailer $k^y(k^y = 1, \ldots, o^y)$. Then, the retailer offers the product to consumers in demand market $l(l = 1, \cdots, r)$. Each market possesses a demand function for an individual product. Product y is transported by freight carrier $h^y(h^y = 1, \cdots, u^y)$. The links in the SCNs represent those for transport/transaction. The inventory costs resulted from the uncertain demands are to be incurred by both the wholesalers and retailers.
The manufacturers, wholesalers, retailers, and demand markets in Y kinds of SCNs exist on the nodes in the transport network. More than one decision-maker cannot deal with the same kind of product at a single node in the transport network. Each node on the transport network can generate and attract the trips of freight vehicles, since the products are transacted and distributed among the decision-makers on it. Other traffic rather than the freight vehicles can also be generated from and attracted at any node in the transport network.

2.1 Notations for the model

The model inherently involves a lot of variables and parameters. Therefore, firstly, the notations majorly used for the model are summarised as below. The equilibrium solution is denoted by "$*$" in the subsequent formulation of the model.

2.1.1 Decision variables

$q^{p^y}_{h^y w^y z^y}$: Amount of product y transacted/transported from entity w^y to entity z^y by freight carrier h^y using path p^y

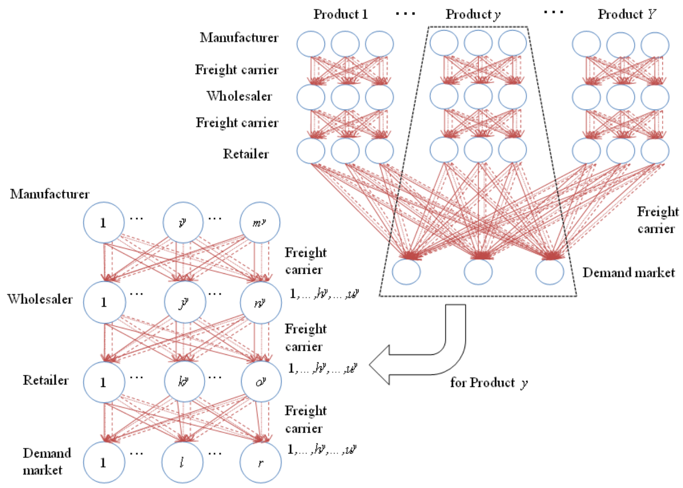

Figure 2: Supply chain network

Q^{1y} : $u^y m^y n^y e^{1y}$-dimensional vector with component $h^y i^y j^y p^{1y}$ denoted by $q_{h^y i^y j^y}^{p^{1y}}$ representing shipments of product y between manufacturers and wholesalers

Q^{2y} : $u^y n^y o^y e^{2y}$-dimensional vector with component $h^y j^y k^y p^{2y}$ denoted by $q_{h^y j^y k^y}^{p^{2y}}$ representing shipments of product y between wholesalers and retailers

Q^{3y} : $u^y o^y r e^{3y}$-dimensional vector with component $h^y k^y l p^{3y}$ denoted by $q_{h^y k^y l}^{p^{3y}}$ representing shipments of product y between retailers and demand markets

Q^1 : S^1-dimensional vector with components: $Q^{11}, \cdots, Q^{1Y}$ $(S^1 = \sum_{y=1}^{Y}(u^y m^y n^y e^{1y}))$

Q^2 : S^2-dimensional vector with components: $Q^{21}, \cdots, Q^{2Y}$ $(S^2 = \sum_{y=1}^{Y}(u^y n^y o^y e^{2y}))$

Q^3 : S^3-dimensional vector with components: $Q^{31}, \cdots, Q^{3Y}$ $(S^3 = \sum_{y=1}^{Y}(u^y o^y r e^{3y}))$

ρ_l^{4y} :Market price of product y at demand market l

ρ^{4y} : r-dimensional vector for product y with component l denoted by ρ_l^{4y}

ρ^4 : rY-dimensional vector with component ly denoted by ρ_l^{4y}

X : $e^5 e^6 e^4$-dimensional vector with component rsp_{rs} denoted by $x_{rs}^{p_{rs}}$

$x_{rs}^{p_{rs}}$: Traffic volume of passenger cars between r and s using path p_{rs}

c_{rs} : Travel cost incurred between r and s

2.1.2 Inventory costs and uncertainty

$s_{w^y z^y}(\bullet)$: Inventory cost generated between entity w^y and entity z^y

$q_{w^y z^y}^s$: Expected inventory quantity generated between entity w^y and entity z^y

$\xi_{w^y z^y}(\bullet)$: Coefficient of inventory cost between w^y and z^y

K_1 : Number of classes in the probability distribution used for estimating $q_{j^y k^y}^s$

K_2 : Number of classes in the probability distribution used for estimating $q_{k^y l}^s$

K_3^y : Number of classes in the probability distribution used for representing the demand of product y at demand markets

L_1 : Class number when $\omega_{j^y k^y}^M = \sum_{h^y=1}^{u^y} \sum_{p^{2y}=E^{2y}} q_{h^y j^y k^y}^{p^{2y}}$.

L_2 : Class number when $\omega_{k^y l}^R = \sum_{h^y=1}^{u^y} \sum_{p^{3y}=E^{3y}} q_{h^y k^y l}^{p^{3y}}$.

$\omega^U_{w^y z^y}$: Class mark of class U in the probability distribution used for estimating $q^s_{w^y z^y}$ (U=M(M=1,$\cdots$,K_1)or$= R(R = 1, \cdots, K_2)$)

$\omega^{V^y}_l$: Class mark of class $V^y (V^y = 1, \cdots, K_{3^y})$ in the probability distribution used for representing the demand of product y at demand market l

$p^U_{w^y z^y}$: Probability of class U in the probability distribution used for estimating $q^s_{w^y z^y}$

$p^{V^y}_l$: Probability of class V^y in the probability distribution used for representing the demand of product y at demand market l

$q^0_{w^y z^y}$: Left endpoint of class 1 in the probability distribution used for estimating $q^s_{w^y z^y}$

$q^{U_1}_{w^y z^y}$: Right endpoint of class U_1 in the probability distribution used for estimating $q^s_{w^y z^y} (U_1 = K_1 or K_2)$

ω^y_l : Random variable representing the demand fluctuations for product y at demand market l

$\bar{\omega}$: Expected value of the demand fluctuations for product y at demand market l

2.1.3 Other variables, functions, sets, and parameters

O : Set of origins for all the traffic in the transport network ($\forall O \subseteq V$)

S : Set of destinations for all the traffic in the transport network ($\forall S \subseteq V$)

$G(V, A)$: Transport network with the set of nodes V and that of links A

$\delta^{w^y z^y}_{a,p^y}$: Binary value of 1 if link a is contained in path p^y between entity w^y and entity z^y; 0 if it is otherwise

$d_{rs}(\bullet)$: Traffic demand function between r and s

$t^{p^y}_{w^y z^y}(\bullet)$: Travel time on path p^y between w^y and z^y

$t_a(x_a)$: Travel time on link a

x_a : Traffic volume on link a

$\rho^1_{i^y j^y}$: Price charged for product y by manufacturer i^y to wholesaler j^y

$\rho^2_{j^y}$: Sales price charged for product y by wholesaler j^y to retailers

$\rho^3_{k^y}$: Sales price charged for product y by retailer k^y to consumers

$\rho^T_{h^y w^y z^y}$: Carriage charged by freight carrier h^y for transporting product y between entity w^y and entity z^y ($T = 5$: between manufacturer i^y and wholesaler j^y, $T = 6$: between wholesaler j^y and retailer k^y, $T = 7$: between retailer k^y and demand market l)

$f_{i^y}(\bullet)$: Production cost of manufacturer i^y for product y

$g_{w^y}(\bullet)$: Facility cost of entity w^y

$c_{w^y z^y}(\bullet)$: Transaction cost for product y incurred between entity w^y and entity z^y (excluding transport cost incurred between w^y and z^y)

$c_{w^y}(\bullet)$: Handling cost of entity w^y

$c^{p^y}_{h^y w^y z^y}(\bullet)$: Unit operation cost (per transport volume) of freight carrier h^y for transporting product y from entity w^y to entity z^y using path p^y

d^y_l : Demand function of product y at demand market l

η : Operation cost for a freight vehicle per unit of time

ζ : Value of time for passenger cars

ι : Capacity of a freight vehicle

κ : Average loading factor of a freight vehicle

ν : Passenger car equivalent

2.2 The behavior of manufacturers and their optimality conditions

The total costs incurred by manufacturer i^y are equal to the sum of his production cost, facility cost, transaction cost, and transport cost. His revenue, in turn, is equal to the price that the manufacturer charges for the product (and the wholesalers are willing to pay) times the total quantity of products obtained/purchased from the manufacturer by the wholesalers.

Let $E^{1y}(= E_{i^y j^y})$ be the set of paths for transporting product y between manufacturer i^y and wholesaler j^y on the transport network, and $\dim p^{1y} = e^{1y}$ is given to path $p^{1y}(= p_{i^y j^y}) \in E^{1y}$ between OD pair $(i^y, j^y)(i^y \in O, j^y \in S)$. The behavior of manufacturer i^y dealing with product y is formulated below as a profit maximization problem.

$$Max \sum_{j^y=1}^{n^y} \rho^{1*}_{i^y j^y} \sum_{h^y=1}^{u^y} \sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y} - f_{i^y}(Q^{1y}) - g_{i^y}(Q^{1y}) - \sum_{j^y=1}^{n^y} c_{i^y j^y}(Q^{1y})$$
$$- \sum_{h^y=1}^{u^y} \sum_{j^y=1}^{n^y} \rho^{5*}_{h^y i^y j^y} \sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y} \quad (1)$$

$$\text{subject to } q^{p^{1y}}_{h^y i^y j^y} \leq 0 \ \forall h^y, j^y, p^{1y} \quad (2)$$

Assuming that production cost functions, facility cost functions and transaction cost functions for each manufacturer are continuously differentiable and convex as well as that the manufacturers compete in a noncooperative fashion (see Nagurney et al., 2002), the optimality conditions for all manufacturers for all kinds of products can simultaneously be expressed as the following variational inequality: determine $Q^{1*} \in R^{S^1}_+$, which satisfies:

$$\sum_{y=1}^{Y} \sum_{h^y=1}^{u^y} \sum_{i^y=1}^{m^y} \sum_{j^y=1}^{n^y} \sum_{p^{1y}\in E^{1y}} \left[\frac{\partial f_{i^y}(Q^{1y*})}{\partial q^{p^{1y}}_{h^y i^y j^y}} + \frac{\partial g_{i^y}(Q^{1y*})}{\partial q^{p^{1y}}_{h^y i^y j^y}} + \frac{\partial c_{i^y j^y}(Q^{1y*})}{\partial q^{p^{1y}}_{h^y i^y j^y}} + \rho^{5*}_{h^y i^y j^y} - \rho^{1*}_{i^y j^y} \right]$$
$$\times \left[q^{p^{1y}}_{h^y i^y j^y} - q^{p^{1y*}}_{h^y i^y j^y} \right] \leq 0 \ \forall Q^1 \in R^{S^1}_+ \quad (3)$$

In this derivation, as in the succeeding derivation of inequalities (8), (13), (17), and (26), the prices charged are not considered variables. Instead, they can be treated as endogenous variables in the complete equilibrium model (i.e., inequality (32)) (Dong et al., 2010; Hammond & Beullens, 2007; Nagurney et al., 2002).

2.3 The behavior of wholesalers with uncertain product demands and their optimality conditions

The wholesalers are involved in transactions both with the manufacturers and with the retailers. Let $E^{2y}(= E_{i^y k^y})$ denote the set of paths for transporting product y between wholesaler j^y and retailer k^y on the transport network, and $p^{2y}(= p_{j^y k^y}) \in E^{2y}$ $(\dim p^{2y} = e^{2y})$ be the path traveled between OD pair (j^y, k^y) in it. The behavior of wholesaler j^y dealing with product y is formulated with the following criterion of profit maximization.

$$\text{Max } \rho^{2*}_{j^y} \sum_{h^y=1}^{u^y} \sum_{k^y=1}^{o^y} \sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} - c_{j^y}(Q^{1y}) - g_{j^y}(Q^{1y}) \quad (4)$$
$$- \sum_{k^y=1}^{o^y} s_{j^y k^y}(Q^{2y}) - \sum_{k^y=1}^{o^y} c_{j^y k^y}(Q^{2y}) - \sum_{h^y=1}^{u^y} \sum_{k^y=1}^{o^y} \rho^{6*}_{h^y j^y k^y} \sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} - \sum_{i^y=1}^{m^y} \rho^{1*}_{i^y j^y} \sum_{h^y=1}^{u^y} \sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y}$$

$$\text{subject to } \sum_{h^y=1}^{u^y} \sum_{k^y=1}^{o^y} \sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} \leq \sum_{h^y=1}^{u^y} \sum_{i^y=1}^{m^y} \sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y} \quad (5)$$

$$q^{p^{1y}}_{h^y i^y j^y} \geq 0 \forall h^y, i^y, p^{1y}, q^{p^{2y}}_{h^y i^y j^y} \geq 0 \forall h^y, k^y, p^{2y} \quad (6)$$

The objective function (4) represents that the difference between the revenues minus all the costs generated (i.e., handling cost, facility cost, inventory cost, transaction cost, and transport cost) and the payout to the manufacturers should be maximized. Constraint (5) simply expresses that the retailers cannot purchase more of the product from a wholesaler than is available in stock.

The handling cost is required for temporal storage of shipments, while the inventory cost includes expenses for inventory-related storage, disposal, and the associated labor, expressed as the product of a coefficient of inventory cost given in advance and the expected amount of inventory. As Fig. 3 indicates, the demand for product y of retailer k^y on wholesaler j^y (i.e., the amount of the product transacted between them) is assumed to follow a probability distribution $\psi_{j^y k^y}$, and an inventory threshold is set at $\sum_{h^y=1}^{u^y} \sum_{p^{2y} \in E^{2y}} q_{h^y j^y k^y}^{p^{2y}}$. Assuming that an excess supply (i.e., inventory) occurs when the product demand between j^y and k^y falls below the inventory threshold (Fig. 4), the expected amount of inventory can be estimated as the expected value of inventory quantity to be generated. Preference for inventory can be changed by relocating the threshold on the probability distribution. The threshold is positioned at a smaller amount of the product transacted (on the horizontal axis in Fig. 3) for inventory-averse wholesalers, while at a larger amount for stockout-averse wholesalers. In the case of the inventory-averse, both the inventory cost and the expected amount of inventory can be diminished, but the amount of the product transacted, $\sum_{h^y=1}^{u^y} \sum_{p^{2y} \in E^{2y}} q_{h^y j^y k^y}^{p^{2y}}$, is likely to decrease due to the lower-positioned inventory threshold, causing the revenues to decline. Although the cost incurred for stockout is not explicitly considered in this model, the stockout-averse wholesalers can increase their revenues through the increased amount of the product transacted, by setting the threshold at a larger amount, which, in turn, lead to the increase in inventory cost in comparison with the inventory-averse case. As such, safety stock can be taken into consideration, if the threshold is set at a larger amount.

The shape of the probability distribution $\psi_{j^y k^y}$ is influenced by the probability distributions of consumers' demands, which are assumed to be continuous, as illustrated in Section 2.5 (also see the examples in Section 4). As is mentioned in Section 2.5, the demands fluctuate even in the same market price, and their variability depends on the quantity of the demands (see Fig. 5 in Section 2.5). Therefore, wholesaler j^y cannot deterministically foresee a single probability distribution of the demand of retailer k^y on wholesaler j^y, but can only perceive that the amount of the product that the retailer will purchase varies in response to the sales price offered and fluctuates even in the same sales price (specifically, parameters of the probability distribution representing the demand of retailer k^y on wholesaler j^y, such as its mean and variance, vary) as well as that the magnitudes of the fluctuation depend on the amount of the retailer's demand. The wholesalers are only able to determine in advance the relative location of the inventory threshold on the probability distribution, since the amount of the products consumed at demand markets results from the behavioral interactions among all economic entities on the SCNs (see variational inequalities (17) and (32) derived hereafter).

Discretely approximated, as shown in Fig. 4, the inventory cost can be derived as follows. Here, the relationship

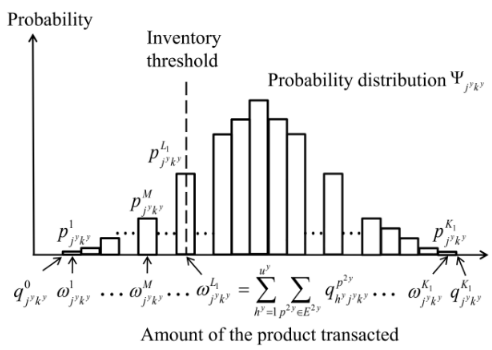

Figure 3: The probability distribution of demand of retailer k^y on wholesaler j^y for product y

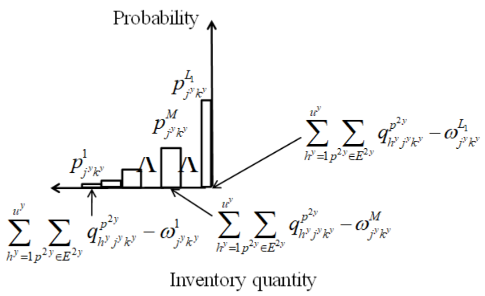

Figure 4: The probability distribution of the amount of inventory generated between j^y and k^y

between $q_{j^y k^y}^0$ and $\sum_{h^y=1}^{u^y} \sum_{p^{2y} \in E^{2y}} q_{h^y j^y k^y}^{p^{2y}}$ in probability distribution $\psi_{j^y k^y}$ is required to be made clear (Fig. 3).

$$s_{j^y k^y}(Q^{2y}) = \xi_{j^y k^y}(q^s_{j^y k^y}) q^s_{j^y k^y} = \xi_{j^y k^y}(q^s_{j^y k^y}) \sum_{M=1}^{L_1} \left[\sum_{h^y=1}^{u^y} \sum_{p^{2y} \in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} - \omega^M_{j^y k^y} \right] p^M_{j^y k^y} \tag{7}$$

$$= \xi_{j^y k^y}(q^s_{j^y k^y}) \sum_{M=1}^{L_1} \left[\sum_{h^y=1}^{u^y} \sum_{p^{2y} \in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} - \left\{ q^0_{j^y k^y} + \left(\frac{2M-1}{2K_1} \right) \left(q^{K_1}_{j^y k^y} - q^0_{j^y k^y} \right) \right\} \right] p^M_{j^y k^y}$$

$$= \xi_{j^y k^y}(q^s_{j^y k^y}) \sum_{M=1}^{L_1} \left[\sum_{h^y}^{u^y} \sum_{p^{2y} \in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} - \left\{ \left(\frac{2K_1 - 2M + 1}{2K_1} \right) q^0_{j^y k^y} + \left(\frac{2M-1}{2K_1} \right) q^{K_1}_{j^y k^y} \right\} \right] p^M_{j^y k^y}$$

$\xi_{j^y k^y}(\bullet)$ is established so that inventory cost function $s_{j^y k^y}(\bullet)$ is continuously differentiable and convex. Assuming that the handling cost functions, facility cost functions, and transaction cost functions are also continuously differentiable and convex as well as that the wholesalers compete with one another in a noncooperative manner, seeking to determine their optimal shipments from the manufacturers and to the retailers, the optimality conditions for all wholesalers for all kinds of products simultaneously coincide with the solution of the following variational inequality: determine $(Q^{1*}, Q^{2*}, \gamma^*) \in R_+^{S^1+S^2+nY}$ satisfying:

$$\sum_{y=1}^{Y} \sum_{h^y=1}^{u^y} \sum_{i^y=1}^{m^y} \sum_{j^y=1}^{n^y} \sum_{p^{1y} \in E^{1y}} \left[\frac{\partial c_{j^y}(Q^{1y*})}{\partial q^{p^{1y}}_{h^y i^y j^y}} + \frac{\partial g_{j^y}(Q^{1y*})}{\partial q^{p^{1y}}_{y^y i^y j^y}} + \rho^{1*}_{i^y j^y} - \gamma^*_{j^y} \right] \times \left[q^{p^{1y}}_{h^y i^y j^y} - q^{p^{1y*}}_{h^y i^y j^y} \right]$$

$$\sum_{y=1}^{Y} \sum_{h^y=1}^{u^y} \sum_{j^y=1}^{n^y} \sum_{k^y=1}^{o^y} \sum_{p^{1y} \in E^{1y}} \left[-\rho^{2*}_{j^y} + \frac{\partial s_{j^y k^y}(Q^{2y*})}{\partial q^{p^{2y}}_{h^y j^y k^y}} + \frac{\partial c_{j^y k^y}(Q^{2y*})}{\partial q^{p^{2y}}_{h^y j^y k^y}} + \rho^{6*}_{h^y j^y k^y} + \gamma^*_{j^y} \right] \times \left[q^{p^{2y}}_{h^y j^y k^y} - q^{p^{2y*}}_{h^y j^y k^y} \right]$$

$$+ \sum_{y=1}^{Y} \sum_{j^y=1}^{n^y} \left[\sum_{h^y=1}^{u^y} \left(\sum_{i^y=1}^{m^y} \sum_{p^{1y} \in E^{1y}} q^{p^{1y*}}_{h^y i^y j^y} - \sum_{k^y=1}^{o^y} \sum_{p^{2y} \in E^{2y}} q^{p^{2*}}_{h^y j^y k^y} \right) \right] \times \left[\gamma_{j^y} - \gamma^*_{j^y} \right] \geq 0$$

$$\forall (Q^1, Q^2, \gamma) \in R_+^{S^1+S^2+nY} \tag{8}$$

Here, the term γ_{i^y} is the Lagrange multiplier associated with constraint (5), and γ^y is an n-dimensional vector with component j^y denoted by γ_{i^y} , whereas γ is an nY-dimensional vector with component y denoted by γ^Y .

2.4 The behavior of retailers with uncertain product demands and their optimality conditions

The retailers, in turn, are involved in transactions with the wholesalers, since they wish to attain the product for their retail outlets, as well as with the consumers who are the ultimate purchasers of the product. Given path $p^{3y}(= p_{k^y l}) \in E^{3y}(\dim p^{3y} = e^{3^y})$ used between OD pair of $(k^y, l)(k^y \in O, l \in S)$ on the transport network, where $E^{3y}(= E_{k^y l})$ is the set of paths between retailer k^y and demand market l, the behavior of retailer k^y who deals with product y and seeks a maximum profit can be formulated as follows:

$$\text{Max } \rho^{3*}_{k^y} \sum_{h^y=1}^{u^y} \sum_{l=1}^{r} \sum_{p^{3y} \in E^{3y}} q^{p^{3y}}_{h^y k^y l} - c_{k^y}(Q^{2y}) - g_{k^y}(Q^{2y})$$

$$- \sum_{l=1}^{r} s_{k^y l}(Q^{3y}) - \sum_{l=1}^{r} c_{k^y l}(Q^{3y}) - \sum_{h^y=1}^{u^y} \sum_{l=1}^{r} \rho^{7*}_{h^y k^y l} \sum_{p^{3y} \in E^{3y}} q^{p^{3y}}_{h^y k^y l} - \sum_{j^y=1}^{n^y} \rho^{2*}_{j^y} \sum_{h^y=1}^{u^y} \sum_{p^{2y} \in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} \tag{9}$$

$$\text{subject to } \sum_{h^y=1}^{u^y} \sum_{l=1}^{r} \sum_{p^{3y} \in E^{3y}} q^{p^{3y}}_{h^y k^y l} \leq \sum_{h=1}^{u^y} \sum_{j^y=1}^{n^y} \sum_{p^{2y} \in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} \tag{10}$$

$$q^{p^{2y}}_{h^y j^y k^y} \geq 0 \ \ \forall h^y, j^y, p^{2y}, \ \ q^{p^{3y}}_{h^y k^y l} \geq 0 \ \ \forall h^y, l, p^{3y} \tag{11}$$

The objective function (9) indicates that the difference between the revenues minus all the costs generated and the payout to the wholesalers should be maximized. Constraint (10) also represents that consumers cannot purchase more of the product from a retailer than is available in stock.
In the same manner as the case of the wholesalers, inventory cost $s_{k^y l}(Q^{3y})$ can also be estimated as the product of a coefficient of inventory cost and the expected amount of inventory as demonstrated in Eq. (12), where the relationship

between $q^0_{k^y l}$ and $\sum_{h^y=1}^{u^y}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l}$ must be identified.

$$s_{k^y l}(Q^{3y}) = \xi_{k^y l}(q^s_{k^y l})q^s_{k^y l} = \xi_{k^y l}(q^s_{k^y l}) \sum_{R=1}^{L_2} \left(\sum_{h^y=1}^{u^y} \sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l} - \omega^R_{k^y l} \right) p^R_{k^y l} \tag{12}$$

$$= \xi_{k^y l}(q^s_{k^y l}) \sum_{R=1}^{L_2} \left[\sum_{h^y=1}^{u^y} \sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l} - \left\{ q^0_{k^y l} + \left(\frac{2R-1}{2K_2} \right) \left(q^{K_2}_{k^y l} - q^0_{k^y l} \right) \right\} \right] p^R_{k^y l}$$

$$= \xi_{k^y l}(q^s_{k^y l}) \sum_{R=1}^{L_2} \left[\sum_{h^y=1}^{u^y} \sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l} - \left\{ \left(\frac{2K_2-2R+1}{2K_2} \right) q^0_{k^y l} + \left(\frac{2R-1}{2K_2} \right) q^{K_2}_{k^l} \right\} \right] p^R_{k^y l}$$

Assuming that the inventory cost coefficient is provided beforehand as well as that the demand for product y in market l follows probability distribution $\psi_{k^y l}$ with its inventory threshold being set at $\sum_{h^y=1}^{u^y}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l}$, an excess supply arises when the demand between k^y and l is lower than $\sum_{h^y=1}^{u^y}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l}$. The position of the threshold in the probability distribution implies the preference for inventory. The probability distribution is assumed to be continuous, which is influenced by the probability distributions of the product demands in markets (see the examples in Section 4), though it is discretely approximated in estimating inventory cost $s_{k^y l}(Q^{3y})$. As is explained in Section 2.5, the demands vary in response to market prices, and their variability is dependent on the amount of the demands (see Fig. 5 in Section 2.5). Thus, like the wholesalers, the information that retailer k^y has is that the probability distribution of the product demand of market l on retailer k^y fluctuates in response to the demand in the market. The retailers can only determine beforehand the relative position of the inventory threshold on the probability distribution.

$\xi_{k^y l}(\bullet)$ is established such that $s_{k^y l}(\bullet)$ is a continuously differentiable and convex function of $q^{p^{3y}}_{h^y k^y l}$. The handling cost functions, facility cost functions, and transaction cost functions are also assumed to be continuously differentiable and convex. The retailers are assumed to compete with one another in a noncooperative manner, seeking to determine their optimal shipments from the wholesalers and to the demand markets. The optimality conditions for all retailers for all kinds of products can simultaneously be formulated as the following variational inequality problem: determine $(Q^{2*}, Q^{3*}, \delta^*) \in R_+^{S^2+S^3+oY}$ which satisfies:

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{j^y=1}^{n^y}\sum_{k^y=1}^{o^y}\sum_{p^{2y}\in E^{2y}} \left[\frac{\partial c_{k^y}(Q^{2y*})}{\partial q^{p^{2y}}_{h^y j^y k^y}} + \frac{\partial g_{k^y}(Q^{2y*})}{\partial q^{p^{2y}}_{h^y j^y k^y}} + \rho^{2*}_{j^y} - \delta^*_{k^y} \right] \times \left[q^{p^{2y}}_{h^y j^y k^y} - q^{p^{2y*}}_{h^y j^y k^y} \right]$$

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{k^y=1}^{o^y}\sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}} \left[-\rho^{3*}_{k^y} + \frac{\partial s_{k^y l}(Q^{3y*})}{\partial q^{p^{3y}}_{h^y k^y l}} + \frac{\partial c_{k^y l}(Q^{3y*})}{\partial q^{p^{3y}}_{h^y k^y l}} + \rho^{7*}_{h^y k^y l} + \delta^*_{k^y} \right] \times \left[q^{p^{3y}}_{h^y k^y l} - q^{p^{3y*}}_{h^y k^y l} \right]$$

$$+ \sum_{y=1}^{Y}\sum_{k=1}^{o^y} \left[\sum_{h^y=1}^{u^y} \left(\sum_{j^y=1}^{n^y}\sum_{p^{2y}\in E^{2y}} q^{p^{2y*}}_{h^y j^y k^y} - \sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}} q^{p^{3*}}_{h^y k^y l} \right) \right] \times \left[\delta_{k^y} - \delta^*_{k^y} \right] \geq 0 \tag{13}$$

$$\forall (Q^2, Q^3, \delta) \in R_+^{S^2+S^3+oY}$$

Here, the term δ_{k^y} is the Lagrange multiplier associated with constraint (10), and δ^y is an o-dimensional vector with component k^y denoted by δ_{k^y} , while δ is an oY-dimensional vector with component y denoted by δ^y .

2.5 The consumers in the demand markets with variable product demands and the equilibrium conditions

The behavior of consumers located at the demand markets is described in this section. The consumers take into account the price charged for the product by the retailers in making their consumption decisions. The demand function is assumed to be continuous, and the following equilibrium (complementarity) conditions hold for demand market l.

$$\rho^{3*}_{k^y} \begin{cases} = \rho^{4y*}_l & \text{if } {q^{p^{3y}}_{h^y k^y l}}^* > 0 \\ \geq \rho^{4y*}_l & \text{if } {q^{p^{3y}}_{h^y k^y l}}^* = 0 \end{cases} \tag{14}$$

$$d^y_l(\rho^{4y*}) \begin{cases} = \sum_{h^y=1}^{u^y}\sum_{k^y=1}^{o^y}\sum_{p^{3y}\in E^{3y}} {q^{p^{3y}}_{h^y k^Y l}}^* & \text{if } \rho^{4y*}_l > 0 \\ \leq \sum_{h^y=1}^{u^y}\sum_{k^y=1}^{o^y}\sum_{p^{3y}\in E^{3y}} {q^{p^{3y}}_{h^y k^Y l}}^* & \text{if } \rho^{4y*}_l = 0 \end{cases} \tag{15}$$

Conditions (14) state that, in equilibrium, if the consumers in demand market l purchase product y from retailer k^y, then the price charged by the retailer for product y is equal to the price that the consumers are willing to pay for it. If

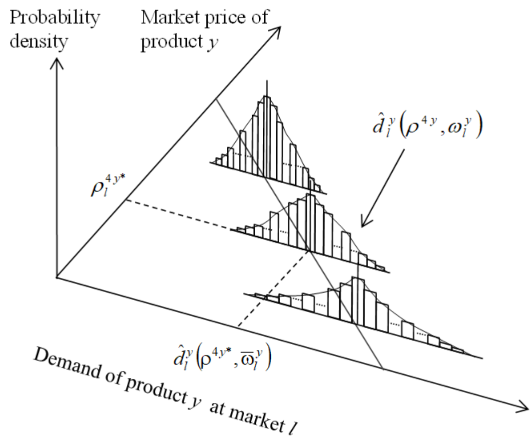

Figure 5: Demand function and its variability in demand market l

the price exceeds the price the consumers are willing to pay at the demand market, then there will be no transaction between the retailer and demand market pair. Conditions (15) state, in turn, that if the equilibrium price the consumers are willing to pay for product y in the demand market is positive, then the quantities purchased of the product from the retailers will be precisely equal to the demand for that product in the market. If the equilibrium price in the demand market is zero, then the shipments to that demand market may exceed the actual demand.

As Fig. 5 illustrates, it is assumed that demand function $d_l^y(\bullet)$ holds for $\hat{d}_l^y(\rho^{4y},\bar{\omega}_l^y)$, namely for the expected value of probability distribution $\hat{d}_l^y(\rho^{4y},\omega_l^y)$, which is a probability distribution of product y in demand market l when the market price is ρ^{4y} .

$$d_l^y(\rho^{4y}) = \hat{d}_l^y(\rho^{4y},\bar{\omega}_l^y) = \sum_{V^y=1}^{K_3^y} \hat{d}_l^y(\rho^{4y},\omega_l^{V^y})p_l^{V^y} \tag{16}$$

The probability distributions of consumers' demands are assumed to be continuous so that their variability hinges on the amount of the products consumed. In equilibrium, conditions (14) and (15) will have to hold for all demand markets for all kinds of products, and these, in turn, can also be expressed as a variational inequality problem, and given by: determine $(Q^{3*},\rho^{4*}) \in R_+^{S^3+rY}$, such that

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{k^y=1}^{o^y}\sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}} \left[\rho_k^{3y*} - \rho_l^{4y*}\right] \times \left[q_{h^yk^yl}^{p^{3y}} - q_{h^yk^yl}^{p^{3y}\,*}\right] + \sum_{y=1}^{Y}\sum_{l=1}^{r}\left[\sum_{h^y=1}^{u^y}\sum_{k^y=1}^{o^y}\sum_{p^{3y}\in E^{3y}} q_{h^yk^yl}^{p^{3y}} - d_l^y(\rho^{4y*})\right]$$
$$\times \left[\rho_l^{4y} - \rho_l^{4y*}\right] \geq 0 \quad \forall (Q^3,\rho^4) \in R_+^{s^3+rY} \tag{17}$$

2.6 The behavior of freight carriers and their optimality conditions

The freight carriers are not only decision-makers in the SCNs but transport network users. A road network is assumed as a transport network with two kinds of user groups: freight vehicles operated by the freight carriers on the SCNs and other vehicles (this can simply be called "passenger car" hereafter). The node for the generation of passenger cars (i.e., node of their origin) is expressed with $r \in O$, and those for their attraction (i.e., node of their destination) with $s \in S$. The path $p_{rs} \in E_{rs}$ between OD pair (r,s) , where E_{rs} is the set of paths between r and s , is given as $\dim p_{rs} = e^4$. The number of origin nodes for passenger cars is represented as e^5 , and that of destination nodes as e^6 .

Travel time on a path (i.e., path cost) varies depending on the traffic volume for each OD pair and the amount of the products transacted (i.e., the amount of the products produced, distributed or transported). The path travel time affects the amount of the products transported on the SCNs through freight carriers' decision-making, and consequently, the demand for freight transport vacillates.

The freight carriers are also profit-maximizers, and the optimization problem for freight carrier h^y is given as below:

$$\text{Max} \sum_{i^y=1}^{m^y}\sum_{j^y=1}^{n^y} \rho^{5*}_{h^y i^y j^y} \sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y} + \sum_{j=1}^{n^y}\sum_{k^y=1}^{o^y} \rho^{6*}_{h^y j^y k^y} \sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^y j^y k^y}$$
$$+ \sum_{k^y=1}^{o^y}\sum_{l=1}^{r} \rho^{7*}_{h^y k^y l} \sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l} - g_{h^y}(Q^{1y}, Q^{2y}, Q^{3y}) - \sum_{i^y=1}^{m^y}\sum_{j^y=1}^{n^y}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y} C^{p^{1y}}_{h^y i^y j^y}(Q^1, Q^2, Q^3, X^*)$$
$$- \sum_{j=1}^{n^y}\sum_{k^y=1}^{o^y}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} C^{p^{2y}}_{h^y j^y k^y}(Q^1, Q^2, Q^3, X^*) - \sum_{k^y=1}^{o^y}\sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l} C^{p^{3y}}_{h^y k^y l}(Q^1, Q^2, Q^3, X^*) \quad (18)$$
$$\text{subject to } q^{p^{1y}}_{h^y i^y j^y} \geq 0 \forall i^y, j^y, p^{1y}, \;\; q^{p^{2y}}_{h^y j^y k^y} \geq 0 \quad \forall j^y, k^y, p^{2y}, \;\; q^{p^{3y}}_{h^y k^y l} \forall k^y, l, p^{3y} \quad (19)$$

The facility cost is required for the operation, improvement, and maintenance of facilities owned by the freight carriers. The operation cost is generated by the operation of freight vehicles, including their fixed costs. The unit operation costs can be formulated as follows:

$$C^{p^{1y}}_{h^y i^y j^y}(Q^1, Q^2, Q^3, X^*) = \frac{\eta t^{p^{1y}}_{i^y j^y}(Q^1, Q^2, Q^3, X^*)}{\iota\kappa} \quad (20)$$
$$C^{p^{2y}}_{h^y j^y k^y}(Q^1, Q^2, Q^3, X^*) = \frac{\eta t^{p^{2y}}_{j^y k^y}(Q^1, Q^2, Q^3, X^*)}{\iota\kappa} \quad (21)$$
$$C^{p^{3y}}_{h^y k^y l}(Q^1, Q^2, Q^3, X^*) = \frac{\eta t^{p^{3y}}_{k^y l}(Q^1, Q^2, Q^3, X^*)}{\iota\kappa} \quad (22)$$

The path travel times can also be derived as below:

$$t^{p^{1y}}_{i^y j^y}(Q^1, Q^2, Q^3, X^*) = \sum_{a\in A} t_a(x_a)\delta^{i^y j^y}_{a,p^{1y}} \quad (23)$$
$$t^{p^{2y}}_{j^y k^y}(Q^1, Q^2, Q^3, X^*) = \sum_{a\in A} t_a(x_a)\delta^{j^y k^y}_{a,p^{2y}} \quad (24)$$
$$t^{p^{3y}}_{k^y l}(Q^1, Q^2, Q^3, X^*) = \sum_{a\in A} t_a(x_a)\delta^{k^y l}_{a,p^{3y}} \quad (25)$$

If the facility cost functions and operation cost functions are continuously differentiable and convex, the optimality conditions for all freight carriers for all kinds of products can simultaneously be formulated as the following variational inequality problem: determine $(Q^{1*}, Q^{2*}Q^{3*}) \in R_+^{S^1+S^2+S^3}$ satisfying:

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{i^y=1}^{m^y}\sum_{j^y=1}^{n^y}\sum_{p^{1y}\in E^{1y}} \Big[\frac{\partial g_{h^y}(Q^{1y^*}, Q^{2y^*}, Q^{3y^*})}{\partial q^{p^{1y}}_{h^y i^y j^y}} + C^{p^{1y}}_{h^y i^y j^y}(Q^{1^*}, Q^{2^*}, Q^{3^*}, X^*)$$
$$+ q^{p^{1y}}_{h^y i^y j^y} \frac{\partial C^{p^{1y}}_{h^y i^y j^y}(Q^{1^*}, Q^{2^*}, Q^{3^*}, X^*)}{\partial q^{p^{1y}}_{h^y i^y j^y}} + \sum_{j^y=1}^{n^y}\sum_{k^y=1}^{o^y}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} \frac{\partial C^{p^{2y}}_{h^y j^y k^y}(Q^{1^*}, Q^{2^*}, Q^{3^*}, X^*)}{\partial q^{p^{1y}}_{h^y i^y j^y}}$$
$$+ \sum_{k^y=1}^{o^y}\sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l} \frac{\partial C^{p^{3y}}_{h^y k^y l}(Q^{1^*}, Q^{2^*}, Q^{3^*}, X^*)}{\partial q^{p^{1y}}_{h^y i^y j^y}} - \rho^{5^*}_{h^y i^y j^y}\Big] \times \Big[q^{p^{1y}}_{h^y i^y j^y} - q^{p^{1y^*}}_{h^y i^y j^y}\Big]$$
$$+ \sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{j^y=1}^{n^y}\sum_{k^y=1}^{o^y}\sum_{p^{2y}\in E^{2y}} \Big[\frac{\partial g_{h^y}(Q^{1y^*}, Q^{2y^*}, Q^{3y^*})}{\partial q^{p^{2y}}_{h^y i^y j^y}} + C^{p^{2y}}_{h^y j^y k^y}(Q^{1^*}, Q^{2^*}, Q^{3^*}, X^*)$$
$$+ q^{p^{2y}}_{h^y j^y k^y} \frac{\partial C^{p^{2y}}_{h^y j^y k^y}(Q^{1^*}, Q^{2^*}, Q^{3^*}, X^*)}{\partial q^{p^{2y}}_{h^y j^y k^y}} + \sum_{i^y=1}^{m^y}\sum_{j^y=1}^{n^y}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y} \frac{\partial C^{p^{1y}}_{h^y i^y j^y}(Q^{1^*}, Q^{2^*}, Q^{3^*}, X^*)}{\partial q^{p^{2y}}_{h^y j^y k^y}}$$
$$+ \sum_{k^y=1}^{o^y}\sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l} \frac{\partial C^{p^{3y}}_{h^y k^y l}(Q^{1^*}, Q^{2^*}, Q^{3^*}, X^*)}{\partial q^{p^{2y}}_{h^y j^y k^y}} - \rho^{6^*}_{h^y j^y k^y}\Big] \times \Big[q^{p^{2y}}_{h^y j^y k^y} - q^{p^{2y^*}}_{h^y j^y k^y}\Big]$$

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{k^y=1}^{o^y}\sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}}\Big[\frac{\partial g_{h^y}(Q^{1y^*},Q^{2y^*},Q^{3y^*})}{\partial q_{h^yk^yl}^{p^{3y}}}+C_{h^yk^yl}^{p^{3y}}(Q^{1^*},Q^{2^*},Q^{3^*},X^*)$$
$$+q_{h^yk^yl}^{p^{3y}}\frac{\partial C_{h^yk^yl}^{p^{3y}}(Q^{1^*},Q^{2^*},Q^{3^*},X^*)}{\partial q_{h^yk^yl}^{p^{3y}}}+\sum_{i^y=1}^{m^y}\sum_{j^y=1}^{n^y}\sum_{p^{1y}\in E^{1y}}q_{h^yi^yj^y}^{p^{1y}}\frac{\partial C_{h^yi^yj^y}^{p^{1y}}(Q^{1^*},Q^{2^*},Q^{3^*},X^*)}{\partial q_{h^yk^yl}^{p^{3y}}}$$
$$+\sum_{j^y=1}^{n^y}\sum_{k^y=1}^{o^y}\sum_{p^{2y}\in E^{2y}}q_{h^yj^yk^y}^{p^{2y}}\frac{\partial C_{h^yj^yk^y}^{p^{2y}}(Q^{1^*},Q^{2^*},Q^{3^*},X^*)}{\partial q_{h^yk^yl}^{p^{3y}}}-\rho_{h^yk^yl}^{7^*}\Big]\times\Big[q_{h^yk^yl}^{p^{3y}}-q_{h^yk^yl}^{p^{3y^*}}\Big]\geq 0 \tag{26}$$
$$\forall(Q^1,Q^2,Q^3)\in R_+^{S^1+S^2+S^3}$$

2.7 The passenger car traffic on the road network and the equilibrium conditions

The behavior of passenger cars is assumed to follow the user equilibrium traffic conditions with variable demands Beckmann, McGuire, and Winsten (1955); Patriksson (1994). Thus, the demand for passenger cars in a given OD pair varies, depending on the travel cost (i.e., travel time) on its shortest path. In this case, the behavior of passenger cars in the road network is formulated as below:

$$\begin{cases}\zeta t_{rs}^{p_{rs}}(Q^{1*},Q^{2*},Q^{3*},X^*)=c_{rs}^* & \text{if } x_{rs}^{p_{rs}*}>0\\ \zeta t_{rs}^{p_{rs}}(Q^{1*},Q^{2*},Q^{3*},X^*)\geq c_{rs}^* & \text{if } x_{rs}^{p_{rs}*}=0\end{cases} \tag{27}$$

$$d_{rs}(c_{rs}^*)\begin{cases}=\sum_{p_{rs}\in E_{rs}}x_{rs}^{p_{rs}*} & \text{if } c_{rs}{}^*>0\\ \leq\sum_{p_{rs}\in E_{rs}}x_{rs}^{p_{rs}*} & \text{if } c_{rs}{}^*=0\end{cases} \tag{28}$$

where

$$t_{rs}^{p_{rs}}(Q^1,Q^2,Q^3,X)=\sum_{a\in A}t_a(x_a)\delta_{a,p}^{rs} \tag{29}$$

$$x_a=\sum_{p_{rs}\in E_{rs}}\delta_{a,p_{rs}}^{rs}x_{rs}^{p_{rs}}+v\left[\sum_{p^{1y}\in E^{1y}}\delta_{a,p^{1y}}^{i^yj^y}\frac{q_{h^yi^yj^y}^{p^{1y}}}{\iota\kappa}+\sum_{p^{2y}\in E^{2y}}\delta_{a,p^{2y}}^{j^yk^y}\frac{q_{h^yj^yk^y}^{p^{2y}}}{\iota\kappa}+\sum_{p^{3y}\in E^{3y}}\delta_{a,p^{3y}}^{k^yl}\frac{q_{h^yk^yl}^{p^{3y}}}{\iota\kappa}\right] \tag{30}$$

Conditions (27) represent the equilibrium conditions known as Wardrop's first principle (Wardrop, 1952). The path costs (i.e., path travel times) spent on all the paths in a given OD are equal and lower than or equivalent to the path costs on any unused path. Conditions (28) show the requirements to be fulfilled for OD traffic demand with price formulations (Nagurney, 1999), implying that traffic volume of passenger cars for an OD will be precisely equal to the traffic demand for it if its minimum travel cost is positive, while the traffic volume for it may exceed the actual demand if its minimum travel cost is zero.

Conditions (27) and (28) hold only for passenger car traffic, while freight traffic follows the product flow determined by variational inequality (26). The traffic volume on a link is obtained by adding the passenger car volume on the link and the converted value of the products transacted among the entities on the SCNs into the passenger car volume (see Eq. (30)). As Eqs. (29) and (30) indicate, the path travel time of passenger cars is subject to the path traffic volume of freight vehicles. Objective function (18) and Eqs. (20)-(25), (29) and (30) demonstrate that the amount of the products distributed in the SCNs (i.e., the amount of the products produced, transacted, or transported) is influenced by the traffic volume of passenger cars. The model likewise allows the freight vehicles and passenger cars to be treated as multiclass users.

Conditions (27) and (28) must hold for all OD pairs in equilibrium. Hence, these conditions are equivalent to obtaining $(X^*,c_{rs}^*)\in R_+^{e^5e^6e^4+e^5e^6}$ which satisfies:

$$\sum_{r\in R}\sum_{s\in S}\sum_{p_{rs}\in E_{rs}}\left[\zeta t_{rs}^{p_{rs}}(Q^{1*},Q^{2*},Q^{3*},X^*)-c_{rs}^*\right]\times\left[x_{rs}^{p_{rs}}-x_{rs}^{p_{rs}*}\right]$$
$$+\sum_{r\in R}\sum_{s\in S}\left[\sum_{p_{rs}\in E_{rs}}x_{rs}^{p_{rs}*}-d_{rs}(c_{rs}^*)\right]\times\left[c_{rs}-c_{rs}^*\right]\geq 0\ \ \forall(X,c_{rs})\in R_+^{e^5e^6e^4+e^5e^6} \tag{31}$$

The SC-T-SNE model adopts a static approach for estimating traffic flow (e.g., daily traffic volume), since such an approach has typically been utilized for traffic flow analyses (Bell & Iida, 1997; Patriksson, 1994). However, unlike passengers, products may be transported during night time. In that case, the model should be extended so that it can incorporate the dynamic nature of SCNs (Daniele, 2010) and transport networks (Friesz, Bernstein, Smith, Tobin, & Wie, 1993).

2.8 The equilibrium conditions of the supply chain-transport supernetwork with uncertain product demands

In equilibrium, all manufacturers have achieved optimality for all kinds of products (cf. (3)); all wholesalers have achieved optimality for all kinds of products (cf. (8)); all retailers have achieved optimality for all kinds of products (cf. (13)); all freight carriers have achieved optimality for all kinds of products (cf. (26)); equilibrium conditions for all demand markets hold for all kinds of products (cf. (17)), and, finally, equilibrium conditions for all OD pairs of passenger car traffic hold (cf. (31)). Moreover, the product flows (as well as that transported) between the distinct tiers of the decision-makers on each SCN coincide. This is explicitly stated in the following definition:

Definition 1 (Supply chain–transport supernetwork equilibrium). The equilibrium state of the supply chain-transport supernetwork is one where the optimality conditions (3), (8), (13), and (26) and the equilibrium conditions (17) and (31) hold simultaneously so that no decision-maker has any incentive to alter his decisions.

Under this definition, since the product flows (as well as that transported), market prices, path traffic volume, and path costs (i.e., path travel time) will have to satisfy the sum of optimality conditions (3), (8), (13) and (26) and conditions (17) and (31), the following theorem can be established.

Theorem 1 (Variational inequality formulation). The equilibrium conditions governing the supply chain-transport supernetwork model are equivalent to the solution to the variational inequality problem given by: determine $(Q^{1*}, Q^{2*}, Q^{3*}, \gamma^*, \delta^*, \rho^{4*}, X^*, c^*_{rs}) \in R^{S^1+S^2+S^3+u^y+o^y+r+e^5e^6e^4+e^5e^6}$ satisfying:

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{i^y=1}^{m^y}\sum_{j^y=1}^{n^y}\sum_{p^{1y}\in E^{1y}}\Big[\frac{\partial f_{i^y}(Q^{1y*})}{\partial q^{p^{1y}}_{h^yi^yj^y}}+\frac{\partial g_{i^y}(Q^{1y*})}{\partial q^{p^{1y}}_{h^yi^yj^y}}+\frac{\partial c_{i^yj^y}(Q^{1y*})}{\partial q^{p^{1y}}_{h^yi^yj^y}}+\frac{\partial c_{j^y}(Q^{1y*})}{\partial q^{p^{1y}}_{h^yi^yj^y}}+\frac{\partial g_{j^y}(Q^{1y*})}{\partial q^{p^{1y}}_{h^yi^yj^y}}$$

$$+\frac{\partial g_{h^y}(Q^{1y*},Q^{2y*},Q^{3y*},X^*)}{\partial q^{p^{1y}}_{h^yi^yj^y}}+C^{p^{1y}}_{h^yi^yj^y}(Q^{1*},Q^{2*},Q^{3*},X^*)+q^{p^{1y}}_{h^yi^yj^y}\frac{\partial C^{p^{1y}}_{h^yi^yj^y}(Q^{1*},Q^{2*},Q^{3*},X^*)}{\partial q^{p^{1y}}_{h^yi^yj^y}}$$

$$+\sum_{j^y=1}^{n^y}\sum_{k^y=1}^{o^y}\sum_{p^{2y}\in E^{2y}}q^{p^{2y}}_{h^yj^yk^y}\frac{\partial C^{p^{2y}}_{h^yj^yk^y}(Q^{1*},Q^{2*},Q^{3*},X^*)}{\partial q^{p^{1y}}_{h^yi^yj^y}}$$

$$+\sum_{k^y=1}^{o^y}\sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}}q^{p^{3y}}_{h^yk^yl}\frac{\partial C^{p^{3y}}_{h^yk^yl}(Q^{1*},Q^{2*},Q^{3*},X^*)}{\partial q^{p^{1y}}_{h^yi^yj^y}}-\gamma^*_{j^y}\Big]\times\Big[q^{p^{1y}}_{h^yi^yj^y}-q^{p^{1y*}}_{h^yi^yj^y}\Big]$$

$$\sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{j^y=1}^{n^y}\sum_{k^y=1}^{o^y}\sum_{p^{2y}\in E^{2y}}\Big[\frac{\partial c_{k^y}(Q^{2y*})}{\partial q^{p^{2y}}_{h^yj^yk^y}}+\frac{\partial g_{k^y}(Q^{2y*})}{\partial q^{p^{2y}}_{h^yj^yk^y}}+\frac{\partial s_{j^yk^y}(Q^{2y*})}{\partial q^{p^{2y}}_{h^yj^yk^y}}+\frac{\partial c_{j^yk^y}(Q^{2y*})}{\partial q^{p^{2y}}_{h^yj^yk^y}}$$

$$+\frac{\partial g_{h^y}(Q^{1y*},Q^{2y*},Q^{3y*})}{\partial q^{p^{2y}}_{h^yj^yk^y}}+C^{p^{2y}}_{h^yj^yk^y}(Q^{1*},Q^{2*},Q^{3*},X^*)+q^{p^{2y}}_{h^yj^yk^y}\frac{\partial C^{p^{2y}}_{h^yj^yk^y}(Q^{1*},Q^{2*},Q^{3*},X^*)}{\partial q^{p^{2y}}_{h^yj^yk^y}}$$

$$+\sum_{j^y=1}^{n^y}\sum_{k^y=1}^{o^y}\sum_{p^{1y}\in E^{1y}}q^{p^{1y}}_{h^yi^yj^y}\frac{\partial C^{p^{1y}}_{h^yi^yj^y}(Q^{1*},Q^{2*},Q^{3*},X^*)}{\partial q^{p^{2y}}_{h^yj^yk^y}}$$

$$+\sum_{k^y=1}^{o^y}\sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}}q^{p^{3y}}_{h^yk^yl}\frac{\partial C^{p^{3y}}_{h^yk^yl}(Q^{1*},Q^{2*},Q^{3*},X^*)}{\partial q^{p^{2y}}_{h^yj^yk^y}}+\gamma^*_{j^y}-\delta^*_{k^y}\Big]\times\Big[q^{p^{2y}}_{h^yj^yk^y}-q^{p^{2y*}}_{h^yj^yk^y}\Big]$$

$$+\sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{k^y=1}^{o^y}\sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}}\Big[\frac{\partial s_{k^yl}(Q^{3y*})}{\partial q^{p^{3y}}_{h^yk^yl}}+\frac{\partial c_{k^yl}(Q^{3y*})}{\partial q^{p^{3y}}_{h^yk^yl}}$$

$$+\frac{\partial g_{k^yl}(Q^{1y*},Q^{2y*},Q^{3y*})}{\partial q^{p^{3y}}_{h^yk^yl}}+C^{p^{3y}}_{h^yk^yl}(Q^{1*},Q^{2*},Q^{3*},X^*)+q^{p^{3y}}_{h^yk^yl}\frac{\partial C^{p^{3y}}_{h^yk^yl}(Q^{1*},Q^{2*},Q^{3*},X^*)}{\partial q^{p^{3y}}_{h^yk^yl}}$$

$$+\sum_{i^y=1}^{m^y}\sum_{j^y=1}^{n^y}\sum_{p^{1y}\in E^{1y}}q^{p^{1y}}_{h^yi^yj^y}\frac{\partial C^{p^{1y}}_{h^yi^yj^y}(Q^{1*},Q^{2*},Q^{3*},X^*)}{\partial q^{p^{3y}}_{h^yk^yl}}$$

$$+\sum_{j^y=1}^{n^y}\sum_{k^y=1}^{o^y}\sum_{p^{2y}\in E^{2y}}q^{p^{2y}}_{h^yj^yk^y}\frac{\partial C^{p^{2y}}_{h^yj^yk^y}(Q^{1*},Q^{2*},Q^{3*},X^*)}{\partial q^{p^{3y}}_{h^yk^yl}}+\delta^*_{k^y}-\rho^{4y*}_{l}\Big]\times\Big[q^{p^{3y}}_{h^yk^yl}-q^{p^{3y*}}_{h^yk^yl}\Big]$$

$$+\sum_{y=1}^{Y}\sum_{j=1}^{n^y}\left[\sum_{h^y=1}^{u^y}\left(\sum_{i^y=1}^{m^y}\sum_{p^{1y}\in E^{1y}} q_{h^y i^y j^y}^{p^{1y*}} - \sum_{k^y=1}^{o^y}\sum_{p^{2y}\in E^{2y}} q_{h^y j^y k^y}^{p^{2y*}}\right)\right]\times\left[\gamma_{j^y}-\gamma_{j^y}^{*}\right]$$
$$+\sum_{y=1}^{Y}\sum_{k^y=1}^{o^y}\left[\sum_{h^y=1}^{u^y}\left(\sum_{j=1}^{n^y}\sum_{p^{2y}\in E^{2y}} q_{h^y j^y k^y}^{p^{2y*}} - \sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}} q_{h^y k^y l}^{p^{3y*}}\right)\right]\times\left[\delta_{k^y}-\delta_{k^y}^{*}\right]$$
$$+\sum_{y=1}^{Y}\sum_{l=1}^{r}\left[\sum_{h^y=1}^{u^y}\sum_{k^y=1}^{o^y}\sum_{p^{3y}\in E^{3y}} q_{h^y k^y l}^{p^{3y*}} - d_l^y(\rho^{4y*})\right]\times\left[\rho_l^{4y}-\rho_l^{4y*}\right]$$
$$+\sum_{r\in R}\sum_{s\in S}\sum_{p_{rs}\in E_{rs}}\left[\zeta t_{rs}^{p_{rs}}(Q^{1*},Q^{2*},Q^{3*},X^*)-c_{rs}^*\right]\times\left[x_{rs}^{p_{rs}}-x_{rs}^{p_{rs}*}\right]$$
$$+\sum_{r\in R}\sum_{s\in S}\left[\sum_{p_{rs}\in E_{rs}} x_{rs}^{p_{rs}*}-d_{rs}(c_{rs}^*)\right]\times\left[c_{rs}-c_{rs}^*\right]\geq 0 \tag{32}$$
$$\forall(Q^1,Q^2,Q^3,\gamma,\delta,\rho^4,X,c_{rs})\in R^{S^1+S^2+S^3+nY+oY+rY+e^5e^6e^4+e^5e^6}$$

The proof to Theorem 1 is similar to that established by Hammond and Beullens (2007); Nagurney et al. (2002). With some algebraic manipulation, variational inequality (32) is noticed to be the sum of inequalities (3), (8), (13), (17), (26) and (31). The converse also needs to be demonstrated to see if the solution to (32) is, in fact, an equilibrium as per Definition 1. This can be undertaken as follows: To inequality (32) add the terms $+\rho_{i^y j^y}^{1*}-\rho_{i^y j^y}^{1*}$ and $+\rho_{h^y i^y j^y}^{5*}-\rho_{h^y i^y j^y}^{5*}$ to the term in the first set of brackets preceding the multiplication sign, add the terms $+\rho_{j^y}^{2*}-\rho_{j^y}^{2*}$ and $+\rho_{h^y j^y k^y}^{6*}-\rho_{h^y j^y k^y}^{6*}$ to the term preceding the second multiplication sign, and add the terms $+\rho_{k^y}^{3*}-\rho_{k^y}^{3*}$ and $+\rho_{h^y k^y l}^{7*}-\rho_{h^y k^y l}^{7*}$ to the term preceding the third multiplication sign. The variational inequality supplemented with these terms turns into the sum of inequalities (3), (8), (13), (17), (26) and (31) without changing the value of variational inequality (32).

2.9 Retrieving the price variables

The price variable $\rho_{i^y j^y}^{1*}$ can be retrieved from the eventual solution by Eq. (33), setting $q_{h^y i^y j^y}^{p^{1y}}>0$ in inequality (3).

$$\rho_{i^y j^y}^{1*}=\frac{\partial f_{i^y}(Q^{1y^*})}{\partial q_{h^y i^y j^y}^{p^{1y}}}+\frac{\partial g_{i^y}(Q^{1y^*})}{\partial q_{h^y i^y j^y}^{p^{1y}}}+\frac{\partial c_{i^y j^y}(Q^{1y^*})}{\partial q_{h^y i^y j^y}^{p^{1y}}}+\rho_{h^y i^y j^y}^{5*} \tag{33}$$

The equilibrium solutions of γ and δ can be derived from inequality (32), and price $\rho_{j^y}^{2*}$ can also be obtained by finding a $q_{h^y j^y k^y}^{p^{2y}}$ in inequality (8) as follows:

$$\rho_{j^y}^{2*}=\frac{\partial c_{j^y k^y}\left(Q^{2y^*}\right)}{\partial q_{h^y j^y k^y}^{p^{2y}}}+\rho_{h^y j^y k^y}^{6*}+\gamma_{j^y}^{*} \tag{34}$$

Also, if $q_{h^y k^y l}^{p^{3y}}$ in inequality (13), the price $\rho_{k^y}^{3*}$ can be derived as Eq. (35).

$$\rho_{k^y}^{3*}=\frac{\partial c_{k^y l}\left(Q^{3y^*}\right)}{\partial q_{h^y k^y l}^{p^{3y}}}+\rho_{h^y k^y l}^{7*}+\delta_{k^y}^{*} \tag{35}$$

Likewise, the carriage charged by freight carrier h^y for transporting product y can be obtained from inequality (26) as follows:

$$\rho^{5*}_{h^y i^y j^y} =$$
$$\frac{\partial g_{h^y}(Q^{1y^*}, Q^{2y^*}, Q^{3y^*})}{\partial q^{p^{1y}}_{h^y i^y j^y}} + C^{p^{1y}}_{h^y i^y j^y}(Q^{1^*}, Q^{2^*}, Q^{3^*}, X^*) + q^{p^{1y}}_{h^y i^y j^y} \frac{\partial C^{p^{1y}}_{h^y i^y j^y}(Q^{1*}, Q^{2*}, Q^{3*}, X^*)}{\partial q^{p^{1y}}_{h^y i^y j^y}}$$
$$\sum_{j^y=1}^{n^y} \sum_{k^y=1}^{o^y} \sum_{p^{2y} \in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} \frac{\partial C^{p^{2y}}_{h^y j^y k^y}(Q^{1*}, Q^{2*}, Q^{3*}, X^*)}{\partial q^{p^{1y}}_{h^y i^y j^y}} + \sum_{k^y=1}^{o^y} \sum_{l=1}^{r} \sum_{p^{3y} \in E^{3y}} \frac{\partial C^{p^{3y}}_{h^y k^y l}(Q^{1*}, Q^{2*}, Q^{3*}, X^*)}{\partial q^{p^{1y}}_{h^y i^y j^y}} \tag{36}$$

$$\rho^{6*}_{h^y j^y k^y} =$$
$$\frac{\partial g_{h^y}(Q^{1y*}, Q^{2y*}, Q^{3y*})}{\partial q^{p^{2y}}_{h^y j^y k^y}} + C^{p^{2y}}_{h^y j^y k^y}(Q^{1*}, Q^{2*}, Q^{3*}, X^*) + q^{p^{2y}}_{h^y j^y k^y} \frac{\partial C^{p^{2y}}_{h^y j^y k^y}(Q^{1*}, Q^{2*}, Q^{3*}, X^*)}{\partial q^{p^{2y}}_{h^y j^y k^y}}$$
$$\sum_{i^y=1}^{m^y} \sum_{j^y=1}^{n^y} \sum_{p^{1y} \in E^{1y}} q^{p^{1y}}_{h^y i^y j^y} \frac{\partial C^{p^{1y}}_{h^y i^y j^y}(Q^{1*}, Q^{2*}, Q^{3*}, X^*)}{\partial q^{p^{2y}}_{h^y j^y k^y}} + \sum_{k^y=1}^{o^y} \sum_{l=1}^{r} \sum_{p^{3y} \in E^{3y}} \frac{\partial C^{p^{3y}}_{h^y k^y l}(Q^{1*}, Q^{2*}, Q^{3*}, X^*)}{\partial q^{p^{2y}}_{h^y j^y k^y}} \tag{37}$$

$$\rho^{7*}_{h^y k^y l} =$$
$$\frac{\partial g_{h^y}(Q^{1y*}, Q^{2y*}, Q^{3y*})}{\partial q^{p^{3y}}_{h^y k^y l}} + C^{p^{3y}}_{h^y k^y l}(Q^{1*}, Q^{2*}, Q^{3*}, X^*) + q^{p^{3y}}_{h^y k^y l} \frac{\partial C^{p^{3y}}_{h^y k^y l}(Q^{1*}, Q^{2*}, Q^{3*}, X^*)}{\partial q^{p^{3y}}_{h^y k^y l}}$$
$$\sum_{i^y=1}^{m^y} \sum_{j^y=1}^{n^y} \sum_{p^{1y} \in E^{1y}} q^{p^{1y}}_{h^y i^y j^y} \frac{\partial C^{p^{1y}}_{h^y i^y j^y}(Q^{1*}, Q^{2*}, Q^{3*}, X^*)}{\partial q^{p^{3y}}_{h^y k^y l}} + \sum_{j^y=1}^{n^y} \sum_{k^y=1}^{o^y} \sum_{p^{2y} \in E^{2y}} \frac{\partial C^{p^{2y}}_{h^y j^y k^y}(Q^{1*}, Q^{2*}, Q^{3*}, X^*)}{\partial q^{p^{3y}}_{h^y k^y l}} \tag{38}$$

3. QUALITATIVE PROPERTIES AND SOLUTION ALGORITHMS

This section highlights the existence and uniqueness of the solution to variational inequality (32). Its solution procedures are also outlined. For ease of reference, assuming that
$Z \equiv \left(Q, Q^2, Q^3, \gamma, \delta, \rho^4, X, c_{rs}\right), F(Z) \equiv \left(F^{p^{1y}}_{h^y i^y j^y}, F^{p^{2y}}_{h^y j^y k^y}, F^{p^{3y}}_{h^y k^y l}, F_{j^y}, F_{k^y}, F^y_l, F^{p_{rs}}_{rs}, F_{rs}\right)$
$h^y = 1, \cdots, u^y;\ i^y = 1, \cdots, m^y;\ j^y = 1, \cdots, n^y;\ k^y = 1, \cdots, o^y;\ l = 1, \cdots, r;\ y = 1, \cdots, Y;$
$p^{1y} = 1, \cdots, e^{1y};\ p^{2y} = 1, \cdots, e^{2y};\ p^{3y} = 1, \cdots, e^{3y};\ r = 1, \cdots, e^5;\ s = 1, \cdots, e^6;\ p_{rs} = 1, \cdots, e^4$, as well as that the specific components of F are given by the functional terms preceding the multiplication signs in (32), variational inequality problem (32) can be rewritten in standard variational inequality form: determine $Z^* \in B$, where

$$B \equiv \left\{\left(Q^1, Q^2, Q^3, \gamma, \delta, \rho^4, X, c_{rs}\right) \mid \left(Q^1, Q^2, Q^3, \gamma, \delta, \rho^4, X, c_{rs}\right) \in R_+^{S^1+S^2+S^3+nY+oY+rY+e^5e^6e^4+e^5e^6}\right\} \tag{39}$$

satisfying

$$\langle F\left(Z^*\right), Z - Z^* \rangle \geq 0, \quad \forall Z \in B \tag{40}$$

Here, the term $\langle \bullet, \bullet \rangle$ represents the inner product in N-dimensional Euclidean space.

3.1 Existence of the solution

The feasible region B is not always compact, even if F in inequality (40) is continuous. Therefore, it is not easy to derive the existence of a solution from the assumption of continuity of the functions. However, it is possible to impose a weak condition on B to guarantee the existence. Let

$$B_b = \left\{ \left(Q^1, Q^2, Q^3, \gamma, \delta, \rho^4, X, c_{rs}\right) | 0 \le Q^1 \le b_1;\ \ 0 \le Q^2 \le b_2;\ \ 0 \le Q^3 \le b_3;\ \ 0 \le \gamma \le b_4; \right.$$
$$\left. 0 \le \delta \le b_5;\ \ 0 \le \rho^4 \le b_6;\ \ 0 \le X \le b_7;\ \ 0 \le c_{rs} \le b_8 \right\} \tag{41}$$

where $b = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8) \ge 0$,
and $Q^1 \le b_1;\ Q^2 \le b_2;\ Q^3 \le b_3;\ \gamma \le b_4;\ \delta \le b_5;\ \rho^4 \le b_6;\ X \le b_7;\ c_{rs} \le b_8$
means that $q^{p^{2y}}_{h^y j^y k^y} \le b_2;\ q^{p^3}_{h^y k^y} \le b_3;\ \gamma_{j^y} \le b_4;\ \delta_{k^y} \le b_5;\ \rho^{4y}_l \le b_6;\ x^{p_n}_{rs} \le b_7;\ c_{rs} \le b_8$
for all $i^y, j^y, k^y, l, h^y, r, s, p^{1y}, p^{2y}, p^{3y}$ and p_{rs} .

B_b is a bounded closed convex subset of $R_+^{S^1+S^2+S^3+nY+oY+rY+e^5e^6e^4+e^5e^6}$, and consequently, the following variational inequality admits at least one solution of $Z^b \in B_b$, as F is continuous:

$$\left\langle F\left(Z^b\right), Z - Z^b \right\rangle \ge 0 \quad \forall Z^b \in B_b \tag{42}$$

Following Theorem 4.2 in Kinderlehrer and Stampacchia (1980) (see also Theorem 1.5 in Nagurney (1999)), Lemma 1 can hold as below:
Lemma 1. Variational inequality (40) (and (32) as well) admits a solution if and only if there exists a $b > 0$ such that variational inequality (42) admits a solution in B_b with

$$Q^1 < b_1;\ \ Q^2 < b_2;\ \ Q^3 < b_3;\ \ \gamma < b_4;\ \ \delta < b_5;\ \ \rho^4 < b_6;\ \ X < b_7;\ \ c_{rs} < b_8 \tag{43}$$

Under the conditions in Theorem 2 described below, the existence of a solution to the original variational inequality problem is guaranteed by Lemma 1, since it is possible to construct $b_1, b_2, b_3, b_4, b_5, b_6, b_7$ and b_8 arge enough so that the restricted variational inequality (42) will satisfy the condition of boundedness implied in (43). (Hammond & Beullens, 2007; Nagurney & Zhao, 1993; Yamada et al., 2011).
Theorem 2. Suppose there exist positive constants H, I and $N (H < I)$ such that,

$$\frac{\partial f_{i^y}\left(Q^{1y}\right)}{\partial q^{p^{1y}}_{h^y i^y j^y}} + \frac{\partial g_{i^y}\left(Q^{1y}\right)}{\partial q^{p^y}_{h^y i^y j^y}} + \frac{\partial c_{i^y j^y}\left(Q^{1y}\right)}{\partial q^{p^{1y}}_{h^y i^y j^y}} + \frac{\partial c_{j^y}\left(Q^{1y}\right)}{\partial q^{p^{1y}}_{h^y i^y j^y}} + \frac{\partial g_{j^y}\left(Q^{1y}\right)}{\partial q^{p^{1y}}_{h^y i^y j^y}}$$
$$+ \frac{\partial g_{h^y}\left(Q^{1y}, Q^{2y}, Q^{3y}\right)}{\partial q^{p^{1y}}_{h^y i^y j^y}} + C^{p^{1y}}_{h^y i^y j^y}\left(Q^1, Q^2, Q^3, X\right) + q^{p^{1y}}_{h^y i^y j^y} \frac{\partial C^{p^{1y}}_{h^y i^y j^y}\left(Q^y, Q^2, Q^3, X\right)}{\partial q^{p^{1y}}_{h^y i^y j^y}}$$
$$+ \sum_{j^y=1}^{n^y} \sum_{k^y=1}^{o^y} \sum_{p^{2y} \in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} \frac{\partial C^{p^{2y}}_{h^y j^y k^y}\left(Q^y, Q^2, Q^3, X\right)}{\partial q^{p^{1y}}_{h^y j^y k^y}}$$
$$+ \sum_{k^y=1}^{o^y} \sum_{l=1}^{r} \sum_{p^{3y} \in E^{3y}} q^{p^{3y}}_{h^y k^y l} \frac{\partial C^{p^{3y}}_{h^y k^y l}\left(Q^1 Q^2, Q^3, X\right)}{\partial q^{p_{1y}}_{h^y k^y l}} \ge I \quad \forall Q^1 \text{ with } q^{p^{1y}}_{h^y i^y j^y} \ge N \quad \forall h^y, i^y, j^y, p^{1y} \tag{44}$$

$$\frac{\partial_{k^y}\left(Q^{2y}\right)}{\partial q^{p_{2y}}_{h^y j^y k^y}} + \frac{\partial g_{k_y}\left(Q^{2y}\right)}{\partial q^{p_{2y}}_{h^y j^y k^y}} + \frac{\partial s_{j^y k^y}\left(Q^{2y*}\right)}{\partial q^{p_{2y}}_{h^y j^y k^y}} + \frac{\partial c_{j^y k^y}\left(Q^{2y}\right)}{\partial q^{p_{2y}}_{h^y j^y k^y}} + \frac{\partial g_{h^y}(Q^{1y}, Q^{2y}, Q^{3y})}{\partial q^{p_{2y}}_{h^y j^y k^y}}$$
$$+ C^{p^{2y}}_{h^y j^y k^y}\left(Q^1, Q^2, Q^3, X\right) + q^{p^{2y}}_{h^y j^y k^y} \frac{\partial C^{p^{2y}}_{h^y j^y k^y}\left(Q^1, Q^2, Q^3, X\right)}{\partial q^{p^{2y}}_{h^y j^y k^y}}$$
$$+ \sum_{i^y=1}^{m^y} \sum_{j^y=1}^{n^y} \sum_{p^{1y} \in E^{1y}} q^{p^{1y}}_{h^y i^y j^y} \frac{\partial C^{p^{1y}}_{h^y i^y j^y j^y}\left(Q^1, Q^2, Q^3, X\right)}{\partial q^{p^{2y}}_{h^y j^y k^y}} + \sum_{k^y=1}^{o^y} \sum_{l=1}^{r} \sum_{p^{3y} \in E^{3y}} \frac{\partial C^{p^{3y}}_{h^y k^y l}(Q^1, Q^2, Q^3, X)}{\partial q^{p^{2y}}_{h^y j^y k^y}} \ge I$$
$$\forall Q^2 \text{ with } q^{p^{2y}}_{h^y j^y k^y} \ge N \quad \forall h^y, j^y, k^y, p^{2y} \tag{45}$$

$$\frac{\partial s_{k^y l}\left(Q^{3y^*}\right)}{\partial q^{p^{3y}}_{h^y k^y l}}+\frac{\partial c_{k^y l}\left(Q^{3y}\right)}{\partial q^{p^{3y}}_{h^y k^y l}}+\frac{\partial g_{h^y}\left(Q^{1y},Q^{2y},Q^{3y}\right)}{\partial q^{p^{3y}}_{h^y k^y l}}+C^{p^{3y}}_{h^y k^y l}\left(Q^1,Q^2,Q^3,X\right)+$$
$$q^{p^{3y}}_{h^y k^y l}\frac{\partial C^{p^{3y}}_{h^y k^y l}\left(Q^1,Q^2,Q^3,X\right)}{\partial q^{p^{3y}}_{h^y k^y l}}+\sum_{i^y=1}^{m^y}\sum_{j^y=1}^{n^y}\sum_{p^{1y}\in E^{1y}}q^{p^{1y}}_{h^y i^y j^y}\frac{\partial C^{p^{1y}}_{h^y i^y j^y}(Q^1,Q^2,Q^3,X)}{\partial q^{p^{3y}}_{h^y k^y l}}$$
$$+\sum_{j^y=1}^{n^y}\sum_{k^y=1}^{o^y}\sum_{p^{2y}\in E^{2y}}q^{p^{2y}}_{h^y j^y k^y}\frac{\partial C^{P^{2y}}_{h^y j^y k^y}(Q^1,Q^2,Q^3,X)}{\partial q^{p^{3y}}_{h^y k^y l}}\geq I\quad\forall Q^3\text{ with }q^{p^y}_{h^y k^y I}\geq N\quad\forall h^y,k^y,l,p^{3y}\tag{46}$$

$$d^y_l\left(\rho^{4y}\right)\leq N\qquad\forall\rho^{4y}\text{ with }\rho^{4y}_l\geq H,\forall l,y\tag{47}$$
$$t^{p_{rs}}_{rs}\left(Q^1,Q^2,Q^3,X\right)\geq R\qquad\forall X\text{ with }x^{p_{rs}}_{rs}>N,\forall r,s,p_{rs}\tag{48}$$
$$d_{rs}\left(c_{rs}\right)\leq N\qquad\forall c_{rs}\geq H,\forall r,s\tag{49}$$

then variational inequality (32) (and (40) as well) admits at least one solution.
Proof. Follows using analogous arguments as the proof of existence in Yamada et al. (2011) (see also Hammond and Beullens (2007); Nagurney and Zhao (1993)).

3.2 The uniqueness of the solution

It is also crucial to investigate the uniqueness of the solution in order to guarantee the convergence of an algorithm used to solve any variational inequality problem. Firstly, the following lemma is established.
Lemma 2. The vector function F that enters the variational inequality (40) (and (32) as well) is strictly monotone, with respect to $\left(Q^1,Q^2,Q^3,\rho^4,X,c_{rs}\right)$, namely

$$\langle F(Z)-F\left(Z'\right),Z-Z'\rangle\geq 0,\quad\forall Z,Z'\in B\tag{50}$$

where

(i) $f_{i^y},g_{i^y},c_{i^y j^y},c_{j^y k^y},c_{j^y},g_{j^y},g_{h^y},c_{k^y},g_{k^y},s_{j^y k^y}$ and $s_{k^y l}$ are convex functions, $C^{p^{1y}}_{h^y i^y j^y},C^{p^{2y}}_{h^y j^y k^y},C^{p^{3y}}_{h^y k^y l}$ are non-decreasing convex functions, d^y_l and d_{rs} are monotone decreasing functions, and $t^{p_{rs}}_{rs}$ is a monotone increasing function; and

(ii) one of the families of these convex functions is a family of strictly convex functions, and d^y_l , d_{rs} and $t^{p_{rs}}_{rs}$ are strictly monotone.

Proof. See the proof in Nagurney et al. (2002). According to Kinderlehrer and Stampacchia (1980); Nagurney (1999), a solution to variational inequality (40) (and (32) as well) is unique if vector function $F(Z)$ is strictly monotone. The theorem can therefore be derived as follows (Nagurney et al., 2002):
Theorem 3. Under the condition of Lemma 2, there is a unique shipment pattern of Q^{1*},Q^{2*},Q^{3*} , a unique market price vector of ρ^{4*} , a unique traffic flow pattern of X^* and a unique travel cost vector of c^*_{rs} satisfying the equilibrium conditions of the supernetwork. In other words, if variational inequality (40) (and (32) as well) admits a solution, then that is the only solution in $\left(Q^1,Q^2,Q^3,\rho^4,X,c_{rs}\right)$.

3.3 The algorithm

A modified projection method can be used to solve variational inequality (32) (e.g., Nagurney et al., 2002). This method is based on Lipschitz continuity condition for $F(Z)$ and involves complicated procedures where the step size must be set in advance with unknown Lipschitz constant. Accordingly, this study applies the solution procedures proposed by Q. Meng, Huang, and Cheu (2007) as outlined below to avoid such difficulties.

STEP 1: As Z is non-negative, variational inequality (32) (and (40) as well) can be converted into an equivalent complementarity problem: determine Z^* satisfying

$$Z^*\geq 0,F\left(Z^*\right)\geq 0,\langle F\left(Z^*\right),Z^*\rangle\geq 0,\quad\forall Z\in B\tag{51}$$

STEP 2: Using Fischer-Burmeister function Fischer (1992) of $\phi:R^2\rightarrow R_+$ (see Eq. (52)), a non-negative real-valued function is defined as Eq. (53).

$$\phi(w,v) = [\sqrt{w^2+v^2} - (w+v)]^2 \tag{52}$$

$$\begin{aligned}
\psi(Z) &= \sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{i^y=1}^{m^y}\sum_{j^y=1}^{n^y}\sum_{p^{1y}\in E^{1y}} \phi\left(q^{p^{1y}}_{h^y i^y j^y}, F^{p^{1y}}_{h^y i^y j^y}(Z)\right) \\
&+ \sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{j^y=1}^{n^y}\sum_{k^Y=1}^{o^y}\sum_{p^{2y}\in E^{2y}} \phi\left(q^{p^{2y}}_{h^y j^y k^y}, F^{p^{2y}}_{h^y j^y k^y}(Z)\right) \\
&+ \sum_{y=1}^{Y}\sum_{h^y=1}^{u^y}\sum_{k^Y=1}^{o^y}\sum_{l=1}^{r}\sum_{p^{3y}\in E^{3y}} \phi\left(q^{p^{3y}}_{h^y k^y l}, F^{p^{3y}}_{h^y k^y l}(Z)\right) + \sum_{y=1}^{Y}\sum_{j^y=1}^{n^y} \phi\left(\gamma_{j^y}, F_{j^y}(Z)\right) \\
&+ \sum_{y=1}^{Y}\sum_{k^y=1}^{o^y} \phi\left(\delta_{k^y}, F_{k^y}(Z)\right) + \sum_{y=1}^{Y}\sum_{l=1}^{r} \phi\left(\rho^{4^y}_l, F^y_l(Z)\right) \\
&+ \sum_{r\in R}\sum_{s\in S}\sum_{p_{rs}\in E_{rs}} \phi\left(X, F^{p_{rs}}_{rs}(Z)\right) + \sum_{r\in R}\sum_{s\in S} \phi\left(c_{rs}, F_{rs}(Z)\right)
\end{aligned} \tag{53}$$

STEP 3: Complementarity conditions (51) can also be converted into an equivalent unconstrained nonlinear optimization problem of min $\psi(Z)$.

STEP 4: The optimization problem is solved using the Quasi-Newton method.

As stated in Q. Meng et al. (2007), the existence of a solution to this optimization problem can be verified with Theorem 2 described above. Geiger and Kanzow (1996) demonstrated that any stationary point of the unconstrained minimization problem is its global minimum under the conditions where $F(Z)$ is monotone and continuously differentiable. The existence of an accumulation point can be guaranteed based on the function form of (52) and (53), and hence, the solution obtained using the Quasi-Newton method turns into a global minimum.

4. NUMERICAL EXAMPLES

4.1 Problem definition and the base case

Numerically tests using the SC-T-SNE model developed are then undertaken with a hypothesized supply chain-transport supernetwork as shown in Fig. 6, for validating its performance and calibrating its parameter values. The model is also numerically tested to investigate the influence of the variable nature of consumers' product demands as well as to explore the effects of information sharing between wholesalers and retailers about the demands. The following assumptions are made for these numerical tests:

- The supernetwork illustrated in Fig. 6 is comprised of a transport network and two kinds of SCNs.
- The transport network is an urban road network.
- The economic activities from production to consumption on the SCNs are completed within this urban area.
- The supernetwork consists of 9 nodes and 36 links, where the link lengths (i.e., distances) are 1 (for link numbers 3, 4, 7, 8, 9, 10, 15, 16, 23, 24, 27, 28, 29, 30, 33, and 34), $\sqrt{2}$ (for link numbers 11, 12, 19, and 20) and (for the remaining links), respectively.
- Two kinds of products (i.e., products 1 and 2) are only manufactured, transacted, and transported.
- The products are independently consumed with each other and transported by trucks traveled on the road network.
- The SCN for each product is comprised of two manufacturers, two wholesalers, two retailers, three demand markets, and one freight carrier.
- The demand markets are located at nodes 3, 5, and 7.
- The SCN for product 1 has the manufacturers at nodes 1 and 9, wholesalers at nodes 2 and 8, and retailers at nodes 4 and 6; while that for product 2 contains the manufacturers at nodes 1 and 9, wholesalers at nodes 4 and 6, and retailers at nodes 2 and 8.

- The generation/attraction nodes for passenger car traffic are nodes 1, 5, and 9.
- Each entity on the SCNs constitutes an origin or destination point for the trucks transporting the products.

The model is also applicable to inter-regional transport networks, since it is revealed that the traffic flow of passenger cars can affect freight flow in such networks (Yamada et al., 2009), even though the tests undertaken in the paper focus only on the urban road network. However, it would be more appropriate to extend the model to intermodal or multimodal situations if applied to the inter-regional case (Arnold, Peeters, & Thomas, 2004; Yamada et al., 2009). Both the functional forms and parameter values of $f_{j^y}, c_{j^y}, c_{k^y}, c_{i^y j^y}, c_{j^y k^y}, c_{k^y l}, g_{j^y}, g_{k^y}, g_{h^y}, d_l^y, \iota, \kappa, \eta, \nu, \zeta$ and t_a are determined so that the existence and uniqueness of the solutions are ensured. These settings are also based on existing studies (Nagurney et al., 2002; Patriksson, 1994; Yamada et al., 2011,?). For simplicity, the functional forms and parameter values of the two kinds of products are set identically (i.e., the same functional forms and parameter values are used for both $y = 1$ and $y = 2$).

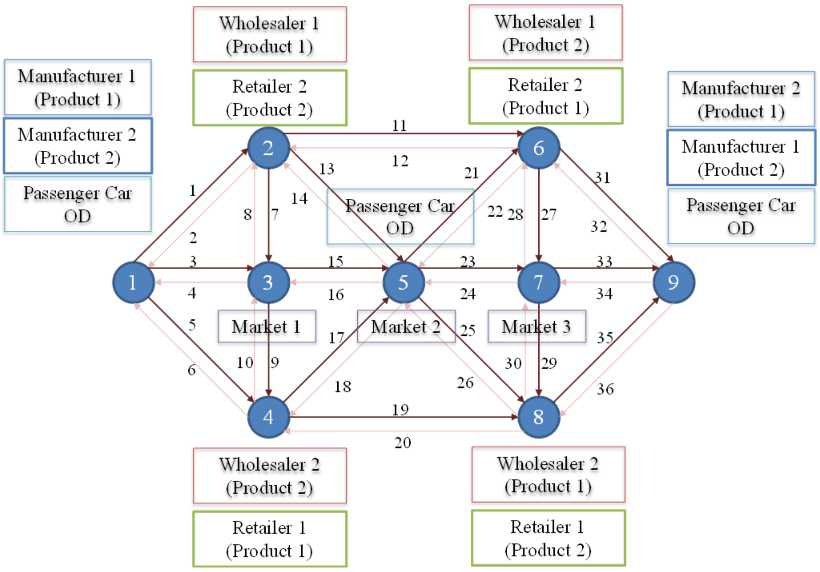

Figure 6: Supernetwork investigated

$$f_{i^y} = 100\left(\sum_{h^y=1}^{1}\sum_{j^y=1}^{2}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y}\right) + 0.2\left(\sum_{h^y=1}^{1}\sum_{j^y=1}^{2}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y}\right)^2 \tag{54}$$

$$c_{j^y} = 0.05\left(\sum_{h^y=1}^{1}\sum_{i^y=1}^{2}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y}\right)^2, c_{k^y} = 0.05\left(\sum_{h^y=1}^{1}\sum_{j^y=1}^{2}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^y j^y k^y}\right)^2 \tag{55}$$

$$c_{i^y j^y} = 5\left(\sum_{h^y=1}^{1}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y}\right), c_{j^y k^y} = 5\left(\sum_{h^y=1}^{1}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^y j^y k^y}\right), c_{k^y l} = 5\left(\sum_{h^y=1}^{1}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l}\right) \tag{56}$$

$$g_{i^y} = 20\sum_{h^y=1}^{1}\sum_{j^y=1}^{2}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y}, g_{j^y} = 20\sum_{h^y=1}^{1}\sum_{i^y=1}^{2}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y}, g_{k^y} = 20\sum_{h^y=1}^{1}\sum_{j^y=1}^{2}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} \tag{57}$$

$$g_{h^y} = 0.4\left(\sum_{i^y=1}^{2}\sum_{j^y=1}^{2}\sum_{p^{1y}\in E^{1y}} q^{p^{1y}}_{h^y i^y j^y} + \sum_{j^y=1}^{2}\sum_{k^y=1}^{2}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} + \sum_{k^y=1}^{2}\sum_{l=1}^{2}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^y k^y l}\right) \tag{58}$$

$$d_l^y = 1000 - 3.0\rho_l^{4y} \tag{59}$$

The other parameters are set as $\iota = 8$; $\kappa = 0.43$; $\eta = 5$; $\nu = 2$; $\zeta = 1$; $C_{0,a} = 120$ for all a; $t_0,a = 0.8$ for $a = 3, 4, 7, 8, 9, 10, 15, 16, 23, 24, 27, 28, 29, 30, 33 and 34$; $t_{0,a} = 1.6$ for $a = 11, 12, 19$ and 20; and $t_{0,a} = 1.13$

for $a = 1, 2, 5, 6, 13, 14, 17, 18, 21, 22, 25, 26, 31, 32, 35$ and 36. For the OD demand of passenger car traffic, $d_{rs} = 120 - 1.0c^*_{rs}$ is given to all OD pairs. The following modified BPR function is employed as a link cost function ta, which is required for estimating the path travel time on the road network:

$$t_a(x_a) = t_{0,a}\left\{1 + 2.62(x_a/C_a)^5\right\} \tag{60}$$

where $t_{0,a}$ denotes the free travel time on link a, and the traffic capacity on link a is represented by C_a.
Passenger car equivalent (PCE) for trucks is utilized for estimating link travel times. Here, it is set at 2 as commonly applied in Japan to the traffic conditions where the percentage of heavy vehicle traffic is 30% of the total in a two-lane road with a gradient of 3% or less. The average loading factor of the trucks is set at 43%, based on an annual statistical report on road-based freight transport in Japan. It is almost impossible to enumerate all possible paths for all OD pairs on the transport network due to the considerable number of combinations of links. Thus, the paths to be used in the calculation are limited to those with a length not exceeding 1.5 times that of the shortest path for each OD pair.
The demand of product y in market l is assumed to comply with normal distribution $\hat{d}^y_l(\rho^{4y}, \bar{\omega}^y_l) \sim N\left(\sum_{h^y=1}^{1}\sum_{k^y=1}^{2}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^yk^yl}, (\sigma^y_l)^2\right)$, where $\sigma^y_l = \iota^y_l \sum_{h^y=1}^{1}\sum_{k^y=1}^{2}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^yk^yl}$.
Here, ι^y_l is a coefficient of variation (CV) for a mean value of $\sum_{h^y=1}^{1}\sum_{k^y=1}^{2}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^yk^yl}$ and exogenously given.
The product demand on retailer k in market l is assumed to obey $N\left(\sum_{h^y=1}^{u^y}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^yk^yl}, (\sigma^y_l)^2\right)$, which is necessary for estimating the inventory cost of retailer k. The demand of retailer k on wholesaler j is also assumed to follow the normal distribution $N\left(\sum_{h^y=1}^{1}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^yj^yk^y}, \sigma^2_{k^y}\right)$ with $\sigma_{k^y} = \iota_{k^y}\sum_{h^y=1}^{1}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^yj^yk^y}$, where ι_{k^y} is a CV for a mean value of $\sum_{h^y=1}^{1}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^yj^yk^y}$ and given as $\iota_{k^y} = \sqrt{\sum_{l=1}^{3}(\iota^y_l)^2}$. Thus, the inventory costs incurred by the wholesalers are affected by the demands in the markets. The discrete approximation method is applied to each case within a range of $\pm 3\sigma^y_l$ (or $\pm 3\sigma_{k^y}$), which is a 99% confidence interval of the normal distribution. Both the functional forms of $\xi_{j^yk^y}(q^s_{j^yk^y})$ and $\xi_{k^yl}(q^s_{k^yl})$ are made such that the uniqueness and existence of the solutions are ensured. Hereafter, the inventory threshold of $\sum_{h^y=1}^{1}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^yj^yk^y}$ (and $\sum_{h^y=1}^{1}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^yk^yl}$ as well) is set at the mean value of the probability distribution used. The functional forms and parameter values required for computing the inventory costs are provided below.

$$K_1 = 31,\ \ K_2 = 31,\ \ \iota^y_l = 0.1 \tag{61}$$

$$L_1 = L_2 = 16 \tag{62}$$

$$q^0_{j^yk^y} = \frac{K_1(1-3\iota_{k^y})}{\{(1-3\iota_{k^y})(K_1 - 2L_1 + 1) + (2L_1 - 1)\}}\sum_{h^y=1}^{1}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^yj^yk^y} = (1-3\iota_{k^y})\sum_{h^y=1}^{1}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^yj^yk^y} \tag{63}$$

$$q^{K_1}_{j^yk^y} = \frac{K_1(1+3\iota_{k^y})}{\{(1-3\iota_{k^y})(K_1 - 2L_1 + 1) + (2L_1 - 1)\}}\sum_{h^y=1}^{1}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^yj^yk^y} = (1+3\iota_{k^y})\sum_{h^y=1}^{1}\sum_{p^{2y}\in E^{2y}} q^{p^{2y}}_{h^yj^yk^y} \tag{64}$$

$$p^M_{j^yk^y} = \frac{6}{K_1\sqrt{2\pi}}\exp\left(-4.5\left(\frac{2M - K_1 - 1}{K_1}\right)^2\right) \tag{65}$$

$$q^0_{k^yl} = (1-3\iota^y_l)\sum_{h^y=1}^{1}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^yk^yl},\ \ q^{K_2}_{k^yl} = (1+3\iota^y_l)\sum_{h^y=1}^{1}\sum_{p^{3y}\in E^{3y}} q^{p^{3y}}_{h^yk^yl} \tag{66}$$

$$p^R_{k^yl} = \frac{6}{K_2\sqrt{2\pi}}\exp\left(-4.5\left(\frac{2R - K_2 - 1}{K_2}\right)^2\right) \tag{67}$$

$$\xi_{j^yk^y}(q^s_{j^yk^y}) = 30q^s_{j^yk^y},\ \ \xi_{k^yl}(q^s_{k^yl}) = 30q^s_{k^yl} \tag{68}$$

The values of ι_{k^y} and ι^y_l are set based on the actual data on retailers (FSA, 2011). The parameters and probability density functions for the expected amount of inventory can be expressed with Eqs. (61)-(67), since the inventory threshold is assigned as the mean value of the demand complying with the normal distribution. Here, on the basis of Eqs. (7), (12), (61)-(68), the inventory costs can be derived as below.

$$s_{j^y k^y}(Q^{2y}) = \left[1.4 \times 10^{-2} \times \sum_{M=1}^{16}(16 - M) \times \exp(-0.019(M-16)^2)\right]^2 \times \left(\sum_{h^y=1}^{1} \sum_{p^{2y} \in E^{2y}} q_{h^y j^y k^y}^{p^{2y}}\right)^2 \tag{69}$$

$$s_{k^y l}(Q^{3y}) = \left[8.3 \times 10^{-3} \times \sum_{R=1}^{16}(16 - R) \times \exp(-0.019(R-16)^2)\right]^2 \times \left(\sum_{h^y=1}^{1} \sum_{p^{3y} \in E^{3y}} q_{h^y k^y l}^{p^{3y}}\right)^2 \tag{70}$$

Since the model entails many functional forms and parameters, unrealistic results might be obtained depending on their settings. Therefore, in order to make the hypothetical numerical examples represent the situation as realistic as possible, the functional forms and parameter values used in the model are calibrated using the results of an interview survey to a logistics company and the data on logistics costs observed by type of industry (JILS, 2006) as well as on road traffic flow (MLIT, 2005).
Based on the adjusted functional forms and parameter values (hereafter, referred to as Case 0), the estimated ratio of logistics costs (i.e., the sum of handling cost, inventory cost and carriage) to the total sales exhibits a good agreement with that actually observed for all types of industries, as can be seen in Fig. 7. The ratio of facility cost to operation cost for the freight carriers is estimated to be 1:11 in Case 0, which is identical to the results of the interview survey to a logistics company in Japan. The same results were confirmed for both products.
The percentage of freight vehicles on the transport network is calculated to be 49% on average for all the links in Case 0, whilst the road traffic census in Japan (MLIT, 2005) reports that those are 47% on national expressways, 41% on urban expressways, and 33% on national highways, respectively. The average congestion rate (i.e., volume- capacity ratio) of all the links is estimated to be 0.69 in this case, whereas those reported by that census are 0.78 for urban expressways and 0.72 for the national highways (MLIT, 2005). These results do not necessarily verify that the case study is capable of fully representing realistic situations, but such attempts are crucial to enhance the reliability of the results estimated.
The amount of the products transacted (i.e., that transported or distributed) in Case 0 is listed in Table 1, which shows the same results for both products 1 and 2 with the total amount of products being 272, except that the values of the retailer-market transaction are switched between retailers 1 and 2. The inventory costs to the wholesalers account for 44% of their total logistics cost (handling for 33% and carriage for 24%), whereas those to the retailers for 19% (handling for 52% and carriage for 29%). The producer surplus gained in both SCNs is 16639, and 4107 is the consumer surplus estimated for both SCNs, making the total surplus be 20746. The producer surplus can be computed as the sum of profits gained by the decision-makers on the SCNs excluding the demand markets, whilst the consumer surplus can be calculated as follows:

$$\int_{\rho_l^{4y*}}^{\rho_l^{4y,0}} d_l^y(\rho_l^{4y}) d\rho_l^{4y} \tag{71}$$

where $\rho_l^{4y,0}$ represents the market price of product y in demand market l when $d_l^y(\rho_l^{4y}) = 0$. The total travel time in

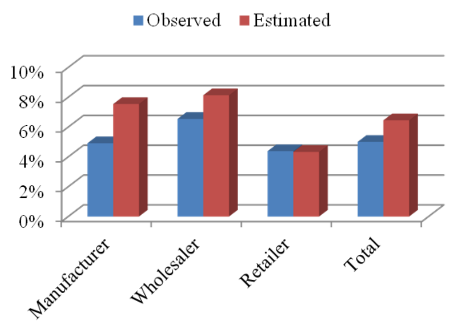

Figure 7: Ratio of logistics costs to sales for product 1 by industries

the transport network in Case 0 is estimated to be 6766, including 3861 for passenger cars and 2905 for trucks.

4.2 Influence of demand variability

This section examines the influence of the variability of product demands on the total surplus as well as on the total amount of the products transacted (i.e., that produced, distributed or transported) among the economic entities involved in. Assuming that both the demand in each market and the retailer's demand on each wholesaler obey the normal

		Wholesaler	
		1	2
Manufacturer	1	110	26
	2	26	110

		Retailer	
		1	2
Wholesaler	1	69	67
	2	67	69

		Market		
		1	2	3
Retailer	1	69	45	22
	2	22	45	69

Table1 Amount of product 1 transacted in Case 0

distribution as in Case 0, the rates of change in the total amount of the products transacted and total surplus are depicted in Fig. 8, with the value of ι_l^y being altered from Case 0 ($\iota_l^y = 0.1$ in Case 0). The results for product 1 are only provided below, but similar results have been obtained for product 2.

The total amount of the product transacted among the entities and the total surplus decrease, as the value of ι_l^y (and ι_{k^y} as well), namely, the variance of consumers' demands, increases. This is consistent with the results indicated by Serel (2009) in the case of price-sensitive stochastic demands. A similar tendency was observed for both the producer surplus and consumer surplus. This implies that the increased demand variability brings about a reduction in the efficiency of SCNs. The price of the product ascended as the value of CV (ι_l^y) increases at any market. The decline in the total amount of the products transacted and the increase in the purchase price lead to a decrease in the consumer surplus. Furthermore, Fig. 9 displays that the inventory costs incurred by the wholesalers and retailers significantly increase as the CV becomes larger.

The total travel time on the transport network decreases, along with an increase in the value of CV. Conversely, with

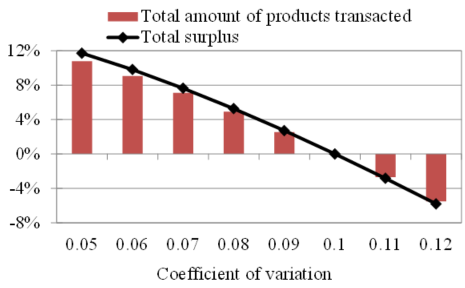

Figure 8: Changes in total surplus and total amount of products due to demand variability

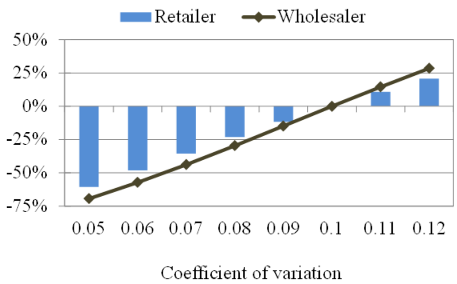

Figure 9: Changes in expected inventory costs due to demand variability

a decrease in the value of CV, the total travel time increases. In the case of $\iota_l^y = 0.05$, the total travel time increases by 9% for passenger cars, by 23% for trucks and by 15% for all kinds of vehicles, respectively, as compared to those in Case 0. In contrast, in the case of $\iota_l^y = 0.12$, it reduces by 4% for passenger cars, by 11% for trucks and by 7% for all kinds of vehicles, respectively, from Case 0. The average congestion rate on the transport network showed a similar trend as the total travel time on it. As the demand fluctuation becomes larger, the amount of the products transacted lessens, which results in the reduced number of truck traffic and thus improves the road traffic environment. Although this improvement enhances the efficiency of SCN due to the reduced operation cost incurred by the freight carriers, the impact of the decrease in the amount of the products transacted is overwhelming; and as a result, the efficiency of SCN declines.

Likewise, a case study is then conducted, where the demands are random (Case 1) (i.e., the case of using the uniform distribution for representing the probability distribution illustrated in Fig. 5). In this case, each class in the probability distribution representing the demand at any market price has an equal probability, unlike the case of using the normal distribution (Case 0). Both the demand in the market and that of the retailer on the wholesaler are assumed to follow the uniform distribution within a range of $\pm 3\sigma_l^y$ (or $\pm 3\sigma_{k^y}$) used for the normal distribution in Case 0, where Eqs.

(61)-(64) and (66) are applied.

$$p^{M}_{j^y k^y} = \frac{1}{K_1}, \;\; p^{R}_{k^y l} = \frac{1}{K_2} \tag{72}$$

$$s_{j^y k^y}(Q^{2y}) = 1.5 \times \left(\sum_{h^y=1}^{1} \sum_{p^{2y} \in E^{2y}} q^{p^{2y}}_{h^y j^y k^y} \right)^2 \tag{73}$$

$$s_{k^y l}(Q^{3y}) = 1.7 \times 10^{-1} \times \left(\sum_{h^y=1}^{1} \sum_{p^{3y} \in E^{3y}} q^{p^{3y}}_{h^y k^y l} \right)^2 \tag{74}$$

As for both kinds of products in Case 1, the amount of the products transacted (i.e., that transported or distributed) between the entities decreases by 56%, and the total surplus reduces by 50%, as compared to Case 0. The results suggest that the amount of the products transacted decreases, and the efficiency of SCNs lowers, if the demands fluctuate more randomly even in the same range of variability.

4.3 Effects of information sharing

Case 0 presumed that the variance in the demands of the retailers on the wholesalers was greater than that of consumers in the markets, that is, the uncertainty suffered by the decision-makers upstream in the SCNs was larger than that by those downstream. Hence, in Case 2, a numerical test is carried out with ι_{k^1} being equal to ι^1_l , which involves that wholesaler 1 and retailer 1 for product 1 share the information about the demands in the markets.

It can be seen from Table 2 that the amount of product 1 transacted between wholesaler 1 and retailer 1, who share the information on the consumers' demands, increases. The SCN for product 1 exhibits a 3% increase in the amount of the product transacted, a 3% increase in producer surplus, a 6% increase in consumer surplus, and a 3% increase in the total surplus, respectively, as compared to Case 0. The distribution channel of "manufacturer 1 $\rightarrow$ wholesaler 1 $\rightarrow$ retailer 1 " appears to dominate. On the other hand, there is no significant change that happened from Case 0 for product 2.

Table 3 compares the amount of the products transacted, producer surplus, consumer surplus and the total surplus of both products combined, among the three cases of Case 0, Case 1 and the case with the increased capacity of two congested links (i.e., links 3 and 34 in Fig. 6) by 10% from Case 0. The effects of information sharing about the demands on the efficiency of SCNs is found to be equal to or greater than the case of a 10% capacity increase in the congested links, even though the partial information of the entire SCNs is only shared.

[Product 1]

		Wholesaler	
		1	2
Manufacturer	1	120	20
	2	34	104

		Retailer	
		1	2
Wholesaler	1	101	53
	2	53	71

		Market		
		1	2	3
Retailer	1	77	50	27
	2	14	45	65

[Product 2]

		Wholesaler	
		1	2
Manufacturer	1	109	26
	2	26	109

		Retailer	
		1	2
Wholesaler	1	68	67
	2	67	68

		Market		
		1	2	3
Retailer	1	23	46	66
	2	66	46	23

Table2 Amount of products transacted in Case 2 (Products 1 and 2 separately)

Table3 Amount of products and surpluses (Products 1 and 2 combined)

	Case 0		Case2
Link Capacity	100%	110%	100%
Amount of Products	544	549	550
Producer Surplus	33278	33560	33727
Consumer Surplus	8214	8387	8419
Total Surplus	41491	41946	42146

5. CONCLUSIONS

Supply chain analyses have been interdisciplinarily undertaken in terms of production, distribution, transport, logistics, and marketing. However, there has been no research on the interactions between the behavior of economic entities on SCNs and the traffic conditions in a transport network, except for the SC-T-SNE model proposed by Yamada et al. (2011). The SC-T-SNE model explicitly integrates the SCNs with the transport network, and consequently, is useful for administrators and planners to comprehend the mechanisms of the generation of goods distribution and investigate the effects of freight transport measures to be implemented as well as for companies to appreciate the influence of such measures.

The model presented in the paper extended the SC-T-SNE model to allow the inventory costs incurred due to the uncertainty of consumers' demands to be taken into consideration. Therefore, the model is more realistic, practical, and useful for companies, administrators, and planners than the existing SC-T-SNE model. Characterized by the availability of a wider variety of probability distributions for representing the demand fluctuations, this model can facilitate the broader application than the existing SCNE models dealing with the uncertain demands. Inventory costs are estimated based on the probability distributions, where wholesalers' (and retailers' as well) preference for inventory can be represented by setting the threshold. For inventory-averse wholesalers, the threshold is set at a smaller amount of the product transacted, while at a larger amount for stockout-averse wholesalers.

The model was then applied to a supernetwork, not only to validate the model but to explore how the demand variability and the information sharing about it affect the amount of the products transacted (and that produced, transported or distributed) and the efficiency of SCNs, as compared to the case of improving the congested links on the road network. Results indicate that the increased variability would lessen the amount of the products transacted, causing deterioration in the efficiency of SCNs while improving the road traffic environment. It is also suggested that the variable demands with higher randomness would lead to a further decline in the efficiency of SCNs even in the same range of fluctuations. In addition, the results show that the information sharing between wholesalers and retailers on the variability of consumers' demands could enhance the efficiency of SCNs with their distribution channels being altered, which is likely to produce the effects almost equal to those brought about by the mitigation of traffic congestion on the road network.

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