International Journal of
Operations Research

International Journal of Operations Research Vol. 17, No. 2, 49-63 (2020)

Interactive Inventory Model under Permissible Delay and Imprecision

P.K. Tripathy [1], Sujata Sukla [2*]

[1,2]P.G. Department of Statistics Utkal University, Bhubaneswar-751004, India.

Received April 2020; Revised June 2020; Accepted June 2020

Abstract: Current study develops a trade credit system between supplier and buyer where demand is assumed to be time dependent and deterioration follows weibull distribution. Shortages are allowed and excess demand is backlogged. The parameters involved in the supply chain system may likely to be varied due to the changing business environment. Therefore, it will be more realistic and market friendly to deal with fuzzy model rather than crisp model. Here the optimal solution is attained for different periods of permissible delay as furnished by the seller to the buyer. Sensitivity analysis is incorporated to investigate the effect of different system parameters in enhancing the profit. The study can make the supply chain strategy of the company more robust by making the most sense through stock prediction and its selection so as to ensure that any disruption in the supply chain would not bring the company to a grinding halt.
Keyword — Inventory, Permissible delay, Credit period, Backlogging, defuzzification.

---

1. INTRODUCTION

Inventory is an integral part of supply chain management system. Supply chain focuses on the main factor like demand which plays a vigorous role in choosing the best inventory policy. In addition, the inventory model has been made quite relevant in the current market scenario by including partial backlogging which warrants it due to the lack of large quantity of stocks. Moreover, product perishability is a critical aspect of inventory system. Products deteriorate during the storage period and thus lose their original value. Researchers have emphasized on various inventory models using aforementioned criteria. Aggrawal and Singh (2017) analysed an Economic Order Quantity(EOQ) model for time dependent deteriorating items assuming demand as a combination of linear and quadratic function of time. Bhojak and Gothi (2015) focused on an inventory model for deteriorating items under time dependent demand and linear holding cost. Jain and Kumar(2010) introduced a more general type of inventory model incorporating ramp type demand and three parameter weibull distribution deterioration. Banu and Mondal (2016) analysed an economic order quantity model with constant deterioration under two level trade credit financing where demand is a function of the length of customer's credit period and the duration of offering the credit period. Giri, Jalan, and Chaudhuri (2003) endeavoured to develop a single item and single period inventory model under completely backlogged situation integrating ramp demand and weibull deterioration. Dharma, Lin, and Lee (2019) carried out an economic order quantity model wherein demand is a function of product price, inventory age, and displayed inventory level. Tripathy and Sukla (2018a) proposed a single item economic order quantity model with ramp demand and Heaviside's deterioration. Karmakar, B. and Choudhuri, K.D. (2013) extracted a partially backlogged inventory model with ramp demand and time dependent holding cost.
Growing economy of today leads the suppliers to accept the delay in payment system to withstand in the competitive business forum. Many researchers have endeavoured to capture business in payment delay scenario. Shah, Jani, and Shah (2015) focused on an inventory model with price sensitive quadratic demand in which the supplier offers a prior mutually agreed fixed credit period to the retailer. Annadurai and Uthayakumar (2015) proposed a delaying inventory model under two level trade credit policy for stock dependent demand. Kaur, Pareek, and Tripathi (2016) developed an inventory model based on trade credit system wherein demand is linear and non-increasing function of time and deterioration also depends on time. Wu and Zhao (2016) established two retailer-supplier uncooperative replenishment models with credit period linked demand and default risk.
Yang (2019) modelled a two-level trade credit inventory problem having ramp demand and credit linked order quantity. Sundararajan and Uthayakumar (2015) dealt with a completely backlogged and deterministic inventory model under delay in payment condition with constant demand.
It has been assumed that the parameters involved in the trade credit inventory are crisp, that is, they possess certain

*Corresponding author's e-mail: suklasujata086@gmail.com.

http://doi.org/10.6886/IJOR.202006_17(2).0002

values but in the diversifying economy as things get changed more quickly, these parameters may deviate more or less from their actual value thus by making the crisp model insufficient to calibrate such changes. Therefore, it is more realistic to consider the system parameters as fuzzy. Mahata and Mahata (2011) analysed an Economic Order Quantity model to reflect the supply chain management situation under two level trade credit in fuzzy environment. Sujatha and Parvathi (2015) developed a fuzzy inventory model for deteriorating items with two parameter weibull demand in partially backlogged situation allowing permissible delay. Tripathy and Sukla (2018b) explored a fuzzy inventory model under delay in payment system where demand is assumed to be a ramp function and deterioration follows weibull distribution. Garai, Chakraborty, and Roy (2019) generated a fuzzy inventory model with price dependent demand and time varying holding cost where the input parameters and decision variables are treated as trapezoidal fuzzy numbers.
The present paper aims at developing a fuzzy inventory model allowing shortages and backlogging under delay in payment system. Demand is assumed to be a linear and quadratic function of time and deterioration follows three parameter weibull distribution. Due to diversification in the retail sector, the parameters involved in the supply chain system may deviate more or less from their actual value. To deal with such type of situation, both triangular and trapezoidal fuzzy numbers have been employed in the current study. Signed distance and graded mean integration methods are used for defuzzification of the total average profit. The model is exemplified by numerical illustrations. This model aids the decision maker in choosing the most suitable economic period which has larger impact on enhancing the profit of the business organisation.
The rest of the article is structured as follows. Section 2 specifies the assumptions and notations required to build up the model. Mathematical model both in crisp and fuzzy is formed and some particular cases have been derived in section 3. The model is numerically tested in section 4 and both sensitivity analysis and comparative analysis are carried out to strengthen the model. Key findings of the model are summarized and suggestions for the decision maker are furnished in section 5. Finally, the practical utility of the model and future research directions are drawn in section 6.

2. ASSUMPTIONS AND NOTATIONS

The following notations and assumptions are employed to build up the model.

2.1 Assumptions

(i) Demand rate is a combination of linear and quadratic function of time.

$$D(t) = \begin{cases} a + bt + ct^2, & t < \mu \\ a + (b + c\mu)t, & t \geq \mu \end{cases}$$

, where $H(t-\mu)$ is a Heaviside's function defined as $H(t-\mu) = \begin{cases} 0, & t < \mu \\ 1, & t \geq \mu \end{cases}$

(ii) Deterioration rate is a three parameter weibull distribution function

$$\theta = \alpha\beta(t-\tau)^{\beta-1}$$

, where α is the shape parameter, β is the scale parameter and τ is the location parameter.

(iii) Excess demand is backlogged at a rate of $e^{-\phi(T-t)}$
, where phi is the backlogging parameter.

2.2 Notations

- A and $\widetilde{A}$: Ordering cost in crisp and fuzzy model.
- h and $\widetilde{h}$: Holding cost in crisp and fuzzy model.
- S and $\widetilde{S}$: Selling price per unit in crisp and fuzzy model.
- P and $\widetilde{P}$: Purchase cost per unit in crisp and fuzzy model.
- l and $\widetilde{l}$: Lost sale cost in crisp and fuzzy model.
- C_s and $\widetilde{C_s}$: Shortage cost in crisp and fuzzy model.

- $I(t)$: Inventory level at time t.

- $I_1(t), I_2(t), I_3(t) \text{and} I_4(t)$: Inventory levels during the time periods $0 \leq t \leq \mu, \mu \leq t \leq \gamma$, $\gamma \leq t \leq T_1$ and $T_1 \leq t \leq T$
- M : Credit period offered by the seller to the buyer.
- γ : Time period of starting deterioration.
- T_1 : Time period of starting of backlogging.
- π and $\widetilde{\pi}$: Total average profit in crisp and fuzzy model.
- $\widetilde{\pi}_{SD}$ and $\widetilde{\pi}_{GM}$ Defuzzified total average profit using signed distance method and graded mean integration method.

3. MODEL FORMULATION

3.1 Crisp Model

Initially the inventory level is Q. During the time period $0 < t < \mu$, the inventory declines due to quadratic demand and during the time period $\mu \leq t < \gamma$, inventory declines due to linear demand. Deterioration of inventory begins at the time point γ and it remains up to T_1. Shortages and backlogging occur during the time period $T_1 \leq t < T$. The model is governed by the following differential equations.

$$\frac{\mathrm{d}}{\mathrm{dt}} I(t) = -(a + bt + ct^2), \quad 0 < t < \mu \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{dt}} I(t) = -(a + (b + c\mu)t), \quad \mu \leq t < \gamma \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{dt}} I(t) + \theta I(t) = -(a + (b + c\mu)t), \quad \gamma \leq t < T_1 \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{dt}} I(t) = -(a + (b + c\mu)t)e^{-\phi(T-t)}, \quad T_1 \leq t < T \tag{4}$$

, with the boundary conditions $I(0) = Q, \quad I(T_1) = 0$
Solution of equations (1), (2), (3) and (4) under the boundary conditions yields

$$I_1(t) = -\left[at + b\frac{t^2}{2} + c\frac{t^3}{3}\right] + Q \tag{5}$$

$$I_2(t) = -at + (b + c\mu)\frac{t^2}{2} + \frac{c\mu^3}{6} + Q \tag{6}$$

$$I_3(t) = \begin{bmatrix} \left[-at - a\alpha\frac{(t-\tau)^{\beta+1}}{(\beta+1)} - (b + c\mu)\frac{t^2}{2} - (b + c\mu)\alpha\left[t\frac{(t-\tau)^{\beta+1}}{(\beta+1)} - \frac{(t-\tau)^{\beta+2}}{(\beta+1)(\beta+2)}\right]\right](1 - \alpha(t - \tau)^{\beta}) \\ + \left[-a\gamma - (b + c\mu)\frac{\gamma^2}{2} + c\mu^3 + Q\right](1 - \mu(t - \tau)^{\beta}) + \alpha(\gamma - \tau)^{\beta}) \\ - \left[-a\gamma - a\alpha\frac{(t-\tau)^{\beta+1}}{(\beta+1)} - (b + c\mu)\frac{\gamma^2}{2} - (b + c\mu)\alpha\left[\gamma\frac{(\gamma-\tau)^{\beta+1}}{(\beta+1)} - \frac{(\gamma-\tau)^{\beta+2}}{(\beta+1)(\beta+2)}\right]\right](1 - \alpha(t - \tau)^{\beta}) \end{bmatrix} \tag{7}$$

$$I_4(t) = \frac{a}{\phi}\left(e^{\phi(T_1-T)} - e^{\phi(t-T)}\right) + \frac{(b + c\mu)}{\phi}\left[e^{\phi(T_1-T)}\left[T_1 - \frac{1}{\phi}\right] - e^{\phi(t-T)}\left[t - \frac{1}{\phi}\right]\right] \tag{8}$$

and the order quantity is

$$Q = \begin{bmatrix} -\left\{-aT_1 - a\alpha\frac{(T_1-\tau)^{\beta+1}}{(\beta+1)} - (b + c\mu)\frac{T_1^2}{2} - (b + c\mu)\alpha\left[T_1\frac{(T_1-\tau)^{\beta+1}}{(\beta+1)} - \frac{(T_1-\tau)^{\beta+2}}{(\beta+1)(\beta+1)}\right]\right\}(1 - \alpha(\gamma - \tau)^{\beta}) + a\gamma \\ +(b + c\mu)\frac{\gamma^2}{2} - \frac{c\mu^3}{6} + \left[-a\gamma - a\alpha\frac{(\gamma-\tau)^{\beta+1}}{(\beta+1)} - (b + c\mu)\frac{\gamma^2}{2} - (b + c\mu)\alpha\left[\gamma\frac{(\gamma-\tau)^{\beta+1}}{(\beta+1)} - \frac{(\gamma-\tau)^{\beta+2}}{(\beta+1)(\beta+2)}\right]\right](1 - \alpha(\gamma - \tau)^{\beta}) \end{bmatrix}$$

The sales revenue: $S\left[\int_0^{\mu} D(t)\mathrm{dt} + \int_{\mu}^{\gamma} D(t)\mathrm{dt} + \int_{\gamma}^{T_1} D(t)\mathrm{dt}\right] = S \times R$
Holding cost: $h\left[\int_0^{\mu} I_1(t)\mathrm{dt} + \int_{\mu}^{\gamma} I_2(t)\mathrm{dt} + \int_{\gamma}^{T_1} I_3(t)\mathrm{dt}\right] = h \times H$

Deterioration cost: $d\int_{\gamma}^{T_1}\alpha\beta(t-\tau)^{\beta-1}I_3(t)\mathrm{dt}=d\times D$
Shortage cost: $-C_s\int_{T_1}^{T}I_4(t)\mathrm{dt}=-C_s\times C$
Lost sale cost: $l\int_{T_1}^{T}D(t)(1-e^{-\phi(T-t)})\mathrm{dt}=l\times L$
Purchase cost: PQ
Ordering cost: A

Case-I: When $T_1\le M\le T$
Since the credit period falls after T_1 , there will be no Interest charges, so $IC_1=0$
Interest earned: $PI_e\left[\int_0^{\mu}tD(t)\mathrm{dt}+\int_{\mu}^{T_1}tD(t)\mathrm{dt}+(M-T_1)\left(\int_0^{\mu}D(t)\mathrm{dt}+\int_{\mu}^{T_1}tD(t)\mathrm{dt}\right)\right]=PI_eE_1$
Total average profit: $\pi=\frac{1}{T}[S\times R-h\times H-d\times D-l\times L-(C_s\times C)+PI_eE_1-PQ-A]$ (9)

Case-II: When $\gamma\le M\le T_1$
Interest charged: $PI_C\int_M^{T_1}I_2(t)\mathrm{dt}=PI_C\times C_2$
Interest earned: $PI_e\int_0^{T_1}tD(t)\mathrm{dt}=PI_e\times E_2$
Total average profit: $\pi=\frac{1}{T}\left[S\times R-h\times H-d\times D-l\times L-(-C_s\times C)-PI_C\times C_2+PI_eE_2-PQ-A\right]$(10)

Case-III: When $\mu\le M\le\gamma$
Interest charged: $PI_C\left(\int_M^{\gamma}I_2(t)\mathrm{dt}+\int_{\gamma}^{T_1}I_3(t)\mathrm{dt}\right)=PI_C\times C_3$
Interest earned: $PI_e\int_0^{T_1}tD(t)\mathrm{dt}=PI_e\times E_3$
Total average profit: $\pi=\frac{1}{T}\left[S\times R-h\times H-d\times D-l\times L-(-C_s\times C)-PI_C\times C_3+PI_eE_3-PQ-A\right]$(11)

Case-IV: When $0\le M\le\mu$
Interest charged: $PI_C\left(\int_M^{\mu}I_1(t)\mathrm{dt}+\int_{\mu}^{\gamma}I_2(t)\mathrm{dt}+\int_{\gamma}^{T_1}I_3(t)\mathrm{dt}\right)=PI_C\times C_4$
Interest earned: $PI_e\int_0^{T_1}tD(t)\mathrm{dt}=PI_e\times E_4$
Total average profit: $\pi=\frac{1}{T}\left[S\times R-h\times H-d\times D-l\times L-(-C_s\times C)-PI_C\times C_4+PI_eE_4-PQ-A\right]$(12)

The optimal values of T_1 and T in equations (9), (10), (11) and (12) which maximize the total average profit can be obtained by solving the equation

$$\frac{\partial\pi}{\partial T_1}=0\quad\text{and}\quad\frac{\partial\pi}{\partial T}=0\tag{13}$$

These values satisfy the sufficient conditions

$$\frac{\partial^2\pi}{\partial T_1^2}<0,\quad\frac{\partial^2\pi}{\partial T^2}<0\quad\text{and}\quad\frac{\partial^2\pi}{\partial T_1^2}\frac{\partial^2\pi}{\partial T^2}-\frac{\partial^2\pi}{\partial T_1\partial T}<0\tag{14}$$

3.2 Fuzzy Model

Due to uncertainty, the cost parameters involved in the model like ordering cost, holding cost, selling price, purchase cost, shortage cost, lost sale cost and deterioration cost are treated as fuzzy in nature.

Cost parameters are Triangular fuzzy numbers

Ordering cost $\widetilde{A}=(A_1,A_2,A_3)$, purchase cost $\widetilde{P}=(P_1,P_2,P_3)$, holding cost $\widetilde{h}=(h_1,h_2,h_3)$, shortage cost $\widetilde{C_s}=(C_{S1},C_{S2},C_{S3})$, lost sale cost $\widetilde{l}=(l_1,l_2,l_3)$ and selling price $S=(S_1,S_2,S_3)$ are triangular fuzzy numbers.

Case-I: When $T_1\le M\le T$
The defuzzified total profit after applying signed distance method is

$$\widetilde{\pi}_{SD}=\frac{1}{4T}\begin{bmatrix}(S_1+2S_2+S_3)\times R-(h_1+2h_2+h_3)\times H-(d_1+2d_2+d_3)\times D-(l_1+2l_2+l_3)\times L\\-(-(C_{S1}+2C_{S2}+C_{S3})\times C)+(P_1+2P_2+P_3)I_eE_1-(P_1+2P_2+P_3)Q-(A_1+2A_2+A_3)\end{bmatrix}\tag{15}$$

The defuzzified total profit after applying graded mean integration method becomes

$$\widetilde{\pi}_{GM}=\frac{1}{6T}\begin{bmatrix}(S_1+4S_2+S_3)\times R-(h_1+4h_2+h_3)\times H-(d_1+4d_2+d_3)\times D-(l_1+2l_2+l_3)\times L\\-(-(C_{S1}+4C_{S2}+C_{S3})\times C)+(P_1+4P_2+P_3)I_eE_1-(P_1+4P_2+P_3)Q-(A_1+4A_2+A_3)\end{bmatrix}\tag{16}$$

Cost parameters are Trapezoidal fuzzy numbers

Fuzzy ordering cost $\widetilde{A} = (A_1, A_2, A_3, A_4)$, fuzzy purchase cost $\widetilde{P} = (P_1, P_2, P_3, P_4)$, fuzzy holding cost $\widetilde{h} = (h_1, h_2, h_3, h_4)$, fuzzy shortage cost $\widetilde{C_s} = (C_{S1}, C_{S2}, C_{S3}, C_{S4})$, fuzzy backlogging cost $\widetilde{l} = (l_1, l_2, l_3, l_4)$ and selling price $S = (S_1, S_2, S_3, S_4)$ are trapezoidal fuzzy numbers.

Case-I: When $T_1 \leq M \leq T$

The defuzzified total profit after applying signed distance method is

$$\tilde{\pi}_{SD} = \frac{1}{4T}\begin{bmatrix} (S_1+S_2+S_3+S_4)\times R - (h_1+h_2+h_3+h_4)\times H - (d_1+d_2+d_3+d_4)\times D - (l_1+l_2+l_3+l_4)\times L \\ -(-(C_{S1}+C_{S2}+C_{S3}+C_{S4})\times C) + (P_1+P_2+P_3+P_4)I_eE_1 - (P_1+P_2+P_3+P_4)Q - (A_1+A_2+A_3+A_4) \end{bmatrix} \tag{17}$$

The defuzzified total profit after applying graded mean integration method becomes

$$\tilde{\pi}_{GM} = \frac{1}{6T}\begin{bmatrix} (S_1+2S_2+2S_3+S_4)\times R - (h_1+2h_2+2h_3+h_4)\times H - (d_1+2d_2+2d_3+d_4)\times D - (l_1+2l_2+2l_3+l_4)\times L \\ -(-(C_{S1}+2C_{S2}+2C_{S3}+C_{S4})\times C) + (P_1+2P_2+2P_3+P_4)I_eE_1 - (P_1+2P_2+2P_3+P_4)Q - (A_1+2A_2+2A_3+A_4) \end{bmatrix} \tag{18}$$

The equations (15), (16), (17) and (18) satisfy the conditions (13) and (14). The defuzzified total profit can also be obtained in other cases and these satisfy the conditions (13) and (14).

4. EMPIRICAL INVESTIGATION

4.1 Illustration-1

The model is evaluated by assuming the following values of the system parameters

$a = 5, b = 8, c = 10, \alpha = 0.0003, \beta = 1.8, h = 0.6, l = 15, C_s = 18, \phi = 0.05, S = 35, A = 200,$
$P = 15, \mu = 3, \gamma = 8, \tau = 0.18, I_C = 0.25$ and $I_e = 0.15$

Case-I: $T_1 \leq M \leq T$: Considering $M = 30$
$T_1 = 16.2245, T = 44.3896, \pi = 118,825$ and $Q = 5,100.34$

Case-II: $\gamma \leq M \leq T_1$: Considering $M = 12$
$T_1 = 17.1386, T = 44.4340, \pi = 121,378$ and $Q = 5,597.87$

Case-III: $\mu \leq M \leq \gamma$: Considering $M = 6$
$T_1 = 17.6482T = 44.6325, \pi = 125,513.0$ and $Q = 6,056.82$

Case-IV: $0 \leq M \leq \mu$: Considering $M = 2$
$T_1 = 18.5210, T = 44.8987, \pi = 129,687$ and $Q = 6,424.32$

4.2 Illustration-2

The model is evaluated using the following values of the system parameters

$a = 3, b = 6, c = 9, \alpha = 0.0009, \beta = 2, h = 0.8, l = 10, C_s = 10, \phi = 0.08, S = 30, A = 120,$
$P = 10, \mu = 4, \gamma = 6, \tau = 1.8, I_C = 0.3$ and $I_e = 0.2$

Case-I: $T_1 \leq M \leq T$: Considering $M = 12$
$T_1 = 10.2055, T = 27.9089, \pi_1 = 16,343.5$ and $Q = 1,287.74$

Case-II: $\gamma \leq M \leq T_1$: Considering $M = 10$
$T_1 = 10.1976, T = 27.8916, \pi_1 = 16,254$ and $Q = 1,286.69$

Case-III: $\mu \leq M \leq \gamma$: Considering $M = 5$
$T_1 = 10.7818T = 28.1485, \pi_1 = 17,570.2$ and $Q = 1,442.8$

Case-IV: $0 \leq M \leq \mu$: Considering $M = 1$
$T_1 = 11.7271, T = 28.5246, \pi_1 = 19,081.3$ and $Q = 1,717.16$

Table 1: Results obtained when cost parameters are fuzzy in illustration-1

Triangular Fuzzy Number								
$\widetilde{h} = (0.3, 0.6, 0.8), \widetilde{l} = (13, 15, 18), \widetilde{C_s} = (15, 18, 20), \widetilde{S} = (30, 35, 39), \widetilde{A} = (150, 200, 220),$ $\widetilde{P} = (10, 15, 18), \widetilde{I_c} = (0.22, 0.25, 0.27)$ and $\widetilde{I_e} = (0.13, 0.15, 0.18)$								
Cases	Signed Distance				Graded Mean Integration			
	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
Case-I	13.1342	40.9930	149,834	3,317.36	11.8884	40.7878	159,251	2,709.79
Case-II	15.2895	40.6865	150,118	4,518.94	14.2093	40.1194	155,596	3,892.74
Case-III	20.2879	42.5513	205,783	8,061.71	16.4012	40.5640	194,389	5,214.36
Case-IV	Infeasible solution				Infeasible solution			
Trapezoidal Fuzzy Number								
$\widetilde{h} = (0.2, 0.5, 0.7, 1), \widetilde{l} = (10, 12, 16, 19), \widetilde{C_s} = (12, 15, 19, 20), \widetilde{S} = (30, 33, 37, 39), \widetilde{A} = (150, 180, 220, 250),$ $\widetilde{P} = (10, 12, 16, 18), \widetilde{I_c} = (0.15, 0.20, 0.30, 0.32)$ and $\widetilde{I_e} = (0.10, 0.12, 0.20, 0.25)$								
Cases	Signed Distance				Graded Mean Integration			
	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
Case-I	12.7763	41.0775	141,273	3,136.3	11.5738	40.9773	150,338	2,565.99
Case-II	14.8428	40.6777	139,172	4,254.1	13.7995	40.1511	14,449	3,667.83
Case-III	18.0174	41.5533	173,210	6,319.06	18.8360	42.1769	210,939	6,921.68
Case-IV	Infeasible solution				Infeasible solution			

Table 2: Results obtained when cost parameters are fuzzy in illustration-2

Triangular Fuzzy Number								
$\widetilde{h} = (0.3, 0.8, 0.9), \widetilde{l} = (5, 10, 13), \widetilde{C_s} = (8, 10, 15), \widetilde{S} = (25, 30, 34), \widetilde{A} = (100, 120, 150),$ $\widetilde{P} = (5, 10, 16), \widetilde{I_c} = (0.1, 0.3, 0.5)$ and $\widetilde{I_e} = (0.1, 0.2, 0.5)$								
Cases	Signed Distance				Graded Mean Integration			
	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
Case-I	8.11302	25.7662	22,445	805.767	7.42765	25.6325	22,989.9	673.782
Case-II	34.5078	47.6261	293,650	20,822	31.1477	43.7551	314,586	15,897.9
Case-III	9.15932	25.7412	25,152.8	1,030.43	8.08291	25.4598	24,690.7	799.707
Case-IV	Infeasible solution				Infeasible solution			
Trapezoidal Fuzzy Number								
$\widetilde{h} = (0.3, 0.6, 0.9, 1), \widetilde{l} = (8, 9, 12, 14), \widetilde{C_s} = (7, 9, 12, 16), \widetilde{S} = (20, 25, 35, 38), \widetilde{A} = (100, 110, 130, 135),$ $\widetilde{P} = (6, 9, 13, 15), \widetilde{I_c} = (0.1, 0.2, 0.4, 0.7)$ and $\widetilde{I_e} = (0.1, 0.15, 0.3, 0.4)$								
Cases	Signed Distance				Graded Mean Integration			
	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
Case-I	8.21552	25.7410	18,696.9	826.579	7.40959	25.6457	23,661	670.471
Case-II	29.2515	42.1493	21,2515	13,536.9	48.4212	41.0749	275,738	12,589.2
Case-III	9.85524	25.8904	27,916.5	1,198.52	9.05417	25.4605	43,274	1,028.4
Case-IV	Infeasible solution				Infeasible solution			

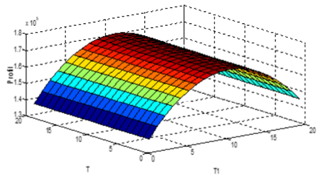

Figure 1: Concavity of profit in case-I of illustration-1

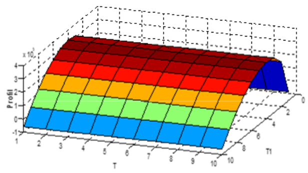

Figure 2: Concavity of profit in case-II of illustration-1

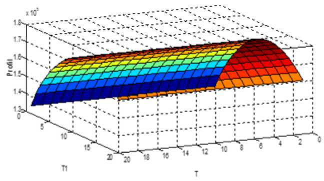

Figure 3: Concavity of profit in case-III of illustration-1

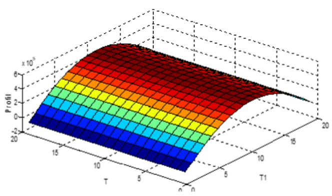

Figure 4: Concavity of profit in case-IV of illustration-1

Total profit also attains concavity in fuzzy model for all cases.

4.2.1 Sensitivity Analysis

The sensitivity analysis is performed to examine the effect of variation in the values of system parameters on T_1 , T , π and Q . For this the values of each parameter is changed by -60%, -40%, -20%, +20%, +40% and +60% at a time and other parameters remain unchanged.

Table 3: Sensitivity analysis in Case-I & Case-II of illustration-1

Parameter	change %	Case-I				Case-II			
		T_1	T	π	Q	T_1	T	π	Q
a	-60 %	16.2847	44.3111	121,259	5,089.82	17.0625	44.3521	122,431	5,601.21
	-40 %	16.2646	44.3371	120,813	5,093.33	17.0878	44.3794	122,132	5,635.52
	-20 %	16.2445	44.3633	120,364	5,096.81	17.1132	44.4067	121,728	5,669.97
	+20 %	16.2047	44.4161	119,460	5,103.97	17.1639	44.4614	121,028	5,739.05
	+40 %	16.1848	44.4427	119,004	5,107.51	17.1893	44.4887	120,678	5,773.74
	+60 %	16.1651	44.4694	118,546	5,111.16	17.2148	44.5162	120,331	5,808.59
b	-60 %	16.2096	44.4082	104,469	4,452.37	17.1569	44.4532	105,813	5,000.39
	-40 %	16.2150	44.4014	109,617	4,668.35	17.1502	44.4462	111,001	5,234.86
	-20 %	16.2200	44.3952	114,765	4,884.37	17.1441	44.4398	116,190	5,469.65
	+20 %	16.2287	44.3844	125,061	5,316.35	17.1334	44.4287	126,566	5,939.26
	+40 %	16.2326	44.3797	130,209	5,532.38	17.1287	44.4237	131,754	6,174.1
	+60 %	16.2320	44.3753	135,356	5,748.4	17.1244	44.4191	136,943	6,408.97
c	-60 %	16.1366	44.5105	58,985.1	2,698.98	17.2529	44.5581	63,067.5	3,090.37
	-40 %	16.1792	44.4512	77,393.2	3,499.37	17.1972	44.4976	82,503	3,961.6
	-20 %	16.2061	44.4146	95,796.5	4,299.87	17.1624	44.4598	101,940	4,833.01
	+20 %	16.2380	44.3716	132,596	5,900.88	17.1213	44.4153	140,816	6,576.06
	+40 %	16.2483	44.3579	150,994	6,701.44	17.1081	44.401	160,254	7,447.61
	+60 %	16.2564	44.3471	169,392	7,502.01	17.0978	44.388	179,693	8,319.25
α	-60 %	16.2030	44.3904	114,166	5,049.16	17.0988	44.4287	121,278	5,628.29
	-40 %	16.2102	44.3901	114,176	5,066.18	17.1120	44.4304	121,311	5,653.51
	-20 %	16.2173	44.3899	114,186	5,083.20	17.1252	44.4322	121,344	5,678.87
	+20 %	16.2318	44.3894	114,180	5,117.61	104.901	130.769	1942,430	3,778.98
	+40 %	16.2390	44.3891	114,219	5,134.87	97.0394	122.022	1658,440	325,071
	+60 %	16.2463	44.3889	114,215	5,151.67	90.7676	115.07	1448,70	285,767
β	-60 %	16.1898	44.3908	114,147	5,011.89	17.0756	44.4255	121,218	5,582.79
	-40 %	16.1923	44.3908	114,151	5,019.07	17.0806	44.4262	121,230	5,592.31
	-20 %	16.2003	44.3905	114,162	5,040.17	17.0952	44.4281	121,266	5,620.71
	+20 %	16.2993	44.3873	114,305	5,279.38	17.2726	44.4537	121,726	5,957.41
	+40 %	28.5442	44.3871	158,664	2,4210.2	17.7517	44.5401	123,023	6,836,24
	+60 %	-	-	-	-	-	-	-	-
h	-60 %	16.0991	44.3633	113,282	4,999.44	16.9733	44.3820	120,236	5,592.61
	-40 %	16.1402	44.3718	113,580	5,032.64	17.0273	44.3987	120,609	5,629.04
	-20 %	16.1827	44.3805	113,885	5,066.53	17.0824	44.4161	120,989	5,666.33
	+20 %	16.2677	44.3990	114,516	5,136.65	17.1959	44.4526	121,775	5,743.56
	+40 %	16.3116	44.4088	114,841	5,172.84	17.2544	44.4719	122,181	5,783.59
	+60 %	16.3562	44.4189	115,175	5,209.95	17.3142	44.4919	122,587	5,824.65
l	-60 %	16.0015	44.4700	112,839	4,956.62	16.8986	44.4679	125,466	5,542.43
	-40 %	16.07625	44.4458	116,908	5,005.03	16.9795	44.4548	124,118	5,596.79
	-20 %	16.1509	44.4167	115,561	5,053.24	17.0595	44.4435	122,755	5,650.82
	+20 %	16.2975	44.3645	112,819	5,148.96	17.2166	44.4263	119,985	5,757.71
	+40 %	16.3698	44.3413	111,426	5,196.46	17.2937	44.4202	118,579	5,810.56
	+60 %	16.4414	44.3199	110,019	5,243.74	17.3698	44.4157	117,159	5,872.97
C_s	-60 %	15.3163	44.8791	43,066	4,473.23	16.8239	44.9825	50,762	5,492.49
	-40 %	15.7681	44.6244	66,951.7	4,789.2	16.9787	44.6910	74,406	5,596.25
	-20 %	16.0407	44.4810	90,630.1	4,976.36	17.0741	44.5333	970,25.3	5,660.71
	+20 %	16.3574	44.3246	137,697	5,190.7	17.1085	44.3657	1454,083	5,736.12
	+40 %	16.458	44.2801	161,151	5,258.13	17.2201	44.3158	168,176	5,760.10
	+60 %	16.5369	44.2448	184,576	5,310.88	17.2476	44.2777	191,545	5,778.63
ϕ	-60 %	121.0130	162.4850	1246,890	168,618.0	124.3600	177.4760	299,440	563741
	-40 %	100.29	126.569	842,858	140,168	29.6120	74.2295	353,789	17727.7
	-20 %	90.1556	109.369	185,881	8,488.97	21.7717	55.5967	194,168	9326.21
	+20 %	13.2787	37.0773	75,581.9	3,366.29	14.0817	36.9948	82,269.7	3821.97
	+40 %	11.1987	31.8539	52,352.9	2,363.34	11.9160	31.6786	58,896.6	2722.35
	+60 %	9.6550	27.9352	17,261.5	1,733.24	10.3027	27.6869	43,834.3	2026.22

Parameter	change %	Case-I				Case-II			
		T_1	T	π	Q	T_1	T	π	Q
S	-60 %	16.6802	44.3027	118,236	5,411.9	17.7357	44.4286	120,321	6,118.44
	-40 %	16.5208	44.3295	113,302	5,301.84	17.5244	44.4252	120,671	5,970.21
	-20 %	16.3692	44.3586	113,756	5,198.47	17.3258	44.4272	121,027	5,832.63
	+20 %	16.0864	44.4224	114,627	5,009.35	16.9614	44.4449	121,694	5,584.6
	+40 %	15.9541	44.4566	116,838	5,573.31	16.7935	44.4594	122,064	5,472.32
	+60 %	15.8272	44.4922	115,453	4,839.93	16.6338	44.4770	122,400	5,366.47
A	-60 %	16.2246	44.3897	114,200	5,101.24	17.1386	44.4341	121,380	5,704.51
	-40 %	16.2246	44.3897	114,199	5,101.17	17.1386	44.4341	121,379	5,704.51
	-20 %	16.2245	44.3896	114,198	5,101.17	17.1386	44.4340	121,379	5,704.51
	+20 %	16.2245	44.3896	114,196	5,101.17	17.1386	44.4340	121,377	5,704.51
	+40 %	16.2245	44.3895	114,195	5,101.17	17.1386	44.4340	121,376	5,704.51
	+60 %	16.2245	44.3895	114,195	5,101.17	17.1386	44.4339	121,376	5,704.51
P	-60 %	16.6344	44.2151	114,131	5,393.58	17.1613	44.2455	116,861	5,719.96
	-40 %	16.4924	44.2721	114,215	5,292.81	17.1535	44.3087	118,376	5,714.65
	-20 %	16.3570	44.3304	114,236	5,195.36	17.1459	44.3715	119,882	5,709.47
	+20 %	16.0979	44.4498	114,103	5,010.05	17.1313	44.4962	122,863	5,699.54
	+40 %	15.9757	44.5107	113,957	4,921.86	17.1243	44.5580	124,340	5,694.78
	+60 %	15.8576	44.5724	113,762	4,836.37	17.1174	44.6195	125,808	5,690.09
τ	-60 %	16.2247	44.3896	114,198	5,102.00	17.1391	44.4341	121,379	5,705.50
	-40 %	16.2247	44.3896	114,198	5,101.71	17.1389	44.4341	121,378	5,705.15
	-20 %	16.2246	44.3896	114,197	5,101.47	17.1388	44.4340	121,378	5,704.86
	+20 %	16.2245	44.3896	114,197	5,100.94	17.1384	44.4340	121,377	5,704.15
	+40 %	16.2244	44.3896	114,197	5,100.70	17.1382	44.4340	121,377	5,703.80
	+60 %	16.2244	44.3896	114,197	5,100.41	17.1380	44.4339	121,377	5,703.44
I_e	-60 %	16.8748	44.2586	117,362	5,526.50	17.8324	44.4676	121,033	6,186.92
	-40 %	16.6448	44.2977	118,255	5,373.72	17.5784	44.4485	121,140	6,007.91
	-20 %	16.4286	44.3416	119,105	5,232.16	17.3486	44.4380	121,256	5,848.34
	+20 %	16.0316	44.4411	120,685	4,977.38	16.9452	44.4354	121,503	5,573.71
	+40 %	15.8486	44.4956	121,423	4,862.71	16.7661	44.4411	121,631	5,454.01
	+60 %	15.6747	44.5527	122,130	5,757.01	16.5993	44.4504	121,760	5,343.76
I_c	-60 %	–	–	–	–	16.8484	44.3254	118,933	5,508.84
	-40 %					16.9395	44.3588	119,700	5,569.88
	-20 %					17.0361	44.3949	120,513	5,634.99
	+20 %					17.2478	44.4766	122,300	5,779.06
	+40 %					17.3647	44.5231	123,289	5,859.45
	+60 %					17.4903	44.5742	124,354	5,946.47

Table 4: Sensitivity analysis in Case-III & Case-IV of illustration-1

Parameter	change %	Case-III				Case-IV			
		T_1	T	π	Q	T_1	T	π	Q
a	-60 %	17.7318	44.5735	126,836	6,061.97	18.3206	44.7775	130,185	6,483.28
	-40 %	17.7038	44.5942	126,397	6,060.19	18.3871	44.8176	130,132	6,550.4
	-20 %	17.6759	44.6133	125,956	6,058.45	18.4540	44.8580	129,848	6,618.14
	+20 %	17.6206	44.4520	125,068	6,055.25	18.5885	44.9398	129,531	6,755.15
	+40 %	17.5932	44.6717	124,620	6,053.79	18.6560	44.9812	129,379	6,824.35
	+60 %	17.5660	44.6916	124,170	6,052.44	18.7238	45.0229	129,232	6,894.11
b	-60 %	17.6279	44.6458	106,360	5,284.86	18.5698	44.9274	113,186	5,879.48
	-40 %	17.6353	44.6409	114,745	5,542.19	18.5520	44.9170	118,686	6,148.38
	-20 %	17.6420	44.6464	120,129	5,799.49	18.5358	44.9074	124,187	6,417.31
	+20 %	17.6539	44.6189	130,897	6,319.16	18.5074	44.8907	135,188	6,955.29
	+40 %	17.6591	44.6255	136,281	6,571.46	18.4949	44.8833	140,689	7,224.33
	+60 %	17.6640	44.6224	141,665	6,828.82	18.4833	44.8765	146,190	7,493.36

Parameter	change %	Case-III				Case-IV			
		T_1	T	π	Q	T_1	T	π	Q
c	-60 %	17.5260	44.7240	65,010.3	3,190.39	18.8259	44.0875	71,238.1	3,689.74
	-40 %	17.5851	44.6784	85,181.9	4,145.70	18.6767	44.9947	88,503.1	7,687.66
	-20 %	17.6225	44.6511	10,5349	5,101.24	18.5841	44.9375	109,092	5,686.69
	+20 %	17.6670	44.6189	14,5676	7,012.48	18.4753	44.8707	150,285	7,686.26
	+40 %	17.6813	44.6091	165,838	7,968.15	18.4406	44.8495	170,884	8,686.41
	+60 %	17.6926	44.6012	185,999	8,923.87	18.4134	44.8329	191,485	9,686.71
α	-60 %	17.5544	44.6147	125,242	5,935.11	18.4134	44.8651	129,232	6,536.21
	-40 %	17.5850	44.6205	125,330	5,974.85	18.4483	44.8760	129,380	6,585.25
	-20 %	17.6163	44.6264	125,420	6,015.44	18.4845	44.8872	129,532	6,635.3
	+20 %	17.6807	44.6389	125,608	6,099.02	18.5583	44.9106	129,846	6,738.4
	+40 %	17.7140	44.6456	125,705	6,142.18	18.5964	44.9228	130,008	6,791.64
	+60 %	17.7479	44.6525	125,805	6,186.19	18.6353	44.9354	130,175	6,846.05
β	-60 %	17.4997	44.6047	125,083	5,963.8	18.3502	44.8461	128,972	6,447.98
	-40 %	17.5107	44.6067	125,113	5,878.18	18.3622	44.8499	129,028	6,465.11
	-20 %	17.5443	44.6128	125,210	5,922.15	18.4001	44.8617	129,191	6,518.64
	+20 %	18.0011	44.7077	126,562	6,507.85	18.9489	44.0366	131,401	7,268.12
	+40 %	-	-	-	-	-	-	-	-
	+60 %								
h	-60 %	17.4122	44.5581	124,076	5,892.28	17.2796	44.7981	124,274	5,800.87
	-40 %	17.4889	44.5817	124,541	5,945.49	17.3579	44.8303	124,788	5,854.75
	-20 %	17.5675	44.6065	125,020	6,000.29	18.4384	44.8638	129,106	6,625.3
	+20 %	17.7310	44.6598	126,022	6,115.12	18.6060	44.9351	130,287	6,749.36
	+40 %	17.8161	44.6885	126,547	6,175.35	18.6935	44.9730	130,906	6,814.62
	+60 %	17.9036	44.7186	127,089	6,237.60	18.7836	44.0127	137,774	6,882.16
l	-60 %	17.3869	44.6401	129,557	5,874.78	18.2573	44.8670	133,797	6,492.62
	-40 %	17.4752	44.6357	128,225	5,935.97	18.3468	44.8761	132,443	6,558.02
	-20 %	17.5623	44.6333	126,876	5,996.65	18.4347	44.8867	131,073	6,622.58
	+20 %	17.7329	44.6335	124,135	6,116.47	18.6059	44.9119	128,288	6,749.29
	+40 %	17.8164	44.6361	122,741	6,175.57	18.6892	44.9264	126,873	6,811.4
	+60 %	17.8986	44.6402	121,334	6,234.04	18.7710	44.9419	125,445	6,872.69
C_s	-60 %	18.4885	45.8036	57,400.6	6,662.26	20.2748	46.8411	64,626.3	8,051.95
	-40 %	17.9924	45.1176	79,624.0	6,301.12	19.2156	45.6514	84,929.8	7,210.95
	-20 %	17.7726	44.8087	102,466	6,144.53	18.7652	45.1641	107,035	6,871.34
	+20 %	17.5681	44.5185	148,654	6,000.71	18.3627	44.7381	152,581	6,569.67
	+40 %	17.5122	44.4387	171,846	5,961.71	18.2528	44.6172	175,602	6,489.35
	+60 %	17.4709	44.3797	195,068	5,932.98	18.1720	44.5336	198,699	6,430.63
ϕ	-60 %	70.3034	125.228	1120,890	124,266	91.5135	143.8540	1652,460	242,620
	-40 %	65.5931	99.4014	734,942	104,997	31.5354	74.8783	370,753	20,259.1
	-20 %	61.8938	86.8186	584,712	91,379.7	23.3345	56.1172	205,189	10,765.8
	+20 %	14.5240	37.1849	85,731.4	4,070.17	15.3493	37.4314	89,055.8	4,555.02
	+40 %	12.3228	31.8707	61,984	2,914.36	13.1007	32.1020	64,732.2	3,300.19
	+60 %	10.6890	27.8871	46,703.3	2,183.45	11.4233	28.1068	49,041.2	2,498.71
s	-60 %	18.3733	44.6885	124,869	6,577.44	19.4117	45.0643	129,599	7,362.91
	-40 %	18.1134	44.6613	125,071	6,388.22	19.0889	44.9950	129,588	7,113.67
	-20 %	17.8725	44.6431	125,287	6,215.44	18.7934	44.9406	129,621	6,889.53
	+20 %	17.4383	44.6285	125,746	5,910.36	18.2686	44.8617	129,780	6,500.86
	+40 %	17.2411	44.6299	125,983	5,774.47	18.0334	44.8442	129,893	6,330.56
	+60 %	17.0552	44.6361	126,224	5,647.91	17.8134	44.8287	130,022	6,173.44
A	-60 %	17.6482	44.6326	125,515	6,056.82	18.5210	44.8988	129,689	6,686.29
	-40 %	17.6482	44.6326	125,514	6,056.82	18.5210	44.8987	129,689	6,686.29
	-20 %	17.6482	44.6326	125,514	6,056.82	18.5210	44.8987	129,689	6,686.29
	+20 %	17.6482	44.6325	125,512	6,056.82	18.5210	44.8987	129,686	6,686.29
	+40 %	17.6482	44.6325	125,512	6,056.82	18.5210	44.8986	129,686	6,686.29
	+60 %	17.6482	44.6325	125,511	6,056.82	18.5210	44.8986	129,685	6,686.29

Parameter	change %	Case-III				Case-IV			
		T_1	T	π	Q	T_1	T	π	Q
P	-60 %	17.2376	44.3076	118,140	5,772.08	17.7525	44.3958	119,461	6,130.31
	-40 %	17.3680	44.4071	120,484	5,861.72	17.9907	44.5480	122,598	6,299.9
	-20 %	17.5046	44.5169	122,938	5,956.42	18.2461	44.7148	125,994	6,484.47
	+20 %	17.7994	44.7547	128,221	6,163.51	18.8187	45.1024	133,730	6,908.57
	+40 %	17.9591	44.8841	131,078	6,277.26	19.1431	45.3295	138,186	7,155.20
	+60 %	18.1284	45.0216	134,101	6,399.06	19.4993	45.5847	143,137	7,431.34
τ	-60 %	17.6494	44.6328	125,516	6,058.41	18.5223	44.8992	129,695	6,688.17
	-40 %	17.6490	44.6327	125,515	6,057.88	18.5219	44.8991	129,692	6,687.57
	-20 %	17.6486	44.6326	125,514	6,057.35	18.5214	44.8988	129,690	6,686.89
	+20 %	17.6278	44.6325	125,512	6,056.30	18.5206	44.8985	129,685	6,685.69
	+40 %	17.6474	44.6342	125,511	6,055.77	18.5202	44.8984	129,682	6,685.09
	+60 %	17.6470	44.6323	125,509	6,055.44	18.5198	44.8982	129,680	6,684.48
I_e	-60 %	18.5273	44.7460	125,734	6,690.95	50.3905	70.0181	357,603	56,655.4
	-40 %	18.1976	44.6936	125,616	6,449.2	19.2322	45.0513	130,301	7,223.54
	-20 %	17.9075	44.6571	125,548	6,420.39	18.8651	44.9636	129,949	6,933.75
	+20 %	17.4137	44.6172	125,503	5,893.31	18.2275	44.8505	129,490	6,470.93
	+40 %	17.1998	44.6090	125,513	5,746.23	17.9639	44.8150	129,342	6,280.70
	+60 %	17.0029	44.6066	125,537	5,612.57	17.7247	44.7896	129,230	6,110.68
I_c	-60 %	16.9074	44.3785	120,172	5,548.33	17.3704	44.4518	121,561	5,863.38
	-40 %	17.1298	44.4502	121,770	5,698.52	17.7050	44.5714	123,898	6,096.79
	-20 %	17.3750	44.4502	123,538	5,866.56	18.0841	44.7174	126,571	6,367.07
	+20 %	17.9566	44.7505	127,751	6,275.47	19.0365	45.1292	133,812	7,073.65
	+40 %	18.3108	44.8942	130,330	6,531.67	19.6651	45.4329	138,024	7,561.79
	+60 %	18.7271	45.0736	133,377	6,839.76	46.1683	67.4447	409,180	46,494.1

4.2.2 Comparative Analysis

Table 5: Comparative analysis in Case-I of illustration-1

Method	Triangular Fuzzy Number							
	Signed Distance				Graded Mean Integration			
Fuzzy Parameters	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
$\widetilde{S}$,$\widetilde{P}$,$\widetilde{C_s}$,$\widetilde{A}$,$\widetilde{I_c}$,$\widetilde{I_e}$,$\widetilde{l}$	13.0448	40.9470	149,418	3,271.64	11.8219	40.8129	158,592	2,678.88
$\widetilde{P}$,$\widetilde{C_s}$,$\widetilde{A}$,$\widetilde{I_c}$,$\widetilde{I_e}$,$\widetilde{l}$	13.3976	40.8338	148,022	3,453.95	12.0937	40.6671	157,471	2,805.40
$\widetilde{C_s}$,$\widetilde{A}$,$\widetilde{I_c}$,$\widetilde{I_e}$,$\widetilde{l}$	16.1938	40.5735	132,625	5,080.66	16.2783	40.1007	134,780	5,134.91
$\widetilde{C_s}$,$\widetilde{A}$,$\widetilde{I_c}$,$\widetilde{I_e}$	15.9203	40.3873	139,699	4,907.14	16.0010	39.8471	142,810	4,958.01
$\widetilde{C_s}$,$\widetilde{A}$,$\widetilde{I_e}$	15.9203	40.3873	139,699	4,907.14	16.0010	39.8471	142,810	4,958.01
$\widetilde{C_s}$,$\widetilde{A}$	17.3899	40.5779	133,747	5,876.85	17.8528	40.2132	135,318	6,201.42
$\widetilde{C_s}$	17.3438	40.5749	124,479	5,845.03	17.8169	40.2073	126,588	6,175.92
Method	**Triangular Fuzzy Number**							
	Signed Distance				**Graded Mean Integration**			
Fuzzy Parameters	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
$\widetilde{S}$,$\widetilde{P}$,$\widetilde{C_s}$,$\widetilde{A}$,$\widetilde{I_c}$,$\widetilde{I_e}$,$\widetilde{l}$	12.6986	41.0951	140,572	3,097.68	11.5198	41.0032	149,717	2,541.75
$\widetilde{P}$,$\widetilde{C_s}$,$\widetilde{A}$,$\widetilde{I_c}$,$\widetilde{I_e}$,$\widetilde{l}$	13.0190	40.9642	139,317	3,258.51	11.7738	40.8449	148,701	2,656.8
$\widetilde{C_s}$,$\widetilde{A}$,$\widetilde{I_c}$,$\widetilde{I_e}$,$\widetilde{l}$	15.9154	40.5518	124,559	4,904.96	16.0081	40.0626	127,230	4,962.5
$\widetilde{C_s}$,$\widetilde{A}$,$\widetilde{I_c}$,$\widetilde{I_e}$	15.6404	40.3839	130,919	4,732.82	15.7232	39.8263	134,538	4,784.05
$\widetilde{C_s}$,$\widetilde{A}$,$\widetilde{I_e}$	15.6404	40.3839	130,919	4,732.82	15.6696	40.3588	198,376	4,750.86
$\widetilde{C_s}$,$\widetilde{A}$	17.3439	40.5748	124,476	5,845.10	17.8169	40.2073	126,584	6,175.92
$\widetilde{C_s}$	17.3438	40.5749	124,479	5,845.03	17.8169	40.2073	126,588	6,175.92

Table 6: Comparative analysis in Case-II of illustration-1

Method	**Triangular Fuzzy Number**							
	Signed Distance				**Graded Mean Integration**			
Fuzzy Parameters	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
$\widetilde{S},\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	15.1009	40.6515	148,347	4,406.11	14.0547	40.1107	153,796	3,807.08
$\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	15.8208	40.7126	149,089	4,844.80	14.6932	40.0999	155,173	4,167.27
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	17.5352	40.8703	133,074	5,977.74	17.7682	40.4886	135,241	6,141.41
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e}$	17.3215	40.6320	140,184	5,829.67	17.5970	40.1928	143,315	6,020.92
$\widetilde{C_s},\widetilde{A},\widetilde{I_e}$	16.7507	40.4716	136,718	5,443.78	16.8342	39.9698	139,238	5,499.36
$\widetilde{C_s},\widetilde{A}$	18.0736	40.7654	133,688	6,359.5	18.5006	40.4276	135,062	6,671.2
$\widetilde{C_s}$	18.0736	40.7654	133,692	6,359.5	18.5006	40.4276	135,066	6,671.2
Method	**Triangular Fuzzy Number**							
	Signed Distance				**Graded Mean Integration**			
Fuzzy Parameters	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
$\widetilde{S},\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	14.6780	40.6585	137,578	4,158.5	13.5013	40.1718	141,999	3,508.5
$\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	17.1219	40.7798	125,381	5,693.15	14.2327	40.1016	143,637	3,905.80
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	15.9154	40.5518	124,559	4,904.06	17.5049	40.4126	127,150	6,743.19
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e}$	15.9154	40.5518	124,559	4,904.06	17.5049	40.4126	127,150	6,743.19
$\widetilde{C_s},\widetilde{A},\widetilde{I_e}$	16.5378	40.4473	127,664	5,303.41	16.9196	39.9332	130,726	5,357.11
$\widetilde{C_s},\widetilde{A}$	18.1043	40.7833	124,432	6,381.65	18.5335	40.4444	126,327	6,695.55
$\widetilde{C_s}$	18.1042	40.7833	124,435	6,381.57	18.5344	40.4445	126,331	6,695.47

Table 7: Comparative analysis in Case-III of illustration-1

Method	**Triangular Fuzzy Number**							
	Signed Distance				**Graded Mean Integration**			
Fuzzy Parameters	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
$\widetilde{S},\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	18.8738	41.8383	189,351	6,950.13	15.9925	40.4491	188,681	4,952.64
$\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	–				17.3496	40.8162	200,446	5,839.03
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	18.5334	41.2363	138,355	6,695.47	18.1440	40.6442	139,803	6,410.34
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e}$	18.3926	40.9863	145,633	6,592.61	18.0176	40.3517	147,950	6,319.21
$\widetilde{C_s},\widetilde{A},\widetilde{I_e}$	16.7327	40.4786	137,648	5,431.84	16.7352	39.9533	139,913	5,433.50
$\widetilde{C_s},\widetilde{A}$	18.1436	40.7973	134,716	6,410.5	18.4512	40.4152	135,844	6,635.03
$\widetilde{C_s}$	18.1436	40.7973	134,719	6,410.05	18.4512	40.4152	135,848	6,634.73
Method	**Triangular Fuzzy Number**							
	Signed Distance				**Graded Mean Integration**			
Fuzzy Parameters	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
$\widetilde{S},\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	17.3838	41.3091	166,478	5,872.62	18.1479	41.1661	204,118	6,413.17
$\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	19.5466	42.1523	181,618	7,468.43	–			
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	18.2286	41.1451	129,520	6,471.73	18.5977	40.8210	132,569	6,743.19
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e}$	18.0585	40.8978	136,054	6,348.62	18.5380	40.5449	140,210	6,698.88
$\widetilde{C_s},\widetilde{A},\widetilde{I_e}$	16.5142	40.4544	128,572	5,287.90	16.5188	39.9186	131,376	5,290.98
$\widetilde{C_s},\widetilde{A}$	18.1481	40.8082	125,473	6,413.31	18.4534	40.4216	127,125	6,636.35
$\widetilde{C_s}$	19.0000	41.8082	125,477	6,413.31	18.4534	40.4216	127,129	6,636.35

Table 8: Comparative analysis in Case-IV of illustration-1

Method	Triangular Fuzzy Number							
	Signed Distance				Graded Mean Integration			
Fuzzy Parameters	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
$\widetilde{S},\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	–				–			
$\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	–				–			
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	20.3124	42.0112	144,503	8,082.75	18.1440	40.6442	139,803	6,410.34
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e}$	20.4924	41.8689	152,705	8,230.94	18.1440	40.6442	139,803	6,410.34
$\widetilde{C_s},\widetilde{A},\widetilde{I_e}$	17.3542	40.6174	138,266	5,852.20	18.1440	40.6442	139,803	6,410.34
$\widetilde{C_s},\widetilde{A}$	19.0706	41.1176	135,141	7,099.68	18.1440	40.6442	139,803	6,410.34
$\widetilde{C_s}$	19.0706	41.1176	135,141	7,099.68	18.1440	40.6442	139,803	6,410.34
Method	**Triangular Fuzzy Number**							
	Signed Distance				**Graded Mean Integration**			
Fuzzy Parameters	T_1	T	$\widetilde{\pi}_{SD}$	Q	T_1	T	$\widetilde{\pi}_{GM}$	Q
$\widetilde{S},\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	–				–			
$\widetilde{P},\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	–				–			
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e},\widetilde{l}$	19.8887	41.8356	135,301	7,739.64	20.5983	43.7341	139,280	8,318.8
$\widetilde{C_s},\widetilde{A},\widetilde{I_c},\widetilde{I_e}$	19.9561	41.6488	142,515	7,793.69	21.0908	41.7016	148,524	8,734.04
$\widetilde{C_s},\widetilde{A},\widetilde{I_e}$	17.1081	40.5756	129,239	5,683.7	17.0930	40.0460	131,666	5,673.52
$\widetilde{C_s},\widetilde{A}$	19.0911	41.1362	125,975	7,115.35	19.4256	40.7961	127,018	7,373.75
$\widetilde{C_s}$	19.0911	41.1361	125,975	7,115.35	17.0930	40.0460	131,670	5,673.52

Sensitivity analysis and comparative analysis are also performed for illustration-2 and the results are exhibited in the Section-5.

5. RESULT & DISCUSSION

5.1 Referring to the results of table 3, the following facts are derived

Case-I

It is observed that as the value of a accelerates, both cycle time and order quantity increase, while total profit and starting time period of backlogging decrease. This allows the backlogging period last for a long period of time and also enhances the order quantity but reduces the total profit. Also acceleration in the value of a, leads to enhancement in demand and to cope-up with the increasing demand order quantity is increased but since the profit decreases, it may be advisable for the decision maker not to enhance the value of the parameter a. The total profit, order quantity and starting time point of backlogging continuously rise but the cycle time declines as the values of the parameters b and c elevate. More precisely, as the value of b and c increase, demand increases and to compensate with the increasing demand, the order quantity is enhanced. The current situation has a significant impact on the growth of the business enterprise because in this case the backlogging period shrinks and the profit enriches. Total profit declines strictly with increase in the values of the parameters φ and β. It enhances as the value of the parameter C_s accelerates and remains unaltered for change in other parameters.

Case-II

As the value of a accelerates, starting time period of backlogging, cycle time and order quantity increase, while total profit decreases. Acceleration in the value of a, leads to enhancement in demand and to cope-up with the increasing demand order quantity is enhanced but since the profit decreases and the backlogging period does not shrink, it may be advisable to the decision maker not to enhance the value of the parameter a. Acceleration in the values of the parameters b and c, enhances the total profit and order quantity by reducing the starting time point of backlogging and cycle time. More specifically, when the parameters b and c increase, the demand rises and to cope-up with increasing demand, order quantity enhances. Since the backlogging period does not shrink, acceleration in order quantity controls the situation. Acceleration in the value of the parameter C_s strengthens the profit. Total profit declines strictly with increase in the values of the parameter φ and remains unaltered for variation in other parameters.

5.2 Referring to the results of table 4, the following facts are derived

Case-III

When the value of the parameter a enhances, keeping other parameters constant, both cycle time and order quantity rise, but total profit and starting time period of backlogging fall. The prevailing circumstance is not economical because the backlogging period lasts for a long time period, total profit is decreasing but order quantity is increasing. So, the decision maker should be careful while choosing the value of the parameter a. The total profit, order quantity and starting time point of backlogging increase, but the cycle time deceases as the value of the parameters b and c increase. This indicates that when b and c increase, demand increases and to compensate with the increasing demand, the order quantity is enhanced. This situation is economical for the business enterprise because here the backlogging time period lasts for a short period of time. Acceleration in the value of the parameter C_s strengthens the profit. Total profit declines strictly with increase in the value of the parameter φ and remains unaltered for variation in other parameters.

Case-IV

As the value of the parameter a accelerates, starting time period of backlogging, cycle time, total profit and order quantity gradually increase. Actually, when a increases, demand increases and to cope-up with the increasing demand, the order quantity is enhanced. The backlogging time period is not reducing, so it is better to enhance the order quantity. When the parameters b and c increase and other parameters remain constant, the total profit and order quantity increase while the starting time point of backlogging and the cycle time decease. This indicates that when b and c increase, demand increases and to compensate with the increasing demand, the order quantity is increased. Since the backlogging time period is not reducing it is better to enhance the order quantity. Total profit declines continuously with increase in the values of the parameters φ and β. It enriches with increase in the values of the parameters I_c,P and C_s and it remains constant for change in other parameters.

5.3 Referring to the results of table 5, table 6, table 7 and table 8, the following facts are derived

It is evident from the result that the fuzzy model acquires more profit as compared to the crisp model. The graded mean integration method for trapezoidal fuzzy number earns maximum profit. Result suggests that both case-II and case-III are more beneficial in achieving our goal. When all the cost parameters involved in the model are treated as fuzzy, the situation earns more profit as compared to the others.

6. CONCLUSION

Trade credit serves as a business supplement for financially constrained firms and financially weaker customers. Moreover, uncertainty in the supply chain is an issue with which every business organiser wrestles for existence. Due to increasing complexity of global supply network, it is difficult to assess the exact values of the parameters involved in the system. The current study contributes an inventory model under fuzzy environment incorporating trade credit policy. Important managerial insights obtained from sensitivity analysis suggest some policies counter to those commonly practised by the retailers. In addition detailed set of relevant equations and testing of concavity have been found to evaluate the functionality of the model. The model is useful for durable consumer goods and perishable products having very high initial demand and after a certain time period it accelerates slowly. This model also aids the decision maker in choosing the most suitable economic period which has larger impact on enhancing the profit of the business organisation. The study encompasses important result about two main characteristics of market research such as prediction of stocks and portfolio selection of stocks. It enables the decision maker to take advantage of opportunities for investment in uncertainty. Further, the model can be extended by considering uncertain demand.

ACKNOWLEDGEMENT

The authors are indebted to the anonymous reviewer for his constructive comments which help improve the quality of the paper.

REFERENCES

Aggrawal, P., & Singh, T. (2017). An eoq model with ramp type demand rate, time dependent deterioration rate and shortages. *Global Journal of Pure and Applied Mathematics*, *13*(7), 3381-3393.

Annadurai, K., & Uthayakumar, R. (2015). Decaying inventory model with stock-dependent demand and shortages under two-level trade credit. *The International Journal of Advanced Manufacturing Technology*, *77*(1-4), 525-543.

Banu, A., & Mondal, S. K. (2016). Analysis of credit linked demand in an inventory model with varying ordering cost. *Springerplus*, *5*(1), 926.

Bhojak, A., & Gothi, U. B. (2015). An eoq model with time dependent demand and weibull distributed deterioration. *International Journal of Engineering Research & Technology (IJERT)*, *4*(9), 109-114.

Dharma, P., Lin, S. W., & Lee, C. S. (2019). Optimal inventory management for perishable goods when the demand depends on product price, inventory age, and displayed inventory level. *International Journal of Operations Research*, *16*(3), 63-76.

Garai, T., Chakraborty, D., & Roy, T. K. (2019). Fully fuzzy inventory model with price-dependent demand and time varying holding cost under fuzzy decision variables. *Journal of Intelligent & Fuzzy Systems*, *36*(4), 3725–3738.

Giri, B. C., Jalan, A. K., & Chaudhuri, K. S. (2003). Economic order quantity model with weibull deterioration distribution, shortage and ramp-type demand. *International Journal of Systems Science*, *34*(4), 237-243.

Kaur, M., Pareek, S., & Tripathi, R. (2016). Optimal ordering policy with non-increasing demand for time dependent deterioration under fixed life time production and permissible delay in payments. *International Journal of Operations Research*, *13*(2), 35-46.

Mahata, G. C., & Mahata, P. (2011). Analysis of a fuzzy economic order quantity model for deteriorating items under retailer partial trade credit financing in a supply chain. *Mathematical and Computer Modelling*, *53*(9-10), 1621-1636.

Shah, N. H., Jani, M. Y., & Shah, D. B. (2015). Economic order quantity model under trade credit and customer returns for price-sensitive quadratic demand. *Investigación Operacional*, *36*(3), 240-248.

Sujatha, J., & Parvathi, P. (2015). Fuzzy inventory model for deteriorating items with weibull demand and time varying holding cost under trade credit. *International Journal of Innovative Research in Computer and Communication Engineering*, *3*(11), 11110-11123.

Sundararajan, R., & Uthayakumar, R. (2015). Eoq model for delayed deteriorating items with shortages and trade credit policy. *International journal of supply and operations management*, *2*(2), 759-783.

Tripathy, P. K., & Sukla, S. (2018a). Decision support model for perishable items impacting ramp type demand in a discounted retail supply chain environment. *Yugoslav Journal of Operations Research*, *28*(3), 371-383.

Tripathy, P. K., & Sukla, S. (2018b). Interactive fuzzy inventory model with ramp demand under trade credit financing. *International Journal of Agricultural and Statistical Sciences*, *14*(1), 377-390.

Wu, C., & Zhao, Q. (2016). Two retailer–supplier supply chain models with default risk under trade credit policy. *SpringerPlus*, *5*(1), 1728.

Yang, H. L. (2019). An inventory model for ramp-type demand with two-level trade credit financing linked to order quantity. *Open Journal of Business and Management*, *7*(2), 427-446.