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On Three-Graded Manpower Model with Non-Homogeneous Poisson Recruitment in first and second grades

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Abstract: Human resource is one of the most important factor which influences the growth and productivity of an organization. Retaining skilled labour is one of the prime requisite for efficient manpower planning in the organization. The manpower planning can be done using mathematical models. This paper addresses the problem of predicting the average number of employees in each grade and their average duration of stay in each grade of a system, in which the recruitment is time dependent and in bulk. Here, the recruitment is carried in first and second grades in a three graded system. The recruitment process in both the grades are characterized by a non-homogeneous compound Poisson processes. Assuming that the promotion/leaving processes follow Poisson processes, the probability generating function of the grade size distribution is derived. The system performance measures such as the mean number of employees in each grade, the average duration of stay of an employee in each grade and the variability of the grade size distribution are derived explicitly. Assuming uniform batch size distribution for bulk recruitment in both the grades, the system performance is evaluated. The sensitivity of the model reveals that the direct recruitment to the second grade is having significant influence on the system performance measures. It is also observed that the batch size distribution parameters influence the duration of stay of employee in the organization, and the mean number of employees in each grade. It is further observed that the non-homogeneous compound Poisson process approximation to the recruitment provides accurate prediction of the performance measures close to the reality. This model includes some of the earlier models as particular cases for specific or limiting values of the parameters.
Keyword — Non-homogeneous compound Poisson process, Multi graded manpower model, Duration of stay of an employee in the organization, Sensitivity analysis, Direct recruitment.

1. INTRODUCTION

The manpower model represents the physical phenomenon associated with manpower flows in an organization. Due to the randomness in the constituent processes of the manpower systems, the stochastic models provide the basic frame work for developing and analysing the optimal operating policies of the systems in an organization. The optimal planning and development of manpower situations is needed for utilization of human resource. The productivity and efficiency of an organization is directly linked to the planning and development of human resources. Hence, the manpower models are much useful in analysing the situations at different government, quasi-government, private and corporate sector organizations (Srinivasa Rao & Mallikharjuna Rao, 2015; Srinivasa Rao, Srinivasa Rao, & Vivekananda murthy, 2006), Govinda Rao and Srinivasa Rao (2013, 2014).

A wide variety of manpower models have been developed and analysing various assumptions in order to analyse practical systems more close to reality. In Seal (1945) has pioneered the mathematical modelling of manpower systems. McClean (1976), has considered a two graded system under Markovian modelling for analysing the personal behavior of the employees in the organization. Tim De Feyter (2009a, 2009b); Ugwuowo and McClean (2000); Wang (2005) have reviewed several manpower models and presented the growth and development of manpower models. Poornima (2018) have reviewed three graded Manpower Models and analyzed the performance measures. Ishwarya and Srinivasan (2016) reviewed various Manpower Models and analyzed the performance measures such as variance and inter decision times as an order statistics. Konda and Govinda (2017) studied various Manpower models and analyzed various performance measures under continuous Truncated Distributions. Samundeeswari, Vidhya, and Chitra Kalarani (2018) analyzed the

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expected time to recruitment for Three graded manpower system under Geometric process. Saral, Sendhamizh Selvi, and Srinivasan (2018a, 2018b) studied various Manpower Models and estimated the mean time to recruitment for a graded manpower system with two thresholds. Anamika and Madhu (2017) analyzed the performance measures in multigraded manpower models. In all these papers the basic assumption is the recruitment was done one person at a time.

However, in many organizations it is a usual phenomenon that there exists a policy decision regarding revision of wages, incentives and revised sales targets, etc,. This in turn leads to depletion of manpower, which can be conceptualized in terms of man hours. It would be uneconomical to go in for frequent recruitments in view of the cost involved for the same. Hence, the organization goes for recruitment as and when the cumulative loss of manpower crosses a random threshold beyond which the organization activities would be adversely affected. Also in other government organizations like defence, banks etc., the recruitment is done in bulk. To have an accurate prediction of manpower situations it is needed to consider manpower models with bulk recruitment.

Realizing this aspect Konda and Srinivasa (2013); Srinivasa Rao and Konda Babu (2014) have developed manpower models with the assumption that the recruitment process follows a compound Poisson process. They assume that the recruitment is homogeneous even though it is done in batches of random size, that is, the recruitment rate is time independent. But in many corporate and government organizations the recruitment is done depending on time. For example, in defence and corporate organizations the recruitment is done in bulk in the first two grades with time dependent recruitment rates. The time dependent recruitment process is a common phenomenon in manpower models in corporate and defence organizations. Recently Srinivasa Rao and Ganapathi Swamy (2019); Srinivasa Rao and Mallikharjuna Rao (2019) have developed and analyzed manpower models with Duane recruitment process. But, to have a close approximation to the practical situation one has to consider non-homogeneous compound Poisson process for characterizing the recruitment process in manpower modelling. Very little work has been reported in Manpower models with non-homogeneous compound Poisson process, this motivated me to develop and analyze a three graded manpower model with non-homogeneous compound Poisson bulk recruitment in the first and the second grades. The non-homogeneous compound Poisson process is the generalization of the Poisson process, which includes a spectra of models. The three graded system with different in and out flow of employees in manpower models is sufficient to analyze a multi graded organization. This is due to the fact that a three graded system will have an intermediatory grade, predeceasing grade and succeeding grades. The explicit expressions for the average number of employees in each grade, average duration of stay of an employee in each grade are needed for efficient control and monitoring of human resource systems in the organization. The rest of the paper is organized as follows: Section 2 deals with the development of 3 graded manpower models with non-homogeneous compound Poisson recruitment under general bulk size distribution. Section 3 is concerned with the derivation of the performance measures of the model. Section 4 is to derive explicit expressions of the performance measures under uniform distribution assumption for batch size recruitment. Section 5 deals with sensitivity analysis of the model which discusses the effect of changes in input parameters on the performance measures. Section 6 presents a comparative study of the developed model with that of Poisson recruitments, under both bulk as well as unit systems and section 7 is concerned with conclusions along with scope for further work in this area.

2. MANPOWER MODEL AND TRANSIENT SOLUTION

In this section, consider a three graded manpower model in which the organization is having three grades namely, grade 1, grade 2, and grade 3. At every recruitment a group of employees are recruited into grade 1 and grade 2. Let us assume that the actual number of employees in grade 1 and grade 2 are random variables 'X' and 'Y' respectively with probability density functions C_x and D_x. If $\lambda_x(t)$ is the recruitment rate of batches of sizes x in grade 1 and $\varepsilon_y(t)$ is the recruitment rate of batches of sizes in grade 2, then the composite recruitment rates are $\lambda(t) = \sum_x \lambda_x(t) =$; $\varepsilon(t) = \sum_y \varepsilon_y(t)$; ,where the composite recruitment process follows a non-homogeneous compound Poisson process. It is further assumed that the recruitment rate is linear functions of time such that $\lambda(t) = \lambda_1 + \lambda_2 t$ and $\varepsilon(t) = \varepsilon_1 + \varepsilon_2 t$, where $\lambda_1, \lambda_2, \varepsilon_1$, and ε_2 are real numbers and also 't' be the time.

It is also assumed that once an employee is recruited in grade-1, after spending a random duration of time in grade-1, he may be promoted to grade-2 with promotion rate β or may leave the organization with a leaving rate α. It is further assumed that an employee after spending a random duration of time in grade-2, he may be promoted to grade 3 with promotion rate δ or he may leave the organization with a leaving rate γ. It is also assumed that an employee after spending a random duration of time in grade 3, an employee leaves the organization with a leaving rate θ. Here, the promotion and the leaving processes follow Poisson processes. With these assumptions the schematic diagram representing the manpower system in the organization is shown in Figure.1 Let $P_{n,m,k}(t)$ be the probability that there are n employees in grade 1, m employees in grade 2, and k employees in grade 3 at time t in the organization. The

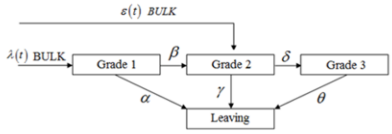

Figure 1: The Schematic diagram of the model

difference differential equations governing the manpower model are

$$\begin{aligned}\frac{\partial}{\partial t}P_{n,m,k}(t) = &\left[\left((\lambda_1+\lambda_2 t)+n\alpha+n\beta+m\gamma+m\delta+(\varepsilon_1+\varepsilon_2 t)+k\theta\right)\right]P_{n,m,k}(t)\\ &+(n+1)\alpha P_{n+1,m,k}(t)+(n+1)\beta P_{n+1,m-1,k}(t)+(m+1)\gamma P_{n,m+1,k}(t)\\ &+(m+1)\delta P_{n,m+1,k-1}(t)+(k+1)\theta P_{n,m,k+1}(t)+(\lambda_1+\lambda_2 t)\left[\sum_{i=1}^{n}P_{n-i,m,k}(t)C_i\right]\\ &+(\varepsilon_1+\varepsilon_2 t)\left[\sum_{j=1}^{m}P_{n,m-j,k}(t)D_j\right];\ \text{for } n,m,k\geq 1\end{aligned} \tag{1}$$

If $\lambda_2 = 0$, and $\varepsilon_2 = 0$ then the equations gives the case of homogeneous recruitment.

$$\begin{aligned}\frac{\partial}{\partial t}P_{n,m,0}(t) = &\left[-\left((\lambda_1+\lambda_2 t)+n\alpha+n\beta+m\gamma+(\varepsilon_1+\varepsilon_2 t)+m\delta\right)\right]P_{n,m,0}(t)\\ &+(n+1)\alpha P_{n+1,m,0}(t)+(n+1)\beta P_{n+1,m-1,0}(t)+(m+1)\gamma P_{n,m+1,0}(t)\\ &+\theta P_{n,m,1}(t)+(\lambda_1+\lambda_2 t)\left[\sum_{i=1}^{n}P_{n-t,m,0}(t)C_i\right]+(\varepsilon_1+\varepsilon_2 t)\left[\sum_{j=1}^{m}P_{n,m-j,0}(t)D_j\right]\\ &\text{for } n,m\geq 1\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial}{\partial t}P_{n,0,k}(t) = &\left[-\left((\lambda_1+\lambda_2 t)+n\alpha+n\beta+(\varepsilon_1+\varepsilon_2 t)+k\theta\right)\right]P_{n,0,k}(t)+(n+1)\alpha P_{n+1,0,k}(t)\\ &+\gamma P_{n,1,k}(t)+\delta P_{n,1,k-1}(t)+(k+1)\theta P_{n,0,k+1}(t)\\ &+(\lambda_1+\lambda_2 t)\left[\sum_{i=1}^{n}P_{n-i,0,k}(t)C_i\right];\ \text{for } n,k\geq 1\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial}{\partial t}P_{0,m,k}(t) = &\left[-\left((\lambda_1+\lambda_2 t)+m\gamma+m\delta+(\varepsilon_1+\varepsilon_2 t)+k\theta\right)\right]P_{0,m,k}(t)+\alpha P_{1,m,k}(t)\\ &+\beta P_{1,m-1,k}(t)+(m+1)\gamma P_{0,m+1,k}(t)+(m+1)\delta P_{0,m+1,k-1}(t)\\ &+(k+1)\theta P_{0,m,k+1}(t)+(\varepsilon_1+\varepsilon_2 t)\left[\sum_{j=1}^{m}P_{0,m-j,k}(t)D_j\right];\ \text{for } m,k\geq 1\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{\partial}{\partial t}P_{n,0,0}(t) = &\left[-\left((\lambda_1+\lambda_2 t)+n\alpha+n\beta+(\varepsilon_1+\varepsilon_2 t)\right)\right]P_{n,0,0}(t)+(n+1)\alpha P_{n+1,0,0}(t)\\ &+\gamma P_{n,1,0}(t)+\theta P_{n,0,1}(t)+(\lambda_1+\lambda_2 t)\left[\sum_{i=1}^{n}P_{n-i,0,0}(t)C_i\right];\ \text{for } n\geq 1\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial}{\partial t}P_{0,m,0}(t) = &\left[-\left((\lambda_1+\lambda_2 t)+m\gamma+(\varepsilon_1+\varepsilon_2 t)+m\delta\right)\right]P_{0,m,0}(t)+\alpha P_{1,m,0}(t)+\beta P_{1,m-1,0}(t)\\ &+(m+1)\gamma P_{0,m+1,0}(t)+\theta P_{0,m,1}(t)+(\varepsilon_1+\varepsilon_2 t)\left[\sum_{j=1}^{m}P_{0,m-j,0}(t)D_j\right];\ \text{for } m\geq 1\end{aligned} \tag{6}$$

$$\begin{aligned}\frac{\partial}{\partial t}P_{0,0,k}(t) = &\left[-\left((\lambda_1+\lambda_2 t)+(\varepsilon_1+\varepsilon_2 t)+k\theta\right)\right]P_{0,0,k}(t)+\alpha P_{1,0,k}(t)+\gamma P_{0,1,k}(t)\\ &+\delta P_{0,1,k-1}(t)+(k+1)\theta P_{0,0,k+1}(t);\ \text{for } k\geq 1\end{aligned} \tag{7}$$

$$\frac{\partial}{\partial t}P_{0,0,0}(t) = [-(\lambda_1 + \lambda_2 t + \varepsilon_1 + \varepsilon_2 t)]\, P_{0,0,0}(t) + \alpha P_{1,0,0}(t) + \gamma P_{0,1,0}(t) + \theta P_{0,0,1}(t) \tag{8}$$

Let $G(Z_1, Z_2, Z_3; t)$ be the joint probability generating function of number of employees in three grades ,$C(Z_1)$ and $D(Z_2)$ is the probability generating functions of batch size distributions in grade 1 and grade 2 respectively. Here,Z_1, Z_2, and Z_3 are real variables such that $|Z_1| \leq 1, |Z_2| \leq 1, |Z_3| \leq 1$. Then

$$G(Z_1, Z_2, Z_3; t) = \sum_n \sum_m \sum_k Z_1^n Z_2^m Z_3^k P_{n,m,k}(t)$$
$$C(Z_1) = \sum_x C_x Z_1^x \text{and} D(Z_2) = \sum_y D_y Z_2^y \tag{9}$$

Multiplying equations (1) to (8) with corresponding ${}^n Z_2^m Z_3^k$ and summing over all $n = 0, 1, 2, 3, ...; m = 0, 1, 2, 3, ...;$
$k = 0, 1, 2, 3, ...$ we get

$$\begin{aligned}\frac{\partial}{\partial t}G(Z_1, Z_2, Z_3; t) = &- \lambda_1 G(Z_1, Z_2, Z_3; t) - \lambda_2 t G(Z_1, Z_2, Z_3; t) - \varepsilon_1 G(Z_1, Z_2, Z_3; t) \\ &- \varepsilon_2 t G(Z_1, Z_2, Z_3; t) - \alpha Z_1 \frac{\partial}{\partial Z_1} G(Z_1, Z_2, Z_3; t) \\ &- \beta Z_1 \frac{\partial}{\partial Z_1} G(Z_1, Z_2, Z_3; t) - \gamma Z_2 \frac{\partial}{\partial Z_2} G(Z_1, Z_2, Z_3; t) \\ &- \delta Z_2 \frac{\partial}{\partial Z_2} G(Z_1, Z_2, Z_3; t) - \theta Z_3 \frac{\partial}{\partial Z_3} G(Z_1, Z_2, Z_3; t) \\ &+ \lambda_1 C(Z_1) G(Z_1, Z_2, Z_3; t) + \lambda_2 t C(Z_1) G(Z_1, Z_2, Z_3; t) \\ &+ \varepsilon_1 C(Z_2) G(Z_1, Z_2, Z_3; t) + \varepsilon_2 t C(Z_2) G(Z_1, Z_2, Z_3; t) \\ &+ \alpha \frac{\partial}{\partial Z_1} G(Z_1, Z_2, Z_3; t) + \beta Z_2 \frac{\partial}{\partial Z_1} G(Z_1, Z_2, Z_3; t) \\ &+ \gamma \frac{\partial}{\partial Z_2} G(Z_1, Z_2, Z_3; t) + \delta Z_3 \frac{\partial}{\partial Z_2} G(Z_1, Z_2, Z_3; t) \\ &+ \theta \frac{\partial}{\partial Z_3} G(Z_1, Z_2, Z_3; t)\end{aligned} \tag{10}$$

Solving the equation (10) by Lagrange's method, the auxiliary equations are

$$\begin{aligned}\frac{dt}{1} &= \frac{-dZ_1}{[\alpha(1 - Z_1) + \beta(Z_2 - Z_1)]} = \frac{-dZ_2}{[\gamma(1 - Z_2) + \delta(Z_3 - Z_2)]} = \frac{-dZ_3}{\theta(1 - Z_3)} \\ &= \frac{dG(Z_1, Z_2, Z_3; t)}{\{\lambda_1 [C(Z_1) - 1] + \lambda_2 t [C(Z_1) - 1] + \varepsilon_1 [C(Z_2) - 1] + \varepsilon_2 t [C(Z_2) - 1]\} G(Z_1, Z_2, Z_3; t)}\end{aligned} \tag{11}$$

The initial conditions of the system are

$$P_{N_0, M_0, K_0}(0) = 1, \text{for} \quad t > 0 \tag{12}$$

That is initially the organization is having employees in grade 1, employees in grade 2, and employees in grade 3. Solving the equation (11), we get

$$A = (Z_3 - 1)e^{-\theta t}$$
$$B = \left[(Z_2 - 1) - \frac{\delta}{(\gamma + \delta) - \theta}(Z_3 - 1)\right] e^{-(\gamma+\delta)t}$$
$$H = \left[(Z_1 - 1) - \frac{\beta}{(\alpha + \beta) - (\gamma + \delta)}(Z_2 - 1) - \frac{\beta\delta}{[\theta - (\alpha + \beta)][(\alpha + \beta) - (\gamma + \delta)]}(Z_3 - 1)\right] e^{-(\alpha+\beta)t}$$

$$D = G(Z_1, Z_2, Z_3; t) \exp\left\{ -(\lambda_1 + \lambda_2 t) \sum_{x=1}^{\infty} \sum_{r=1}^{x} \sum_{s=0}^{r} \sum_{f=0}^{s} (-1)^{s-f} C_x \begin{pmatrix} x \\ r \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix} \begin{pmatrix} s \\ f \end{pmatrix} A^f B^{(s-f)} H^{(r-s)} \right.$$

$$\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{f}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)}$$
$$\frac{e^{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]}+\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x\binom{x}{r}\binom{r}{s}\binom{s}{f}$$

$$A^fB^{(s-f)}H^{(r-s)}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right]^{f}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)}$$
$$\left.\frac{e^{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]^2}\right\}-(\varepsilon_1+\varepsilon_2 t)\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}D_y\binom{y}{u}\binom{u}{v}$$
$$A^vB^{(u-v)}\left(\frac{\delta}{\theta-\gamma-\delta}\right)^{v}\frac{e^{[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]}+\varepsilon_2\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}D_y$$

$$\binom{y}{u}\binom{u}{v}A^vB^{(u-v)}\left(\frac{\delta}{\theta-\gamma-\delta}\right)^{v}\frac{e^{[\theta v+(\gamma+\delta)(u-v)]}\,t}{[\theta v+(\gamma+\delta)(u-v)]^2} \tag{13}$$

where A,B,H and D are arbitrary constants. Using the initial conditions and substituting the value of H in equation (10), we get the joint probability generating function of $P_{n,m,k}(t)$ as

$$G\left(Z_1,Z_2,Z_3;t\right)=\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x\binom{x}{r}\binom{r}{s}\binom{s}{f}(Z_3-1)^f\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)}\right.$$
$$\left[(Z_2-1)-\frac{\delta\left(Z_3-1\right)}{(\gamma+\delta)-\theta}\right]^{(s-f)}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{f}$$
$$\left[(Z_1-1)-\frac{\beta\left(Z_2-1\right)}{(\alpha+\beta)-(\gamma+\delta)}-\frac{\beta\delta(Z_3-1)}{[\theta-(\alpha+\beta)]\left[(\alpha+\beta)-(\gamma+\delta)\right]}\right]^{(r-s)}$$
$$\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x\binom{x}{r}\binom{r}{s}\binom{s}{f}$$

$$(Z_3-1)^f\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)}$$
$$\left[(Z_2-1)-\frac{\delta(Z_3-1)}{(\gamma+\delta)-\theta}\right]^{s-f}$$
$$\left[(Z_1-1)-\frac{\beta\left(Z_2-1\right)}{(\alpha+\beta)-(\gamma+\delta)}-\frac{\beta\delta\left(Z_3-1\right)}{[\theta-(\alpha+\beta)][(\alpha+\beta)-(\gamma+\delta)]}\right]^{(r-s)}$$
$$\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{f}\frac{1}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]}-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x$$
$$\binom{x}{r}\binom{r}{s}\binom{s}{f}(Z_3-1)^f\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)}\left[(Z_2-1)-\frac{\delta(Z_3-1)}{(\gamma+\delta)-\theta}\right]^{(s-f)}$$
$$\left[(Z_1-1)-\frac{\beta(Z_2-1)}{(\alpha+\beta)-(\gamma+\delta)}-\frac{\beta\delta(Z_3-1)}{[\theta-(\alpha+\beta)][(\alpha+\beta)-(\gamma+\delta)]}\right]^{(r-s)}$$
$$\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{f}\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]^2}$$
$$+\varepsilon_1\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}D_y\binom{y}{u}\binom{u}{v}\left[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-(\gamma+\delta)}\right]^{(u-v)}$$
$$\left(\frac{\delta(Z_3-1)}{\theta-\gamma-\delta}\right)^{v}\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]}+\varepsilon_2 t\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}D_y$$
$$\binom{y}{u}\binom{u}{v}[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-(\gamma+\delta)}]^{(u-v)}\left(\frac{\delta(Z_3-1)}{\theta-\gamma-\delta}\right)^{v}\frac{1}{[\theta v+(\gamma+\delta)(u-v)]} \tag{14}$$

$$- \varepsilon_2 \sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v} D_y \binom{y}{u}\binom{u}{v}[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-(\gamma+\delta)}]^{(u-v)}$$
$$(\frac{\delta(Z_3-1)}{\theta-\gamma-\delta})^{v}\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]^2}\}\Big[1-(1-Z_1)e^{-(\alpha+\beta)t}$$
$$-\frac{\beta(1-Z_2)[e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}]}{[(\gamma+\delta)-(\alpha+\beta)]}-\frac{\beta\delta(1-Z_3)[e^{-(\alpha+\beta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]}$$
$$+\frac{\beta\delta(1-Z_3)[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\Big]^{N_0}\Big[1-(1-Z_2)e^{-(\gamma+\delta)t}$$
$$-\frac{\delta(1-Z_3)[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[\theta-(\gamma+\delta)]}]^{M_0}[1-(1-Z_3)e^{-\theta t}]^{K_0};$$
$$|Z_1|<1;|Z_2|<1;|Z_3|<1$$

3. CHARACTERISTICS OF THE MODEL

The characteristics of the model are obtained by using the equation (14). The probability generating function of the number of employees in grade 1, grade 2, and grade 3 at time $'t'$ when there are initially N_0 employees , M_0 employees, and K_0 employees in grade 1 , grade 2, and grade 3 respectively is

$$G\left(Z_1,Z_2,Z_3;t\right)=\exp\Bigg\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x\binom{x}{r}\binom{r}{s}\binom{s}{f}(Z_3-1)^f\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)}$$
$$\left[(Z_2-1)-\frac{\delta\left(Z_3-1\right)}{(\gamma+\delta)-\theta}\right]^{(s-f)}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{f}$$
$$\left[(Z_1-1)-\frac{\beta\left(Z_2-1\right)}{(\alpha+\beta)-(\gamma+\delta)}-\frac{\beta\delta(Z_3-1)}{[\theta-(\alpha+\beta)]\left[(\alpha+\beta)-(\gamma+\delta)\right]}\right]^{(r-s)}$$
$$\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x\binom{x}{r}\binom{r}{s}\binom{s}{f}$$
$$(Z_3-1)^f\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)}\left[(Z_2-1)-\frac{\delta(Z_3-1)}{(\gamma+\delta)-\theta}\right]^{s-f}$$
$$\left[(Z_1-1)-\frac{\beta\left(Z_2-1\right)}{(\alpha+\beta)-(\gamma+\delta)}-\frac{\beta\delta\left(Z_3-1\right)}{[\theta-(\alpha+\beta)][(\alpha+\beta)-(\gamma+\delta)]}\right]^{(r-s)}$$
$$\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{f}\frac{1}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]}-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x$$
$$\binom{x}{r}\binom{r}{s}\binom{s}{f}(Z_3-1)^f\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)}\left[(Z_2-1)-\frac{\delta\left(Z_3-1\right)}{(\gamma+\delta)-\theta}\right]^{(s-f)}$$
$$\left[(Z_1-1)-\frac{\beta\left(Z_2-1\right)}{(\alpha+\beta)-(\gamma+\delta)}-\frac{\beta\delta\left(Z_3-1\right)}{[\theta-(\alpha+\beta)][(\alpha+\beta)-(\gamma+\delta)]}\right]^{(r-s)}$$
$$\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{f}\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]^2}$$
$$+\varepsilon_1\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}D_y\binom{y}{u}\binom{u}{v}\left[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-(\gamma+\delta)}\right]^{(u-v)}$$
$$\left(\frac{\delta\left(Z_3-1\right)}{\theta-\gamma-\delta}\right)^{v}\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]}+\varepsilon_2 t\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}D_y$$
$$\binom{y}{u}\binom{u}{v}[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-(\gamma+\delta)}]^{(u-v)}(\frac{\delta(Z_3-1)}{\theta-\gamma-\delta})^{v}\frac{1}{[\theta v+(\gamma+\delta)(u-v)]} \tag{15}$$

$$- \varepsilon_2 \sum_{y=1}^{\infty} \sum_{u=1}^{y} \sum_{v=0}^{u} (-1)^{2u-v} D_y \begin{pmatrix} y \\ u \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \left[(Z_2 - 1) + \frac{\delta(Z_3 - 1)}{\theta - (\gamma + \delta)} \right]^{(u-v)}$$

$$\left(\frac{\delta(Z_3 - 1)}{\theta - \gamma - \delta} \right)^{v} \frac{1 - e^{-[\theta v + (\gamma+\delta)(u-v)]t}}{[\theta v + (\gamma + \delta)(u - v)]^2} \Bigg\} \Big[1 - (1 - Z_1) e^{-(\alpha+\beta)t}$$

$$- \frac{\beta(1 - Z_2)[e^{-(\alpha+\beta)t} - e^{-(\gamma+\delta)t}]}{[(\gamma + \delta) - (\alpha + \beta)]} - \frac{\beta\delta(1 - Z_3)[e^{-(\alpha+\beta)t} - e^{-\theta t}]}{[(\gamma + \delta) - (\alpha + \beta)][\theta - (\alpha + \beta)]}$$

$$+ \frac{\beta\delta(1 - Z_3)[e^{-(\gamma+\delta)t} - e^{-\theta t}]}{[(\gamma + \delta) - (\alpha + \beta)][\theta - (\gamma + \delta)]} \Bigg]^{N_0} \Big[1 - (1 - Z_2) e^{-(\gamma+\delta)t}$$

$$- \frac{\delta(1 - Z_3)[e^{-(\gamma+\delta)t} - e^{-\theta t}]}{[\theta - (\gamma + \delta)]} \Bigg]^{M_0} \Big[1 - (1 - Z_3) e^{-\theta t} \Big]^{K_0} ; |Z_1| < 1; |Z_2| < 1; |Z_3| < 1$$

Expanding $G(Z_1, Z_2, Z_3; t)$ and collecting the constant terms, we get the probability that there is no employee in the organization by considering that $Z_1 = 0, Z_2 = 0$, and $Z_3 = 0$ in $G(Z_1, Z_2, Z_3; t)$. Thus,

$$G_{0,0,0}(t) = \exp \Bigg\{ \lambda_1 \sum_{x=1}^{\infty} \sum_{r=1}^{x} \sum_{s=0}^{r} \sum_{f=0}^{s} (-1)^s C_x \begin{pmatrix} x \\ r \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix} \begin{pmatrix} s \\ f \end{pmatrix} \left(\frac{\theta - \gamma}{(\gamma + \delta) - \theta} \right)^{(s-f)}$$

$$\left[\frac{\beta\delta}{[\theta - (\gamma + \delta)][\theta - (\alpha + \beta)]} \right]^{f} \left(\frac{\beta}{(\gamma + \delta) - (\alpha + \beta)} \right)^{(s-f)}$$

$$\left[-1 - \frac{\beta}{(\gamma + \delta) - (\alpha + \beta)} - \frac{\beta\delta}{[\theta - (\alpha + \beta)] \, [(\gamma + \delta) - (\alpha + \beta)]} \right]^{(r-s)}$$

$$\frac{1 - e^{-[\theta f + (\gamma+\delta)(s-f) + (\alpha+\beta)(r-s)]t}}{[\theta f + (\gamma + \delta)(s - f) + (\alpha + \beta)(r - s)]} + \lambda_2 t \sum_{x=1}^{\infty} \sum_{r=1}^{x} \sum_{s=0}^{r} \sum_{f=0}^{s} (-1)^s C_x \begin{pmatrix} x \\ r \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix} \begin{pmatrix} s \\ f \end{pmatrix}$$

$$\left(\frac{\theta - \gamma}{(\gamma + \delta) - \theta} \right)^{(s-f)} \left[\frac{\beta\delta}{[\theta - (\gamma + \delta)][\theta - (\alpha + \beta)]} \right]^{f} \left(\frac{\beta}{(\gamma + \delta) - (\alpha + \beta)} \right)^{(s-f)}$$

$$\left[-1 - \frac{\beta}{(\gamma + \delta)) - (\alpha + \beta)} - \frac{\beta\delta}{[\theta - (\alpha + \beta)][(\gamma + \delta) - (\alpha + \beta)]} \right]^{(r-s)}$$

$$\frac{1}{[\theta f + (\gamma + \delta)(s - f) + (\alpha + \beta)(r - s)]} - \lambda_2 \sum_{x=1}^{\infty} \sum_{r=1}^{x} \sum_{s=0}^{r} \sum_{f=0}^{s} (-1)^s C_x \begin{pmatrix} x \\ r \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix} \begin{pmatrix} s \\ f \end{pmatrix} \left(\frac{\theta - \gamma}{(\gamma + \delta) - \theta} \right)^{(s-f)}$$

$$\left[\frac{\beta\delta}{[\theta - (\gamma + \delta)][\theta - (\alpha + \beta)]} \right]^{f} \left(\frac{\beta}{(\gamma + \delta) - (\alpha + \beta)} \right)^{(s-f)}$$

$$\left[-1 - \frac{\beta}{(\gamma + \delta)) - (\alpha + \beta)} - \frac{\beta\delta}{[\theta - (\alpha + \beta)][(\gamma + \delta)) - (\alpha + \beta)]} \right]^{(r-s)}$$

$$\frac{1 - e^{-[\theta f + (\gamma+\delta)(s-f) + (\alpha+\beta)(r-s)]t}}{[\theta f + (\gamma + \delta)(s - f) + (\alpha + \beta)(r - s)]^2} + \varepsilon_1 \sum_{y=1}^{\infty} \sum_{u=1}^{y} \sum_{v=0}^{u} (-1)^{2u-v} D_y \begin{pmatrix} y \\ u \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

$$\left[\frac{\gamma - \theta}{\theta - (\gamma + \delta)} \right]^{(u-v)} \left(\frac{\delta}{\theta - \gamma - \delta} \right)^{v} \frac{1 - e^{[\theta v + (\gamma+\delta)(u-v)]t}}{[\theta v + (\gamma + \delta)(u - v)]}$$

$$+ \varepsilon_2 t \sum_{y=1}^{\infty} \sum_{u=1}^{y} \sum_{v=0}^{u} (-1)^{2u-} D_y \begin{pmatrix} y \\ u \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \left[\frac{\gamma - \theta}{\theta - (\gamma + \delta)} \right]^{(u-v)} \left(\frac{\delta}{\theta - \gamma - \delta} \right)^{v}$$

$$\frac{1}{[\theta v + (\gamma + \delta)(u - v)]} - \varepsilon_2 \sum_{y=1}^{\infty} \sum_{u=1}^{y} \sum_{v=0}^{u} (-1)^{2u} D_y \begin{pmatrix} y \\ u \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \left(\frac{\delta}{\theta - \gamma - \delta} \right)^{v}$$

$$\left(\frac{\gamma - \theta}{\theta - \gamma - \delta} \right)^{u-v} \frac{1 - e^{-[\theta v + (\gamma+\delta)(u-v)]t}}{[\theta v + (\gamma + \delta)(u - v)]^2} \Bigg\} \Big[1 - e^{-(\alpha+\beta)t}$$

$$- \frac{\beta[e^{-(\alpha+\beta)t} - e^{-(\gamma+\delta)t}]}{[(\gamma + \delta) - (\alpha + \beta)]} - \frac{\beta\delta[e^{-(\alpha+\beta)t} - e^{-\theta t}]}{[(\gamma + \delta) - (\alpha + \beta)][\theta - (\alpha + \beta)]} \tag{16}$$

$$+\frac{\beta\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\Big]^{N_0}\Big[1-e^{-(\gamma+\delta)t}-\frac{\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[\theta-(\gamma+\delta)]}]^{M_0}$$
$$[1-e^{-\theta t}]^{K_0};$$
$$|Z_1|<1;|Z_2|<1;|Z_3|<1$$

Taking $Z_2=1$ and $Z_3=1$ in equation (15), we get the probability generating function of the number of employees in grade 1 in the organization as

$$G\left(Z_1;t\right)=\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\left(\begin{array}{c}x\\r\end{array}\right)\left(Z_1-1\right)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{(\alpha+\beta)r}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\left(\begin{array}{c}x\\r\end{array}\right)\frac{\left(Z_1-1\right)^r}{(\alpha+\beta)r}\right.$$
$$\left.-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\left(\begin{array}{c}x\\r\end{array}\right)\left(Z_1-1\right)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\right\}\left[1-\left(1-Z_1\right)e^{-(\alpha+\beta)t}\right]^{N_0};|Z_1|<1 \tag{17}$$

Expanding $G(Z_1,t)$ and collecting the constant terms, we get the probability that there is no grade 1 employee in the organization by taking $Z_1=0, Z_2=1$ and $Z_3=1$ in $G(Z_1,Z_2,Z_3;t)$. Thus,

$$G_{0\bullet\bullet}=\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\left(\begin{array}{c}x\\r\end{array}\right)(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{(\alpha+\beta)r}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\left(\begin{array}{c}x\\r\end{array}\right)\frac{(-1)^r}{(\alpha+\beta)r}\right.$$
$$\left.-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\left(\begin{array}{c}x\\r\end{array}\right)(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0} \tag{18}$$

Similarly, taking $Z_1=1$ and $Z_3=1$ equation (15), we get the probability generating function of the number of employees in grade 2 in the organization as

$$G\left(Z_2;t\right)=\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^s C_x\left(\begin{array}{c}x\\r\end{array}\right)\left(\begin{array}{c}r\\s\end{array}\right)\left(\frac{\beta\left(Z_2-1\right)}{(\gamma+\delta)-(\alpha+\beta)}\right)^r\frac{1-e^{-[(\gamma+\delta)s+(\alpha+\beta)(r-s)]t}}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]}\right.$$
$$+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^s C_x\left(\begin{array}{c}x\\r\end{array}\right)\left(\begin{array}{c}r\\s\end{array}\right)\left(\frac{\beta\left(Z_2-1\right)}{(\gamma+\delta)-(\alpha+\beta)}\right)^r\frac{1}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]}$$
$$-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^s C_x\left(\begin{array}{c}x\\r\end{array}\right)\left(\begin{array}{c}r\\s\end{array}\right)\left(\frac{\beta\left(Z_2-1\right)}{(\gamma+\delta)-(\alpha+\beta)}\right)^r\frac{1-e^{-[(\gamma+\delta)s+(\alpha+\beta)(r-s)]t}}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]^2}$$
$$+\varepsilon_1\sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{2u}D_y\left(\begin{array}{c}y\\u\end{array}\right)\left(Z_2-1\right)^u\frac{1-e^{-(\gamma+\delta)ut}}{(\gamma+\delta)\,u}+\varepsilon_2 t\sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{2u}D_y\left(\begin{array}{c}y\\u\end{array}\right)$$
$$\left(Z_2-1\right)^u\frac{1}{(\gamma+\delta)\,u}-\varepsilon_2\sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{2u}D_y\left(\begin{array}{c}y\\u\end{array}\right)\left(Z_2-1\right)^u\frac{1-e^{-(\gamma+\delta)ut}}{[(\gamma+\delta)\,u]^2}\}$$
$$\left[1-\frac{\beta(1-Z_2)[e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}]}{[(\gamma+\delta)-(\alpha+\beta)]}\right]^{N_0}\left[1-(1-Z_2)e^{-(\gamma+\delta)t}\right]^{M_0};|Z_2| \tag{19}$$

Expanding $G(Z_2;t)$ and collecting the constant terms, we get the probability that there is no grade 2 employee in the organization by taking $Z_1 = 1, Z_2 = 0$ and $Z_3 = 1$ in $G(Z_1, Z_2, Z_3; t)$. Thus,

$$
\begin{aligned}
G_{\bullet 0 \bullet} = &\exp\Bigg\{\lambda_1 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}C_x \binom{x}{r}\binom{r}{s}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r \frac{1-e^{-[(\gamma+\delta)s+(\alpha+\beta)(r-s)]t}}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]} \\
&+\lambda_2 t \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}C_x \binom{x}{r}\binom{r}{s}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r \frac{1}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]} \\
&-\lambda_2 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}C_x \binom{x}{r}\binom{r}{s}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r \frac{1-e^{-[(\gamma+\delta)s+(\alpha+\beta)(r-s)]t}}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]^2} \\
&+\varepsilon_1 \sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}D_y \binom{y}{u}\frac{1-e^{-(\gamma+\delta)ut}}{(\gamma+\delta)\,u} + \varepsilon_2 t \sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}D_y \binom{y}{u} \\
&\frac{1}{(\gamma+\delta)\,u} - \varepsilon_2 \sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}D_y \binom{y}{u}\frac{1-e^{-(\gamma+\delta)ut}}{[(\gamma+\delta)\,u]^2}\Bigg\} \\
&[1-\frac{\beta[e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}]}{[(\gamma+\delta)-(\alpha+\beta)]}]^{N_0}[1-e^{-(\gamma+\delta)t}]^{M_0}; |Z_2|<1
\end{aligned}
\tag{20}
$$

Similarly, taking $Z_1 = 1$ and $Z_2 = 1$ equation (15), we get the probability generating function of the number of employees in grade 3 in the organization as

$$
\begin{aligned}
G(Z_3;t) = &\exp\Bigg\{\lambda_1 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x \binom{x}{r}\binom{r}{s}\binom{s}{f}(Z_3-1)^r\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f \\
&\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)}[\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}]^{r-s} \\
&\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)\,(s-f)+(\alpha+\beta)\,(r-s)]} + \lambda_2 t \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x \binom{x}{r}\binom{r}{s}\binom{s}{f} \\
&(Z_3-1)^r\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)} \\
&[\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}]^{r-s}\frac{1}{[\theta f+(\gamma+\delta)\,(s-f)+(\alpha+\beta)\,(r-s)]} \\
&-\lambda_2 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x \binom{x}{r}\binom{r}{s}\binom{s}{f}(Z_3-1)^r\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f \\
&\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)}[\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}]^{r-s} \\
&\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)\,(s-f)+(\alpha+\beta)\,(r-s)]^2} + \varepsilon_1 \sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}D_y \binom{y}{u}\binom{u}{v} \\
&[\frac{\delta}{\theta-(\gamma+\delta)}]^u \frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)\,(u-v)]} + \varepsilon_2 t \sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}D_y \binom{y}{u}\binom{u}{v} \\
&[\frac{\delta}{\theta-(\gamma+\delta)}]^{(u-v)}\frac{1}{[\theta v+(\gamma+\delta)\,(u-v)]} - \varepsilon_2 \sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}C_y \binom{y}{u}\binom{u}{v} \\
&[\frac{\delta}{\theta-(\gamma+\delta)}]^u \frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)\,(u-v)]^2}[1-\frac{\beta\delta\,(1-Z_3)\,[e^{-(\alpha+\beta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]} \\
&+\frac{\beta\delta\,(1-Z_3)\,[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}]^{N_0}[1-\frac{\delta\,(1-Z_3)\,[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[\theta-(\gamma+\delta)]}]^{M_0} \\
&[1-(1-Z_3)\,e^{-\theta t}]^{K_0}; |Z_3|<1
\end{aligned}
\tag{21}
$$

Expanding $G(Z_3; t)$ and collecting the constant terms, we get the probability that there is no grade 3 employee in the organization by taking $Z_1 = 1, Z_2 = 1$ and $Z_3 = 0$ in $G(Z_1, Z_2, Z_3; t)$. Thus,

$$
\begin{aligned}
G_{\bullet\bullet 0} = &\exp\Bigg\{\lambda_1 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{r+s-f}C_x\binom{x}{r}\binom{r}{s}\binom{s}{f}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f \\
&\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)}\left[\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}\right]^{r-s} \\
&\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]} + \lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{r+s-f}C_x\binom{x}{r}\binom{r}{s}\binom{s}{f} \\
&\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)} \\
&\left[\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}\right]^{r-s}\left[\frac{1}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]}\right] \\
&-\lambda_2 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x\binom{x}{r}\binom{r}{s}\binom{s}{f}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f \\
&\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}\right)^{(r-s)} \\
&\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]^2} + \varepsilon_1\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{3u-v}D_y\binom{y}{u}\binom{u}{v} \\
&\left[\frac{\delta}{\theta-(\gamma+\delta)}\right]^u\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]} + \varepsilon_2 t\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{3u-v}D_y\binom{y}{u}\binom{u}{v} \\
&\left[\frac{\delta}{\theta-(\gamma+\delta)}\right]^{(u-v)}\frac{1}{[\theta v+(\gamma+\delta)(u-v)]} - \varepsilon_2\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{3u-v}C_y\binom{y}{u}\binom{u}{v} \\
&\left[\frac{\delta}{\theta-(\gamma+\delta)}\right]^u\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]^2}\Bigg\}\left[1-\frac{\beta\delta[e^{-(\alpha+\beta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]}\right. \\
&\left.+\frac{\beta\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right]^{N_0}\left[1\frac{\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[\theta-(\gamma+\delta)]}\right]^{M_0} \\
&[1-e^{-\theta t}]^{K_0}; |Z_3|<1
\end{aligned}
\tag{22}
$$

The mean number of employees in grade 1 of the organization is

$$
\begin{aligned}
L_1 = &\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=1}^{\infty}xC_x\left[1-e^{-(\alpha+\beta)t}\right] + \frac{\lambda_2 t}{(\alpha+\beta)}\sum_{x=1}^{\infty}xC_x - \frac{\lambda_2}{(\alpha+\beta)^2}\sum_{x=1}^{\infty}xC_x\left[1-e^{-(\alpha+\beta)t}\right] \\
&+\mathrm{N}_0 e^{-(\alpha+\beta)t}
\end{aligned}
\tag{23}
$$

The probability that there is at least one employee in grade 1 is

$$
\begin{aligned}
U_1 = 1 - G_{0\bullet\bullet}(t) = &1-\exp\Bigg\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\tbinom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{(\alpha+\beta)r} + \lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\tbinom{x}{r}\frac{(-1)^r}{(\alpha+\beta)r} \\
&-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(-1)^r\frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\Bigg\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0}
\end{aligned}
\tag{24}
$$

The mean number of employees in grade 2 of the organization is

$$
\begin{aligned}
L_2 =& \frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty} xC_x\right)\left[\left(\frac{1-e^{-(\gamma+\delta)t}}{\gamma+\delta}\right)-\left(\frac{e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}}{(\alpha+\beta)-(\gamma+\delta)}\right)\right]+\frac{\lambda_2\beta t}{(\gamma+\delta)(\alpha+\beta)}\left(\sum_{x=1}^{\infty} xC_x\right)\\
&-\frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty} xC_x\right)\left[\frac{(1-e^{-(\gamma+\delta)t})(\alpha+\beta+\gamma+\delta)}{(\gamma+\delta)^2}+\left(\frac{e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}}{(\gamma+\delta)-(\alpha+\beta)}\right)\right]\\
&+\frac{\varepsilon_1}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty} yD_y\right)\left(1-e^{-(\gamma+\delta)t}\right)+\frac{\varepsilon_2 t}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty} yD_y\right)-\frac{\varepsilon_2 t}{(\gamma+\delta)^2}\left(\sum_{y=1}^{\infty} yD_y\right)\left(1-e^{-(\gamma+\delta)t}\right)\\
&+\mathrm{N}_0\left(\frac{\beta}{(\alpha+\beta)-(\gamma+\delta)}\right)\left(e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}\right)+\mathrm{M}_0 e^{-(\gamma+\delta)t}
\end{aligned} \tag{25}
$$

The probability that there is at least one employee in grade 2 is

$$
\begin{aligned}
U_2 &= 1-G_{\bullet 0\bullet}(t)\\
&= 1-\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r\frac{1-e^{-[(\gamma+\delta)s+(\alpha+\beta)(r-s)]t}}{[(\gamma+\delta)s+(\alpha+\beta)(r-s)]}\right.\\
&+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r\frac{1}{[(\gamma+\delta)s+(\alpha+\beta)(r-s)]}\\
&-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r\frac{1-e^{-[(\gamma+\delta)s+(\alpha+\beta)(r-s)]t}}{[(\gamma+\delta)s+(\alpha+\beta)(r-s)]^2}\\
&+\varepsilon_1\sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}D_y\binom{y}{u}\frac{1-e^{-(\gamma+\delta)ut}}{(\gamma+\delta)u}+\varepsilon_2 t\sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}D_y\binom{y}{u}\\
&\frac{1}{(\gamma+\delta)u}-\varepsilon_2\sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}D_y\binom{y}{u}\frac{1-e^{-(\gamma+\delta)ut}}{[(\gamma+\delta)u]^2}\}\\
&[1-\frac{\beta[e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}]}{[(\gamma+\delta)-(\alpha+\beta)]}]^{N_0}[1-e^{-(\gamma+\delta)t}]^{M_0};|Z_2|<1
\end{aligned} \tag{26}
$$

The mean number of employees in grade 3 of the organization is

$$
\begin{aligned}
L_3 =& \lambda_1\left(\sum_{x=1}^{\infty} xC_x\right)\left[\frac{\beta\delta\left[1-e^{-(\gamma+\delta)t}\right]]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)](\alpha+\beta)}+\frac{\beta\delta\left[1-e^{-\theta t}\right]}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]\theta}\right.\\
&\left.-\frac{\beta\delta\left[1-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)](\gamma+\delta)}\right]+\lambda_2 t\left(\sum_{x=1}^{\infty} xC_x\right)\left[\frac{\beta\delta}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]\theta}\right.\\
&\left.+\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)](\alpha+\beta)}-\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)](\gamma+\delta)}\right]\\
&-\lambda_2\left(\sum_{x=1}^{\infty} xC_x\right)\left[\frac{\beta\delta\left[1-e^{-(\alpha+\beta)t}\right]]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)](\alpha+\beta)^2}+\frac{\beta\delta\left[1-e^{-\theta t}\right]}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]\theta^2}\right.\\
&\left.-\frac{\beta\delta\left[1-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)](\gamma+\delta)^2}\right]+\mathrm{M}_0\left(\frac{\delta\left[e^{-(\gamma+\delta)t}-e^{-\theta t}\right]}{[\theta-(\gamma+\delta)]}\right)+\mathrm{K}_0 e^{-\theta t}\\
&+\frac{\varepsilon_1\delta}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty} yD_y\right)\left[(\frac{1-e^{-\theta t}}{\theta})(\frac{\left[e^{-\theta t}-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-\theta]})\right]+\frac{\varepsilon_2 t\delta}{(\gamma+\delta)}(\sum_{y=1}^{\infty} yD_y)(\frac{1}{\theta})\\
&-\frac{\varepsilon_2\delta}{(\gamma+\delta)^2}(\sum_{y=1}^{\infty} yD_y)\left[(\frac{(1-e^{-\theta t})(\gamma+\delta+\theta)}{\theta^2})-(\frac{\left[e^{-\theta t}-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-\theta]})\right]\\
&+\mathrm{N}_0\left[\frac{\beta\delta\left[e^{-(\alpha+\beta)t}-e^{-\theta t}\right]]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]}-\frac{\beta\delta\left[e^{-(\gamma+\delta)t}-e^{-\theta t}\right]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right]
\end{aligned} \tag{27}
$$

The probability that there is at least one employee in grade 3 is

$$
\begin{aligned}
U_3 &= 1 - G_{\bullet\bullet 0}(t) \\
&= 1 - \exp\Bigg\{ \lambda_1 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{r+s-f} C_x \binom{x}{r}\binom{r}{s}\binom{s}{f}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{f} \\
&\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)} [\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}]^{r-s} \\
&\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]} + \lambda_2 t \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{r+s-f} C_x \binom{x}{r}\binom{r}{s}\binom{s}{f} \\
&\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{f}\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)} \\
&[\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}]^{r-s}\frac{1}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]} \\
&-\lambda_2 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f} C_x \binom{x}{r}\binom{r}{s}\binom{s}{f}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{f} \\
&\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}\right)^{(r-s)} \\
&\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]^2} + \varepsilon_1 \sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{3u-v} D_y \binom{y}{u}\binom{u}{v} \\
&[\frac{\delta}{\theta-(\gamma+\delta)}]^{u}\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]} + \varepsilon_2 t \sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{3u-v} D_y \binom{y}{u}\binom{u}{v} \\
&[\frac{\delta}{\theta-(\gamma+\delta)}]^{(u-v)}\frac{1}{[\theta v+(\gamma+\delta)(u-v)]} - \varepsilon_2 \sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{3u-v} C_y \binom{y}{u}\binom{u}{v} \\
&[\frac{\delta}{\theta-(\gamma+\delta)}]^{u}\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]^2}\Bigg\}[1-\frac{\beta\delta[e^{-(\alpha+\beta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]} \\
&+\frac{\beta\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}]^{N_0}[1-\frac{\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[\theta-(\gamma+\delta)]}]^{M_0} \\
&[1-e^{-\theta t}]^{K_0}; |Z_3| < 1
\end{aligned} \tag{28}
$$

The mean number of employees in the organization is $L = L_1 + L_2 + L_3$. where the values of L_1, L_2, and L_3 from the equations (23), (25), and (27).

The average duration of stay of an employee in grade 1 is

$$W_1 = \frac{L_1}{(\alpha+\beta)(1-G_{0\bullet\bullet}(t))} \tag{29}$$

The average duration of stay of an employee in grade 2 is

$$W_2 = \frac{L_2}{\gamma(1-G_{\bullet 0\bullet}(t))} \tag{30}$$

The average duration of stay of an employee in grade 3 is

$$W_3 = \frac{L_3}{\theta(1-G_{\bullet\bullet 0}(t))} \tag{31}$$

The variance of the number of employees in grade 1 is

$$
\begin{aligned}
V_1 =& \frac{\lambda_1}{2(\alpha+\beta)}\sum_{x=1}^{\infty} x(x-1)C_x\left[1-e^{-2(\alpha+\beta)t}\right] + \frac{\lambda_1}{(\alpha+\beta)}\sum_{x=1}^{\infty} xC_x\left[1-e^{-(\alpha+\beta)t}\right] \\
&+ \frac{\lambda_2 t}{2(\alpha+\beta)}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right) - \frac{\lambda_2}{[2(\alpha+\beta)]^2}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left[1-e^{-2(\alpha+\beta)t}\right] \\
&+ \frac{\lambda_2 t}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty} xC_x\right) - \frac{\lambda_2}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty} xC_x\right)\left[1-e^{-(\alpha+\beta)t}\right] + N_0\left[e^{-(\alpha+\beta)t}-e^{-2(\alpha+\beta)t}\right]
\end{aligned}
\tag{32}
$$

The variance of the number of employees in grade 2 is

$$
\begin{aligned}
V_2 =& \frac{\lambda_1\beta^2}{[(\gamma+\delta)-(\alpha+\beta)]^2}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left[\left(\frac{1-e^{-2(\alpha+\beta)t}}{2(\alpha+\beta)}\right) - 2\left(\frac{1-e^{-(\alpha+\beta+\gamma+\delta)t}}{(\alpha+\beta+\gamma+\delta)}\right)\right. \\
&\left.+\left(\frac{1-e^{-2(\gamma+\delta)t}}{2(\gamma+\delta)}\right)\right] + \frac{\lambda_2 t\beta^2}{[(\gamma+\delta)-(\alpha+\beta)]^2}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left[\left(\frac{1}{2(\alpha+\beta)}\right)\right. \\
&\left.-\left(\frac{2}{(\alpha+\beta+\gamma+\delta)}\right)+\left(\frac{1}{2(\gamma+\delta)}\right)\right] - \frac{\lambda_2\beta^2}{[(\gamma+\delta)-(\alpha+\beta)]^2}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right) \\
&\left[\left(\frac{1-e^{-2(\alpha+\beta)t}}{(2(\alpha+\beta))^2}\right) - 2\left(\frac{1-e^{-(\alpha+\beta+\gamma+\delta)t}}{(\alpha+\beta+\gamma+\delta)^2}\right) + \left(\frac{1-e^{-2(\gamma+\delta)t}}{[2(\gamma+\delta)]^2}\right)\right] \\
&+ \frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty} xC_x\right)\left[\left(\frac{1-e^{-(\gamma+\delta)t}}{\gamma+\delta}\right) - \left(\frac{e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}}{(\gamma+\delta)-(\alpha+\beta)}\right)\right] + \frac{\lambda_2 t\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty} xC_x\right) \\
&\left(\frac{1}{\gamma+\delta}\right) - \frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty} xC_x\right)\left[\left(\frac{(\alpha+\beta+\gamma+\delta)(1-e^{-(\gamma+\delta)t})^2}{\gamma+\delta}\right) - \left(\frac{e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}}{(\gamma+\delta)-(\alpha+\beta)}\right)\right] \\
&+ \varepsilon_1\left(\sum_{y=1}^{\infty} y(y-1)D_y\right)\left(\frac{1-e^{-2(\gamma+\delta)t}}{[2(\gamma+\delta)]}\right) + \varepsilon_2 t\left(\sum_{y=1}^{\infty} y(y-1)D_y\right)\left(\frac{1}{[2(\gamma+\delta)]}\right) \\
&- \varepsilon_2\left(\sum_{y=1}^{\infty} y(y-1)D_y\right)\left(\frac{1-e^{-2(\gamma+\delta)t}}{[2(\gamma+\delta)]^2}\right) + \varepsilon_1\left(\sum_{y=1}^{\infty} yD_y\right)\left(\frac{1-e^{-(\gamma+\delta)t}}{(\gamma+\delta)}\right) \\
&+ \varepsilon_2 t\left(\sum_{y=1}^{\infty} yD_y\right) - \varepsilon_2\left(\sum_{y=1}^{\infty} yD_y\right)\left(\frac{1-e^{-(\gamma+\delta)t}}{(\gamma+\delta)^2}\right) \\
&+ \frac{N_0\beta}{(\alpha+\beta)-(\gamma+\delta)}\left[e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}\right]\left[1-\frac{\beta}{[(\alpha+\beta)-(\gamma+\delta)]^2}\left(e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}\right)^2\right] \\
&+ M_0\left(e^{-(\gamma+\delta)t}-e^{-2(\gamma+\delta)t}\right)
\end{aligned}
\tag{33}
$$

The variance of the number of employees in grade 3 is

$$
\begin{aligned}
V_3 =& \lambda_1\beta\delta\sum_{x=1}^{\infty} xC_x\left\{\frac{1-e^{-(\alpha+\beta)t}}{[(\gamma+\delta)-(\alpha+\beta)]\,[\theta-(\alpha+\beta)]\,(\alpha+\beta)}\right. \\
&\left.+\frac{1-e^{-\theta t}}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]\theta} - \frac{1-e^{-(\gamma+\delta)t}}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)](\gamma+\delta)}\right\} \\
&+ \lambda_2 t\beta\delta\sum_{x=1}^{\infty} xC_x\left\{\frac{1}{[(\gamma+\delta)-(\alpha+\beta)]\,[\theta-(\alpha+\beta)]\,(\alpha+\beta)}\right. \\
&\left.+\frac{1}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]\theta} - \frac{1}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)](\gamma+\delta)}\right\} \\
&- \lambda_2\beta\delta\sum_{x=1}^{\infty} xC_x\left\{\frac{1-e^{-(\alpha+\beta)t}}{[(\gamma+\delta)-(\alpha+\beta)]\,[\theta-(\alpha+\beta)]\,(\alpha+\beta)^2}\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1-e^{-\theta t}}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]\theta^2}-\frac{1-e^{-(\gamma+\delta)t}}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)](\gamma+\delta)^2}\Bigg\}\\
&+N_0\left[\frac{\beta\delta[e^{-(\alpha+\beta)t}-e^(-\theta t)]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]}-\frac{\beta\delta[e^{-(\gamma+\delta)t}-e^(-\theta t)]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right]\\
&+M_0\frac{\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[\theta-(\gamma+\delta)]}+K_0e^{-\theta t}+\lambda_1(\beta\delta)^2\sum_{x=1}^{\infty}x(x-1)C_x\\
&\Bigg\{\frac{1-e^{-2(\alpha+\beta)t}}{[(\gamma+\delta)-(\alpha+\beta)]^2[\theta-(\alpha+\beta)]^2 2(\alpha+\beta)}-2\left(\frac{1}{[(\gamma+\delta)-(\alpha+\beta)]^2}\right)\\
&\frac{1}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]}\left(\frac{1-e^{-(\alpha+\beta+\gamma+\delta)t}}{\alpha+\beta+\gamma+\delta}\right)+2\left(\frac{1}{\theta-(\alpha+\beta)}\right)^2\left(\frac{1-e^{-(\alpha+\beta+\theta)t}}{\alpha+\beta+\theta}\right)\\
&\frac{1}{[\theta-(\gamma+\delta)][(\gamma+\delta)-(\alpha+\beta)]}+\frac{1}{[\theta-(\gamma+\delta)]^2[(\gamma+\delta)-(\alpha+\beta)]^2}\left(\frac{1-e^{-2(\gamma+\delta)t}}{2(\gamma+\delta)}\right)\\
&-2\left(\frac{1}{\theta-(\gamma+\delta)}\right)^2\left(\frac{1-e^{-(\gamma+\delta+\theta)t}}{\gamma+\delta+\theta}\right)\frac{1}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}+\left(\frac{1-e^{-2\theta t}}{2\theta}\right)\\
&\frac{1}{[\theta-(\gamma+\delta)]^2[\theta-(\alpha+\beta)]^2}\Bigg\}+\lambda_2 t(\beta\delta)^2\sum_{x=1}^{\infty}x(x-1)C_x\\
&\Bigg\{\frac{1}{[(\gamma+\delta)-(\alpha+\beta)]^2[\theta-(\alpha+\beta)]^2 2(\alpha+\beta)}-2\left(\frac{1}{[(\gamma+\delta)-(\alpha+\beta)]^2}\right)\\
&\frac{1}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]}\left(\frac{1}{\alpha+\beta+\gamma+\delta}\right)+2\left(\frac{1}{\theta-(\alpha+\beta)}\right)^2\left(\frac{1}{\alpha+\beta+\theta}\right)\\
&\frac{1}{[\theta-(\gamma+\delta)][(\gamma+\delta)-(\alpha+\beta)]}+\frac{1}{[\theta-(\gamma+\delta)]^2[(\gamma+\delta)-(\alpha+\beta)]^2}\left(\frac{1}{2(\gamma+\delta)}\right)\\
&-2\left(\frac{1}{\theta-(\gamma+\delta)}\right)^2\left(\frac{1}{\gamma+\delta+\theta}\right)\frac{1}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}+\left(\frac{1}{2\theta}\right) \qquad (34)\\
&\frac{1}{[\theta-(\gamma+\delta)]^2[\theta-(\alpha+\beta)]^2}\Bigg\}-\lambda_2(\beta\delta)^2\sum_{x=1}^{\infty}x(x-1)C_x\\
&\Bigg\{\frac{1-e^{-2(\alpha+\beta)t}}{[(\gamma+\delta)-(\alpha+\beta)]^2[\theta-(\alpha+\beta)]^2[2(\alpha+\beta)]^2}-2\left(\frac{1}{[(\gamma+\delta)-(\alpha+\beta)]^2}\right)\\
&\frac{1}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]}\left(\frac{1-e^{-(\alpha+\beta+\gamma+\delta)t}}{[\alpha+\beta+\gamma+\delta]^2}\right)+2\left(\frac{1}{\theta-(\alpha+\beta)}\right)^2\left(\frac{1-e^{-(\alpha+\beta+\theta)t}}{[\alpha+\beta+\theta]^2}\right)\\
&\frac{1}{[\theta-(\gamma+\delta)][(\gamma+\delta)-(\alpha+\beta)]}+\frac{1}{[\theta-(\gamma+\delta)]^2[(\gamma+\delta)-(\alpha+\beta)]^2}\\
&\left(\frac{1-e^{-2(\gamma+\delta)t}}{[2(\gamma+\delta)]^2}\right)-2\left(\frac{1}{\theta-(\gamma+\delta)}\right)^2\left(\frac{1-e^{-(\gamma+\delta+\theta)t}}{[\gamma+\delta+\theta]^2}\right)\frac{1}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}\\
&+\left(\frac{1-e^{-2\theta t}}{(2\theta)^2}\right)\frac{1}{[\theta-(\gamma+\delta)]^2[\theta-(\alpha+\beta)]^2}\Bigg\}-N_0\Bigg[\frac{\beta\delta[e^{-(\alpha+\beta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]}\\
&-\frac{\beta\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\Bigg]^2-M_0\left(\frac{\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{\theta-(\gamma+\delta)}\right)^2-K_0e^{-2\theta t}\\
&+\varepsilon_1\left(\frac{\delta}{\theta-(\gamma+\delta)}\right)^2\left(\sum_{y=1}^{\infty}y(y-1)D_y\right)\Bigg\{\frac{(1-e^{-2(\gamma+\delta)t})}{2(\gamma+\delta)}-\frac{2[1-e^{-(\theta+\gamma+\delta)t}]}{(\theta+\gamma+\delta)}\\
&+\frac{1-e^{-2\theta t}}{2\theta}\Bigg\}+\varepsilon_2 t\left(\frac{\delta}{\theta-(\gamma+\delta)}\right)^2\left(\sum_{y=1}^{\infty}y(y-1)D_y\right)\\
&\left\{\frac{1}{2(\gamma+\delta)}-\frac{2}{(\theta+\gamma+\delta)}+\frac{1}{2\theta}\right\}-\varepsilon_2\left(\frac{\delta}{\theta-(\gamma+\delta)}\right)^2\left(\sum_{y=1}^{\infty}y(y-1)D_y\right)\\
&\left\{\frac{(1-e^{-2(\gamma+\delta)t})}{[2(\gamma+\delta)]^2}-\frac{2[1-e^{-(\theta+\gamma+\delta)t}]}{(\theta+\gamma+\delta)^2}+\frac{1-e^{-2\theta t}}{(2\theta)^2}\right\}
\end{aligned}
$$

The coefficient of variation of the number of employees in grade 1 is

$$CV_1 = \frac{\sqrt{V_1}}{L_1} \tag{35}$$

Where L_1 and V_1 are as given in equations (23) and (32). The coefficient of variation of the number of employees in grade 2 is

$$CV_2 = \frac{\sqrt{V_2}}{L_2} \tag{36}$$

Where L_2 and V_2 are as given in equations (25) and (33). The coefficient of variation of the number of employees in grade 3 is

$$CV_3 = \frac{\sqrt{V_3}}{L_3} \tag{37}$$

Where L_3 and V_3 are as given in equations (27) and (34).

4. CHARACTERISTICS OF THE MODEL WITH UNIFORM BATCH SIZE DISTRIBUTION

To study the performance of the model one has to specify the batch size distribution of the bulk recruitment. That is the number of employees recruited at a time is a random variable and follows a specific distribution. Let us assume that the number of employees in a batch of recruitment follows uniform distribution with parameters a and b of grade 1 and with parameters of grade 2.

The probability mass function of the batch size distribution of grade 1 is

$$C_x = \frac{1}{b-a+1}; \quad x = a, a+1, \cdots, b$$

The mean number of employees in each batch of grade 1 is

$$\frac{a+b}{2}$$

The variance of the batch size of grade 1 is

$$\frac{1}{12}[(b-a+1)^2 - 1]$$

The probability mass function of the batch size distribution of grade 2 is

$$D_y = \frac{1}{d-c+1}; \quad y = c, c+1, \cdots, d$$

The mean number of employees in each batch of grade 2 is

$$\frac{c+d}{2}$$

The variance of the batch size of grade 2 is

$$\frac{1}{12}[(d-c+1)^2 - 1]$$

Substituting the values of C_x and D_y in (15), we get the joint probability generating function of the number of employees in grade 1 and grade 2 is obtained as employees in grade 1, grade 2, and grade 3 respectively is

$$
\begin{aligned}
G(Z_1,Z_2,Z_3;t) =\exp\Bigg\{ & \lambda_1 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x \binom{x}{r}\binom{r}{s}\binom{s}{f}(Z_3-1)^f\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)} \\
& \left(\frac{1}{b-a+1}\right)\left[(Z_2-1)-\frac{\delta(Z_3-1)}{(\gamma+\delta)-\theta}\right]^{(s-f)}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f \\
& \left[(Z_1-1)-\frac{\beta(Z_2-1)}{(\alpha+\beta)-(\gamma+\delta)}-\frac{\beta\delta(Z_3-1)}{[\theta-(\alpha+\beta)]\,[(\alpha+\beta)-(\gamma+\delta)]}\right]^{(r-s)} \\
& \frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x\binom{x}{r}\binom{r}{s}\binom{s}{f} \\
& (\frac{1}{b-a+1})(Z_3-1)^f\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)}\left[(Z_2-1)-\frac{\delta(Z_3-1)}{(\gamma+\delta)-\theta}\right]^{s-f} \\
& \left[(Z_1-1)-\frac{\beta(Z_2-1)}{(\alpha+\beta)-(\gamma+\delta)}-\frac{\beta\delta(Z_3-1)}{[\theta-(\alpha+\beta)][(\alpha+\beta)-(\gamma+\delta)]}\right]^{(r-s)} \\
& \left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f\frac{1}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]}-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{s-f}C_x \\
& \binom{x}{r}\binom{r}{s}\binom{s}{f}(Z_3-1)^f\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{(s-f)}\left[(Z_2-1)-\frac{\delta(Z_3-1)}{(\gamma+\delta)-\theta}\right]^{(s-f)} \\
& \left[(Z_1-1)-\frac{\beta(Z_2-1)}{(\alpha+\beta)-(\gamma+\delta)}-\frac{\beta\delta(Z_3-1)}{[\theta-(\alpha+\beta)][(\alpha+\beta)-(\gamma+\delta)]}\right]^{(r-s)} \\
& (\frac{1}{b-a+1})\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]^2} \\
& +\varepsilon_1\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}\left(\frac{1}{d-c+1}\right)\binom{y}{u}\binom{u}{v}\left[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-(\gamma+\delta)}\right]^{(u-v)} \\
& \left(\frac{\delta(Z_3-1)}{\theta-\gamma-\delta}\right)^v\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]}+\varepsilon_2 t\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}\left(\frac{1}{d-c+1}\right) \\
& \binom{y}{u}\binom{u}{v}\left[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-(\gamma+\delta)}\right]^{(u-v)}\left(\frac{\delta(Z_3-1)}{\theta-\gamma-\delta}\right)^v\frac{1}{[\theta v+(\gamma+\delta)(u-v)]} \\
& -\varepsilon_2\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{2u-v}\left(\frac{1}{d-c+1}\right)\binom{y}{u}\binom{u}{v}\left[(Z_2-1)+\frac{\delta(Z_3-1)}{\theta-(\gamma+\delta)}\right]^{(u-v)} \\
& \left(\frac{\delta(Z_3-1)}{\theta-\gamma-\delta}\right)^v\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]^2}\Bigg\}\Big[1-(1-Z_1)e^{-(\alpha+\beta)t} \\
& -\frac{\beta(1-Z_2)[e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}]}{[(\gamma+\delta)-(\alpha+\beta)]}-\frac{\beta\delta(1-Z_3)[e^{-(\alpha+\beta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]} \\
& +\frac{\beta\delta(1-Z_3)[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\Bigg]^{N_0}\Big[1-(1-Z_2)e^{-(\gamma+\delta)t} \\
& -\frac{\delta(1-Z_3)[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[\theta-(\gamma+\delta)]}\Bigg]^{M_0}[1-(1-Z_3)e^{-\theta t}]^{K_0};|Z_1|<1;|Z_2|<1;|Z_3|<1
\end{aligned}
\tag{38}
$$

The mean number of employees in grade 1 of the organization is

$$
\begin{aligned}
L_1 =&\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=1}^{\infty}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{(\alpha+\beta)}\sum_{x=1}^{\infty}x\left(\frac{1}{b-a+1}\right) \\
&-\frac{\lambda_2}{(\alpha+\beta)^2}\sum_{x=1}^{\infty}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]+\mathrm{N}_0e^{-(\alpha+\beta)t}
\end{aligned}
\tag{39}
$$

The probability that there is at least one employee in grade 1 is

$$
\begin{aligned}
U_1 =& 1 - G_{0\bullet\bullet}(t) = 1 - \exp\left\{\lambda_1 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(-1)^r \frac{1-e^{-[(\alpha+\beta)rt]}}{(\alpha+\beta)r}\right. \\
&+ \lambda_2 t \sum_{x=1}^{\infty}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\left(\sum_{r}^{x}\right)\frac{(-1)^r}{(\alpha+\beta)r} - \lambda_2 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right) \\
&\left.\binom{x}{r}(-1)^r \frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0}
\end{aligned}
\tag{40}
$$

The mean number of employees in grade 2 of the organization is

$$
\begin{aligned}
L_2 =& \frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty} x \frac{1}{b-a+1}\right)\left[\left(\frac{1-e^{-(\gamma+\delta)t}}{\gamma+\delta}\right) - \left(\frac{e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}}{(\alpha+\beta)-(\gamma+\delta)}\right)\right] \\
&+ \frac{\lambda_2\beta t}{(\gamma+\delta)(\alpha+\beta)}\left(\sum_{x=1}^{\infty} x\frac{1}{b-a+1}\right) - \frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty} x\frac{1}{b-a+1}\right) \\
&\left[\frac{(1-e^{-(\gamma+\delta)t})(\alpha+\beta+\gamma+\delta)}{(\gamma+\delta)^2} + \left(\frac{e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}}{(\gamma+\delta)-(\alpha+\beta)}\right)\right] + \frac{\varepsilon_1}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty} y\frac{1}{d-c+1}\right) \\
&\left(1-e^{-(\gamma+\delta)t}\right) + \frac{\varepsilon_2 t}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty} y\frac{1}{d-c+1}\right) - \frac{\varepsilon_2}{(\gamma+\delta)^2}\left(\sum_{y=1}^{\infty} y\frac{1}{d-c+1}\right)\left(1-e^{-(\gamma+\delta)t}\right) \\
&+ \mathrm{N}_0\left(\frac{\beta}{(\alpha+\beta)-(\gamma+\delta)}\right)\left(e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}\right) + \mathrm{M}_0 e^{-(\gamma+\delta)t}
\end{aligned}
\tag{41}
$$

The probability that there is at least one employee in grade 2 is

$$
\begin{aligned}
U_2 &= 1 - G_{\bullet 0\bullet} \\
&= 1 - \exp\left\{\lambda_1 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r \frac{1-e^{-[(\gamma+\delta)s+(\alpha+\beta)(r-s)]t}}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]}\right. \\
&(\frac{1}{b-a+1}) + \lambda_2 t \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}(\frac{1}{b-a+1})\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r \\
&\frac{1}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]} - \lambda_2 \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}(\frac{1}{b-a+1})\binom{x}{r}\binom{r}{s}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r \\
&\frac{1-e^{-[(\gamma+\delta)s+(\alpha+\beta)(r-s)]t}}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]^2}(\frac{1}{b-a+1}) + \varepsilon_1 \sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}(\frac{1}{d-c+1})\binom{y}{u} \\
&\frac{1-e^{-(\gamma+\delta)ut}}{(\gamma+\delta)\,u} + \varepsilon_2 t \sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}(\frac{1}{d-c+1})\binom{y}{u}\frac{1}{(\gamma+\delta)u} - \varepsilon_2 \sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u} \\
&\frac{1}{(d-c+1)}\binom{y}{u}\frac{1-e^{-(\gamma+\delta)ut}}{[(\gamma+\delta)\,u]^2}\}[1-\frac{\beta[e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}]}{[(\gamma+\delta)-(\alpha+\beta)]}]^{N_0} \\
&[1-e^{-(\gamma+\delta)t}]^{M_0};|Z_2|<1
\end{aligned}
\tag{42}
$$

The mean number of employees in grade 3 of the organization is

$$
\begin{aligned}
L_3 =&\lambda_1 \left(\sum_{x=1}^{\infty} x\frac{1}{b-a+1}\right)\left[\frac{\beta\delta\left[1-e^{-(\alpha+\beta)t}\right]]}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\alpha+\beta)\right](\alpha+\beta)}\right.\\
&\left.+\frac{\beta\delta\left[1-e^{-\theta t}\right]}{[\theta-(\gamma+\delta)]\left[\theta-(\alpha+\beta)\right]\theta}-\frac{\beta\delta\left[1-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\gamma+\delta)\right](\gamma+\delta)}\right]\\
&+\lambda_2 t\left(\sum_{x=1}^{\infty} x\frac{1}{b-a+1}\right)\left[\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\alpha+\beta)\right](\alpha+\beta)}\right.\\
&\left.-\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\gamma+\delta)\right](\gamma+\delta)}+\frac{\beta\delta}{[\theta-(\gamma+\delta)]\left[\theta-(\alpha+\beta)\theta\right](\gamma+\delta)}\right]\\
&-\lambda_2\left(\sum_{x=1}^{\infty} x\frac{1}{b-a+1}\right)\left[\frac{\beta\delta\left[1-e^{-(\alpha+\beta)t}\right]]}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\alpha+\beta)\right](\alpha+\beta)^2}\right.\\
&\left.+\frac{\beta\delta\left[1-e^{-\theta t}\right]}{[\theta-(\gamma+\delta)]\left[\theta-(\alpha+\beta)\right]\theta^2}-\frac{\beta\delta\left[1-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\gamma+\delta)\right](\gamma+\delta)^2}\right]\\
&+\mathrm{M}_0\left(\frac{\delta\left[e^{-(\gamma+\delta)t}-e^{-\theta t}\right]}{[\theta-(\gamma+\delta)]}\right)+\mathrm{K}_0 e^{-\theta t}+\frac{\varepsilon_1\delta}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty} y\frac{1}{d-c+1}\right)\\
&\left[\left(\frac{1-e^{-\theta t}}{\theta}\right)\left(\frac{\left[e^{-\theta t}-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-\theta]}\right)\right]+\frac{\varepsilon_2 t\delta}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty} y\frac{1}{d-c+1}\right)\left(\frac{1}{\theta}\right)\\
&-\frac{\varepsilon_2\delta}{(\gamma+\delta)^2}\left(\sum_{y=1}^{\infty} y\frac{1}{d-c+1}\right)\left[\left(\frac{(1-e^{-\theta t})(\gamma+\delta+\theta)}{\theta^2}\right)-\left(\frac{\left[e^{-\theta t}-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-\theta]}\right)\right]\\
&+\mathrm{N}_0\left[\frac{\beta\delta\left[e^{-(\alpha+\beta)t}-e^{-\theta t}\right]]}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\alpha+\beta)\right]}-\frac{\beta\delta\left[e^{-(\gamma+\delta)t}-e^{-\theta t}\right]}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\gamma+\delta)\right]}\right]
\end{aligned}
\tag{43}
$$

The probability that there is at least one employee in grade 3 is

$$
\begin{aligned}
U_3 &= 1-G_{\bullet\bullet 0}(t)\\
&= 1-\exp\left\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{r+s-f}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\binom{s}{f}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f\right.\\
&\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)}\left[\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}\right]^{r-s}\\
&\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]}+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{r+s-f}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\binom{s}{f}\\
&\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)}\\
&\left[\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}\right]^{r-s}\frac{1}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]}\\
&-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}\sum_{f=0}^{s}(-1)^{r+s-f}\binom{x}{r}\binom{r}{s}\binom{s}{f}\left(\frac{1}{b-a+1}\right)\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^f\\
&\left(\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right)^{(s-f)}\left(\frac{\beta\delta}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}\right)^{(r-s)}\\
&\frac{1-e^{-[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]t}}{[\theta f+(\gamma+\delta)(s-f)+(\alpha+\beta)(r-s)]^2}+\varepsilon_1\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{3u-v}\left(\frac{1}{d-c+1}\right)\binom{y}{u}\binom{u}{v}\\
&\left[\frac{\delta}{\theta-(\gamma+\delta)}\right]^u\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)(u-v)]}+\varepsilon_2 t\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{3u-v}\left(\frac{1}{d-c+1}\right)\binom{y}{u}\binom{u}{v}
\end{aligned}
\tag{44}
$$

$$[\frac{\delta}{\theta-(\gamma+\delta)}]^{(u-v)}\frac{1}{[\theta v+(\gamma+\delta)\,(u-v)]}-\varepsilon_2\sum_{y=1}^{\infty}\sum_{u=1}^{y}\sum_{v=0}^{u}(-1)^{3u-v}(\frac{1}{d-c+1})\left(\begin{array}{c}y\\u\end{array}\right)\left(\begin{array}{c}u\\v\end{array}\right)$$
$$[\frac{\delta}{\theta-(\gamma+\delta)}]^{u}\frac{1-e^{-[\theta v+(\gamma+\delta)(u-v)]t}}{[\theta v+(\gamma+\delta)\,(u-v)]^2}\}[1-\frac{\beta\delta[e^{-(\alpha+\beta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]}$$
$$+\frac{\beta\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}]^{N_0}[1-\frac{\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[\theta-(\gamma+\delta)]}]^{M_0}$$
$$[1-e^{-\theta t}]^{K_0};|Z_3|<1$$

The mean number of employees in the organization is $L=L_1+L_2+L_3$. Substituting the values of L_1, L_2 and L_3 from the equations (40), (42), and (44), we get

$$\begin{aligned}
L=&\frac{\lambda_1}{(\alpha+\beta)}(\sum_{x=1}^{\infty}x\frac{1}{b-a+1})\left[1-e^{-(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{(\alpha+\beta)}\sum_{x=1}^{\infty}x\left(\frac{1}{b-a+1}\right)\\
&-\frac{\lambda_2}{(\alpha+\beta)^2}(\sum_{x=1}^{\infty}x\frac{1}{b-a+1})\left[1-e^{-(\alpha+\beta)t}\right]+\mathrm{N}_0e^{-(\alpha+\beta)t}+\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)\\
&\left[\left(\frac{1-e^{-(\gamma+\delta)t}}{\gamma+\delta}\right)-\left(\frac{e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}}{(\alpha+\beta)-(\gamma+\delta)}\right)\right]-\frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)\\
&\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)+\mathrm{N}_0\left(\frac{\beta}{(\alpha+\beta)-(\gamma+\delta)}\right)\left(e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}\right)+\mathrm{M}_0e^{-(\gamma+\delta)t}\\
&+\frac{\varepsilon_1}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty}y\frac{1}{d-c+1}\right)\left(1-e^{-(\gamma+\delta)t}\right)+\frac{\varepsilon_2 t}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty}y\frac{1}{d-c+1}\right)\\
&-\frac{\varepsilon_2}{(\gamma+\delta)^2}\left(\sum_{y=1}^{\infty}y\frac{1}{d-c+1}\right)\left(1-e^{-(\gamma+\delta)t}\right)+\lambda_1(\sum_{x=1}^{\infty}x\frac{1}{b-a+1})\\
&\left[\frac{(1-e^{-(\gamma+\delta)t})(\alpha+\beta+\gamma+\delta)}{(\gamma+\delta)^2}+\left(\frac{e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}}{(\gamma+\delta)-(\alpha+\beta)}\right)\right]+\frac{\lambda_2\beta t}{(\gamma+\delta)(\alpha+\beta)}\\
&(\sum_{x=1}^{\infty}x\frac{1}{b-a+1})\mathrm{N}_0\left(\frac{\beta}{(\alpha+\beta)-(\gamma+\delta)}\right)\left(e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}\right)+\mathrm{M}_0e^{-(\gamma+\delta)t}\\
&+\frac{\varepsilon_1}{(\gamma+\delta)}(\sum_{y=1}^{\infty}y\frac{1}{d-c+1})(1-e^{-(\gamma+\delta)t})+\frac{\varepsilon_2 t}{(\gamma+\delta)}(\sum_{y=1}^{\infty}y\frac{1}{d-c+1}) \qquad (45)\\
&-\frac{\varepsilon_2}{(\gamma+\delta)^2}(\sum_{y=1}^{\infty}y\frac{1}{d-c+1})(1-e^{-(\gamma+\delta)t})+\lambda_1(\sum_{x=1}^{\infty}x\frac{1}{b-a+1})\\
&\left[\frac{\beta\delta\left[1-e^{-(\alpha+\beta)t}\right]]}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\alpha+\beta)\right](\alpha+\beta)}+\frac{\beta\delta\left[1-e^{-\theta t}\right]}{[\theta-(\gamma+\delta)]\left[\theta-(\alpha+\beta)\right]\theta}\right.\\
&\left.-\frac{\beta\delta\left[1-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\gamma+\delta)\right](\gamma+\delta)}\right]+\lambda_2 t\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)\\
&\left[\frac{\beta\delta}{[\theta-(\gamma+\delta)]\left[\theta-(\alpha+\beta)\right]\theta}+\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\alpha+\delta)\right](\alpha+\delta)}\right.\\
&\left.-\frac{\beta\delta}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\gamma+\delta)\right](\gamma+\delta)}\right]+\frac{\varepsilon_1\delta}{(\gamma+\delta)}(\sum_{y=1}^{\infty}y\frac{1}{d-c+1})\\
&\left[(\frac{1-e^{-\theta t}}{\theta})-(\frac{\left[e^{-\theta t}-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-\theta]})\right]+\frac{\varepsilon_2 t\delta}{(\gamma+\delta)}(\sum_{y=1}^{\infty}y\frac{1}{d-c+1})(\frac{1}{\theta})\\
&-\frac{\varepsilon_2\delta}{(\gamma+\delta)^2}(\sum_{y=1}^{\infty}y\frac{1}{d-c+1})\left[(\frac{(1-e^{-\theta t})(\gamma+\delta+\theta)}{\theta^2})-(\frac{\left[e^{-\theta t}-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-\theta]})\right]\\
&-\lambda_2\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)\left[\frac{\beta\delta\left[1-e^{-(\alpha+\beta)t}\right]]}{[(\gamma+\delta)-(\alpha+\beta)]\left[\theta-(\alpha+\beta)\right](\alpha+\beta)^2}\right.
\end{aligned}$$

$$+\frac{\beta\delta\left[1-e^{-\theta t}\right]}{[\theta-(\gamma+\delta)]\,[\theta-(\alpha+\beta)]\,\theta^2}-\frac{\beta\delta\left[1-e^{-(\gamma+\delta)t}\right]}{[(\gamma+\delta)-(\alpha+\beta)]\,[\theta-(\gamma+\delta)]\,(\gamma+\delta)^2}\Bigg]$$

$$+\mathrm{M}_0\left(\frac{\delta\left[e^{-(\gamma+\delta)t}-e^{-\theta t}\right]}{[\theta-(\gamma+\delta)]}\right)+\mathrm{K}_0e^{-\theta t}+\mathrm{N}_0\Bigg[\frac{\beta\delta\left[e^{-(\alpha+\beta)t}-e^{-\theta t}\right]]}{[(\gamma+\delta)-(\alpha+\beta)]\,[\theta-(\alpha+\beta)]}$$

$$-\frac{\beta\delta\left[e^{-(\gamma+\delta)t}-e^{-\theta t}\right]}{[(\gamma+\delta)-(\alpha+\beta)]\,[\theta-(\gamma+\delta)]}\Bigg]$$

The average duration of stay of an employee in grade 1 is

$$W_1=\frac{L_1}{(\alpha+\beta)(1-G_{0\bullet\bullet(t)})}=\frac{A}{B}\quad \text{where,} \tag{46}$$

$$A=\Bigg\{\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=1}^{\infty}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{(\alpha+\beta)}\sum_{x=1}^{\infty}x\left(\frac{1}{b-a+1}\right)$$
$$-\frac{\lambda_2}{(\alpha+\beta)^2}\sum_{x=1}^{\infty}x\left(\frac{1}{b-a+1}\right)\left[1-e^{-(\alpha+\beta)t}\right]+\mathrm{N}_0e^{-(\alpha+\beta)t}\Bigg\}$$

$$B=(\alpha+\beta)\Bigg(1-\exp\Bigg\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\begin{pmatrix}x\\r\end{pmatrix}(-1)^r\frac{1-e[-[(\alpha+\beta)rt]}{(\alpha+\beta)r}$$
$$+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\begin{pmatrix}x\\r\end{pmatrix}\frac{(-1)^r}{(\alpha+\beta)^r}-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\begin{pmatrix}x\\r\end{pmatrix}(-1)^r$$
$$\frac{1-e^{-[(\alpha+\beta)rt]}}{[(\alpha+\beta)r]^2}\Bigg\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0}\Bigg)$$

The average duration of stay of an employee in grade 2 is

$$W_2=\frac{L_2}{\gamma(1-G_{\bullet 0\bullet(t)})}=\frac{C}{D}\quad \text{where,} \tag{47}$$

$$C=\Bigg\{\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)\left[\left(\frac{1-e^{-(\gamma+\delta)t}}{\gamma+\delta}\right)-\left(\frac{e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}}{(\alpha+\beta)-(\gamma+\delta)}\right)\right]$$
$$+\frac{\lambda_2\beta t}{(\gamma+\delta)(\alpha+\beta)}\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)-\frac{\lambda_2\beta}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)$$
$$\left[\frac{\left(1-e^{-(\gamma+\delta)t}\right)(\alpha+\beta+\gamma+\delta)}{(\gamma+\delta)^2}+\left(\frac{e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}}{(\gamma+\delta)-(\alpha+\beta)}\right)\right]$$
$$+\frac{\varepsilon_1}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty}y\frac{1}{d-c+1}\right)\left(1-e^{-(\gamma+\delta)t}\right)+\frac{\varepsilon_2 t}{(\gamma+\delta)}\left(\sum_{y=1}^{\infty}y\frac{1}{d-c+1}\right)$$
$$-\frac{\varepsilon_2}{(\gamma+\delta)^2}\left(\sum_{y=1}^{\infty}y\frac{1}{d-c+1}\right)\left(1-e^{-(\gamma+\delta)t}\right)+\mathrm{N}_0\left(\frac{\beta}{(\alpha+\beta)-(\gamma+\delta)}\right)$$
$$\left(e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}\right)+\mathrm{M}_0e^{-(\gamma+\delta)t}\Bigg\}$$

$$D=\gamma\Bigg(1-\exp\Bigg\{\lambda_1\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}\begin{pmatrix}x\\r\end{pmatrix}\begin{pmatrix}r\\s\end{pmatrix}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r\frac{1-e^{-[(\gamma+\delta)s+(\alpha+\beta)(r-s)]t}}{[(\gamma+\delta)s+(\alpha+\beta)(r-s)]}$$
$$(\frac{1}{b-a+1})+\lambda_2 t\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}(\frac{1}{b-a+1})\begin{pmatrix}x\\r\end{pmatrix}\begin{pmatrix}r\\s\end{pmatrix}\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^r$$

$$\frac{1}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]}-\lambda_2\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{r+s}(\frac{1}{b-a+1})\left(\begin{array}{c}x\\r\end{array}\right)\left(\begin{array}{c}r\\s\end{array}\right)\left(\frac{\beta}{(\gamma+\delta)-(\alpha+\beta)}\right)^{r}$$
$$\frac{1-e^{-[(\gamma+\delta)s+(\alpha+\beta)(r-s)]t}}{[(\gamma+\delta)\,s+(\alpha+\beta)\,(r-s)]^2}(\frac{1}{b-a+1})+\varepsilon_1\sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}(\frac{1}{d-c+1})\left(\begin{array}{c}y\\u\end{array}\right)$$
$$\frac{1-e^{-(\gamma+\delta)ut}}{(\gamma+\delta)\,u}+\varepsilon_2 t\sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}(\frac{1}{d-c+1})\left(\begin{array}{c}y\\u\end{array}\right)\frac{1}{(\gamma+\delta)u}-\varepsilon_2\sum_{y=1}^{\infty}\sum_{u=1}^{y}(-1)^{3u}$$
$$\left.\left(\frac{1}{d-c+1}\right)\left(\begin{array}{c}y\\u\end{array}\right)\frac{1-e^{-(\gamma+\delta)ut}}{[(\gamma+\delta)u]^2}\right\}\left[1-\frac{\beta[e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}]}{[(\gamma+\delta)-(\alpha+\beta)]}\right]^{N_0}\left[1-e^{-(\gamma+\delta)t]M_0}\right]\Bigg)$$

The average duration of stay of an employee in grade 3 is

$$W_3=\frac{L_3}{\theta(1-G_{\bullet\bullet 0(t)})} \tag{48}$$

The variance of the number of employees in grade 1 is

$$\begin{aligned}V_1=&\frac{\lambda_1}{2(\alpha+\beta)}\sum_{x=1}^{\infty}x(x-1)\left(\frac{1}{b-a+1}\right)\left[1-e^{-2(\alpha+\beta)t}\right]+\frac{\lambda_1}{(\alpha+\beta)}\sum_{x=1}^{\infty}x\left(\frac{1}{b-a+1}\right)\\&\left[1-e^{-(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{2(\alpha+\beta)}\left(\sum_{x=1}^{\infty}x(x-1)\left(\frac{1}{b-a+1}\right)\right)-\frac{\lambda_2}{[2(\alpha+\beta)]^2}\\&\left(\sum_{x=1}^{\infty}x(x-1)\right)\left(\frac{1}{b-a+1}\right)\left[1-e^{-2(\alpha+\beta)t}\right]+\frac{\lambda_2 t}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty}x\left(\frac{1}{b-a+1}\right)\right)\\&-\frac{\lambda_2}{(\alpha+\beta)^2}\left(\sum_{x=1}^{\infty}x\left(\frac{1}{b-a+1}\right)\right)\left[1-e^{-(\alpha+\beta)t}\right]+N_0\left[e^{-(\alpha+\beta)t}-e^{-2(\alpha+\beta)t}\right]\end{aligned} \tag{49}$$

The variance of the number of employees in grade 2 is

$$\begin{aligned}V_2=&\frac{\lambda_1\beta^2}{[(\gamma+\delta)-(\alpha+\beta)]^2}\left(\sum_{x=1}^{\infty}x(x-1)\frac{1}{b-a+1}\right)\left[\left(\frac{1-e^{-2(\alpha+\beta)t}}{2(\alpha+\beta)}\right)\right.\\&\left.-2\left(\frac{1-e^{-(\alpha+\beta+\gamma+\delta)t}}{(\alpha+\beta+\gamma+\delta)}\right)+\left(\frac{1-e^{-2(\gamma+\delta)t}}{2(\gamma+\delta)}\right)\right]+\frac{\lambda_2 t\beta^2}{[(\gamma+\delta)-(\alpha+\beta)]^2}\\&\left(\sum_{x=1}^{\infty}x(x-1)\frac{1}{b-a+1}\right)\left[\left(\frac{1}{2(\alpha+\beta)}\right)\right.\\&\left.-\left(\frac{2}{(\alpha+\beta+\gamma+\delta)}\right)+(\frac{1}{2(\gamma+\delta)})\right]-\frac{\lambda_2 t\beta^2}{[(\gamma+\delta)-(\alpha+\beta)]^2}(\sum_{x=1}^{\infty}x(x-1)C_x)\\&\left[(\frac{1-e^{-2(\alpha+\beta)t}}{(2(\alpha+\beta))^2})-2(\frac{1-e^{-(\alpha+\beta+\gamma+\delta)t}}{(\alpha+\beta+\gamma+\delta)^2})+(\frac{1-e^{-2(\gamma+\delta)t}}{[2(\gamma+\delta)]^2})\right]\\&+\frac{\lambda_1\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)\left[\left(\frac{1-e^{-(\gamma+\delta)t}}{\gamma+\delta}\right)-\left(\frac{e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}}{(\gamma+\delta)-(\alpha+\beta)}\right)\right]\\&+\frac{\lambda_2 t\beta}{(\alpha+\beta)}\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)\left(\frac{1}{\gamma+\delta}\right)-\frac{\lambda_2\beta}{(\alpha+\beta)^2}\left[\frac{(\alpha+\beta+\gamma+\delta)\left(1-e^{-(\gamma+\delta)t}\right)}{(\gamma+\delta)^2}\right.\\&\left.-\left(\frac{e^{-(\alpha+\beta)t}-e^{-(\gamma+\delta)t}}{(\gamma+\delta)-(\alpha+\beta)}\right)\right]\left(\sum_{x=1}^{\infty}x\frac{1}{b-a+1}\right)+\varepsilon_1\left(\sum_{y=1}^{\infty}y(y-1)\frac{1}{d-c+1}\right)\left(\frac{1-e^{-2(\gamma+\delta)t}}{[2(\gamma+\delta)]}\right)\end{aligned} \tag{50}$$

$$+\varepsilon_2 t\left(\sum_{y=1}^{\infty} y(y-1)\frac{1}{d-c+1}\right)\left(\frac{1}{[2(\gamma+\delta)]}\right)+\varepsilon_2 t\left(\sum_{y=1}^{\infty} y\frac{1}{d-c+1}\right)$$
$$-\varepsilon_2\left(\sum_{y=1}^{\infty} y(y-1)\frac{1}{d-c+1}\right)\left(\frac{1-e^{-2(\gamma+\delta)t}}{[2(\gamma+\delta)]^2}\right)+\varepsilon_1 t\left(\sum_{y=1}^{\infty} y(\frac{1}{d-c+1}\right)(\frac{1-e^{-(\gamma+\delta)t}}{(\gamma+\delta)})$$
$$-\varepsilon_2\left(\sum_{y=1}^{\infty} y\frac{1}{d-c+1}\right)\left(\frac{1-e^{-(\gamma+\delta)t}}{(\gamma+\delta)^2}\right)+\frac{N_0\beta}{(\alpha+\beta)-(\gamma+\delta)}\left[e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t}\right]$$
$$\left[1-\frac{\beta}{[(\alpha+\beta)-(\gamma+\delta)]^2}(e^{-(\gamma+\delta)t}-e^{-(\alpha+\beta)t})^2\right]+M_0(e^{-(\gamma+\delta)t}-e^{-2(\gamma+\delta)t})$$

The variance of the number of employees in grade 3 is

$$\begin{aligned}
V_3 =&\lambda_1\beta\delta\sum_{x=1}^{\infty} x(\frac{1}{b-a+1})\left[\frac{1-e^{-(\alpha+\beta)t}}{[(\gamma+\delta)-(\alpha+\beta)]\,[\theta-(\alpha+\beta)]\,(\alpha+\beta)}\right.\\
&\left.+\frac{1-e^{-\theta t}}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]\theta}-\frac{1-e^{-(\gamma+\delta)t}}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)](\gamma+\delta)}\right]\\
&+\lambda_2 t\beta\delta\sum_{x=1}^{\infty} x(\frac{1}{b-a+1})\left[\frac{1}{[(\gamma+\delta)-(\alpha+\beta)]\,[\theta-(\alpha+\beta)]\,(\alpha+\beta)}\right.\\
&\left.+\frac{1}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]\theta}-\frac{1}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)](\gamma+\delta)}\right]\\
&-\lambda_2\beta\delta\sum_{x=1}^{\infty} x(\frac{1}{b-a+1})\left[\frac{1-e^{-(\alpha+\beta)t}}{[(\gamma+\delta)-(\alpha+\beta)]\,[\theta-(\alpha+\beta)]\,(\alpha+\beta)^2}\right.\\
&\left.+\frac{1-e^{-\theta t}}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]\theta^2}-\frac{1-e^{-(\gamma+\delta)t}}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)](\gamma+\delta)^2}\right]\\
&+N_0\left[\frac{\beta\delta[e^{-(\alpha+\beta)t}-e^(-\theta t)]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]}-\frac{\beta\delta[e^{-(\gamma+\delta)t}-e^(-\theta t)]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right]\\
&+M_0\frac{\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[\theta-(\gamma+\delta)]}+K_0e^{-\theta t}+\lambda_1(\beta\delta)^2\sum_{x=1}^{\infty} x(x-1)(\frac{1}{b-a+1})\\
&\left[\frac{1-e^{-2(\alpha+\beta)t}}{[(\gamma+\delta)-(\alpha+\beta)]^2[\theta-(\alpha+\beta)]^2 2(\alpha+\beta)}-2(\frac{1}{[(\gamma+\delta)-(\alpha+\beta)]^2})\right.\\
&\frac{1}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]}(\frac{1-e^{-(\alpha+\beta+\gamma+\delta)t}}{\alpha+\beta+\gamma+\delta})+2(\frac{1}{\theta-(\alpha+\beta)})^2(\frac{1-e^{-(\alpha+\beta+\theta)t}}{\alpha+\beta+\theta})\\
&\frac{1}{[\theta-(\gamma+\delta)][(\gamma+\delta)-(\alpha+\beta)]}+\frac{1}{[\theta-(\gamma+\delta)]^2[(\gamma+\delta)-(\alpha+\beta)]^2}(\frac{1-e^{-2(\gamma+\delta)t}}{2(\gamma+\delta)})\\
&-2(\frac{1}{\theta-(\gamma+\delta)})^2(\frac{1-e^{-(\gamma+\delta+\theta)t}}{\gamma+\delta+\theta})\frac{1}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}+(\frac{1-e^{-2\theta t}}{2\theta})\\
&\left.\frac{1}{[\theta-(\gamma+\delta)]^2[\theta-(\alpha+\beta)]^2}\right]+\lambda_2 t(\beta\delta)^2\sum_{x=1}^{\infty} x(x-1)(\frac{1}{b-a+1})\\
&\left[\frac{1}{[(\gamma+\delta)-(\alpha+\beta)]^2[\theta-(\alpha+\beta)]^2 2(\alpha+\beta)}-2(\frac{1}{[(\gamma+\delta)-(\alpha+\beta)]^2})\right.\\
&\frac{1}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]}(\frac{1}{\alpha+\beta+\gamma+\delta})+2(\frac{1}{\theta-(\alpha+\beta)})^2(\frac{1}{\alpha+\beta+\theta})\\
&\frac{1}{[\theta-(\gamma+\delta)][(\gamma+\delta)-(\alpha+\beta)]}+\frac{1}{[\theta-(\gamma+\delta)]^2[(\gamma+\delta)-(\alpha+\beta)]^2}(\frac{1}{2(\gamma+\delta)})\\
&-2(\frac{1}{\theta-(\gamma+\delta)})^2(\frac{1}{\gamma+\delta+\theta})\frac{1}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}+(\frac{1}{2\theta})\\
&\left.\frac{1}{[\theta-(\gamma+\delta)]^2[\theta-(\alpha+\beta)]^2}\right]-\lambda_2(\beta\delta)^2\sum_{x=1}^{\infty} x(x-1)(\frac{1}{b-a+1})
\end{aligned} \tag{51}$$

$$\left[\frac{1-e^{-2(\alpha+\beta)t}}{[(\gamma+\delta)-(\alpha+\beta)]^2[\theta-(\alpha+\beta)]^2[2(\alpha+\beta)]^2} - 2(\frac{1}{[(\gamma+\delta)-(\alpha+\beta)]^2})\right.$$
$$\frac{1}{[\theta-(\gamma+\delta)][\theta-(\alpha+\beta)]}(\frac{1-e^{-(\alpha+\beta+\gamma+\delta)t}}{[\alpha+\beta+\gamma+\delta]^2}) + 2(\frac{1}{\theta-(\alpha+\beta)})^2(\frac{1-e^{-(\alpha+\beta+\theta)t}}{[\alpha+\beta+\theta]^2})$$
$$\frac{1}{[\theta-(\gamma+\delta)][(\gamma+\delta)-(\alpha+\beta)]} + \frac{1}{[\theta-(\gamma+\delta)]^2[(\gamma+\delta)-(\alpha+\beta)]^2}$$
$$(\frac{1-e^{-2(\gamma+\delta)t}}{[2(\gamma+\delta)]^2} - 2(\frac{1}{\theta-(\gamma+\delta)})^2(\frac{1-e^{-(\gamma+\delta+\theta)t}}{[\gamma+\delta+\theta]^2})\frac{1}{[\theta-(\alpha+\beta)][(\gamma+\delta)-(\alpha+\beta)]}$$
$$\left. + (\frac{1-e^{-2\theta t}}{(2\theta)^2})\frac{1}{[\theta-(\gamma+\delta)]^2[\theta-(\alpha+\beta)]^2}\right] - N_0\left[\frac{\beta\delta[e^{-(\alpha+\beta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\alpha+\beta)]}\right.$$
$$\left. - \frac{\beta\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{[(\gamma+\delta)-(\alpha+\beta)][\theta-(\gamma+\delta)]}\right]^2 - M_0(\frac{\delta[e^{-(\gamma+\delta)t}-e^{-\theta t}]}{\theta-(\gamma+\delta)})^2 - K_0e^{-2\theta t}$$
$$+\varepsilon_1(\frac{\delta}{\theta-(\gamma+\delta)})^2(\sum_{y=1}^{\infty} y(y-1)\frac{1}{d-c+1})\left[\frac{(1-e^{-2(\gamma+\delta)t})}{2(\gamma+\delta)} - \frac{2[1-e^{-(\theta+\gamma+\delta)t}]}{(\theta+\gamma+\delta)}\right.$$
$$\left. + \frac{1-e^{-2\theta t}}{2\theta}\right] + \varepsilon_2 t(\frac{\delta}{\theta-(\gamma+\delta)})^2(\sum_{y=1}^{\infty} y(y-1)\frac{1}{d-c+1})$$
$$\left\{\frac{1}{2(\gamma+\delta)} - \frac{2}{(\theta+\gamma+\delta)} + \frac{1}{2\theta}\right\} - \varepsilon_2(\frac{\delta}{\theta-(\gamma+\delta)})^2(\sum_{y=1}^{\infty} y(y-1)\frac{1}{d-c+1})$$
$$\left\{\frac{(1-e^{-2(\gamma+\delta)t})}{[2(\gamma+\delta)]^2} - \frac{2[1-e^{-(\theta+\gamma+\delta)t}]}{(\theta+\gamma+\delta)^2} + \frac{1-e^{-2\theta t}}{(2\theta)^2}\right\}$$

The coefficient of variation of the number of employees in grade 1 is

$$CV_1 = \frac{\sqrt{V_1}}{L_1} \tag{52}$$

Where L_1 and V_1 are as given in equations (39) and (49). The coefficient of variation of the number of employees in grade 2 is

$$CV_2 = \frac{\sqrt{V_2}}{L_2} \tag{53}$$

Where L_2 and V_2 are as given in equations (41) and (50). The coefficient of variation of the number of employees in grade 3 is

$$CV_3 = \frac{\sqrt{V_3}}{L_3} \tag{54}$$

where L_3 and V_3 are as given in equations (43) and (51).

4.1 NUMERICAL ILLUSTRATION AND RESULTS

In this section, the behavior of the model is discussed through a numerical illustration. Different values of the parameters are considered for recruitment, promotion rate and leaving rates of the system. Since the performance characteristics of the manpower model are highly sensitive with respect to the time, the transient behavior of the model is studied through computing the performance measures with the following set of values for the model parameters.

Using the equations (39), (41) and (43) the average number of employees in grade 1, in grade 2 and in grade 3 are computed and presented in Table 1. The relationship between the change in parameters and the average number of employees in each grade are shown in figures 2. It is observed that the average number of employees in the grade 1, grade 2 and grade 3 in the organization is highly sensitive with respect to changes in time. As time (t) varies from 0.1 to 1 units, the average number of employees in the grade 1 decreasing from 981.134 to 831.102, when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization decreases from 210.434 to 11.372 and also the average number of employees in grade 3 in the organization decreases from 186.631 to 11.024 for given values of other parameters and the average number of employees in grade 1, grade 2, and grade 3 are simultaneously decreasing with respect to time. As the leaving rate of grade 1 employee (α) varies from 0.1 to 0.7 units, the average number of employees in grade 1 reduces from 843.474 to 468.731 when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization reduces from 13.380 to 9.508 and in grade 3 in the organization reduces from 12.601 to 9.661 for given values of other parameters. The average number of employees in grade 1, grade 2 and grade 3 are simultaneously decreasing with respect to the leaving rate of grade 1 employee (α) increasing.

The recruitment parameter in grade 1 (λ_1) varies from 3 to 6 units; the average number of employees in grade 1 is increasing from 831.102 to 839.259, when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization increases from 11.3772 to 11.453, and also in grade 3 in the organization increases from 11.024 to 11.083 for given values of other parameters. The average number of employees in grade 1, grade 2 and grade 3 are simultaneously increasing with respect to the recruitment parameter λ_1 increases.

The other recruitment parameter in grade 1 (λ_2) varies from 3 to 6 units; the average number of employees in grade 1 is increasing from 839.259 to 843.474, when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization increases from 11.453 to 11.491, and also in grade 3 in the organization increases from 11.083 to 11.107 for given values of other parameters. The average number of employees in grade 1, grade 2 and grade 3 are simultaneously increasing with respect to the recruitment parameter λ_2 increases.

When the promotion rate from grade 1 to grade $2(\beta)$ varies from 0.1 to 0.7, the average number of employees in grade 1 reduces from 468.731 to 262.222, when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization increases from 9.508 to 27.632, and also in grade 3 in the organization increases from 9.661 to 29.085 for given values of other parameters.

The recruitment parameter in grade $2(\varepsilon_1)$ varies from 3 to 6 units; the average number of employees in grade 1 remains unchanged, when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization increases from 11.491 to 12.491, and also in grade 3 in the organization increases from 11.107 to 11.961 for given values of other parameters. The average number of employees in grade 2 and grade 3 are simultaneously increasing with respect to the recruitment parameter ε_1 increases.

The other recruitment parameter in grade $2(\varepsilon_2)$ varies from 3 to 6 units; the average number of employees in grade 1 remains unchanged, when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization increases from 12.491 to 13.380, and also in grade 3 in the organization increases from 11.961 to 12.601 for given values of other parameters. The average number of employees in grade 2 and grade 3 are simultaneously increasing with respect to the recruitment parameter ε_2 increases.

As the leaving rate of grade 2 employee (γ) varies from 3 to 7 units, the average number of employees in grade 1 remains unchanged when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization reduces from 27.632 to 18.327, and also in grade 3 in the organization reduces from 29.085 to 19.061, for given values of other parameters.

When the promotion rate from grade 2 to grade 3 (δ) varies from 6 to 9, the average number of employees in grade 1 remains unchanged, when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization decreases from 18.327 to 14.648, and also in grade 3 in the organization increases from 19.061 to 22.740 for given values of other parameters.

As the leaving rate of grade 3 employee (θ) varies from 7 to 10 units, the average number of employees in grade 1 remains unchanged when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization remains unchanged, and also in grade 3 in the organization reduces from 22.740 to 14.790, for given values of other parameters.

It is further observed that the initial number of employees in grade 1 (N_0) , grade 2 (M_0) , and grade 3 (K_0) in the organization have a vital influence on the average number of employees in grade 1, grade 2 and grade 3. As the initial number of employees in grade $1(N_0)$ increases from 500 to 1000, the average number of employees in grade 1 increases from 188.402 to 311.700, the average number of employees in grade 2 is also increases from 17.680 to 23.591, and in grade 3 the average number of employees increases from 16.154 to 22.337, when the other parameters are fixed. As the initial number of employees in grade 2 (M_0) increases from 250 to 750, the average number of employees in grade 1 and in grade 2 in the organization have no influence, but in grade 3 the average number of employees increases from 22.320 to 22.354, when other parameters are fixed, the initial number of employees in grade 3 increases from 100 to 700, the average number of employees in grade 1 and in grade 2 remains unchanged, but in grade 3 increases from 22.364 to 22.382 for given values of other parameters. As the uniform batch size distribution parameter of grade 1 (a) varies from 1 to 5 units the average number of employees in the grade 1 is increasing from 262.222 to 272.638, when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization is increasing from 14.648 to 15.077 and in grade 3 also increasing from 14.790 to 15.138 for given values of other parameters. It is further observed that the batch size distribution parameter of grade 1 (b) varies from 5 to 20 units the average number of employees in the grade 1 is increasing from 272.638 to 311.700, in grade 2 from 15.077 to 16.689 and also in grade 3 from 15.138 to 16.446, when other parameters are fixed.

As the uniform batch size distribution parameter of grade 2 (c) varies from 1 to 5 units the average number of employees in the grade 1 remains unchanged, when other parameters are fixed. Similarly, the average number of employees in grade 2 in the organization is increasing from 16.689 to 18.142 and in grade 3 also increasing from 16.446

to 17.687 for given values of other parameters. It is further observed that the batch size distribution parameter of grade 2 (d) varies from 5 to 20 units the average number of employees in the grade 1 remains unchanged, in grade 2 from 18.142 to 23.591 and also in grade 3 from 17.687 to 22.337, when other parameters are fixed.

Using the equations (46), (47), and (48) the average duration of stay of an employee in grade 1, in grade 2, and in grade 3 in the organization at different values of the parameters are computed and presented in Table 2. The relationship between the change in parameters and the average duration of stay of an employee in each grade are shown in figure 3.

It is observed that the average duration of stay of an employee in grade 1, grade 2 and grade 3 in the organization are highly sensitive with respect to changes in time. As time varies from 0.1 to 1.0 units, the average duration of stay of an employee in the grade 1 decreasing from 4905.672 to 4155.511, when other parameters are fixed. The average duration of stay of an employee in the grade 2 in the organization decreasing from 23.498 to 1.264 for given values of other parameters. Similarly, the average duration of stay of an employee in the grade 3 in the organization decreasing from 26.701 to 1.575 for given values of other parameters.

As the leaving rate of grade 1 employee (α) varies from 0.1 to 0.7 units, the average duration of stay of an employee in grade 1 reduces from 4217.369 to 585.914 when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization reduces from 1.487 to 1.056 and in grade 3 in the organization reduces from 1.800 to 1.380 for given values of the other parameters. The average duration of stay of an employee in grade 1, grade 2 and grade 3 are decreasing with respect to the leaving rate of grade 1 employee (α).

The recruitment parameter of grade $1(\lambda_1)$ varies from 3 to 6 units; the average duration of stay of an employee in grade 1 is increasing from 4155.511 to 4196.297, when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization increases from 1.264 to 1.273, and also in grade 3 in the organization increases from 1.575 to 1.583 for given values of other parameters. The average duration of stay of an employee in grade 1, grade 2 and grade 3 are increasing with respect to the recruitment parameter λ_1 increases.

The recruitment parameter of grade $1(\lambda_2)$ varies from 3 to 6 units; the average duration of stay of an employee in grade 1 is increasing from 4196.297 to 4217.369, when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization increases from 1.273 to 1.277, and also in grade 3 in the organization increases from 1.583 to 1.587 for given values of other parameters. The average duration of stay of an employee in grade 1, grade 2 and grade 3 are increasing with respect to the recruitment parameter λ_2 increases.

The recruitment parameter of grade $2(\varepsilon_1)$ varies from 3 to 6 units; the average duration of stay of an employee in grade 1 remains unchanged, when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization increases from 1.277 to 1.388, and also in grade 3 in the organization increases from 1.587 to 1.709 for given values of other parameters. The average duration of stay of an employee in grade 2 and grade 3 are increasing with respect to the recruitment parameter ε_1 increases.

The recruitment parameter of grade $2(\varepsilon_2)$ varies from 3 to 6 units; the average duration of stay of an employee in grade 1 remains unchanged, when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization increases from 1.388 to 1.487, and also in grade 3 in the organization increases from 1.709 to 1.800 for given values of other parameters. The average duration of stay of an employee in grade 2 and grade 3 are increasing with respect to the recruitment parameter ε_2 increases.

When the promotion rate from grade 1 to grade $2(\beta)$ varies from 0.1 to 0.7, the average duration of stay of an employee in grade 1 reduces from 585.914 to 264.951, when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization increases from 1.056 to 2.663, and also in grade 3 in the organization increases from 1.380 to 0.000 for given values of other parameters.

As the leaving rate of grade 2 employee (γ) varies from 6 to 10 units, the average duration of stay of an employee in grade 1 remains unchanged when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization reduces from 1.455 to 0.803, and also in grade 3 in the organization reduces from 2.680 to 1.856, for given values of other parameters.

When the promotion rate from grade 2 to grade $3(\delta)$ varies from 8 to 12, the average duration of stay of an employee in grade 1 unchanged, when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization decreases from 0.631 to 0.418, and also in grade 3 in the organization increases from 2.175 to 2.635 for given values of other parameters.

As the leaving rate of grade 3 employee (θ) varies from 5 to 7 units, the average duration of stay of an employee in grade 1 remains unchanged when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization remains unchanged, and also in grade 3 in the organization reduces from 5.884 to 2.635, for given values of other parameters.

It is further observed that the initial number of employees in grade $1(N_0)$, grade $2(M_0)$, and grade $3(K_0)$ in the organization have a vital influence on the average duration of stay of an employee in grade 1, grade 2 and grade 3. As the initial number of employees in grade 1 (N_0) increases from 500 to 1000, the average duration of stay of an employee in grade 1 increases from 195.277 to 320.774, the average duration of stay of an employee in grade 2 is also increases from 0.611 to 0.775, but in grade 3 the average duration of stay of an employee remains unchanged, when the other parameters are fixed. As the initial number of employees in grade 2 (M_0) increases from 250 to 750, the average duration of stay of an employee in grade 1, in grade 2, and in grade 3 have no influence when the other parameters are fixed, the initial number of employees in grade 3 increases from 100 to 700, the average duration of stay of an employee in grade 1, in grade 2, and in grade 3 remains unchanged for given values of other parameters.

As the uniform batch size distribution parameter of grade 1 (a) varies from 1 to 5 units the average duration of stay of an employee in the grade 1 is increasing from 264.951 to 274.255, when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization is increasing from 0.418 to 0.429 and in grade 3 also increasing from 2.635 to 3.365 for given values of other parameters. It is further observed that the batch size distribution parameter of grade $1(b)$ varies from 5 to 25 units the average duration of stay of an employee in the grade 1 is increasing from 274.255 to 320.774, in grade 2 from 0.429 to 0.485 and in grade 3 is zero, when other parameters are fixed.

As the uniform batch size distribution parameter of grade 2 (c) varies from 1 to 5 units the average duration of stay of an employee in the grade 1 remains unchanged, when other parameters are fixed. Similarly, the average duration of stay of an employee in grade 2 in the organization is increasing from 0.485 to 0.533 and in grade 3 zero for given values of other parameters. It is further observed that the batch size distribution parameter of grade $2(d)$ varies from 5 to 25 units the average duration of stay of an employee in the grade 1 remains unchanged, in grade 2 increasing from 0.533 to 0.775 and in grade 3 is zero, when other parameters are fixed.

Using the equations (49), (50), and (51) the variance of the number of employees in grade 1, in grade 2, and in grade 3 in the organization at different values of the parameters are computed and presented in Table 3. The relationship between the change in parameters and the variance of the number of employees in each grade are shown in figure 4.

It is observed that the variance of the number of employees in the grade 1, grade 2 and grade 3 in the organization is highly sensitive with respect to changes in time. As time (t) varies from 0.1 to 1 units, the variance of the number of employees in the grade 1 increasing from 22.816 to 191.111, when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization decreases from 128.927 to 13.881 and also the variance of the number of employees in grade 3 in the organization decreases from 125.597 to 11.687 for given values of other parameters and the variance of the number of employees in grade 1, grade 2, and grade 3 are simultaneously decreasing with respect to time.

As the leaving rate of grade 1 employee (α) varies from 0.1 to 0.7 units, the variance of the number of employees in grade 1 reduces from 305.814 to 228.905 when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization increasing from 14.667 to 32.430 and in grade 3 in the organization increasing from 11.166 to 30.569 for given values of other parameters. The variance of the number of employees in grade 1 decreasing where as in grade 2 and grade 3 increasing with respect to the leaving rate of grade 1 employee (α).

The recruitment parameter of grade $1(\lambda_1)$ varies from 3 to 6 units; the variance of the number of employees in grade 1 is increasing from 191.111 to 219.049, when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization increases from 13.881 to 13.964, and also in grade 3 in the organization increases from 11.687 to 11.748 for given values of other parameters. The variance of the number of employees in grade 1, grade 2 and grade 3 are simultaneously increasing with respect to the recruitment parameter λ_1 increases. The recruitment parameter of grade $1(\lambda_2)$ varies from 3 to 6 units; the variance of the number of employees in grade 1 is increasing from 219.049 to 233.811, when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization increases from 13.964 to 14.003, and also in grade 3 in the organization increases from 11.748 to 11.772 for given values of other parameters. The variance of the number of employees in grade 1, grade 2 and grade 3 are increasing with respect to the recruitment parameter λ_2 increases.

The recruitment parameter of grade $2(\varepsilon_1)$ varies from 3 to 6 units; the variance of the number of employees in grade 1 remains unchanged, when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization increases from 14.003to 16.336, and also in grade 3 in the organization increases from 11.772 to 13.055 for given values of other parameters. The variance of the number of employees in grade 2 and grade 3 are simultaneously increasing with respect to the recruitment parameter ε_1 increases. The recruitment parameter of grade $2(\varepsilon_2)$ varies from 3 to 6 units; the variance of the number of employees in grade 1 remains unchanged, when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization increases from 16.336 to 18.484, and also in grade 3 in the organization increases from 13.055 to 14.042 for given values of other parameters. The variance of the number of employees in grade 2 and grade 3 are increasing with respect to the recruitment parameter ε_2 increases.

When the promotion rate from grade 1 to grade $2(\beta)$ varies from 0.1 to 0.7, the variance of the number of employees in grade 1 reduces from 305.814 to 228.905, when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization increases from 14.667 to 32.430, and also in grade 3 in the organization increases from 11.166 to 30.569 for given values of other parameters. As the leaving rate of grade 2 employee (γ) varies from 3 to 7 units, the variance of the number of employees in grade 1 is remains unchanged when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization reduces from 32.430 to 21.793, and also in grade 3 in the organization reduces from 30.569 to 19.912, for given values of the other parameters.

When the promotion rate from grade 2 to grade $3(\delta)$ varies from 6 to 9, the variance of the number of employees in grade 1 unchanged. Similarly, the variance of the number of employees in grade 2 in the organization decreases from 21.793 to 17.506, and in grade 3 in the organization increases from 19.912 to 24.139 for given values of other parameters. As the leaving rate of grade 3 employee (θ) varies from 7 to 10 units, the variance of the number of employees in grade 1 remains unchanged when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization remains unchanged, and in grade 3 in the organization reduces from 24.139 to 15.675, for given values of other parameters.

It is further observed that the initial number of employees in grade $1(N_0)$, grade $2(M_0)$, and grade $3(K_0)$ in the organization have a vital influence on the variance of the number of employees in grade 1, grade 2 and grade 3. As the initial number of employees in grade 1 (N_0) increases from 500 to 1000, the variance of the number of employees in grade 1 increases from 725.042 to 817.935, the variance of the number of employees in grade 2 is also increases from 79.453 to 85.295, and also in grade 3 the variance of the number of employees increases from 34.897 to 41.069, when the other parameters are fixed. As the initial number of employees in grade 2 (M_0) increases from 250 to 750, the variance of the number of employees in grade 1 and in grade 2 in the organization have no influence, but in grade 3 the variance of the number of employees increases from 39.834 to 41.086, when other parameters are fixed, the initial number of employees in grade 3 increases from 100 to 700, the variance of the number of employees in grade 1 and in grade 2 remains unchanged, but in grade 3 increases from 41.086 to 41.113 for given values of other parameters. As the uniform batch size distribution parameter of grade 1(a) varies from 1 to 5 units the variance of the number of employees in the grade 1 is increasing from 228.905 to 280.561, when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization is increasing from 17.506 to 18.004 and in grade 3 also increasing from 15.675 to 16.069 for given values of other parameters. It is further observed that the batch size distribution parameter of grade 1(b) varies from 5 to 20 units the variance of the number of employees in the grade 1 is increasing from 280.561 to 817.935, in grade 2 from 18.004 to 20.438 and also in grade 3 from 16.069 to 17.934, when other parameters are fixed. As the uniform batch size distribution parameter of grade 2(c) varies from 1 to 5 units the variance of the number of employees in the grade 1 remains unchanged, when other parameters are fixed. Similarly, the variance of the number of employees in grade 2 in the organization is increasing from 20.438 to 26.328 and in grade 3 also increasing from 17.934 to 20.492 for given values of other parameters. It is further observed that the batch size distribution parameter of grade 2 (d) varies from 5 to 20 units the variance of the number of employees in the grade 1 remains unchanged, in grade 2 increasing from 26.328 to 85.295 and also in grade 3 from 20.492 to 41.069, when other parameters are fixed.

5. COMPARATIVE STUDY OF THE MODEL

A comparative study of the developed model with that of compound Poisson bulk arrivals is carried Table 4, shows the points study of having models with homogeneous and non-homogeneous compound Poisson bulk arrivals. From the Table 4, it can also be observed that as time increases, the percentage variation of the performance measures between the models also increases. The model with non-homogeneous compound Poisson bulk arrivals has higher utilisation than the model with homogeneous compound Poisson bulk arrivals. It is also observed that the assumption of non-homogenous compound Poisson arrivals has a significant influence on all the performance measures of the model. Time also has a significant effect on the system performance measures, and this model can predict the performance measures more accurately. This model also includes some of the earlier models as particular cases.

6. CONCLUSIONS

This paper discusses a novel methodology for predicting the manpower situation in an organization. The manpower models provide the basic frame for analysis of manpower systems. Here, the constituent process of the manpower model namely recruitment process is characterized by a non-homogeneous compound Poisson process. Since, it is closely matches the postulates of the system. The non-homogeneous compound Poisson bulk recruitment assumptions provides close approximation to predictions of the characteristics of a manpower system such as mean number of employees in the organization and mean duration of an employee in the organization. The sensitivity analysis of the model carried under uniform bulk size distribution revealed that the bulk size distribution parameters and the direct recruitment in second grade have significant influence on the performance measures. This model also includes some of the earlier models as particular cases. The direct recruitment strategy which is adopted in several organizations under transient conditions is effective for efficient manpower planning in the organization. The HR managers can suitably schedule the promotion and other welfare measures by accurately predicting the duration of stay of an employee in the organization and average number of employees in each grade. It is possible to consider perspective modelling of the manpower model systems by assuming suitable cost associated with the human resource, which will be taken up elsewhere.

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