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Modern challenges of multiobjective optimization as a decision support discipline

Dmitry Podkopaev[1*]

[1]Systems Research Institute, Polish Academy of Sciences
Newelska 6, 01-447 Warsaw, Poland

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Abstract: Multiobjective optimization started as a mathematical discipline, is gradually becoming a multidisciplinary field of study, finding its place in operational research, multiple criteria decision making and decision analytics. This transformation is accompanied with update of the methodology, strengthening its focus on human-related aspects of decision support.

In this paper we take a look at multiobjective optimization as a framework for decision making and support in complex systems. We identify some challenges of its methodology, that need to be addressed in order to harness the practical potential of this research discipline. We review some our past works addressing those challenges.
Keyword — multiobjective optimization, multiple criteria decision making, decision support, human aspects.

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1. THE FIELD OF STUDY

1.1 Decision making problems with multiple criteria

A *decision making* problem arises, when an individual or a group of stakeholders needs to choose an alternative among a given set of alternatives, which serves their interests in a best way. Such problems appear in many, if not all, areas of organized human activity: industry (e.g. selecting best parameters of a production process), business (e.g. choosing a best business strategy for a new venture), engineering (e.g. finding a best design of a new product), public administration (e.g. selecting a best portfolio of public investments), supply chain management (e.g. constructing a supply chain network in a best possible way), space exploration (e.g. choosing a best trajectory of a spacecraft), and so on.

In the case where alternatives are evaluated and compared based on multiple aspects and the aspects can be quantified, the problem is often framed as a *multiple criteria decision making* (MCDM) problem (see e.g. Greco, Ehrgott, and Figueira (2016); C. Hwang and Yoon (1981); Miettinen (1999); Steuer (1986), and also Kaliszewski, Miroforidis, and Podkopaev (2016)). In terms of MCDM methodology, the entity making the choice is referred to as the *decision maker* (DM), and the considered aspects as *criteria* or *objectives*. It is assumed that for each alternative, the numerical value of each criterion is known or can be calculated. It is also assumed that the DM compares alternatives only based on their values of criteria. In this respect, the set of criteria is divided into *maximization criteria* (the DM prefers higher values to lower values) and *minimization criteria* (the DM prefers lower values to higher values).

In the ideal case, a best alternative is one maximizing or minimizing, respectively, all criteria values on the given set of alternatives. The existence of such an alternative in practice is very rare, therefore the condition of simultaneous optimality with respect to all criteria is relaxed to the condition of Pareto optimality. A given alternative is called *Pareto optimal* (also *efficient* or *non-dominated*)[1], if there does not exist another alternative which dominates it in the following sense: all its criteria values are preferred at least as much as of the given alternative, and at least one criterion value is more preferred. The best alternative is selected among the set of Pareto optimal alternatives.

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*Corresponding author's e-mail: dmitry.podkopaev@ibspan.waw.pl

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[1]These two expressions are often considered as synonyms of "Pareto optimal", although sometimes they have different meanings depending on context. Efficiency often refers to a modification of Pareto optimality, while non-dominance has a special meaning in the context of population-based heuristics.

The set of Pareto optimal alternatives (the Pareto optimal set for short) has the following important property: for any pair of alternatives, if one alternative has a more preferred value of one criterion, then the other alternative has more preferred value of at least one other criterion. This property is often referred to in MCDM as *the conflict of criteria*. Thus, in most real-life problems, instead of aspiring for an ideal solution, the DM has to settle for an acceptable *compromise* between achieving good enough values of different criteria.

For many complicated MCDM problems, mathematical methods and their computer implementations are the only way of solving the problem. However, the extent of formalization of such problems is limited. Since the Pareto optimal set usually contains more than one element (sometimes many or infinitely many elements), it is impossible to uniquely define a solution of an MCDM problem in a formal way. Moreover, different DMs may regard different Pareto optimal alternatives as best ones, since all humans may have different views on what is best compromise between criteria. Therefore, any MCDM method (with very few exceptions) requires additional information in order to select one among the set of Pareto optimal alternatives. This information is related to preferences of the specific DM with regard to the criteria of the specific MCDM problem, and is called *preference information* or *DM's preferences*. An alternative whose criteria values satisfy the DM's preferences in a best possible way is called a *most preferred alternative*, or a *most preferred solution* to the decision making problem. Thus, a typical MCDM problem cannot be solved by a formal method, without the involvement of a DM.

1.2 Multiobjective optimization problems

Multiobjective optimization (MOO) is the part of MCDM that deals with situations, where the set of alternatives and criteria values cannot be presented explicitly. In MOO problems, alternatives are referred to as *feasible solutions*, and their set is defined via constraints or combinatorial structures. The criteria are referred to as *objective functions* and are often represented by mathematical functions (closed-form expressions). It is worth noting that in MOO problems arising in practice, objective functions can also be defined via open-form expressions (Peitz, Ober-Blöbaum, & Dellnitz, 2019) or black-boxes such as simulations (Tabatabaei, Hakanen, Hartikainen, Miettinen, & Sindhya, 2015).

A general MOO problem is formulated as follows:

$$\begin{array}{ll} \text{maximize} & f_1(x), \dots, f_k(x) \\ \text{subject to} & x = (x_1, \dots, x_n)^T \in X, \end{array} \tag{1}$$

where $X \subseteq \mathbb{R}^n$ is a nonempty compact set of *feasible solutions* (also called *decision vectors*), and $k \geq 2$ is the number of *objective functions*. The latter are defined as $f_i: \ X \to \mathbb{R}, \ i = 1, \cdots, k$ and sometimes are referred to as *objectives* for short.

For any $\mathbf{x} \in X$, the vector $f(x) := (f_1(x), \dots, f_k(x))^T \in \mathbb{R}^k$ consisting of objective function values is called an *objective vector*, $\mathbb{R}^k$ is referred to as the *objective space*, while $\mathbb{R}^n$ as the *decision space*. The image of X in the objective space is defined by $f(X) = \{f(x): \ x \in X\}$.

The word "maximize" refers to the fact that the DM prefers larger objective values to smaller values. We assume that all objectives are maximized for the simplicity of notation. Note than any minimization objective can be transformed into a maximization objective by multiplying it by -1.

The MOO problem formulation differs from a single objective optimization problem only with the number of objective functions. However, the difference in approaches to solving the problems in the decision making context is drastic. The definitions below are refined from the ones presented above for MCDM problems. *Solving the MOO problem* means finding a feasible solution which is most preferred for the DM. As noted earlier, problem solution cannot be formally defined without knowing preferences of the specific DM, in whose interest the given problem instance is being solved. Let us remind that by assumption, the DM compares feasible solutions solely based on their objective vectors. It follows that from the DM's point of view, solving the problem is equivalent to finding a most preferred objective vector z in the set $f(X)$, and selecting any feasible solution x such that $f(x) = z$.

The search of the most preferred solution is limited to the set of *Pareto optimal solutions* defined as follows:

$$P = \{x \in X: \ \ \{\bar{x} \in X: f(x') \geq f(x) \ \text{ and } \ f(x') \neq f(x)\} = \emptyset\},$$

where an inequality between vectors is defined as inequalities between all the corresponding components. The objective vector $f(x)$ of a Pareto optimal solution x is called a *Pareto optimal objective vector*. The image of P in the objective space defined as $\{f(x): \ x \in P\}$ is referred to as the *Pareto front*.

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1.3 Interactive methods

Unlike in single-objective optimization, research challenges in MOO are not limited to mathematical and computational aspects. Additional challenges arise from the necessity to obtain preference information from a human DM and use this information in the solution process. The following three main approaches of including a DM in the solution process are distinguished in the literature (Buchanan, 1986; C.-L. Hwang & Masud, 1979; Miettinen, 1999).

- *A priori approach*: first the DM expresses preference information, then a preference model is constructed and the most preferred Pareto optimal solution is derived according to this model.
- *A posteriori approach*: first the set of Pareto optimal solutions (or its subset) is derived, then the DM selects the most preferred solution from this set.
- *Interactive approach*: the DM expresses preferences gradually during solution process and gets corresponding Pareto optimal solutions as feedback from the method.

In the case of computationally and cognitively complex MOO problems with many objectives, the first two approaches are rarely used due to their evident flaws. The a priori approach should not be used because in practice, one cannot expect the DM to have deep knowledge about the problem and preference expression mechanisms. Therefore, a solution obtained in a single attempt is not likely to represent what the DM really wants to achieve in the problem. The a posteriori approach cannot be used in a situation, when a large number of objectives dictates that many Pareto optima has to be derived in order to represent the Pareto front accurately[2]. Firstly, one cannot expect that a DM is able to compare many multidimensional objective vectors and secondly, deriving many Pareto optimal solutions may be impossible due to high computational complexity of the problem.

The interactive methods are devoid of the above disadvantages. In contrast to a priori methods, interactive methods allow the DM to correct mistakes arising from incomplete knowledge about the problem and preference expression mechanisms thanks to feedback. During the interactive solution process, the DM can learn about the problem which contributes to the quality of solution (Belton et al., 2008). Finally, interactive methods conserve the computing power by focusing only on those Pareto optima which are relevant to the solution process. Therefore, interactive approach is the de facto standard of solving computationally and cognitively complex MOO problems with many objectives.

The general scheme of an interactive method can be outlined as follows (Miettinen, Ruiz, & Wierzbicki, 2008):

Step 0. Initialization of the method. Present to the DM some information about the problem.

Step 1. Ask the DM to specify preference information.

Step 2. Generate one or several Pareto optimal solutions according to the preference information and present them to the DM.

Step 3. Ask the DM if one of solutions is satisfactory enough to be considered as a most preferred solution of the problem. If yes, stop; otherwise, go to Step 1.

In this paper we concentrate on interactive methods as the dominant approach to decision support in real-life applications of MOO. In the next section we offer some critical reflections on the current state of interactive MOO from the practical perspective. We refine them as challenges that need to be addressed at the methodological level. In Section 3, we review some our works that address them.

2. MODERN CHALLENGES OF MULTIOBJECTIVE OPTIMIZATION

The research discipline of MOO has been formed as a continuation of scalar optimization, goal and parametric programming. It is a relatively young discipline, appeared under the names *multiple objective programming* and *vector optimization* in the second half of the XX century (Köksalan, Wallenius, & Zionts, 2013). First interactive methods have been published in early 70s (Dyer, 1972; Geoffrion, Dyer, & Feinberg, 1972). The basic principles and research methodologies did not change much since then.

The foundation of MOO has been laid by mathematicians. Early publications as well as the majority of later papers focus on formal issues while paying less attention to human aspects. The interactive methods are presented in the form of pseudocode programs and/or flowcharts. The DM is called at certain steps to provide preference information, playing the role of an oracle or an external black box (Dyer, 1972; Geoffrion et al., 1972; Ghaznavi, Ilati, & Khorram, 2016; Ruiz, Sindhya, Miettinen, Ruiz, & Luque, 2015). Such a presentation is convenient from developer's point of view, however it implicitly assigns a passive role to the DM in the interactive solution processes. On the other hand, the

[2]According to some estimates (see e.g. Sen & Yang, 2012), for a given density or resolution of Pareto front representation, the number of Pareto optimal solutions required in order to achieve this density grows exponentially with the number of objectives.

only goal of solving an MOO problem is satisfying DM's needs. Therefore, the DM should become the central part of interactive MOO methods.

The narrow definition of DM's role can be traced back to the origins of MOO discipline. This brings challenges to researchers aiming at uncovering the full potential of MOO methodology in practical applications. Those challenges can be broadly divided into two types, which are described in Subsections 2.1 and 2.2, respectively. Some works addressing them are presented in Subsections 3.1 and 3.2, respectively.

2.1 Connecting the DM with the method through preference expression mechanisms

In many MCDM methods and the vast majority of MOO methods, DM's preferences are modeled by parametric scalarizing functions where parameters (often referred to as weights) are provided directly by the DM. A solution corresponding to DM's preference information is calculated by minimizing or maximizing such a function over the set of feasible solutions. Some MCDM methods also produce ranking of solutions according to the function values (Greco et al., 2016).

In order to claim that a solution or ranking obtained in the above way satisfies DM's preferences, one needs to ensure that (i) the DM accepts the ordering of the feasible solution set imposed by the scalarizing function, and (ii) the DM understands the correspondence between provided parameters and the resulted solution. Neither of that is addressed in publications describing the scalarizing functions or their applications in interactive optimization. One has to take into account that real DMs who need to solve practical MOO problems are not necessary mathematicians, but rather experts in their problem domains. Thus, the preference expressing mechanisms can be disconnected from real DM's.

The most popular class of preference models used in interactive MOO are achievement scalarizing functions (ASFs). First papers presenting the concept of ASF (Wierzbicki, 1980, 1982) are devoted to studying their mathematical properties, e.g. characterizing Pareto optimal solutions in terms of ASFs. A part of ASF's parameters is the vector of aspiration levels (a reference point) that can be easily understood by a typical DM. However, the rest of parameters as well as the inner mechanics of ASF remain unexplained. Usually the ASF is interpreted in terms of minimizing a distance to a reference point in the objective space, which may sound too abstract to non-mathematicians. Later papers developing advanced ASF modifications (see e.g. Luque, Miettinen, Ruiz, and Ruiz (2012); Nikulin, Miettinen, and Mäkelä (2012)) continue the tradition of presenting ASF from the mathematical perspective, while mentioning the needs of the DM in motivational parts of the papers. Only few papers explicitly address the issue of interpreting ASFs in decision making terms (Kaliszewski, 2004, 2015).

There are many other types of scalarizing functions, where the DM is supposed to provide parameters playing the role of weights, but the interpretation of parameters and the mechanics of deriving solutions is not explained (Brans & De Smet, 2016; C. Hwang & Yoon, 1981; Miettinen, Eskelinen, Ruiz, & Luque, 2010; Opricovic, 1998). A typical interpretation of weights provided by authors sounds as follows: *the higher is the weight associated with a criterion, the more important is this criterion for the DM*. However, in order to set weights in a meaningful way, the DM would need to understand, *how large weight corresponds to how much importance of each criterion*, and what does "importance" mean in this context.

Explaining the mechanics behind a scalarizing function to a DM and providing a clear interpretation of its parameters would allow the DM to connect the values of parameters with his/her preferences. Besides that, such explanations and interpretations provided for different methods would help the DM to choose a method, whose preference model better suits DM's understanding of the solution process.

2.2 Taking into account the human nature of the DM

As discussed earlier, the MOO methodology treats the DM as an oracle which is called at certain steps of the solution process and asked for certain information. One type of information is method-specific preferences which are used to derive new Pareto optimal solutions. Another type of information is the selection of the most preferred solution among previously derived Pareto optimal solutions. It is used in two ways: selecting the most preferred solution of the problem and in some methods (Jaszkiewicz & Słowiński, 1999; Miettinen & Mäkelä, 2006), expressing preference information with respect to a previously derived solution.

Almost no publications address possible imperfections of the oracle, i.e. the limitations and needs of human DMs. In practice, this means that interactive methods are not adapted to be used by real people. Below are three potential problems which can be encountered in regard to that.

1. In the case of many objective functions and many derived solutions, selecting one among previously derived solutions may be cognitively difficult for the DM. One can see that by looking at complexity of visualizations in the case of moderate numbers of objectives and Pareto optimal solutions (see e.g. Miettinen, 2014).

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2. In the case of a computationally complex problem, waiting times between iterations can be unacceptable. The DM may have limitations related to time spent with the method, as well as requirements to solve the problem by a certain deadline.

3. In the presence of a large variety of interactive methods, it is unclear how to choose the most suitable method for the specific DM in the specific circumstances. The real DMs who are not experts in MOO cannot make this choice by themselves due to the lack of guidance and comparative studies.

Although MOO is relatively young as for a mathematical discipline, it is old as a computer science discipline. The foundation of MOO has been laid before big data, user-friendly interfaces and widespread parallelization came into practice. The scale and complexity of decision making problems has been rapidly increasing in recent decades as a response to the growing scale of information systems (Filip, 2007; Tang & Liao, 2019). This makes the first two of the mentioned issues more pronounced. At the same time, information technologies become more accessible to general public. As a result, there are more opportunities for DMs to use decision support systems by themselves. This raises the importance of the third issue.

3. ADDRESSING THE CHALLENGES

The challenges described in the previous section are rather broad. They call for thorough reconsideration of MOO methodologies and approaches. Many researchers in the MCDM community work in this direction (see e.g. Branke, Deb, Miettinen, & Slowinski, 2008; P. Korhonen & Wallenius, 1996; Ogryczak & Vetschera, 2004). Our papers presented in the next two subsections contribute to addressing the issues formulated in Subsections 2.1 and 2.2, respectively.

3.1 Explainable preference modeling

The works Miettinen, Podkopaev, Ruiz, and Luque (2015); Podkopaev (2007, 2008); Podkopaev and Miettinen (2011) deal with achievement scalarizing functions (ASFs) first introduced in Wierzbicki (1980, 1982), which became the most popular class of preference models used in interactive MOO. The goal of our study is to make the mechanism and parameters of ASFs understandable for DMs, which would contribute to transparency and accuracy of expressing preferences.

The basic and most common form of an ASF is the following:

$$\begin{aligned} \text{minimize} \quad & \max_{i=1,\cdots,k} \lambda_i \left(z_i^{asp} - f_i(x)\right) + \rho(x) \\ \text{subject to} \quad & x \in X, \end{aligned} \tag{2}$$

where $z^{asp} = (z_1^{asp}, \ldots, z_k^{asp})$ is the reference point composed of aspiration levels; $\lambda_1, \ldots, \lambda_k$ are weights corresponding to objective functions, and $\rho(x)$ is a linear augmentation term ensuring Pareto optimality of any derived solution. The aspiration levels have clear interpretation in decision making terms: they are objective function values, which would satisfy the DM if being achieved. However, the weights, the augmentation term and the overall correspondence between the ASF parameters and derived solutions cannot be easily understood.

Papers by Kaliszewski (2004, 2015) contribute to better understanding of weights and propose a holistic interpretation of the ASF. The works by Kaliszewski (1994); Kaliszewski and Michalowski (1997) connect k parameters of the linear augmentation term with $k(k-1)$ bounds on pairwise trade-off coefficients between objective functions. Bounds on trade-offs are a good way of representing partial preference information in a situation, where the DM is not confident about preferences and therefore cannot express information in a complete way, e.g. as rates of substitution between objectives.

We go further by proposing a new, simple linear augmentation term with $k(k-1)$ parameters, which have a direct interpretation in terms of corresponding bounds on trade-off coefficients. Bounding trade-offs using this augmentation term is reduced to a linear transformation of the objective space (Podkopaev, 2007). In the paper Podkopaev (2008), we prove the rationality of DM's preferences represented by the proposed linear model. Our approach allows any DM to freely express partial information about pairwise preferences between criteria in understandable terms.

In the paper Podkopaev and Miettinen (2011), we combine the above model of partial preference information with an ASF preference model. The latter is interpreted in terms of a direction of improvement of objectives. As a result, we obtain a holistic ASF-based preference model fully interpreted in decision making terms. Not only parameters but also the whole mechanism of deriving solutions are explained in common language. In addition to the interpretation, we propose an interactive procedure of forming the direction of improvement which can be used by inexperienced DMs. This model and the procedure are suitable for interactive methods where a DM or a group of DMs wants to avoid conflicts between objectives while concentrating on their synergy. We integrated the above developments with an interactive method based on the same philosophy, called NAUTILUS (Miettinen et al., 2010). The new NAUTILUS

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method Miettinen et al. (2015) is suitable for both individual and group decision making in conflict-free environment. This method was created in an attempt to avoid cognitive biases caused by the conflict (Kahneman & Tversky, 1979), where the DM is forced to give up in some objectives in order to improve other objectives.

Beside ASF, there are many other parametric scalarizing functions serving as DM's preference models, mainly in the context of multi-attribute decision making (Greco et al., 2016; C. Hwang & Yoon, 1981; Shih, Shyur, & Lee, 2007). Only additive scalarizing functions have been studied in terms of interpreting parameters (weights) in the decision making context (see e.g. Saaty (1986) and references in P. J. Korhonen, Silvennoinen, Wallenius, and Öörni (2013)). The papers Kaliszewski and Podkopaev (2016); Podkopaev (2016) contribute to explaining non-additive preference models to the DM.

In Podkopaev (2016), we conduct experimental study of three among most popular non-additive scalarizing methods: TOPSIS, VIKOR and PROMETHEE, and compare them with the simple additive weighting (SAW) method. The experiments involved comparing rankings produced by all four methods for randomly generated problem data. We observed significant differences between rankings, implying that the method selection should not be made arbitrarily. We provided weights interpretations in the considered methods, helping potential DMs to make the choice of the most suitable method reflecting their preferences.

Besides that, in Podkopaev (2016) we analyzed how much the adjustment of weights can reduce the differences between rankings of the non-additive methods and SAW. The achieved reduction is multifold, which suggests that the different interpretation of weights in the methods significantly contributes to the difference in rankings (comparing to the contribution by different shapes of the non-linear functions). This fact justifies our proposition of a posteriori interpretation of solutions presented in the paper Kaliszewski and Podkopaev (2016). We proposed to adjust weights of SAW function in order to obtain the same or similar ranking as the one based on a complex scalarizing function, and to interpret the ranking process in more simple terms of the linear model.

3.2 Taking into account DM's needs and limitations

Long waiting times between iterations can be costly for a busy DM and discourage him/her from participation in the decision making process. The following scheme, named in Tabatabaei et al. (2015) the *adaptive surrogate-based framework*, has been exploited by several papers as an approach to overcoming that issue:

1. *Construction phase.* A representation of the Pareto front (so called surrogate problem) is constructed. This may take time for a computationally expensive problem, however the DM is not involved in this phase and therefore does not spend his/her valuable time.

2. *Decision phase.* The interactive method is employed to solve the surrogate problem in communication with the DM. The Pareto front representation allows calculating approximate solutions corresponding to DM's preferences instantly, thereby eliminating waiting times.

3. *Projection phase.* The solution of the original problem is obtained based on the solution of the surrogate problem. The DM needs to wait before the most preferred solution is delivered, however his/her involvement in the solution process is no longer required.

As concluded in Tabatabaei et al. (2015), the adaptive framework is more advantageous in the case of MOO problems compared to the traditional approach, where surrogate functions are created for individual computationally complex objective functions.

The paper Kaliszewski, Miroforidis, and Podkopaev (2012) presents a novel surrogate-based approach in the adaptive framework. Unlike other approaches, here instead of approximate Pareto optimal solutions in the decision phase, the DM obtains information about lower and upper bounds on their objective function values. Thus, the DM knows the guaranteed accuracy of the approximate objective vector corresponding to any instance of preference information. This feature is made possible by creating a special type of representation of the Pareto front in the construction phase. It consists of so called lower and upper shells, which are sets of feasible and non-feasible points, respectively, enclosing the Pareto front in between like a sandwich.

If the accuracy of a solution is not satisfactory for the DM, the density of Pareto set representation near this solution can be improved by another algorithm. However the DM has to pay for this improvement with waiting time. Both the construction phase algorithm and the improvement algorithm proposed in Kaliszewski et al. (2012) are evolutionary-based heuristics. The paper is a continuation of earlier works by Kaliszewski (2004, 2006); Kaliszewski and Miroforidis (2009, 2010).

The described scheme of surrogate-based interactive MOO can be further improved by employing techniques of parallel computing and artificial intelligence. In the paper Ojalehto, Podkopaev, and Miettinen (2015), we propose a concept of an agent-based framework extending the adaptive surrogate-based framework as follows. Several intelligent modules called *agents* work in parallel during the decision phase. They exchange information, observe the behavior

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of the DM and learn his/her preferences. Using this information, they try to predict the areas of the objective space which are most interesting for the DM. They derive new Pareto optimal solutions in those areas and update the Pareto front representation, thereby improving the accuracy of solutions derived during the decision phase. The agents work seamlessly for the DM, utilizing time the DM spends with the computer for improving the quality of solutions in background.

The agent-assisted concept was implemented on top of the surrogate-based method PAINT developed earlier (M. Hartikainen, Miettinen, & Wiecek, 2012; M. E. Hartikainen & Lovison, 2014). This method represents Pareto front by linear interpolation between derived Pareto optimal solutions using Delaunay triangulation. We demonstrated the benefits of the agent-assisted algorithm by applying it to solving a real-life problem of chemical engineering. During the decision phase, we utilized information obtained from a real DM in an industrial project. It is worth noting that the concept of the agent-assisted framework can be implemented with any type of surrogate-based method including the one from Kaliszewski et al. (2012).

The issue of cognitive difficulty of interactive MOO has been addressed in the paper Filatovas, Podkopaev, and Kurasova (2015). We consider the class of interactive methods where the DM has to express information with respect to one of previously derived Pareto optimal solutions. The Pareto optimal solutions are accumulated in each iteration forming the so called solution pool. In order to express preference information in each iteration, the DM has to select one of them. Comparing a dozen of objective vectors may be cognitively difficult in the case of many objectives. We propose and implement the concept of a visualization tool assisting the DM in this task.

The visualization tool is based on reducing the dimensionality of objective vectors from k to 2 or 3 using multidimensional scaling, and visualizing them in an interactive plot. It helps the DM to understand how different solutions are clustered in space, as well as interconnected in time throughout the iterations. We describe the algorithm implementing the visualization tool in detail, as well as a general scheme of connecting it with any method of the considered class. We demonstrate its work with NIMBUS method (Miettinen & Mäkelä, 2006) using an example problem.

The papers Miettinen, Hakanen, and Podkopaev (2016); Ojalehto, Podkopaev, and Miettinen (2016) address the lack of guidance for DMs related to selecting appropriate interactive MOO methods. Usually authors of any interactive method recommend to use it, emphasizing method's advantages. On the other hand, different DMs may prefer different styles of expressing preferences which vary across methods. Besides that, experimental studies comparing interactive methods are scarce due to numerous difficulties of involving humans in such experiments.

In the survey of interactive MOO methods Miettinen et al. (2016), we classify all well-known interactive methods based on the type of preference information required from the DM. The survey contains 277 references. We identified five main types of preference information:

- *the vector of aspiration levels* also known as the reference point – the desirable levels of objective function values;
- *classification of objectives* – after selecting one among previously derived Pareto optimal solutions, the DM describes the desirable improvement of this solution objective-wise;
- *comparison of solutions* – the DM is required to compare given Pareto optimal solutions with each other;
- *marginal rates of substitution* – desirable proportions between improvement of some objectives and deterioration of other objectives when moving from one Pareto optimal solution to another one;
- *navigation* – free navigation among Pareto optimal solutions using graphical user interface.

One way of overcoming the difficulties related to conducting interactive methods tests with human DMs is to construct artificial DMs simulating such interaction. There are no formal theories of preferences which could be used to set aspiration levels in justified ways. Despite reference points are the most popular type of preference information used in interactive MOO, only few works propose artificial DMs for testing reference point-based methods. Our work Ojalehto et al. (2016) is among the first ones in this direction. We propose a holistic structure of an artificial DM mimicking how humans interact with MOO methods in our understanding. Generation of reference points is based on adjusting the aspiration levels in each iteration, taking into account priorities of objectives and involving randomness. We demonstrate the universality of our approach by comparing two interactive reference point-based methods of different nature: the classical interaction procedure by setting the reference point and the evolutionary method R-NSGA-II.

Last but not least, we contribute to educating DMs of various backgrounds by publishing a textbook Kaliszewski et al. (2016) and presenting it as the first human-centric decision making approach available to general public (Kaliszewski, Miroforidis, & Podkopaev, 2018). It is based on our course of computer-aided decision making taught to managers at the level of high school. The textbook presents a simple but complete methodology allowing a DM to solve an MCDM problem by himself/herself. It is self-sufficient, meaning that it describes all theoretical concepts required for mastering the methodology. Accompanying electronic materials illustrate the implementation of this methodology in Microsoft Excel. Our textbook can help closing the gap between the MOO methods and the DMs with non-mathematical background.

4. CONCLUSION

In this short paper we attempted at introducing the MOO as a framework for decision making and support. The developments of MOO have great potential for meeting the needs of real-life decision makers in the modern world of growing complexity. However, important challenges need to be addressed in the methodology of MOO for enabling that. We concentrated on the challenges related to strengthening human aspects, and reviewed our works addressing them. The scope of future work which has to be done is much broader than presented here. In order to keep up with the demands of evolving world, MOO should strengthen its integration with such disciplines as cognitive psychology, behavioral science, user interface design, artificial intelligence, and others. The first step could be the systematization of research challenges in the broader context and the elaboration of a roadmap for future studies.

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