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Availability Enhancement of a Complex Hybrid System Consisting of Three Subsystems

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Abstract: This paper deals with the modelling and enhancement of availability of hybrid system containing three subsystems I, II and III. Subsystems I and III each has two processors while subsystem II has two unit in active parallel system. Subsystem I is linked to unit I while subsystem II is linked to unit II for the smooth operation of the system. It is assumed that time-to-failure and the time-to-repair of the unit and processors are exponentially distributed. Explicit expression for the system availability is developed using the system state transition diagram and differential difference equations. Graphical illustrations are given to highlight impact of failure rate on availability for different repair scenarios and vice vasa. The results have shown that availability can be enhanced with minor failure and major repair.
Keyword — Availability; parallel; devices, hybrid; subsystem; units; repair; failure

1. INTRODUCTION

Redundancy, repair, inspection and replacement are techniques used to improve system reliability, availability, production output and generated revenue. Redundancy is a technique for increasing system effectiveness by reducing failure and maintenance cost. Provision of standby unit is vital towards achieving high reliability. System reliability is improved through a standby unit support which is capable of performing similar function with the operational unit but with different degree and desirability. Cold standby redundancy is a form of redundancy used to raise the system availability Sinaki (1994). Space exploration and satellite systems, manufacturing textile and carbon recovery systems used in fertilizer are some of the systems that uses cold standby redundancy to achieve high system reliability and availability [see for instance Sinaki (1994), Pandey, Jacob, and Yadav (1996), Kumar, Kumar, and N.P. (1996) and some reference therein] . Many researchers have identified maintenance models for enhancing reliability and availability, reducing operating costs and the risk of a catastrophic breakdown of different systems. Garg and Sharma (2012a), Garg and Sharma (2012b) analyzed the performance of synthesis unit and urea decomposition system in fertilizer plant. Garg, Rani, and Sharma. (2013) studied preventive maintenance scheduling of the pulping unit in a paper plant. Niwas and Garg (2018) presents an approach for the analysis of reliability and profit of an industrial system under free warranty policy. Garg (2015) and Garg (2016) analyzed the reliability of series-parallel system using credibility and fuzzy numbers and fuzzy Kolmogrov's differential equations respectively. Garg (2014) analyzed approaches to reliability, availability and maintainability of industrial systems.

Extensive researches have been carried out on reliability analysis for systems that cannot work without the aid of supporting device. Singh, Singh, Ram, and Goel (2012) dealt with comparison of some reliability characteristics between redundant systems requiring supporting units for their operations. Yusuf (2013) performed a comparative analysis of some reliability characteristics between redundant systems requiring supporting units for their operations. Yusuf and Bala (2013) studied Analysis of reliability characteristics of a parallel system with external supporting devices for operation. Yusuf, Yusuf, and Lawan (2014) studied Reliability modelling and analysis of redundant systems connected to supporting external device for operation attended by a repairman and repairable service station. Yusuf, Babagana, Yusuf, and Lawan (2016) studied Reliability analysis of a linear consecutive 2-out-of-3 system in the Presence of supporting device and repairable service station while Yusuf (2016) studied reliability modelling of a parallel system with a supporting device and two types of preventive maintenance. While the following investigates mean time to system failure and cost of repairing systems with repair policies. Fagge, Ali, and Yusuf (2016) presents the mean time to system

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failure assessment of a single unit system connected to two types of redundant supporting devices for operation under the assumption that the unit always works with both types of supporting devices. Fagge, Yusuf, and Ali (2017) presents availability evaluation of a single unit system connected to two types of redundant supporting devices for operation. Yusuf and Fagge (2017) deal with evaluation of some reliability characteristics of a single unit system connected to two types of redundant supporting devices for operation. Singh et al. (2012) studied the availability, MTTF and Cost Analysis of a system having Two Units in Series Configuration with Controller. Singh and Rawal (2014) discussed the availability, MTTF and cost analysis of complex system under Preemptive resume repair policy using copula distribution. In this paper, two units active parallel system with two types of dissimilar cold standby processors for operation is considered and derived its corresponding mathematical model. Furthermore, we study the system availability of the proposed system using differential difference equations. The focus of our analysis is primarily to capture the effect of both hardware and software failure and repair rates on availability. Example such system can be seen in computer systems, Telecom systems, oil refineries, food processing plants, chemical process plant, etc. The organization of the paper is as follows. Section 2 contains a description of the system under study. Section 3 presents formulations of the models. The results of our numerical simulations are presented and discussed in section 4. Finally, we make some concluding remarks in Section 5.

2. DESCRIPTION OF THE SYSTEM

The system composed of three subsystems I, II and III. Subsystem I has two identical units, unit I and unit II in active parallel. Subsystems I and III each has two processors P_1 and P_2 which support the units I and II for the system operation as shown in Figure 1 below. Each unit is connected to P_1 and P_2. Each of the device P_1 and P_2 fails with exponential failure distribution with parameter λ_1 and λ_2 , exponential repair distribution with parameter μ_1 and μ_2 respectively. When one of the P_1 fails, which occurs with failure rate λ_1, it is repaired with the rate μ_1 and the corresponding P_2 then carries out the function of the failed P_1. When both P_1 and P_2 on the same unit fails, the unit is resting and the other unit will continue operating with its P_1 and P_2 devices in a manner described above. It is assumed that switching from standby to operation is perfect. System failure results from the failure of two unit or both types of supporting devices. System failure occurred when both units or P_1 and P_2 have failed.

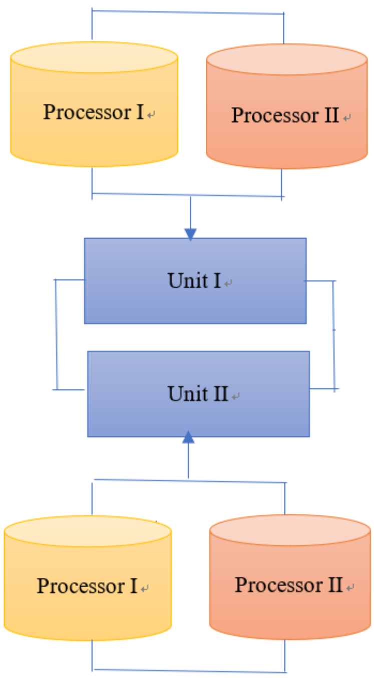

Figure 1: Reliability block diagram of the system

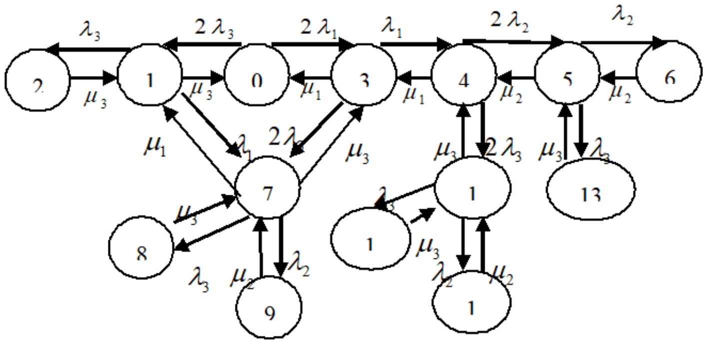

Figure 2: Transition diagram of the system

Table 1: State of the system

State	Description of the System Components						System Status
	Unit I	Processor I	Processor II	Unit II	Processor I	Processor II	
S_0	Working	Working	Standby	Working	Working	Standby	Operational
S_1	Failed	Resting	Standby	Working	Working	Standby	Operational
S_2	Failed	Resting	Standby	Failed	Resting	Standby	Down
S_3	Working	Failed	Working	Working	Working	Standby	Operational
S_4	Working	Failed	Working	Working	Failed	Working	Operational
S_5	Working	Failed	Working	Resting	Failed	Failed	Operational
S_6	Resting	Failed	Failed	Resting	Failed	Failed	Down
S_7	Working	Failed	Working	Failed	Resting	Resting	Down
S_8	Failed	Failed	Resting	Failed	Failed	Resting	Down
S_9	Failed	Failed	Failed	Failed	Failed	Resting	Down
S_{10}	Working	Failed	Working	Failed	Failed	Resting	Operational
S_{11}	Failed	Failed	Resting	Failed	Failed	Resting	Down
S_{12}	Failed	Failed	Resting	Resting	Failed	Failed	Down
S_{13}	Failed	Failed	Resting	Failed	Failed	Resting	Down

3. FORMULATION OF THE MODEL

The corresponding differential difference equations associated with the transition diagram in Figure 2 are:

$$\frac{\mathrm{d}}{\mathrm{dt}}p_0(t) = -(2\lambda_3 + 2\lambda_1)p_0(t) + \mu_3 p_1(t) + \mu_1 p_3(t)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}p_1(t) = -(\lambda_3 + \lambda_1 + \mu_3)p_1(t) + 2\lambda_3 p_0(t) + \mu_3 p_2(t) + \mu_1 p_7(t)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}p_2(t) = -\mu_3 p_2(t) + \lambda_3 p_1(t)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}p_3(t) = -(2\lambda_3 + \lambda_1 + \mu_1)p_3(t) + 2\lambda_1 p_0(t) + \mu_1 p_4(t) + \mu_3 p_7(t)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}p_4(t) = -(2\lambda_3 + 2\lambda_2 + \mu_1)p_3(t) + 2\lambda_1 p_3(t) + \mu_2 p_5(t) + \mu_3 p_{10}(t)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}p_5(t) = -(\lambda_3 + \lambda_2 + \mu_2)p_3(t) + 2\lambda_2 p_4(t) + \mu_2 p_6(t) + \mu_3 p_{13}(t)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}p_6(t) = -\mu_2 p_6(t) + \lambda_2 p_5(t)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}p_7(t) = -(\lambda_3 + \lambda_2 + \mu_3 + \mu_1)p_7(t) + \lambda_1 p_1(t) + 2\lambda_3 p_3(t) + \mu_2 p_8(t) + \mu_3 p_9(t)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}p_8(t) = -\mu_2 p_8(t) + \lambda_2 p_7(t)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}p_9(t) = -\mu_3 p_9(t) + \lambda_3 p_7(t)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}p_{10}(t) = -(\lambda_3+\lambda_2+\mu_3)p_{10}(t) + 2\lambda_3 p_4(t) + \mu_3 p_1 1(t) + \mu_2 p_1 2(t)$$
$$\frac{\mathrm{d}}{\mathrm{dt}}p_{11}(t) = -\mu_3 p_1 1(t) + \lambda_3 p_{10}(t)$$
$$\frac{\mathrm{d}}{\mathrm{dt}}p_{12}(t) = -\mu_2 p_{12}(t) + \lambda_2 p_{10}(t)$$
$$\frac{\mathrm{d}}{\mathrm{dt}}p_{13}(t) = -\mu_3 p_{13}(t) + \lambda_3 p_7(t) \tag{1}$$

The initial condition for in this study is:

$$p_i(0) = \begin{cases} 1, & i = 0 \\ 0, & i = 1, 2, 3, \cdots, 13 \end{cases} \tag{2}$$

The differential difference equation in (1) above is expressed as

$$\begin{pmatrix} p_0'(t) \\ p_1'(t) \\ p_2'(t) \\ p_3'(t) \\ p_4'(t) \\ p_5'(t) \\ p_6'(t) \\ p_7'(t) \\ p_8'(t) \\ p_9'(t) \\ p_{10}'(t) \\ p_{11}'(t) \\ p_{12}'(t) \\ p_{13}'(t) \end{pmatrix} = \begin{pmatrix} -q_0 & \mu_3 & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\lambda_3 & -q_1 & \mu_3 & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_3 & -\mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\lambda_1 & 0 & 0 & -q_2 & \mu_1 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & -q_3 & \mu_2 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\lambda_2 & -q_4 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 \\ 0 & 0 & 0 & 0 & 0 & \lambda_2 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 2\lambda_3 & 0 & 0 & 0 & -q_5 & \mu_2 & \mu_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & -\mu_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\lambda_3 & 0 & 0 & 0 & 0 & 0 & -q_6 & \mu_3 & \mu_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & -\mu_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_3 \end{pmatrix} \begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \\ p_9(t) \\ p_{10}(t) \\ p_{11}(t) \\ p_{12}(t) \\ p_{13}(t) \end{pmatrix} \tag{3}$$

Where $q_0 = (2\lambda_3 + 2\lambda_1)$, $q_1 = (\lambda_3 + \lambda_1 + \mu_3)$, $q_2 = (2\lambda_3 + \lambda_1 + \mu_1)$, $q_3 = (2\lambda_3 + 2\lambda_2 + \mu_1)$, $q_4 = (\lambda_3 + \lambda_2 + \mu_2)$, $q_5 = (\lambda_3 + \lambda_2 + \mu_3 + \mu_1)$, $q_6 = (\lambda_3 + \lambda_2 + \mu_3)$
From Figure 2, the steady-state availability of the system is given by

$$A_V(\infty) = p_0(\infty) + p_1(\infty) + p_3(\infty) + p_4(\infty) + p_5(\infty) + p_7(\infty) + p_{10}(\infty) \tag{4}$$

In the steady state, (3) is now

$$\begin{pmatrix} -q_0 & \mu_3 & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\lambda_3 & -q_1 & \mu_3 & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_3 & -\mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\lambda_1 & 0 & 0 & -q_2 & \mu_1 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & -q_3 & \mu_2 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\lambda_2 & -q_4 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 \\ 0 & 0 & 0 & 0 & 0 & \lambda_2 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 2\lambda_3 & 0 & 0 & 0 & -q_5 & \mu_2 & \mu_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & -\mu_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\lambda_3 & 0 & 0 & 0 & 0 & 0 & -q_6 & \mu_3 & \mu_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & -\mu_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_3 \end{pmatrix} \begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \\ p_9(t) \\ p_{10}(t) \\ p_{11}(t) \\ p_{12}(t) \\ p_{13}(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

Combining (5) with the following normalizing conditions:

$$\sum_{k=0}^{13} p_k(\infty) = 1 \tag{6}$$

to give

$$\begin{pmatrix} -q_0 & \mu_3 & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\lambda_3 & -q_1 & \mu_3 & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_3 & -\mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\lambda_1 & 0 & 0 & -q_2 & \mu_1 & 0 & 0 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_1 & -q_3 & \mu_2 & 0 & 0 & 0 & 0 & \mu_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\lambda_2 & -q_4 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_3 \\ 0 & 0 & 0 & 0 & 0 & \lambda_2 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 2\lambda_3 & 0 & 0 & 0 & -q_5 & \mu_2 & \mu_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & -\mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & 0 & -\mu_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\lambda_3 & 0 & 0 & 0 & 0 & 0 & -q_6 & \mu_3 & \mu_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_3 & -\mu_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \\ p_9(t) \\ p_{10}(t) \\ p_{11}(t) \\ p_{12}(t) \\ p_{13}(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{7}$$

and solving (7) using MATLAB package so as to obtain the state probabilities $p_k(\infty)$ which enable the derivation of (4) above as $A_V(\infty) = \frac{N_0}{D_0}$ where

$$\begin{aligned} N_0 =& \mu_1^2\mu_2^2\mu_3^2(\mu_1 + \mu_3 + \lambda_1 + 2\lambda_3) + 2\mu_1\mu_2^2\mu_3^2\lambda_1(\mu_1 + \mu_3 + \lambda_1 + \lambda_3) + 2\mu_2^2\mu_3^2\lambda_1^2(\mu_1 + \mu_3 + \lambda_1 + \lambda_3) \\ &+ 4\mu_2\mu_3^2\lambda_1^2\lambda_2(\mu_1 + \mu_3 + \lambda_1 + \lambda_3) + 2\mu_1\mu_2^2\mu_3\lambda_1\lambda_3(\mu_1 + \mu_3 + \lambda_1 + \lambda_3) + 4\mu_2^2\mu_3\lambda_1^2\lambda_3(\mu_1 + \mu_3 + \lambda_1 + \lambda_3) \\ D_0 =& 4\mu_2\mu_3^2\lambda_1^3\lambda_2 + 4\mu_2^2\lambda_1^3\lambda_3^2 + 4\mu_2^2\lambda_1^3\lambda_3^2 + 4\mu_3^3\lambda_1^2\lambda_2^2 + 4\mu_2\mu_3^3\lambda_1^2\lambda^2 + 6\mu_2^2\mu_3^2\lambda_1^2\lambda_3 + 8\mu_1^2\mu_3\lambda_1^2\lambda_3^2 + 4\mu_2^2\lambda_1^2\lambda_3^3 \\ &+ 2\mu_1^3\mu_2^2\lambda_3^2 + 4\mu_1^2\mu_2^2\mu_3^2\lambda_3 + 6\mu_1^2\mu_2^2\mu_3\lambda_3^2 + 4\mu_1^2\mu_2^2\lambda_3^3 + 4\mu_1\mu_2^2\mu_3^2\lambda_1^2 + 2\mu_1\mu_2^2\mu_3^3\lambda_1 + 4\mu_2^2\mu_3\lambda_1^3\lambda_3 + 3\mu_1^2\mu_2^2\mu_3^2\lambda_1 \\ &+ 4\mu_3^2\lambda_3^3\lambda_2^2 + 2\mu_1^3\mu_2^2\mu_3\lambda_3 + 16\mu_2\mu_3^2\lambda_1^2\lambda_2\lambda_3 + 20\mu_2\mu_3\lambda_1^2\lambda_2\lambda_3^2 + 12\mu_2\mu_3\lambda_1^3\lambda_2\lambda_3 + 4\mu_1\mu_2\mu_3^2\lambda_1\lambda_2\lambda_3 \\ &+ 4\mu_1\mu_2\mu_3\lambda_1\lambda_2\lambda_3^2 + 8\mu_2\lambda_1^2\lambda_2\lambda_3^3 + 16\mu_1\mu_2\mu_3\lambda_1^2\lambda_2\lambda_3 + 8\mu_1\mu_2\lambda_1^2\lambda_2\lambda_3^2 + 2\mu_2^2\mu_3^3\lambda_1^3 + 2\mu_2^2\mu_3^3\lambda_1^2 + \mu_1^3\mu_2^2\mu_3^2 \\ &+ \mu_1^2\mu_2^2\mu_3^3 + 8\mu_3\lambda_1^3\lambda_2^2\lambda_3 + 6\mu_1\mu_2^2\mu_3^2\lambda_1\lambda_3 + 8\mu_1\mu_2^2\mu_3\lambda_1\lambda_3 + 8\mu_1\mu_2^2\mu_3\lambda_1\lambda_3^2 + 4\mu_1\mu_2^2\lambda_1\lambda_3^3 + 8\mu_2\lambda_1^3\lambda_2\lambda_3^2 \\ &+ 12\mu_3^2\lambda_1^2\lambda_2^2\lambda_3 + 8\mu_3\lambda_1^2\lambda_2^2\lambda_3^2 + 2\mu_1^2\mu_2\mu_3\lambda_1\lambda_2\lambda_3 + 6\mu_1^2\mu_2^2\mu_3\lambda_1\lambda_3 + 6\mu_1^2\mu_2^2\lambda_1\lambda_3^2 + 4\mu_1\mu_3^2\lambda_1^2\lambda_2^2 \\ &+ 8\mu_1\mu_3\lambda_1^2\lambda_2^2\lambda_3 + 4\mu_1\mu_2\mu_3^2\lambda_1^2\lambda_2 + 8\mu_1\mu_2^2\mu_3\lambda_1^2\lambda_3 + 8\mu_1\mu_2^2\lambda_1^2\lambda_3^2 \end{aligned}$$

4. NUMERICAL EXAMPLE AND DISCUSSION

To validate the model, this section provide numerical example using MATLAB package by considering the following parameter values for consistency:
$\mu_1 = 0.12, \quad \mu_2 = 0.1, \quad \mu_3 = 0.3, \quad \lambda_1 = 0.2, \quad \lambda_2 = 0.1$ and $\lambda_3 = 0.2$
In this section, failure or repair is

(i) Minor whenever $\mu_j, \lambda_j < 0.5$

(ii) Medium whenever $\mu_j, \lambda_j = 0.5$

(iii) Major whenever $\mu_j, \lambda_j > 0.5$

In Figure 2, $\mu_2 = 0, 1$,$\mu_3 = 0.3$, $\lambda_2 = 0.1$ and $\lambda_3 = 0.2$ are fixed and vary $\lambda_1 \in [0, 1]$ for different values of $\mu_1 \in [0.1 : 0.4 : 0.9]$ and vary $\mu_1 \in [0, 1]$ for different values of $\lambda_1 \in [0.1 : 0.4 : 0.9]$ in Figure 3.
In Figure 4,$\mu_1 = 0.12$,$\mu_3 = 0.3$,$\lambda_1 = 0.2$ and $\lambda_3 = 0.2$ are fixed and vary $\lambda_2 \in [0, 1]$ for different values of $\mu_2 \in [0.1 : 0.4 : 0.9]$ and vary $\mu_2 \in [0, 1]$ for different values of $\lambda_2 \in [0.1 : 0.4 : 0.9]$ in Figure 5.
In Figure 6,$\mu_2 = 0.1$,$\mu_1 = 0.12$,$\lambda_2 = 0.1$ and $\lambda_1 = 0.2$ are fixed and vary $\lambda_3 \in [0, 1]$ for different values of $\mu_3 \in [0.1 : 0.4 : 0.9]$ and vary $\mu_1 \in [0, 1]$ for different values of $\lambda_3 \in [0.1 : 0.4 : 0.9]$ in Figure 7.

Simulations depicted in Figures 3, 5 and 7 displayed the impact of λ_1 , λ_2 and λ_3 on availability for different values of μ_1 , μ_2 and μ_3 respectively. From these figures, availability decreases whenever λ_j , $j = 1, 2, 3$ increase. It is observed from the figures that availability is less for minor repair $u_j = 0.1$, moderate for medium repair $u_j = 0.5$ and higher for major repair $u_j = 0.9$. From these simulations, it is worthwhile if every failure is treated with perfect repair to revert the system to its position when new. On the other hand, graphs presented in Figures 4, 6 and 8 depicts the impact of μ_1 , μ_2 and μ_3 on availability for different values of λ_1 , λ_2 and λ_3 respectively. From these graphs, availability increases as each μ_j increase different values of λ_1 , λ_2 and λ_3 respectively. Though availability increases in these graphs, it is less for $\lambda_j = 0.9$, moderate for $\lambda_j = 0.5$ and higher for $\lambda_j = 0.1$.

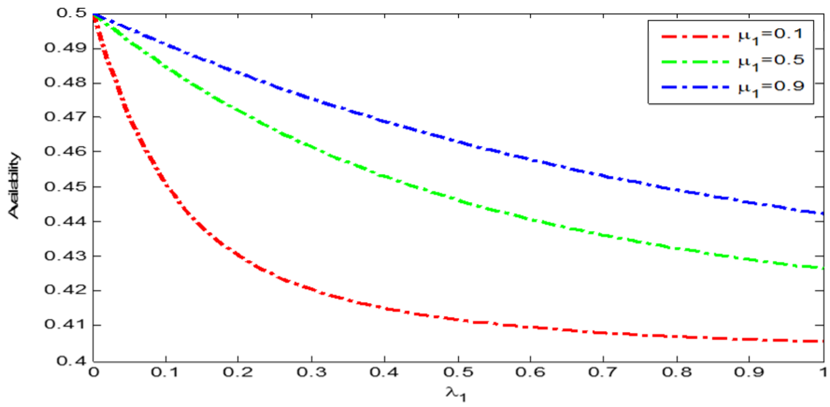

Figure 3: Availability against λ_1 for different values of $\mu_1 \in [0.1 : 0.4 : 0.9]$

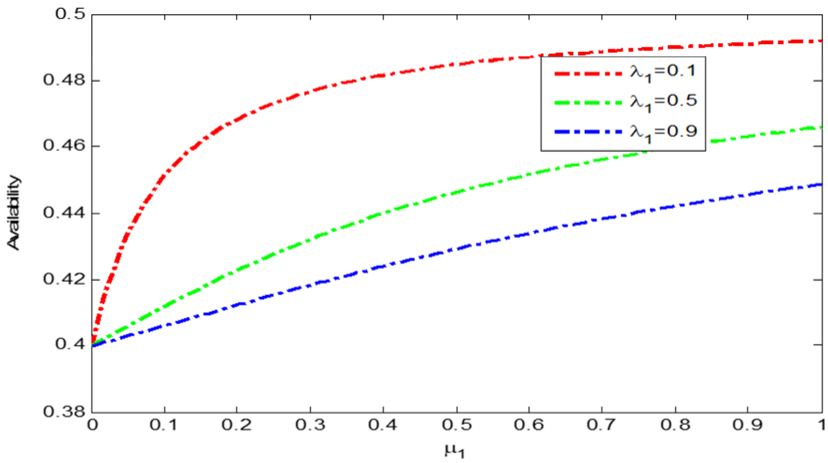

Figure 4: Availability against μ_1 for different values of $\lambda_2 \in [0.1 : 0.4 : 0.9]$

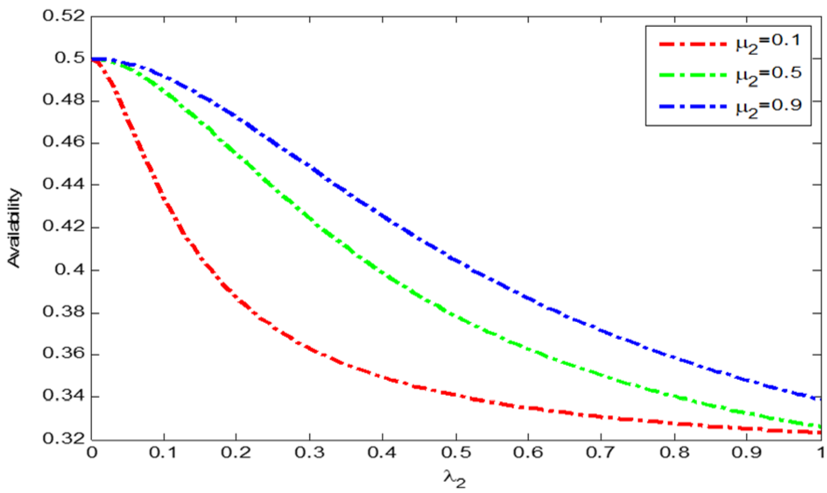

Figure 5: Availability against λ_2 for different values of $\mu_2 \in [0.1 : 0.4 : 0.9]$

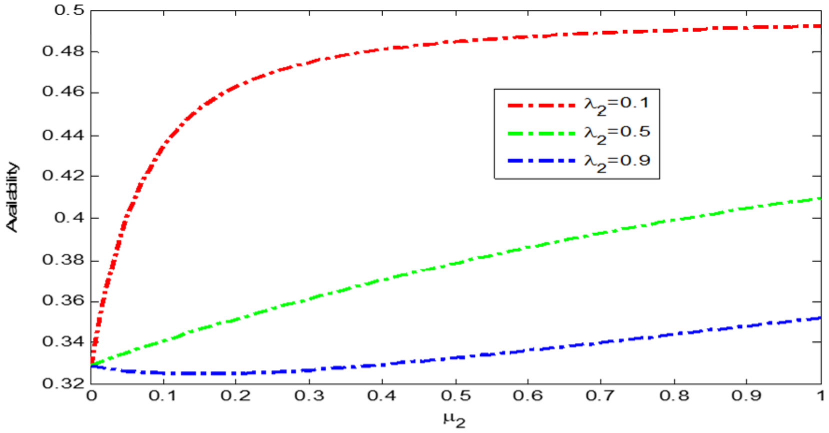

Figure 6: Availability against μ_2 for different values of $\lambda_2 \in [0.1 : 0.4 : 0.9]$

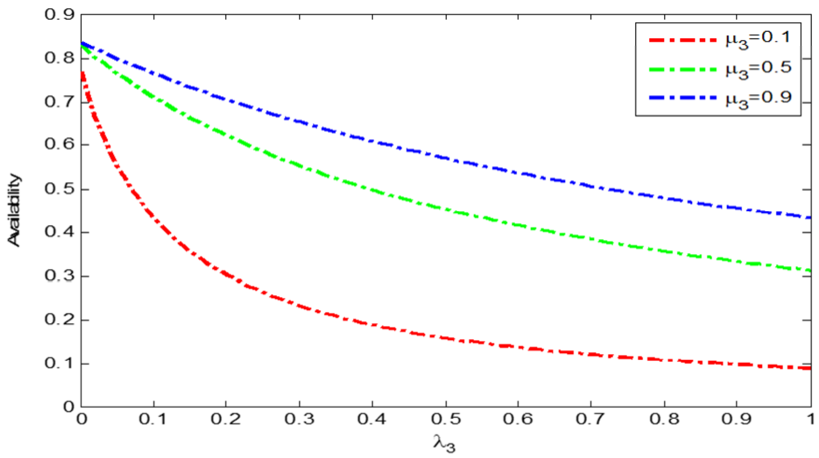

Figure 7: Availability against λ_3 for different values of $\mu_3 \in [0.1 : 0.4 : 0.9]$

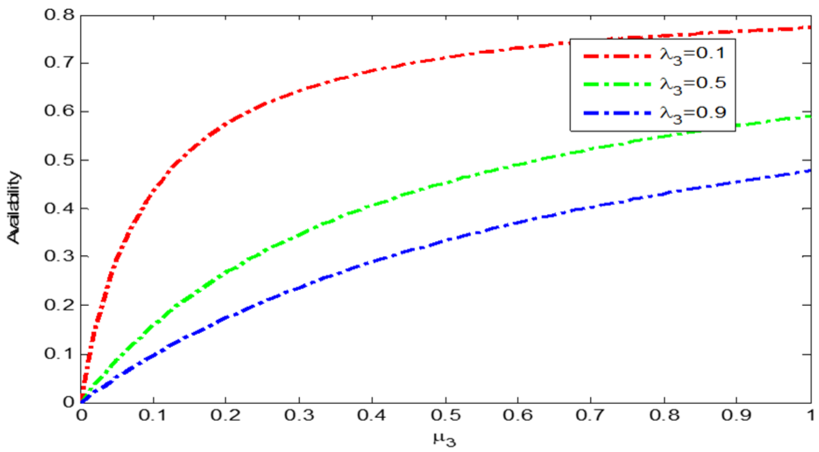

Figure 8: Availability against μ_3 for different values of $\lambda_3 \in [0.1 : 0.4 : 0.9]$

From the simulations presented in this section, it is suggested that adequate maintenance action should be introduced to prevent the occurrence of major failure and adoption of perfect repair return the system to its position when new.

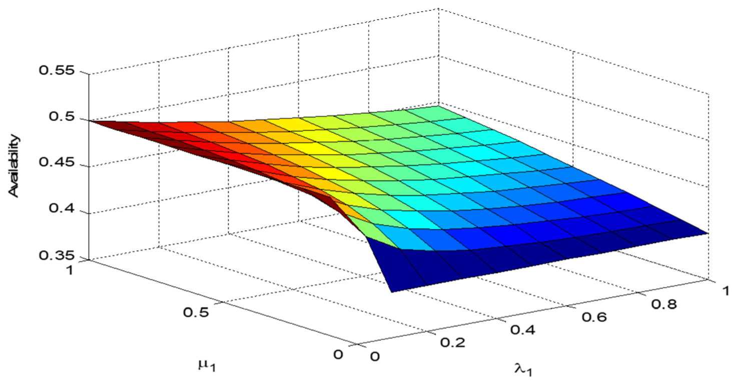

Figure 9: Surface plot of Availability against μ_1 and λ_1

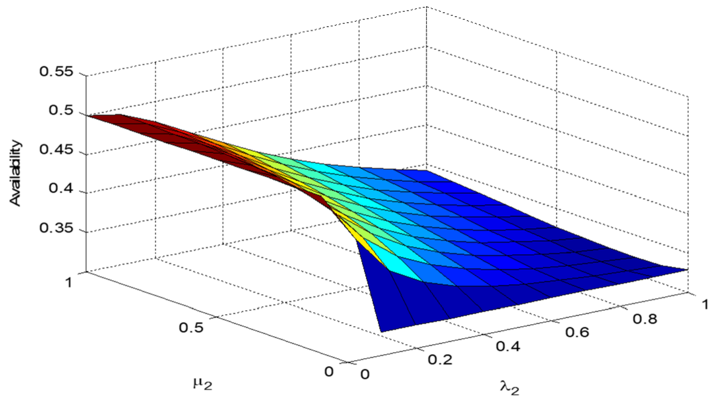

Figure 10: Surface plot of Availability against μ_2 and λ_2

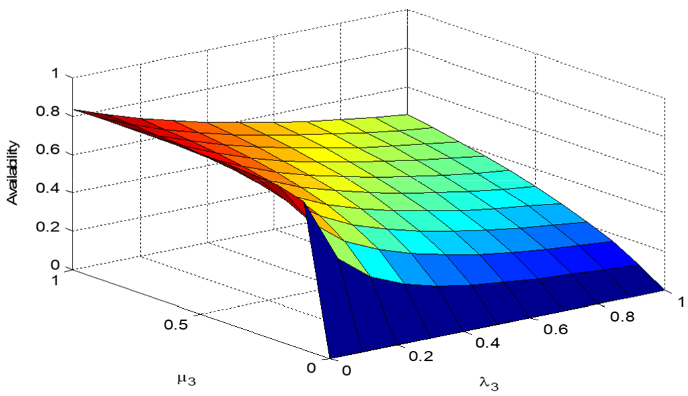

Figure 11: Surface plot of Availability against μ_3 and λ_3

The surface plots in Figures 9, 10 and 11 displayed the trend of availability against failure and repair rates. In these figures, availability increases with increase in μ_j and λ_j decreases as increase.

This sensitivity analyses implies that maintenance strategies be adopted to keep the system strong, improve and maximize the system availability as well as production output.

5. CONCLUSION

This paper studied a hybrid system containing three subsystems I, II and III. Subsystem I is linked to unit I while subsystem II is linked to unit II for the operation of the system. Subsystem II has two units in active parallel system. Explicit expression for the steady-state availability is derived. The system is studied for different failure and repair scenarios. Failure and repair scenarios are assumed to be minor, medium or major. Impact of failure rate on availability for different repair scenario is studied in which the system availability is better when the repair is major. On the other hand, impact of repair rate on system availability is studied for different failure scenario in which availability is better with minor failure. From the simulations presented in the study, it can be concluded that maintenance strategies that will keep the system as failure free should be invoked to maximize availability, product quality, and output and revenue generation. The present work can be extended further for a system to connect to multi standby devices.

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