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Reverse Supply Chain Coordinations for Multi-collectors, Multi-retailers and Single Remanufacturer with Symmetric Information

Atmaram Nigwal[1*], Uttam Kumar Khedleker[2], Nishant Kumar Khedleker[3]

[1*]Department. of Mathematics, Ujjain Engineering.College Ujjain Sanwer, Road, M. P., India 456010

[2]Department of Mathematics and Statistics,Dr. Harising Gour Vishwavidhyalaya, Sagar (MP) 470003

[3] Department. of Management Studies(DOM), Indian Institute of Technology(IIT), Madras, India

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Abstract: This study develops a three-layer, multi-channel coordination policy for reverse supply chain consisting of collectors, remanufacturer, and retailers. It is concerned about the problems, how to decide collecting, retailing, and wholesale pricing decisions in a reverse supply chain of a single used product. A decentralized mathematical model is established under the complete symmetric information scheme by considering price dependent demand of remanufactured product incorporating different holding cost at various stages. It is a generalized, systematic study of collection of a single used product and distribution of remanufactured product. Firstly we have formulated the profit functions, output parameters of all supply chain members, and then we have analyzed the model by simulation study with respect to some key parameters. Model is also illustrated by analytically and numerically in all dimensions.
Keyword — Reverse supply chain, symmetric information, multi-channel, net profit, holding cost.

1. INTRODUCTION

Nowadays, increasing awareness of customers for environmental quality and more tough provisions and regulations introduced by government heavily affects the marketing and operation strategies of firms. To fulfill these necessities, many companies have increasingly showed interest on installing RSCs and developing remanufacturing exercises in order to find more economic benefits. Generally, in the RSCM, due to lack of coordinations among RSC members (collectors, retailers, and remanufacturers) there are two types of problems may arise. The first one is either oversupply or inadequate supply of the used product at the remanufacturer's end, and the another one is either oversupply or inadequate supply of remanufactured product at the retailer's end. Consequently, the upcoming profit of each RSC members, and service providing capacity may be affected. For removing/reducing these types of difficulties the large scale industries, corporate researchers, practitioners have developed a lot of mathematical models in this field.

In (2004), two echelon closed-loop supply chain model is developed by Savaskan, Bhattacharya, and Van Wassenhove, for remanufacturing of used product by considering downwards sloping linear demand function of price. In this article they use three collection methods of used products, in which the first one is manufacturer, the second one is retailer and the third one is third party collecting methods and they proved which method is better. Berastin and Federgruen (2005) suggested a decentralized supply chain model with a competitive environment for retailers in the uncertain demand of the product. **?** proposed a two layer supply chain inventory model, in which they assumed that a single manufacturer works as a Stackelberg leader, and a single retailer works as a follower. They optimized the profit function of each supply chain members for various deterministic demand pattern with symmetric information to asymmetric information structures. **?** presented a two-echelon supply chain model in which they assumed that, the manufacturer is a Stackelberg leader, and two competitive retailers are followers. They analyzed the competitive behavior among retailers in three environments, which are (1) Cournot, (2) Collusion, (3) Stackelberg, and concluded which environment is suitable for competitive behavior.

? designed a RSC model in which they examined the effects of recovery yield rate on pricing strategies. The authors are assumed that at the end of product life, a particular part of a product can be remanufactured/recycled for reuse. For this, they developed a model to find an optimal acquisition price at the end of life of the product and the

[*]Corresponding author's e-mail: arnw@rediffmail.com

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retailing price of remanufactured parts. It is a model for the auto parts recycling industry. **?** suggested a model for two collections and distributions structures of a RSC for collecting and remanufacturing of the used product, in which the first one is by manufacture directly collects the used product from the end consumer, and the second one is by retailer works as a collector of the used product. For this they have considered the demand is a linear function of price.

? studied the problem of supplier selection for manufacturer who acquires a single product from various capacitate suppliers. They made a strategic decision policy to select suitable supplier, and they also determined how much quantity, and how much frequency of the product to purchase from each elected suppliers. **?** developed a closed-loop supply chain model for remanufacturing of used product. The article is based on various collection methods of used products, which are as follows: (1) ownership-based, (2) service-contract, (3) direct-order, (4) deposit-based, (5) credit-based, (6) buy-back, and (7) voluntary based. Furthermore they derived relationships among collection methods and they also described advantages and disadvantages of these methods.

? studied the impact of green production design on the remanufacturing of deteriorating products. For this, they developed an integrated production inventory model in which they derived the replenishment policy and distribution policy for deteriorating products. **?** presented a two-period closed-loop supply chain (CLSC) game model in which they assumed the collection of used product is done by two members, the first one is a retailer, and the second one is a third party. **?** examined the trade-off of acquisition and scrapping costs vs remanufacturing costs, when the used product condition is widely varying and uncertain. Authors have derived optimal acquisition quantity of a used product that optimized the total expected costs. This model is based on the experience of the remanufacturing industry of the cell phone.

? presented an integrated production inventory model in which green product design and remanufacturing efforts are investigated for the product with short lifetime. **?** developed a closed-loop supply chain model for remanufacturing of the used product. In this article, they assumed that the demand of used product is linear function of price and can be collected by either retailer or third-party or both. **?** proposed a reverse supply chain framework to recycle the used computers. They studied and identified the critical factors, which are a barrier to implementing the recycling of computers. They also determined the causal relationship between the factors, which influence the recycling of computers.

? designed a RSC coordination policy for the personal computer industry by using a revenue-sharing contract. The study has presented an analytical structure for two and three-layer echelon stages.

? developed two echelon RSC by considering the demand function is a linear function of price for both products (fresh new one and remanufactured). In this study they also considered the manufacturer, remanufacturers their own product. They analyzed three collection models which are as follow: (1)Retailer managed (2) manufacturer managed and (3) third party managed. Finally, they concluded, which method is better. **?** studied the impact of channel leadership in the RSC. In this paper, they consider a supply chain formed by a single retailer, a single collector, and a single manufacturer. In this supply chain model, they also examined various channel leadership structures assuming demand is a linear decreasing function of price, and finally, they suggested that the retailer led-channel model gives better results. **?** have developed two competing supply chains in which they considered these two supply chains are formed by a single retailer and a single manufacturer, which compete with each other. Further, they considered a manufacturer produces a new product while another one remanufacturer, remanufacturers, the used product introducing some new and advanced features. The retailer demand function taken as a linear function of price.

? developed an optimal decision model for a closed-loop supply chain under symmetric and asymmetric information. They suggested how the manufacturer and the retailer make their own decision about the wholesale price, the retail price, and the collection rate under the retail price dependent demand function. **?** extended the model of **?** by examining the impact of supply chain competition on the RSC in which remanufacturer, remanufacturers their own used product. They designed two chains, the first one is the retailer managed RSC, and the second one is the manufacturer managed RSC. **?** suggested a supply chain model on remanufacturing, in which they applied two strategies, the first one is First remanufacturing, then the pricing process, and the second one is first pricing, then remanufacturing process. In this model, they assumed the demand function is an exponential decreasing function of price.

? developed a closed-loop supply chain model using retail price and quality dependent demand furthermore, they also studied a hybrid manufacturing and remanufacturing process with retail price as well as product quality dependent market demand. **?** focused on power structures of various channel viz. manufacture Stackelberg, vertical structure and retailers Stackelberg in closed-loop supply chain for a centralized and decentralized scenario. They considered price and efforts level dependent demand.

? developed a two-layer supply chain inventory model comprising a single manufacturer and a single retailer using selling price, quality, and warranty dependent demand of a single product. They also derived profit functions for both supply chain members in the centralized and decentralized scenario. **?** developed a multi-channel, multi-echelon supply chain for a single product consisting of a single manufacturer, distributors, and retailers as the supply chain members. In this model, the demand rate is taken as a function of the retail and suggested retail price dependent. In the centralized and decentralized scenario, pricing strategies under a two-part tariff and bargaining process are suggested by authors.

? developed a three-layer reverse supply chain consisting of a single collector, a single re-manufacturer, and a single retailer by using the retail price-dependent demand of a remanufactured product. In this study pricing, collecting, and contract designing problems are highlighted by authors under the complete, and incomplete information. Finally, they have shown that incomplete and asymmetric information might be caused to an efficiency loss of reverse supply chain.

In this article, we have developed a three-layer multi-channel and multi-echelon RSC model consisting of multi-collector, a single remanufacturer, and multi-retailers, (shown in the Fig.1.) for a single used product with symmetric information in the decentralized scenario. It is a generalized, systematic study of collection of a single used product and distribution of remanufactured product (based on RSC model with CI of ?.

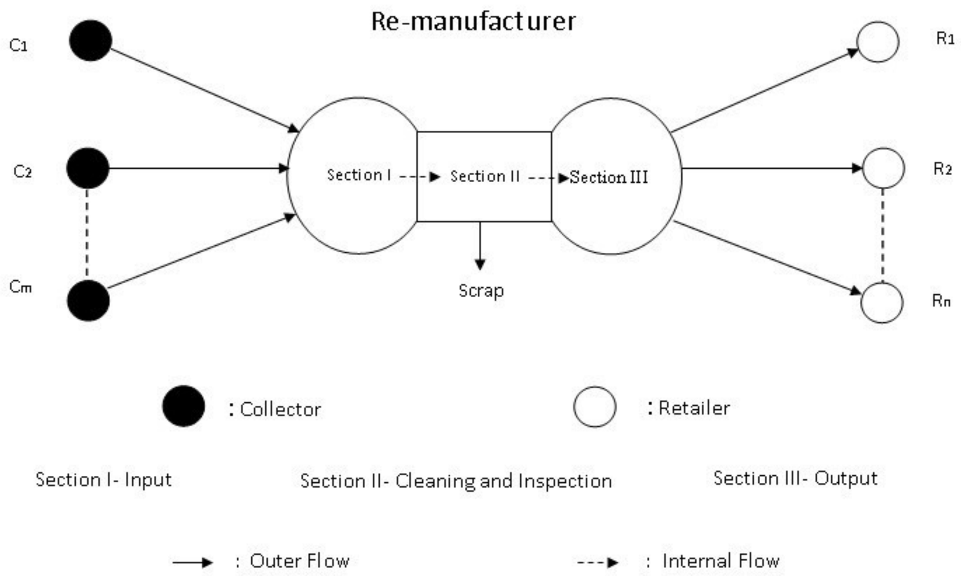

Figure 1: Flow Chart

The market analysis demonstrates that the collection volume of the used product is directly impacted by the acquisition price, which is paid by a collector to buy a single unit of used product. Moreover, the collector's efforts level also increases the collection volume of the used product. Let e_i^c be the effort level, and p_i^c be the acquisition price of the i^{th} collector, we consider the acquisition linear function $s_i^c = a_i + bp_i^c + \gamma e_i^c$ for supply of the used product (where $i = 1, 2, 3, ...m$). Furthermore, we consider the collection effort level cost is quadratic function e_i. i.e. $C(e) = \frac{de_i^2}{2\eta}$.

Market survey reveals that the retailing price has a direct impact on the demand of the remanufactured product, therefore, we can assume that the market demand of the remanufactured is a linear decreasing function of the retail price. Let p_j^r and α_j, respectively, be the retailing price and base demand of remanufactured product for $j^{th}(j = 1, 2, 3, ...n)$ retailer. Without any hassle, we can consider $D_j^r = \alpha_j - \beta p_j^r$ as a linear function of price.

The objective of this research is to find an optimal acquisition price p_i^c, an optimum effort level e_i for all collectors, an optimum acquisition wholesale price w^c, an optimal wholesale price w^m for remanufacturer and an optimal retailing price p_j^r for all retailers in the decentralized scenario with complete informations.

2. NOTATIONS AND ASSUMPTIONS

For this article, we have made the following notations:

p_i^c : Acquisition price per unit of used product for the i^{th} collector $(i = 1, 2, 3, ...m)$,

e_i^c : Per unit collection effort level of the used product for the i^{th} collector $(i = 1, 2, 3, ...m)$,

s_i^c : Per unit time supply of the used product at i^{th} collector's end $(i = 1, 2, 3, ...m)$,

w^c : Acquisition wholesale price per unit of the remanufactured product for all collectors with symmetric informations,

p_j^r : Selling price of the remanufactured product at j^{th} retailer's end $(j = 1, 2, 3, ...n)$,

D_j^r : Demand rate per unit time of the remanufactured product at j^{th} retailer's end $(j = 1, 2, 3, ...n)$,

w^m : Per unit wholesale price of the remanufactured product for the j^{th} retailer's end $(j = 1, 2, 3, ...n)$,

c : Remanufacturing cost per unit of the used product,

c_c : Collection cost per unit of the used product at i^{th} collector's end $(i = 1, 2, 3, ...m)$,

h : Per hulk per unit salvage value of the used product,

s_r : Unit salvage value per unit of manufacturable part of the used product,

c_l : Per unit inspection, cleaning and sorting cost of the used product,

n : Total number of retailers,

m : Total number of collectors,

g : Coefficient of the collection effort level,

$C(e)$: Per unit effort level cost $a(e) = \frac{g e_i^{c2}}{2\eta}$,

Π_i^c : The i^{th} collector's net profit,

Π^m : Net profit of remanufacturer,

Π_J^r : j^{th} retailer net profit,

h^r : Holding cost per unit per unit time for retailers,

h^c : Holding cost per unit per unit time for collectors,

h^m : Holding cost per unit per unit time for remanufacturer.

2.1 Assumptions

For this article, we have made the following assumptions:

- Supply of the used product from end-user at i^{th} collector's is $s_i^c = a_i + bp_i^c + \gamma e_i^c$, a linear function of acquisition price and effort level,
- Demand rate of the remanufactured product from the market at j^{th} retailer's end is $D_j^r = \alpha_j - \beta p_j^r$, a linearly decreasing function of the retail price, where $\alpha_j, \beta > 0$,
- $a = \sum_{i=1}^{m} a_i$(as **?**), where a_i represents on-hand collection volume of the used product at i^{th} $(i = 1, 2, 3, ...m)$ collector's end, 'a' represents, total on-hand collection volume of the used product at remanufacturer's end. $\alpha = \sum_{j=1}^{n} \alpha_j$, (as **?**), where, α_j represents the initial demand of the remanufactured product at j^{th} $(j = 1, 2, 3, ...n)$ retailer's end and α represents the total initial demand for the remanufactured product at remanufacturer's end.
- The competition among collectors and retailers are not allowed, because the situations of collectors and retailers have been considered in different geographical areas.
- Holding cost of all RSC members are different, where, $h^c < h^m < h^r$,
- Acquisition wholesale price of the used product is equal for all collectors and the wholesale price of the remanufactured product is equal for all retailers.
- Effort level cost assumed as $C(e) = \frac{ge^2}{2\eta}$, is a linear-quadratic function of the effort level (as **?**, where, η is effort level expenditure cost controlling parameter, which can be either increased or decreased by remanufacturer in case of either oversupply or poor supply of the used product,
- Manufacturing cost of fresh product from raw material is more costly than remanufacturing cost of a new one product from the used product.
- Remanufacturer is a Stackelberg leader of the whole supply chain.
- During in the process of optimization of all supply chain member's objectives, it had been assumed that all RSC members have the uniform informations about market.
- Shortages are not allowed at any stage.
- Deterioration rate is zero.

2.2 Proposed Model for Collectors

The collector who buys the used product from customers. According to assumptions, there are m-collectors provide the collection facility in the different geographical areas. A collector can use his efforts to collect more and more collection of the used product to optimize his profit.

Based on the above information and assumptions, the net profit function Π_i^c of the i^{th} collector can be defined as follows:

$$\Pi_i^c = (w^c - p_i^c - c_c - h^c)(a_i + bp_i^c + \gamma e_i^c) - \frac{g e_i^{c2}}{2\eta}, (\text{where } i = 1, 2, 3, \ldots m). \tag{1}$$

Lemma2.1 *The net profit function* Π_i^c *of the* i^{th} *collector is a jointly concave with respect to effort level* e_i^c *and collector's acquisition price* p_i^c*, if* $2bg - \gamma^2\eta > 0$.

Proof: The first-order partial derivatives of the net profit function Π_i^c of the i^{th} collector with respect to e_i^c and p_i^c respectively are:

$$\frac{\partial \Pi_i^c}{\partial e_i^c} = (w^c - p_i^c - c_c - h^c)\gamma - \frac{g e_i^c}{\eta}, (\text{where } i = 1, 2, 3, \ldots m). \tag{2}$$

$$\frac{\partial \Pi_i^c}{\partial p_i^c} = (w^c - p_i^c - c_c - h^c)b - (a_i + bp_i^c + \gamma e_i^c), (\text{where } i = 1, 2, 3, \ldots m). \tag{3}$$

To prove that the profit function Π_i^c is a jointly concave with respect to e_i^c , and p_i^c , the Hessian matrix of the profit Π_i^c, must be negative semi definite

$$HM = \begin{bmatrix} \frac{\partial^2 \Pi_i^c}{\partial e_i^{c2}} & \frac{\partial^2 \Pi_i^c}{\partial p_i^c \partial e_i} \\ \frac{\partial^2 \Pi_i^c}{\partial p_i^c \partial e_i} & \frac{\partial^2 \Pi_i^c}{\partial p_i^{c2}} \end{bmatrix} = \begin{bmatrix} -2b & -\gamma \\ -\gamma & -\frac{g}{\eta} \end{bmatrix}. \tag{4}$$

Hence, if $b > 0, \eta > 0$ and $2bg - \gamma^2\eta > 0$. then the Hessian matrix of the profit function Π_i^c, is negative semi definite, and therefore the profit Π_i^c is a jointly concave with respect to e_i^c and p_i^c.

Proposition2.1 *The* i^{th} *collector's optimal collection effort level* e_i^c *and optimal acquisition price* p_i^c*, respectively are given by* p_i^{c*} *and* ${e_i}^*$ *, if* $2bg - \gamma^2\eta > 0$*, where*

$$e_i^{c*} = \frac{(w^c - c^c - h^c)b\gamma\eta + \gamma a_i \eta}{2bg - \gamma^2\eta} \tag{5}$$

$$p_i^{c*} = \frac{(w^c - c^c - h^c)(bg - \gamma^2\eta) - a_i g}{2bg - \gamma^2\eta}. \tag{6}$$

Proof: On equating to zero, the above equations (2) and (3), respectively give the following paired simultaneous linear equations:

$$(w^c - p_i^c - c_c - h^c)b - (a_i + bp_i^c + \gamma e_i^c) = 0 \tag{7}$$

$$(w^c - p_i^c - c_c - h^c)\gamma - \frac{g e_i^c}{\eta} = 0. \tag{8}$$

On solving these simultaneous linear equations, we get the optimal effort level and the acquisition price of the used product for the i^{th} collector, which are given by equations (5) and (6), respectively.

Proposition2.2 *The* i^{th} *collector's acquisition price* p_i^c *increases, when the remanufacturer's acquisition wholesale price* w^c *increases, if* b *is sufficiently large.*

Proof: From the equation (5) collector's acquisition price p_i^c depends on remanufacturer's acquisition wholesale price w^c. Hence the first order partial derivative of p_i^c with respect to w^c is

$$\frac{\partial p_i^c}{\partial w^c} = \frac{bg - \gamma^2\eta}{2bg - \gamma^2\eta} > 0, \text{where } i = 1, 2, 3, \ldots m,$$

if when $2bg-\gamma^2\eta > 0$, then p_i^c is an increasing function with respect to w^c. It means the acquisition price p_i^c is increases, when the acquisition wholesale price w^c increases. It is possible only when acquisition price sensitive parameter b is sufficiently large. In other words, it is possible only when customers are more sensitive with the acquisition price.

Proposition2.3 *The i^{th} collector's effort levels e_i^c increases, when the remanufacturer's acquisition wholesale price w^c increases.*

Proof: Using the equation (6) the first order partial derivative with respect to w^c is

$$\frac{\partial e_i^c}{\partial w^c} = \frac{b\gamma\eta}{2bg-\gamma^2\eta} > 0, \text{where } i = 1,2,3,\ldots m.$$

The above equation shows that the effort level e_i^c is a increasing function of the wholesale acquisition price w^c, this reveals that when the wholesale acquisition price increases, then the i^{th} collector's effort level e_i increases, because all collectors are engaged for getting high margin of individual profit.

2.3 Proposed Model for Retailers

A retailer that sells remanufactured products in the secondary market. When remanufacturer supplies the remanufactured product to the $j^{th}(j = 1,2,3,...n)$ retailer at a rate of w^m per unit and the demand of remanufactured product at j^{th} retailer's end is $\alpha_j - \beta p_j^r$. According to the assumptions, the j^{th} retailer's profit function can be defined as follows:

$$\Pi_j^r = (\alpha_j - \beta p_j^r)(p_j^r - w^m - h^r), \text{where } j = 1,2,3,\ldots n. \tag{9}$$

Lemma2.2: For $\beta > 0$, the j^{th} retailer's net profit function Π_j^r is concave with respect to the retail price and an optimal retail price p_j^{r*} is:

$$p_j^{r*} = \frac{\alpha_j + (w^m + h^r)\beta}{2\beta}. \tag{10}$$

Proof: The first order partial derivative of the equation (9) is

$$\frac{\partial \Pi_j^r}{\partial p_j^r} = (\alpha_j - \beta p_j^r) - \beta(p_j^r - w^m - h^r). \tag{11}$$

For showing concavity with respect to a single variable, the second order partial derivative of equation (9) is

$$\frac{\partial^2 P_j^r}{\partial p_j^{r2}} = -2\beta, \tag{12}$$

hence, if $\beta > 0$, Π_j^r shows concavity with respect to $p_j^r, (j = 1,2,3,...n)$. Further, at an optimum value of p_j^r, the first order partial derivative will be zero. By using the equation (11) we have

$$(\alpha_j - \beta p_j^r) - \beta(p_j^r - w^m - h^r) = 0 \tag{13}$$

on solving the equation (13), yields optimal retailing price p_j^{r*} for j^{th} retailer, i.e.,

$$p_j^{r*} = \frac{\alpha_j}{2\beta} + \frac{w^m}{2} + \frac{h^r}{2}. \tag{14}$$

2.4 Proposed Model for Remanufacturer

Before the formulation of remanufacturer's profit function, it is required to formulate the total supply rate of used product from m-collectors, and the total demand rate of remanufactured product from n-retailers. Therefore, the total supply rate of the used product and the total demand rate of the re-manufactured product respectively can be defined by the following Lemmas:

2.4.1 Supply for Remanufacturer

Lemma2.3 *If the total supply of the remanufacturer's end is equal to the sum of m-collector's supply of the used product, then the total supply of the remanufacturer's end can be given by equation (15) and it is denoted by U^m.*

$$U^m = \frac{abg + b^2gm(w^c - c^c - h^c)}{2bg - \gamma^2\eta}, \text{where } a = a_1 + a_2 + a_3 + \cdots + a_m. \tag{15}$$

Proof: According to the assumptions, a_i be the basic supply, e_i^c is the effort level and p_i^c is the acquisition price of the used product at the i^{th} collector's end $(i = 1, 2, 3, ...m)$. If the supply of the used product at j^{th} collector's end is s_i^c , then the total supply of remanufacturer's end could be formulated as follow:

$$U^m = \sum_{i=1}^{m} s_i^c = \sum_{i=1}^{m}(a_i + bp_i^c + \gamma e_i) = a + b\sum_{i=1}^{m} p_i^c + \gamma\sum_{i=1}^{m} e_i. \tag{16}$$

On inserting the above values of p_i^c and e_i from equations (5) and (6) respectively into the equation (19), yields.

$$U^m = \frac{abg + b^2gm(w^c - c^c - h^c)}{2bg - \gamma^2\eta}, \text{where } a = a_1 + a_2 + a_3 + \cdots + a_m. \tag{17}$$

2.4.2 Demand for Remanufacturer

Lemma2.4 *If the total demand of the remanufacturer's end is equal to the sum of all the n-retailer's demand of the remanufactured product, then the total demand of remanufacturer's end can be given by the equation (18), and it is denoted by D^m.*

$$D^m = \frac{\alpha - (w^m + h^c)\beta n}{2}, \text{where } \alpha = \alpha_1 + \alpha_2 + \alpha_3 + \cdots + \alpha_n. \tag{18}$$

Proof: According to the assumptions, α_i be the basic demand and p_j^r be the retailing price of the remanufactured product at $j^{th}(j = 1, 2.3, ...n)$ retailer's end, and if D_j^r be the demand of the remanufactured product at j^{th} retailer's end, then the total demand of the remanufactured product at the remanufacturer's end could be formulated as follow:

$$D^m = \sum_{j=1}^{m} D_j^r = \sum_{j=1}^{n}(\alpha_j - \beta p_j^r) = \alpha - \beta\sum_{j=1}^{n} p_j^r. \tag{19}$$

On inserting the value of p_j^r from equation (14) into the equation (19), yields

$$D^m = \frac{\alpha - (w^m + h_c^c)\beta n}{2}, \text{where } \alpha = \alpha_1 + \alpha_2 + \alpha_3 + \cdots + \alpha_n. \tag{20}$$

2.5 Remanufacturer's Profit Function

By using results of the above propositions the remanufacturer's profit function can be formulated as:

$$\begin{aligned} \max \Pi^m =& \frac{\alpha - w^m\beta n - h^c\beta n}{2}(w^m - s_r - c - h^m) \\ &+ \frac{abg + b^2gm(w^c - c^c - h^c)}{2bg - \gamma^2\eta}(h + s_r - w^c - c_l) \end{aligned} \tag{21}$$

subject to :

$$\frac{\alpha - w^m\beta n - h^c\beta n}{2} \leq \frac{abg + b^2gm(w^c - c^c - h^c)}{2bg - \gamma^2\eta} \tag{22}$$

where, the term of right-hand side shows the supply of used products and the left hand side shows the demand for the remanufacturerd product.

The equation (21) shows the profit of the remanufacturer, who dispatch the remanufactured product to the n-retailers at different geographical locations. The constraints shown in the equation (22) provides the information that the volume of the remanufactured product can not exceed the supply of the used product. Therefore, from the above constraints, there are two situations may be possible, the first one is

$$\frac{\alpha - w^m\beta n - h^c\beta n}{2} < \frac{abg + b^2gm(w^c - c^c - h^c)}{2bg - \gamma^2\eta} \tag{23}$$

and the second one is

$$\frac{\alpha - w^m\beta n - h^c\beta n}{2} = \frac{abg + b^2gm(w^c - c^c - h^c)}{2bg - \gamma^2\eta} \tag{24}$$

According to the first situation which is given in equation (23), remanufacturer's decisions consist of two facts:

- What should be the optimum acquisition wholesale price, when the collection amount of the used product is sufficient for the demand of remanufactured product?
- What should be the optimum wholesale price of remanufactured product for maximizing the profit?

In the second situation in the equation (24) remanufacturer's decision consists only one fact that what will be the optimum wholesale price of the remanufactured product for maximization of the profit, because remanufacturing of the used product depends on the supply of the used product.

Our objective is to determine max Π^m (given in the equation 21) under the situation (22). For this, we developed the following function under the situation (22) by using the method of Lagrange's multipliers

$$\begin{aligned} L(w^m, w^c, \lambda) = &\left(\frac{\alpha - w^m\beta n - h^c\beta n}{2}\right)(w^m - s_r - c - h^m) \\ &+ \left[\frac{bga + b^2g(w^c - c_c - h^c)m}{2bg - \gamma^2\eta}\right](h + s_r - w^c - c_l) \\ &- \lambda\left[\frac{\alpha - w^m\beta n - h^c\beta n}{2} - \frac{bga + b^2g(w^c - c_c - h^c)m}{2bg - \gamma^2\eta} + Z^2\right], \end{aligned} \tag{25}$$

where, λ is a Lagrangian multiplier.

Lemma2.5 *Remanufacturer's profit function* $\Pi^m(i)$ *shows jointly concavity in* w^c *and* w^m *in the situation of the equation (23) (i.e* $Z^2 > 0$ *or* $f < o$*) and (ii) shows concavity in* w^c *in the situation of equation(24) (as* **?***) (i.e* $Z^2 \leq 0$ *or* $f \geq o$*), when* $2b^2gm + bn(2bg - \gamma^2 n) > 0$*,where* $f = \phi(\alpha - \beta nh_c^r - \beta n(s_r + c + h^m)) - 2bga - 2b^2gm(h + s_r - c_l - c_c - h^r)$.

Proof: (i) According to the first situation given in the equation (23), the remanufacturer's profit is not restricted by the collection amount of the used product, and the remanufacturer can made his own strategies about the acquisition collection price w^c and the wholesale price w^m. Hence, on applying the Kuhn Tucker optimization conditions on $L(w^m, w^c, \lambda)$ (as You (2005)).

$$\frac{\partial L}{\partial w^c} = -\frac{bga + b^2gm(w^c - c_c - h^c)}{2bg - \gamma^2\eta} + \frac{b^2gm}{2bg - \gamma^2\eta}(h + s_r - w^c - c_l) + \frac{\lambda b^2gm}{2bg - \gamma^2\eta} = 0 \tag{26}$$

$$\frac{\partial L}{\partial w^m} = \frac{\alpha - w^m\beta n - h^r\beta n}{2} - \frac{\beta n}{2}(w^m - s_r - c - h^m) - \frac{\lambda\beta n}{2} = 0 \tag{27}$$

$$\frac{\partial L}{\partial \lambda} = -\left[\frac{\alpha - w^m\beta n - h^c\beta n}{2} - \frac{bga + b^2gm(w^c - c_c - h^c)}{2bg - \gamma^2\eta} + Z^2\right] = 0 \tag{28}$$

$$\frac{\partial L}{\partial Z} = -2\lambda Z = 0 \tag{29}$$

if $Z^2 > 0$, then from the equation (29), $\lambda = 0$ and hence, we obtained

$$\begin{aligned} \Pi^m = &\frac{\alpha - w^m\beta n - h^c\beta n}{2}(w^m - s_r - c - h^m) \\ &+ [\frac{abg + b^2gm(w^c - c^c - h^c)}{2bg - \gamma^2\eta}](h + s_r - w^c - c_l) \end{aligned} \tag{30}$$

and second order partial derivatives of the equation (30) are:

$$\frac{\partial^2\Pi^m}{\partial w^{c2}} = -\frac{2b^2gm}{2bg - \gamma^2\eta}$$

$$\frac{\partial^2\Pi^m}{\partial w^{m2}} = -\beta n$$

$$\frac{\partial^2\Pi^m}{\partial w^c\partial w^m} = 0$$

To ensure that the profit Π^m is a jointly concave with respect to w^m, and w^c, the Hessian matrix of profit Π^m, must be negative semi definite,

$$HM = \begin{bmatrix} \frac{\partial^2 \Pi^m}{\partial w^{c2}} & \frac{\partial^2 \Pi^m}{\partial w^m \partial w^c} \\ \frac{\partial^2 \Pi^m}{\partial w^m \partial w^c} & \frac{\partial^2 \Pi^m}{\partial w^{m2}} \end{bmatrix} = \begin{bmatrix} -\frac{2b^2 gm}{2bg-\gamma^2\eta} & 0 \\ 0 & -\beta n \end{bmatrix}. \tag{31}$$

Hence, if when $\beta > 0, \eta > 0$ and $2bg-\gamma^2\eta > 0$, then the Hessian Matrix of profit Π^m, is negative semi definite and the profit Π^m is a jointly concave with respect to w^m, and w^c .
For the proof of the second part (i.e, $Z \leq 0$, or $f \geq 0$), see next proposition.

Proposition2.4 *An optimal acquisition wholesale price and an optimal wholesale price for remanufacturer's are* w^{c*} *and* w^{m*} *respectively, where*

$$w^{c*} = \frac{(c_c + h^c + h + s_r - c_l)}{2} - \frac{\alpha}{2bm} \qquad , if\ f < 0 \tag{32}$$

$$w^{c*} = \frac{(\alpha - h^r\beta n - \theta)\phi bn\beta m - (a - bc_c m - bh^c m)(4b^2 gm + \beta n\phi)}{2bm(2b^2 gm + \beta n\phi)} \qquad , if\ f \geq 0 \tag{33}$$

$$w^{m*} = \frac{\alpha - h^r\beta n}{2\beta n} + \frac{(s_r + c + h^m)}{2} \qquad , if\ f < 0 \tag{34}$$

$$w^{m*} = \frac{(\alpha - h^r\beta n)}{\beta n} - \frac{2bga + 2b^2 g(w^c - c_c - h^c)m}{\phi\beta n} \qquad , if\ f \geq 0 \tag{35}$$

where, $\theta = (c + h^m - h + c_l)$, $\phi = (2bg-\gamma^2\eta)$ and $f = \phi(\alpha - \beta n h_c^r - \beta n(s_r + c + h^m)) - 2bga + 2b^2 g(h + s_r - c_l - c_c - h^r)m$.
Proof: (i) If $f < 0$, the partial derivative of Π^m with respect to w^c and w^m respectively are:

$$\frac{\partial \Pi^m}{\partial w^c} = -\frac{bga + b^2 gm(w^c - c_c - h^c)}{\phi} + \frac{b^2 dm}{\phi}(h + s_r - w^c - c_l) \tag{36}$$

$$\frac{\partial \Pi^m}{\partial w^m} = \frac{a - w^m\beta n - h^r\beta n}{2} - \frac{\beta n}{2}(w^m - s_r - c - h^m) \tag{37}$$

where, $\phi = (2bg-\gamma^2\eta)$, for the optimum w^c and w^m at which both the first order partial derivatives of Π^m must be zero, therefor using the equations (36) and (37) respectively and put them equal to zero,i.e.,

$$-\frac{bga + b^2 gm(w^c - c_c - h^c)}{\phi} + \frac{b^2 dm}{\phi}(h + s_r - w^c - c_l) = 0 \tag{38}$$

$$\frac{a - w^m\beta n - h^r\beta n}{2} - \frac{\beta n}{2}(w^m - s_r - c - h^m) = 0. \tag{39}$$

On solving the equations (38) and (39) yield, the acquisition wholesale price w^{c*} and the wholesale price w^{m*} which are:

$$w^{c*} = \frac{(c_c + h^c + h + s_r - c_l)}{2} - \frac{\alpha}{2bm} \qquad , if\ f < 0 \tag{40}$$

$$w^{m*} = \frac{\alpha - h^r\beta n}{2\beta n} + \frac{(s_r + c + h^m)}{2} \qquad , if\ f < 0 \tag{41}$$

(ii) If $f \geq 0$, in this situation the remanufacturer's decisions depend on the collection amount of the used product, and therefore, the wholesale price of remanufactured product w^m can be represented in terms of the acquisition wholesale price of the used product. Hence, from the constraint (24) on assuming both, sides are equal, the wholesale price is given by

$$w^m = \frac{\alpha - h^r\beta n}{\beta n} - \frac{2bga + 2b^2 g(w^c - c_c - h^c)m}{\phi\beta n}. \tag{42}$$

Then, inserting the value of w^m from the equation (42) into equation (21), redefined remanufacturer's profit function is

$$\begin{aligned}\Pi^m =& \frac{bd(a+b\Gamma m)}{\phi^2 \beta n}[(\alpha - h^r \beta n)\phi - 2bd(a+b\Gamma m)] \\ &- \frac{bd(a+b\Gamma m)}{\phi^2 bn}[(s_r + c + h^m)\phi\beta n + (h + s_r - w^c - c_l)\phi\beta n],\end{aligned} \tag{43}$$

where, $\phi = (2bg - \gamma^2\eta)$, $\Gamma = (w^c - c_c - h^c)$.
Using the equation (43) the first order partial derivative of Π^m with respect to w^c is

$$\begin{aligned}\frac{\partial \Pi^m}{\partial w^c} =& \frac{bga + b^2gm(w^c - c_c - h^c)}{\phi^2 bn}(-2b^2gm - \beta n\phi) \\ &+ \frac{b^2gm}{\phi^2\beta n}[(\alpha - h^r\beta n)\phi - 2bga + b^2gm(w^c - c_c - h^c)] \\ &- \frac{b^2gm}{\phi}[(s_r + c + h^m) + (h + s_r - w^c - c_l)].\end{aligned} \tag{44}$$

According to the optimality condition, for the optimum value of w^c, $\frac{\partial \Pi^m}{\partial w^c} = 0$ i.e.,

$$\begin{aligned}\frac{\partial \Pi^m}{\partial w^c} =& \frac{bga + b^2gm(w^c - c_c - h^c)}{\phi^2 bn}(-2b^2gm - \beta n\phi) \\ &+ \frac{b^2gm}{\phi^2\beta n}[(\alpha - h^r\beta n)\phi - 2bga + b^2gm(w^c - c_c - h^c)] \\ &- \frac{b^2gm}{\phi}[(s_r + c + h^m) + (h + s_r - w^c - c_l)] = 0\end{aligned} \tag{45}$$

solution of the above equation gives, the optimal acquisition wholesale price

$$w^{c*} = \frac{(\alpha - h^r\beta n - \theta)\phi bn\beta m - (a - bc_c m - bh^c m)(4b^2dm + \beta n\phi)}{2bm(2b^2dm + \beta n\phi)}, \tag{46}$$

where, $2bm(2b^2gm + bn\phi) > 0$, and $\theta = (c + h^m - h + c_l)$, $\phi = (2bg - \gamma^2\eta)$.

At last w^{m*} is obtained after inserting the value of w^c from equation (33) into equation (35).
Now the proof of second part of Lemma (2.5), using the equation (43) the second order partial derivative of Π^m is

$$\frac{\partial^2 \Pi^m}{\partial w^{c2}} = -\frac{2b^2dm}{\phi^2\beta n}(2b^2dm + \beta n\phi) \tag{47}$$

if $(2b^2dm + \beta n\phi) > 0$, and $\phi > 0$ where, $\phi = (2\beta v - \gamma^2\eta)$. Hence, remanufacturer's profit function Π^m shows concavity with respect to wholesale collection price w^c.

3. ALL DECISION COMPONENT

Using the results of proposition (2.1) all decision components of every RSC members are as follows:

3.1 Collector's Decision Components

Collector's profit depends on the acquisition price p_i^c and the effort level e_i, where $i = 1, 2, 3, \ldots, m$.Therefore, on inserting the optimal acquisition wholesale price w^c, from the equation (32) (for $f < 0$) and equation (33) (for $f \geq 0$) into equations (5) and (6) respectively yield, for the i^{th} collector's acquisition price and effort level, which are given by the following equations:

$$\begin{aligned}
p_i^c &= \frac{(h+s_r-c_l-c_c-h^c-\frac{a}{mb})(bg-\gamma^2\eta)-a_i g}{2\phi} && , if\ f < 0 \\
p_i^c &= \frac{[((\alpha-h^r\beta n)-\kappa)\phi bn\beta m-(a-bc_c m-bh^c m)(4b^2gm+\beta n\phi)-\xi(c_c+h^c)](bg-\gamma^2\eta)-a_i d\xi}{\phi\xi} && , if\ f \geq 0 \\
e_i &= \frac{(h+s_r-c_l-c_c-h^c-\frac{a}{mb})b\gamma\eta+2\gamma a_i\eta}{2\phi} && , if\ f < 0 \\
e_i &= \frac{[((\alpha-h^r\beta n)-\kappa)\phi bn\beta m-(a-b(c_c+h^c)m)(4b^2gm+\beta n\phi)-\xi(c_c+h^c)]b\gamma a+\gamma a_i a\xi}{\phi\xi} && , if\ f \geq 0
\end{aligned}$$

where, $\kappa = (c + h^m - h + c_l)$, $\xi = 2bm(2b^2gm + \beta n(2bg - \gamma^2\eta))$ and $\phi = (2bg - \gamma^2\eta)$.

3.2 Remanufacturer's Decision Components

Remanufacturer's decision components are: (i) the total supply of the used product, which is the sum of all m-collector's individual supply, (ii) the total demand of the remanufactured product, which is the sum of all n-retailer's individual demand, (iii) the acquisition wholesale price of the used product, and (iv) the wholesale price of the remanufactured product. The optimal acquisition wholesale price w^c and the optimal wholesale price w^m for $f < 0$, respectively are given by the equations (32) and (34). The optimal acquisition wholesale price w^c and optimal wholesale price w^m for $f \geq 0$, respectively are given by the equations (33) and (35).The total supply and total demand are given by the following equations:

$$\begin{aligned}
U^m &= \frac{2bga+bg(h+s_r-c_l-c_c-h^c-\frac{a}{mb})}{2\phi} && , if\ f < 0 \\
U^m &= \frac{bg[a\xi+bm((\alpha-h^r\beta n)-\kappa)\phi bn\beta m-(a-b(c_c+h^c)m)(4b^2gm+\beta n\phi)-\xi(c_c+h^c)]}{\phi\xi} && , if\ f \geq 0 \\
D^m &= \frac{\alpha-h^r\beta n-\beta n(s_r+c+h_c^m)}{4} && , if\ f < 0 \\
D^m &= \frac{bg[a\xi+bm((\alpha-h^r\beta n)-\kappa)\phi bn\beta m-(a-b(c_c+h^c)m)(4b^2gm+\beta n\phi)-\xi(c_c+h^c)]}{\phi\xi} && , if\ f \geq 0
\end{aligned}$$

where, $\phi = (2bg - \gamma^2\eta)$.

3.3 Retailer's Decision Components

The j^{th} retailer's has only one decision variable, which is the retailing price of the remanufactured product.
The j^{th} retailer's optimum decision variable $p_j^r (j = 1, 2, 3, ...n)$ is given by the following equations:

$$p_j^r = \frac{[2n\alpha_j + \alpha + \beta n(s_r + c + h^m + h^r)]}{4\beta n} \quad , if\ f < 0 \tag{48}$$

$$p_j^r = \frac{[(\alpha_j n + \alpha)\phi - 2bd(a + b(w^c - c_c - h^c)m)]}{4\beta n\phi} \quad , if\ f \geq 0 \tag{49}$$

where, $\phi = (2bg - \gamma^2\eta)$, and w^c is given by equation (33). The $i^{th}(i = 1, 2, 3, ...m)$ collector's optimum profit function can be obtained (for $f < 0$, $f \geq 0$) by inserting the corresponding acquisition price, and effort level from components subsection.

The remanufacturer's optimum profit function can be obtained (for $f < 0$, $f \geq 0$) by inserting the corresponding acquisition wholesale price and wholesale price from the concerning subsection.

The j^{th} retailer's optimum profit can be obtained (for $f < 0$, $f \geq 0$) by inserting the j^{th} retail price from equation (48), and (49) into the equation (9), respectively.

3.4 Numerical Example

For numerically illustration, we assumed that the RSC contains a single remanufacturer M, two collectors (C_1, C_2) and two retailers (R_1, R_2). According to the Fig.1., each retailer is associated with a certain geographical situation. We considered the following data set of input parameters for the situation $f < 0$:

Table 1: Input data

a_1	a_2	c_c	h^c	b	η	γ	g	α_1	α_2	β	h^r	h^m	s_r	c_l	c	h
80	85	1.3	0.12	15	3	1.2	2	125	120	0.5	0.7	0.9	11	1	75	15

the optimum results is shown in the following Table 2:

Table 2: Numerical Values

Situation	Optimal	C_1	C_2	R_1	R_2	M	S^m	D^m
$f < 0$	Acquisition price	1.30	1.12	-	-	-	235	39.35
	Effort level	13.94	14.26	-	-	-	235	39.35
	Acq.W. S. price	-	-	-	-	10.46	235	39.35
	wholesale price	-	-	-	-	165.60	235	39.35
	Retailing price	-	-	208	203	-	235	39.35
	Profit	834	874	875	678	6514	235	39.35

We also considered the following data set of input parameters for the situation $f \geq 0$:

Table 3: Input data

a_1	a_2	c_c	h^c	b	η	γ	g	α_1	α_2	β	h^r	h^m	s_r	c_l	c	h
12	13	1.3	0.12	15	3	1.2	2	500	502	0.5	0.70	0.9	11	1	75	15

the optimum results is shown in following table 4:

Table 4: Numerical Values

Situ.	Optimal	C_1	C_2	R_1	R_2	M	S^m	D^m
$f \geq 0$	Acq. price	5.68	5.64	-	-	-	227.66	227.66
	Effort level	13.63	13.68	-	-	-	227.66	227.66
	Acq.W. S. price	-	-	-	-	14.67	227.66	227.66
	wholesale price	-	-	-	-	545.97	227.66	227.66
	Retailing price	-	-	773.33	774.34	-	227.66	227.66
	Profit	797.84	805.42	25687	26142	106865	227.66	227.66

3.5 Analysis of Decision Components

The present section illustrates optimal decisions and profits of all supply chain members. We have observed how optimal decisions and profits of collectors, remanufacturer, and retailers are affected with respect to positive changes in remanufacturing cost.

3.5.1 Effects of Remanufacturing Cost ‘c’

Tables 5 and 6 show the effects of unit remanufacturing cost on optimal decisions and profits of all supply chain members.
The above results shown in table 5 is valid for the interval $0 \leq \mathbf{c} \leq 232$, and for $\mathbf{c} > 232$ model gives adverse results for profits of the remanufacturer, and retailers.

Table 5: Sensitivity analysis with remanufacturing cost '**c**' (Range: $0 \leq \mathbf{c} \leq 232$)

Situ.	cost	p_1^c	p_2^c	p_1^r	p_2^r	w^c	w^m	π_1^c	π_2^c	π_1^r	π_2^r	π^m
$f < 0$	75	1.30	1.12	208	203	10.46	165	834	874	875	678	6514
	110	1.30	1.12	216	211	10.46	183	834	874	547	394	5289
	145	130	1.12	225	220	10.46	200	834	874	296	187	4372
	180	1.30	1.12	234	229	10.46	218	834	874	121	56.18	3760
	215	1.30	1.12	243	238	10.46	235	834	874	23.46	1.71	3455
	250	1.30	1.12	251	246	10.46	253	834	874	1.80	23.81	3455
	285	1.30	1.12	260	255	10.46	270	834	874	56.71	122	3763

Table 6: Sensitivity analysis with remanufacturing cost **c** (Range: $0 \leq \mathbf{c} \leq 850$)

Situ.	cost	p_1^c	p_2^c	p_1^r	p_2^r	w^c	w^m	π_1^c	π_2^c	π_1^r	π_2^r	π^m
$f \geq 0$	100	5.51	5.47	779	781	14.30	558	755	763	24331	24775	101249
	225	4.64	4.61	809	811	12.42	618	562	569	18104	18486	75444
	350	3.78	3.74	840	842	10.55	679	398	403	12795	13117	53428
	475	2.91	2.88	870	872	8.67	740	261	266	8406	8667	35201
	600	2.05	2.01	900	902	6.79	800	154	157	4935	5135	20762
	725	1.18	1.15	931	933	4.92	861	74.70	77	2383	2523	10113
	850	0.32	0.28	961	963	3.04	921	23.75	25	749	829	3253

The above results shown in table 6 is valid for the interval $0 \leq \mathbf{c} \leq 850$ and for $\mathbf{c} > 850$ model gives adverse results for profits of collectors, retailers and the remanufacturer.

When the collection amount of used product is sufficient for remanufacturing i.e. remanufacturing is not bounded by the supply of the used products, then the collectors' decisions and the acquisition wholesale price are not affected by the changes in the remanufacturing cost, because the collected volume of the used product is sufficient for covering the demand of the re-manufactured product. However, the increment of remanufacturing cost, increases the wholesale price, the retailing price, and decreases the profits of the re-manufacturer, and retailers.

When remanufacturing is bounded by the supply of the used product, then the remanufacturer's decisions depend on collection volume of the used product. In this situation, marginal positive changes in remanufacturing cost reduce the acquisition price, the effort level cost, the acquisition wholesale price, profits of retailers, collectors and remanufacturer and increase the wholesale price and the retailing price. In short, what will effects of 'c' on various outputs, which are shown in Table 7. Where, $\times$, $+$ and $-$ respectively denotes no changes, positive changes, and negative changes.

Table 7: Sensitivity analysis with various costs

positive changes	situation	p_i^c	w^c	w^m	p_j^r	π_i^c	π^m	π_j^r
c	$f < 0$	$\times$	$\times$	$+$	$+$	$\times$	-	-
	$f \geq 0$	-	-	$+$	$+$	-	-	-

3.5.2 Effects of Parameter 'b'

Now we have studied how optimal decisions and profits of collectors, remanufacturer, and retailers are affected with respect to positive changes in price-sensitive demand parameter 'b'. Table 8 and 9 show the effect of price-sensitive demand parameter 'b' on optimal decisions and profits of all supply chain members. The results shown in the Table 8 is valid for the interval $0.21 \leq b \leq 0.56$, and for $b > 0.56$ model gives the adverse results for profits of remanufacturer, and retailers.

The results shown in Table 9 is valid for the interval $0.07 \leq \mathbf{b} \leq 0.87$ and for $\mathbf{b} > 0.87$, model gives the adverse results for profits of remanufacturer and collectors.

Table 8: Analysis with price-sensitive demand parameter **b** (Range: 0.21 ≤ '**b**' ≤ 0.56)

Situ.	b	p_1^c	p_2^c	p_1^r	p_2^r	w^c	w^m	π_1^c	π_2^c	π_1^r	π_2^r	π^m
$f < 0$	0.21	1.30	1.12	327	315	10.46	59.05	834	874	15054	13745	355
	0.28	1.30	1.12	264	255	10.46	82.12	834	874	9254	8368	2942
	0.35	1.30	1.12	232	225	10.46	106	834	874	5450	4844	5111
	0.42	1.30	1.12	215	209	10.46	133	834	874	2810	2416	6490
	0.49	1.30	1.12	208	203	10.46	161	834	874	1058	838	6626
	0.56	1.30	1.12	207	203	10.46	191	834	874	136	69.75	4982
	0.63	1.30	1.12	210	206	10.46	2228	834	874	98.93	171	932

Table 9: Analysis with price demand parameter **b**(Range: 0.07 ≤ **b** ≤ 0.87)

Situ.	b	p_1^c	p_2^c	p_1^r	p_2^r	w^c	w^m	π_1^c	π_2^c	π_1^r	π_2^r	π^m
$f \geq 0$	0.07	0.12	0.08	6916	6930	2.61	6688	16.03	17.13	3597	4065	216863
	0.18	1.57	1.53	2548	2553	5.76	2318	106	109	9476	9941	188148
	0.33	3.52	3.48	1286	1289	9.98	1057	354	359	17237	17697	149637
	0.48	5.43	5.39	814	816	14.12	587	736	744	24712	25167	111850
	0.63	7.30	7.27	568	570	18.19	342	1247	1257	31911	32362	74768
	0.78	9.15	9.11	417	419	22.19	194	1879	1891	38845	39293	38373
	0.93	10.96	10.92	316	317	26.11	94	2625	2639	45526	45970	2647

By observation of the above data tables, when the supply amount of the used product is enough for re-manufacturing, then the collectors' decisions, profits, acquisition price and acquisition wholesale price are not affected by the changes in acquisition price-sensitive parameter '**b**'. However, the increment of '**b**' reduces the retailing price , the wholesale price, and profits of all retailers and re-manufacturer.

When the re-manufacturing depends on the supply of the used product i.e., the supply of the used product is not enough than the demand of the remanufactured product, then the remanufacturer's decisions depend on collection volume of the used product. In this situation, marginal positive changes in '**b**' increase the acquisition price, the acquisition wholesale price, profits of retailers and collectors, however, reduce the manufacturer's profit along with the retailing price. In short, what will effects of **b** on various output, which are shown in Table 10. Where, '$\times$', '$+$', '$-$' and '$\frown$' respectively denotes, no changes, positive changes, negative changes, and positive then negative changes.

Table 10: Sensitivity analysis with various costs

positive changes	situation	p_i^c	w^c	w^m	p_j^r	π_i^c	π^m	π_j^r
b	$f < 0$	$\times$	$\times$	$+$	-	$\times$	$\frown$	-
	$f \geq 0$	$+$	$+$	-	-	$+$	-	$+$

Similarly, we can also analyze the effect of other associated costs like c_c, h, s_r, c_l, h^r, h^m, h^r and parameters like η, β, γ, v on the model's outputs.

4. CONCLUSION

This article studies the collecting, remanufacturing and retailing decision in three-layer multi-echelon reverse supply chain for remanufacturing of a single used product. Two decision strategies have been established by considering the availability of the used product and the optimal decisions and maximal profits of reverse supply chain members have been derived in above two strategies. We have presented a comparison and analysis of the equilibrium solutions of two strategies. We have also performed the sensitivity analysis with respect to some key parameters through analytical and numerical studies.

Based on the analytical and numerical studies we have derived some managerial insights. First, propositions 3.2 and 3.3 show that the end-user of the product is more sensible with respect to acquisition price, therefore the collected volume of used product is affected enough by acquisition price and effort level and both depend on acquisition wholesale price. Hence, management should have to take optimum decision carefully about acquisition wholesale price. Second, Lemma 4.1 shows retail price p_j^r is inversely proportional to β, therefore, as far as possible management should have to keep constant retail price for optimal profit.Third, management should have to increase the value of parameter η because, $f \geq 0$ is better strategies for all the reverse supply chain members. Forth, due to less profit of collectors, management should have to make revenue-sharing contract with collectors for betterment.

Our results are based upon some assumptions about used product thus several extensions of this paper are possible. First, one can consider the competitive environment among either retailers or collectors or both. Second, one can consider the different acquisition supply function of used product and demand function of remanufactured product. Forth, This study also considered that all the used-product units that are collected are usable for remanufacturing in the new product, future research can be analysed the effect of the degree of product stability in the re-manufacturing product.

REFERENCES