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Retailer's Ordering and Pricing Strategy for New Product and Buyback Strategy for Used Product with Deterioration

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Abstract: To prolong limited natural resources and persuade green production not only manufacturers and government but consumer is also concerned about the effects of manufacturing on the environment. As a result, consumers have shown their curiosity to buy used or refurbished product. This paper presents a methodology to provide optimal pricing and ordering strategy for new product and buy back strategy for used product for retailer. The rate of demand is assumed to be nonlinear function of price and time for new product, and linear function of price and time for buy back used product. The objective is to maximize total profit per time unit for retailer with respect to optimal price and ordering quantity for new product and optimal buy back quantity for used product. The model is illustrated with numerical examples and sensitivity analysis is performed for key parameters.
Keyword — Inventory, Price dependent demand, Replenishment quantity, used buyback Product, deterioration.

1. INTRODUCTION

Broad utilization of innovation helps all the monetary segments to prevail with regards to getting the purchasers from any edge of the world. Consequently, the manufacturing sectors are fabricating more to fulfil the needs and enterprises are contending with one another to catch the market. And so, industries are using more natural resources for excess production which generates more trash. Now, the key issue is to improve strategies for ecological support to preserve our limited natural resources and reduce the generation of trash as specified government regulations. Customers are worried about ecological issues and they like to buy items from producers having a green image. That is why many manufacturers have started collecting used products which are discarded by the customers. After refurbishing or re-cycling those products, manufacturer sells the product to a new customer at lower price. The process of recycling or refurbishing product is not a novel idea. It has been a common practice for the products like, aeronautics, metal, glass, ornaments, paper etc. in the last two three decades. Recently, it has been observed for many other products like plastic bags, water bottles, mobile phones, marker pens, etc. Reuse in deterministic model is introduced by Schrady (1967) with a constant rate of demand. Two cost components fixed cost and holding cost were considered in Schrady's model (1967). Mabini, Pintelon, and Gelders (1992) extended Schrady's model for multi-items having same repair facility. Koh, Hwang, Sohn, and Ko (2002), Richter and Dobos (2004), Kannan, Sasikumar, and Devika (2010), Govindan, Soleimani, and Kannan (2015), Chen, Weng, and Lo (2016) etc. provided an enriched literature review of the recent past papers and also identified research gap too, and its cited references for more about recycling or reusable item. Other motivating work in this area are batteries recycling, Daniel, Pappis, and Voutsinas (2003), electronic waste recy-cling, Nagurney and Toyasaki (2005), glass recycling, González-Torre and Adenso-Díaz (2006), paper recycling, Pati, Vrat, and Kumar (2008), etc.

In recent time, consumers are price sensitive and purchase the product after looking to the price only. Selling price of an item is prime factor to influence demand; studies on optimal pricing policies have received considerable attention these days. Most commonly, it is observed that the demand of an item is inversely proportionate to its retail selling price. Therefore, price varying demand pattern needs to be highlighted and considered. Whitin (1955) was the first to discuss inventory system with price dependent demand. Thereafter, many inventory models are formulated with price dependent demand like, Mondal, Bhunia, and Maiti (2003), Mukhopadhyay, Mukherjee, and Chaudhuri (2004), Jaggi,

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Tiwari, and Goel (2017), N. Shah and Vaghela (2017), N. H. Shah and Vaghela (2018), Sundararajan, Palanivel, and Uthayakumar (2019), Dey, Sarkar, Sarkar, and Pareek (2019), Suthar and Shukla (2019), Shaikh and Cárdenas-Barrón (2020), Giri and Masanta (2020) and many more.

Again, in profit-making business environment, the effect of deterioration on items is required to be taken in to account while formulating inventory system mathematically. The phenomenon which reduces the present value or usefulness of an item and hinders it from being used from its actual use is termed as deterioration. This may because of continual spoilage, degradation or evaporation etc. It may be observed for items like fruits, vegetables or any other food stuffs, medicine, electronic goods, batteries, volatile liquids and others. Ghare (1963) were first to incorporate effect of deterioration in inventory system. They proposed exponentially decaying inventory system. Covert and Philip (1973), Philip (1974) and Tadikamalla (1978) extended the model of Ghare (1963) by using Weibull distribution and Gamma distribution. An up to date review on inventory systems for deteriorating items is presented by N. H. Shah, Chaudhari, and Cárdenas-Barrón (2020), Goyal and Giri (2001), Bakker, Riezebos, and Teunter (2012) and Janssen, Claus, and Sauer (2016) cited. Thereafter many researchers have formulated inventory systems for deteriorating items by assuming constant, linear and non-linear rate of deterioration like N. H. Shah and Shukla (2009), Shukla and Suthar (2016), N. H. Shah et al. (2020),etc. Cárdenas-Barrón and Sana (2015) developed a concavity by Eigen values of Hassian matrix.

In this article, with assumption that retailer sells the new product to the customers as well as collects the used product to resell it, product have a deteriorating nature, the optimal pricing and ordering policy for new product and optimal policy for used buyback product is formulated in order to maximize retailer's total profit per time unit. The demand of a product is considered to be price and time responsive. All the assumptions and notations required to formulate problem mathematically are given in section 2. Mathematical formulation is discussed in section 3. To demonstrate the methodology numerical example is given in section 4 and 5 and sensitivity analysis is carried out to discuss strategic implications in section 6. We summarize the article in section 7.

2. ASSUMPTIONS AND NOTATIONS

2.1 Assumptions

1. The inventory system deals with single product.
2. The replenishment is instantaneous and planning horizon is infinite.
3. The holding cost is considered to be constant for new product as well as used buyback product with $h > h_u$.
4. The rate of demand for new product is taken as a
   $R_n(p,t) = ap^{-b}e^{-\epsilon t}, 0 \leq t \leq T$,
   where $a > 0$ denotes the scale demand, $0 < b < 1$ denotes the price elasticity and $0 < \epsilon < 1$.
5. The rate of demand for used buyback product is taken as a
   $R_u(p,t) = \alpha(1-\beta t) - p(1-p_0), \tau \leq t \leq T$,
   where $\alpha > 0$ denotes the scale demand and $0 < \beta < 1$.
6. The Lead time is negligible or zero and shortages are not allowed.
7. A retailer sells the new product to customers as well as collects and sells the used products again. Rework or repairing of used buyback product is not considered.
8. A retailer sell the new product during $0 \leq t \leq T$ and collects the used product at time τ and
   Sell the used buyback product during $\tau \leq t \leq T$.
9. The product is deteriorating nature with constant rate of deterioration for both type of new and used buyback product. Rate of deterioration is θ for new product and γ is the rate of deterioration for used buyback product with $\gamma \geq \theta$.
10. There is no replacement or repair of deteriorating items during the period under consideration.

2.2 Notations

Table 1: Notations used in model

A	Retailer ordering cost (in ₹/order).
C	Purchase cost (Constant) (in ₹/unit).
p	Selling Price (in ₹/unit) (a decision variable).$p > C$
h	Inventory holding cost (in ₹/unit) for new product.
h_u	Inventory holding cost (in ₹/unit) for used buy back product.
Q	The replenishment quantity for new product.
Q_u	The quantity of used buy back product.
T	The length of ordering cycle (a decision variable) (years).
τ	The point of time when collection and sell of used buy back products starts (years).
$R_n(p,t)$	Demand rate for new product at $0 \leq t \leq T$ (units).
$R_u(p,t)$	Demand rate for new product at $t \geq \tau$ (units).
$I(t)$	Inventory level at time $0 \leq t \leq T$ for new product (units).
$I_u(t)$	Inventory level at time $t \geq \tau$ for used buy back product (units).
P_0	Rate of discount on selling price for used buy back product.
d	Rate of depreciation on purchase cost for used buy back product.
θ	Constant rate of deterioration for new product.
γ	Constant rate of deterioration for used buyback product.
$\pi(p,T)$	Total profit of the retailer during cycle time (in₹).

3. MATHEMATICAL FORMULATION

The inventory level of the new product at time t over the period $[0, T]$ can be represented by the following differential equation,

$$\frac{dI(t)}{dt} + \theta I(t) = -R_n(p,t), 0 \leq t \leq T. \tag{1}$$

At time $t = T$, the inventory level reaches zero i.e. $I(T) = 0$.
The solution of the differential Eq. (1) is given by

$$I(t) = \frac{ap^{-b}e^{-\theta t}}{\epsilon - \theta}\left(e^{-t(\epsilon-\theta)} - e^{-T(\epsilon-\theta)}\right). \tag{2}$$

But $I(0) = Q$ gives the ordering quantity of new product is,

$$Q = \frac{ap^{-b}}{\epsilon - \theta}\left(1 - e^{-T(\epsilon-\theta)}\right). \tag{3}$$

Now for the used product during the period $[\tau, T]$, the inventory level is affected by the return rate of the used product, so the governing differential equation for inventory level $I_u(t)$ at time t,

$$\frac{dI_u(t)}{dt} + \gamma I_u(t) = -R_u(p,t), \tau \leq t \leq T. \tag{4}$$

But used buy back product inventory level also reached zero at time $t = T$ i.e. $I_u(T) = 0$.
The solution of the differential Eq. (4) is given by

$$I_u(t) = \frac{\alpha\beta t}{\gamma} - \frac{\alpha\beta}{\gamma^2} - \frac{\alpha}{\gamma} + \frac{p}{\gamma}(1-p_0) - e^{\gamma(T-t)}\left(\frac{\alpha\beta t}{\gamma} - \frac{\alpha\beta}{\gamma^2} - \frac{\alpha}{\gamma} + \frac{p}{\gamma}(1-p_0)\right). \tag{5}$$

Thus, the quantity of used buyback product given by

$$Q_u(t) = e^{\gamma(T-\tau)}\left(\frac{\alpha\beta T}{\gamma} - \frac{\alpha\beta}{\gamma^2} - \frac{\alpha}{\gamma} + \frac{p}{\gamma}(1-p_0)\right) - \left(\frac{\alpha\beta\tau}{\gamma} - \frac{\alpha\beta}{\gamma^2} - \frac{\alpha}{\gamma} + \frac{p}{\gamma}(1-p_0)\right). \tag{6}$$

Now to calculate total profit for new product, we calculate all the components as below,
Sales revenue from new product is

$$SR_n = \frac{p}{T}\left(\int_0^T ap^{-b}e^{-\epsilon t}dt\right). \tag{7}$$

Purchase cost for new product is

$$PC_n = \frac{CQ}{T}. \tag{8}$$

Holding cost for new product

$$HC_n = \frac{1}{T}\int_0^T [h \cdot I(t)]dt. \tag{9}$$

Ordering cost

$$OC_n = \frac{A}{T}. \tag{10}$$

Total profit for new product during the cycle is from Eq. (7) to Eq. (10),

$$\pi_n(p,t) = SR_n - OC_n - HC_n - PC_n. \tag{11}$$

Now, to calculate total profit from used buy back product, we calculate all the components as below
Sales revenue from used buyback product

$$SR_u = \frac{p(1-p_0)}{T}\left(\int_\tau^T (\alpha(1-\beta t) - p(1-p_0))dt\right). \tag{12}$$

Purchase cost for used buyback product

$$PC_u = \frac{C(1-d)Q_u}{T-\tau}. \tag{13}$$

Holding cost for used buyback product

$$HC_u = \frac{1}{T}\int_\tau^T [h_u \cdot I_u(t)]dt. \tag{14}$$

Total profit from used buyback product during the cycle is

$$\pi_u(p,t) = SR_u - HC_u - PC_u. \tag{15}$$

Therefore, the total profit from the both type of product is given by Eq. (11) and Eq. (15),

$$\begin{aligned}\pi(p,T) = &\left(\frac{p}{T}\int_0^T [ap^{-b}e^{-\epsilon t}]dt - \frac{CQ}{T} - \frac{1}{T}\int_0^T [h \cdot I(t)]dt - \frac{A}{T}\right)\\ &+ \left(\frac{p(1-p_0)}{T}\int_\tau^T [(\alpha(1-\beta t) - p(1-p_0))]dt - \frac{C(1-d)Q_u}{T-\tau} - \frac{1}{T}\int_\tau^T [h_u \cdot I_u(t)]dt\right).\end{aligned} \tag{16}$$

The total profit function is a function of selling price p and the replenishment cycle time T. The objective is to find the optimal selling price and the replenishment cycle time such that the retailer's total profit is maximized.

4. SOLUTION PROCEDURE

To obtain the optimal selling price that corresponds to maximising the total profit, for given , we first check necessary and sufficient conditions. (Sundararajan et al. (2019))
The necessary condition for finding the optimal selling price p^* for fix value of T is given as follows:

$$\begin{aligned}\frac{\partial \pi(p,T)}{\partial p} =& \frac{ap^{-b}(e^{-T\epsilon}-1)}{T\epsilon}[b-1] - \frac{Cap^{-b}b}{Tp(\epsilon-\theta)}[1-e^{-T(\epsilon-\theta)}] + h\frac{abp^{-b}e^{-T\epsilon}(e^{T\epsilon}\theta - e^{T\theta}\epsilon + \epsilon - \theta)}{(\epsilon-\theta)Tp\theta\epsilon}\\ &+ \frac{(1-p_0)}{T}\left(-\frac{1}{2}\alpha\beta(T^2-\tau^2) + \alpha(T-\tau) - p(1-p_0)(T-\tau)\right) - \frac{p(1-p_0)^2(T-\tau)}{T}\\ &+ \frac{C(1-d)}{T-\tau}\left(\frac{p_0}{\gamma} - \frac{1}{\gamma} + e^{\gamma(T-\tau)}(-\frac{p_0}{\gamma} + \frac{1}{\gamma})\right) - \frac{h_u(1-p_0)}{2T\gamma^3}[2\gamma^2(T-\tau) - 2e^{\gamma(T-\tau)}\gamma + 2\gamma] = 0.\end{aligned} \tag{17}$$

Theorem 4.1: For a given value of T, we have

(i) The Eq. (17) has a unique solution.

(ii) The solution in (i) satisfies the second-order conditions for the maximum.

Proof: To check the sufficient condition for optimal value of selling price, it is enough to show second order derivative of $\pi(p,T)$ with respect to p, is less than zero.

$$\begin{aligned}\frac{\partial^2 \pi(p,T)}{\partial p^2} =& \frac{abp^{-b-1}(e^{-T\epsilon}-1)}{T\epsilon}[1-b] - \frac{Cap^{-b-2}}{T(\epsilon-\theta)}(b^2+b)[1-e^{-T(\epsilon-\theta)}]\\ &- h\frac{ap^{-b-2}e^{-T\epsilon}(e^{T\epsilon}\theta - e^{T\theta}\epsilon + \epsilon - \theta)}{(\epsilon-\theta)T\theta\epsilon}(b^2+b) - \frac{2(1-p_0)^2(T-\tau)}{T}.\end{aligned}$$

In above expression, $e^{-T\epsilon} - 1 < 0, 0 < b < 1, T \geq 0, T \geq \tau, p > C$ and $\epsilon \neq \theta$
Clearly, $\frac{\partial^2 \pi(p,T)}{\partial p^2} < 0$.
Hence, the Eq. (17) has a unique solution and satisfies the sufficient condition for the maximum.
Now, to obtain the optimal cycle time that correspond to maximising the total profit, for given fix selling price, we first check necessary and sufficient conditions. (Sundararajan et al. (2019)).

The necessary condition for finding the optimal cycle time $T*$ for fix value of p is given as follows:

$$\begin{aligned}\frac{\partial \pi(p,T)}{\partial T} =& \frac{ap^{-b+1}}{T}[(\frac{e^{-T\epsilon}-1}{T\epsilon}+e^{-T\epsilon}]+\frac{A}{T^2}\\ &+\frac{Cap^{-b}}{T(\epsilon-\theta)}[(-\epsilon+\theta)e^{-T(\epsilon-\theta)}+\frac{1}{T}-e^{-T(\epsilon-\theta)}]\\ &+\frac{hap^{-b}e^{-T\epsilon}}{T\theta(\epsilon-\theta)}\begin{bmatrix}\frac{(e^{T\epsilon}-1)\theta-(e^{\theta T}-1)\epsilon}{T\epsilon}-\theta(e^{T\epsilon}-e^{\theta T})\\ +(e^{T\epsilon}-1)\theta-(e^{\theta T}-1)\epsilon\end{bmatrix}\\ &+\frac{p(1-p_0)}{T^2}\begin{bmatrix}-\frac{1}{2}\alpha\beta(T^2-\tau^2)+\alpha(T-\tau)\\ -p(1-p_0)(T-\tau)+T(-\alpha\beta T+\alpha-p(1-p_0))\end{bmatrix}\\ &+\frac{C(1-d)}{T-\tau}\begin{bmatrix}\frac{1}{T-\tau}\left(-\frac{\alpha\beta\tau}{\gamma}+\frac{pp_0}{\gamma}+\frac{\alpha\beta}{\gamma^2}+\frac{\alpha}{\gamma}-\frac{p}{\gamma}\right)\\ -\frac{\alpha\beta}{\gamma}e^{\gamma(T-\tau)}+e^{\gamma(T-\tau)}\left(\frac{\alpha\beta T}{\gamma}-\frac{pp_0}{\gamma}-\frac{\alpha\beta}{\gamma^2}-\frac{\alpha}{\gamma}+\frac{p}{\gamma}\right)\left(\frac{1}{T-\tau}-\gamma\right)\end{bmatrix}\\ &-\frac{h_u}{2T^2\gamma^3}\begin{bmatrix}-\alpha\beta\gamma^2(T^2-\tau^2)+2pp_0(T-\tau)+2\alpha\beta\gamma Te^{\gamma(T-\tau)}\\ +2T\alpha\gamma^2-2p\gamma^2(T-\tau)-2\alpha\beta\tau\gamma-2\alpha\tau\gamma^2\\ +2pp_0\gamma(1-e^{-\gamma\tau})-2\alpha e^{-\gamma\tau}(\gamma+\beta)+2e^{-\gamma\tau}p\gamma+2\alpha(\beta-\gamma)-2p\gamma\end{bmatrix}\\ &+\frac{h_u}{2T\gamma^2}\begin{bmatrix}\gamma^3\alpha\beta(T^2-\tau^2)-2\alpha\beta\gamma^2(T-\tau)-2\gamma^3pp_0(T-\tau)\\ +2\alpha\beta\gamma(e^{(T-\tau)}-1)+2\gamma^3T(p-\alpha)-2\gamma^3\tau(p-\alpha)\end{bmatrix}\\ &+\frac{h_u}{2T\gamma^2}\begin{bmatrix}-\gamma^2\alpha\beta(T^2-\tau^2)+2\gamma^2pp_0(T-\tau)+2\alpha\beta\gamma(Te^{(T-\tau)}-\tau)\\ +2T\gamma^2(p-\alpha)+2\tau\gamma^2(p-\alpha)+2pp_0\gamma(1-e^{-\gamma\tau})\\ +2\alpha\beta(1-e^{-\gamma\tau})+2\alpha\gamma(1-e^{-\gamma\tau})-p\gamma(1-e^{-\gamma\tau})\end{bmatrix}=0.\end{aligned} \tag{18}$$

Theorem4.2: For a given value of p, we have

(i) The Eq. (18) has unique solution.

(ii) The solution in (i) satisfies the second-order conditions for the maximum.

Proof: See Appendix A.

5. NUMERICAL EXAMPLE

The proposed models are illustrated below by considering the following example.
The numerical values of the parameter in proper unit were considered as input for numerical, graphical and sensitivity analysis of the model, the scale demand of new product $a = 255$ units, price elasticity of new product $b = 0.4$, $\epsilon = 0.9$, scale demand of used buyback product $\alpha = 100$ units, $\beta = 0.3$, purchasing cost $C = 55$₹/unit, ordering cost $A = 100$₹/order, holding cost of new product $h = 0.5$₹/unit/year, holding cost of used buyback product $h_u = 0.2$₹/unit/year, rate of depreciation of buyback product $d = 0.15$, $\tau = \frac{30}{365}$ year, price discount on selling price of used buyback product $p_0 = 0.5$, rate of deterioration of new product and used buyback product $\theta = 0.01$ and $\gamma = 0.02$ respectively.

Using mathematical software like, MATLAB or Mathematica or Maple 18 software, the optimal values of decision variables are obtained as $p^* = 103.7220$₹ and $T^* = 0.37085$ Year.

The optimum ordering quantity of new product is $Q^* = 12.58$ units and and used optimal buyback product quantity is $Q_u^* = 11.97$ units. The maximum profit of retailer is **4980.21₹**.

The concavity of the profit function is developed by the well-known Hessian matrix, Consider Hessian Matrix as following,

$$H(p,T) = \begin{pmatrix}\frac{\partial^2\pi(p,T)}{\partial p^2} & \frac{\partial^2\pi(p,T)}{\partial p\partial T}\\ \frac{\partial^2\pi(p,T)}{\partial T\partial p} & \frac{\partial^2\pi(p,T)}{\partial T^2}\end{pmatrix} \tag{19}$$

$$H(p^*,T^*) = \begin{pmatrix}-0.5648122516 & -20.48479785\\ -20.48479785 & -11351.81445\end{pmatrix}$$

As per Cárdenas-Barrón and Sana (2015), If the Eigen values of the Hessian matrix at the solution (p^*, T^*) are all negative then the profit function $\pi(p^*, T^*)$ is maximum at the solution. Here, eigenvalues of the above Hessian matrix are $\lambda_1 = -11351.85$ and $\lambda_2 = -0.53$.Therefore, the profit function $\pi(p^*, T^*)$ is maximum.

From above Hessian Matrix, define that $\Delta_{11} = \frac{\partial^2 \pi(p,T)}{\partial p^2}$, $\Delta_{22} = \frac{\partial^2 \pi(p,T)}{\partial T^2}$ and $\Delta_{12} = \frac{\partial^2 \pi(p,T)}{\partial p \partial T}$ for optimal value of p^* and T^*, it is clear that $\Delta_{11} = -0.56 < 0$, $\Delta_{22} = -11351.81 < 0$ and $\Delta_{11}\Delta_{22} - (\Delta_{12})^2 > 0$ then the optimal value of p^* and T^* satisfies the Eqs. (17) and (18) and value of p^* and T^* is unique and maximize $\pi(p, T)$.

The concavity of profit function is also shown in Figure.1, Figure.2 and Figure.3 as below:

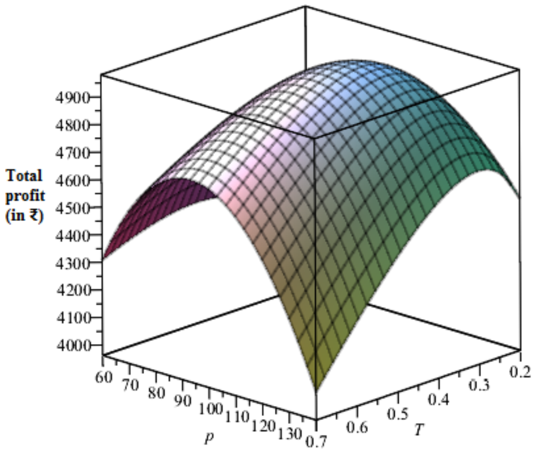

Figure 1: Concavity behaviour of the Total profit function

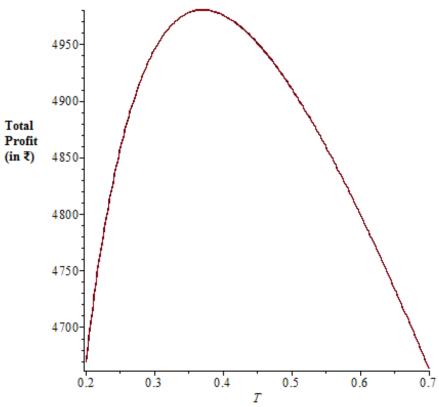

Figure 2: Total Profit Vs Cycle time

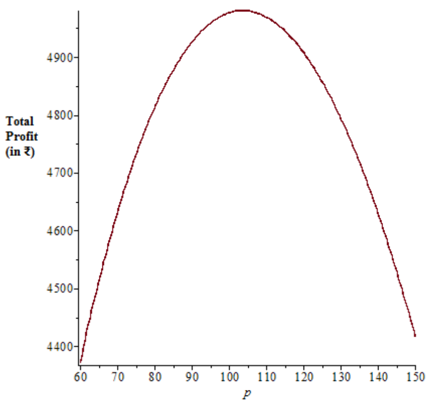

Figure 3: Total Profit Vs Selling Price

6. SENSITIVITY ANALYSIS

Table 2 shown sensitivity analysis is performed by changing each parameter values in relative steps of -20%, -10%, 10%, 20%, taking one parameter at a time and the remaining values of the parameters are unchanged.

Table 2: Sensitivity with respect to key parameter

Inventory Parameter	Change %	Value	T^*	p^*	Q^*	Q_u^*	Profit (in ₹)	Eigen Values of (19) (λ_1, λ_2)
a	-20	204	0.4038	92.68	11.31	14.96	4684.57	(-8845.88,-0.54)
	-10	229.5	0.3873	98.23	12.00	13.42	4824.05	(-10015.90,-0.54)
	0	255	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	280.5	0.3546	109.19	13.05	10.61	5152.64	(-12874.72,-0.52)
	20	306	0.3387	114.66	12.31	9.35	5144.76	(-14610.24,-0.52)
b	-20	0.32	0.2860	135.16	13.39	5.49	5947.61	(-22413.16,-0.41)
	-10	0.36	0.3340	116.86	13.27	8.92	5359.55	(-16870.10,-0.48)
	0	0.4	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.44	0.3998	93.69	11.64	14.63	4729.78	(-9233.21,-0.55)
	20	0.48	0.4225	85.76	10.60	16.92	4562.41	(-7687.54,-0.58)
α	-20	80	0.3672	91.52	13.12	8.24	3399.65	(-8763.60,-0.55)
	-10	90	0.3686	97.51	10.10	10.10	4178.73	(-10057.40,-0.54)
	0	100	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	110	0.3737	110.16	12.36	13.86	5805.05	(12852.29,-0.54)
	20	120	0.3770	116.80	12.16	15.77	6654.21	(-14012.40,-0.50)
β	-20	0.24	0.4012	103.64	13.44	7.16	5103.21	(-8918.77,-0.53)
	-10	0.27	0.3851	103.69	12.99	12.71	5040.58	(-10143.23,-0.53)
	0	0.3	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.33	0.3581	103.74	12.21	11.30	4921.89	(-12690.10,-0.52)
	20	0.36	0.3466	103.76	11.88	10.71	4865.44	(-14110.52,-0.51)
C	-20	44	0.3574	110.33	11.90	10.55	4970.51	(-12486.98,-0.48)
	-10	49.5	0.3642	106.93	12.24	11.27	4975.59	(-11918.24,-0.51)
	0	55	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	60.5	0.3774	100.69	12.92	12.66	4989.97	(-10830.21,-0.55)
	20	66	0.3839	97.83	13.26	13.34	5004.50	(-10347.50,-0.57)
A	-20	80	0.3569	104.24	12.15	11.37	5035.18	(-11892.53,-0.54)
	-10	90	0.3639	103.98	12.37	11.67	5007.4	(-11513.12,-0.53)
	0	100	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	110	0.3777	103.47	12.79	12.26	4953.50	(10998.11,-0.52)
	20	120	0.3844	103.23	12.99	12.54	4927.23	(-10866.13,-0.51)
d	-20	0.12	0.3714	102.24	12.67	12.20	5049.23	(-11319.50,-0.53)
	-10	0.135	0.3711	102.98	12.62	12.09	5014.55	(-11335.07,-0.53)
	0	0.15	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.165	0.3705	104.47	12.54	11.85	4946.16	(-11366.19,-0.53)
	20	0.18	0.3702	105.22	12.49	11.73	4912.41	(-11387.14,-0.53)
h	-20	0.4	0.3710	103.71	12.58	11.97	4980.81	(-11341.60,-0.53)
	-10	0.45	0.3709	103.72	12.58	11.97	4980.49	(-11346.21,-0.53)
	0	0.5	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.55	0.3708	103.73	12.58	11.96	4979.91	(-11355.92,-0.53)
	20	0.6	0.3707	103.73	12.58	11.96	4979.63	(-11361.80,-0.53)
h_u	-20	0.16	0.3709	103.72	12.58	11.97	4980.39	(-11345.60,-0.53)
	-10	0.18	0.3709	103.72	12.58	11.97	4980.28	(-11348.23,-0.53)
	0	0.2	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.22	0.3708	103.73	12.58	11.97	4980.12	(-11354.11,-0.53)
	20	0.24	0.3708	103.73	12.58	11.96	4980.05	(-11357.70,-0.53)
τ	-20	0.0658	0.3437	104.46	11.76	11.60	5101.48	(-12416.54,-0.54)
	-10	0.0740	0.3575	104.09	12.18	11.79	5039.67	(-11883.50,-0.53)
	0	0.0822	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.0904	0.3838	103.36	12.97	12.12	4922.86	(-10932.02,-0.52)
	20	0.0986	0.3964	103.01	13.34	12.26	4867.43	(-10485.44,-0.52)
p_0	-20	0.4	0.3986	83.38	14.58	13.57	4480.58	(-9232.77,-0.78)
	-10	0.45	0.3851	92.48	13.59	12.80	4714.00	(-10189.88,-0.65)
	0	0.5	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)

	10	0.55	0.3557	117.94	11.54	11.07	5288.79	(12988.38,-0.50)
	20	0.6	0.3393	136.43	10.45	10.08	5653.79	(-14638.19,-0.47)
ϵ	-20	0.72	0.3789	105.01	13.17	12.07	5034.12	(-10737.55,-0.53)
	-10	0.81	0.3747	104.36	12.86	12.02	5006.61	(-11058.12,-0.53)
	0	0.9	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.99	0.3674	103.09	12.32	11.93	4954.86	(-11618.79,-0.53)
	20	1.08	0.3643	102.47	12.07	11.90	4930.46	(-11860.58,-0.53)
θ	-20	0.008	0.3710	103.71	12.58	11.97	4980.88	(-11340.13,-0.53)
	-10	0.009	0.3709	103.72	12.58	11.97	4980.53	(-11345.75,-0.53)
	0	0.01	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.011	0.3708	103.73	12.58	11.96	4979.90	(-11357.29,-0.53)
	20	0.012	0.3707	103.73	12.58	11.96	4979.54	(-11364.12,-0.53)
γ	-20	0.016	0.3705	103.76	12.57	11.94	4979.12	(-11380.87,-0.53)
	-10	0.018	0.3707	103.74	12.57	11.95	4979.66	(-11366.13,-0.53)
	0	0.02	0.3708	103.72	12.58	11.97	4980.21	(-11351.85,-0.53)
	10	0.022	0.3710	103.70	12.59	11.98	4980.76	(-11337.29,-0.53)
	20	0.024	0.3712	103.69	12.59	11.99	4981.30	(-11322.77,-0.53)

Observations with managerial insights

To observe the sensitivity of the inventory parameters on the optimal solution, the data provided in the numerical example are considered.

1. Here observed that Eigen values of Hessian Matrix at corresponding value of p^* and T^* all are negative, means that profit is maximize at (p^*, T^*).

2. The scale demand a and α have positive impact on selling price and total profit of retailer. This finding implies that a higher scale demand inspires a retailer to set a high selling price and gain more profit.

3. An increasing the ordering cost A lead to gradually decrease the selling price and increase the cycle time, while the total profit will be decreases. This finding implies that the high ordering cost may negative impact on retailer total profit.

4. Higher value of holding cost for new product and buyback used product which negative impacts on retailer's total profit. So, retailer should try to reduce holding cost for new product and buyback used product for reduces the loss.

5. Higher rate of depreciation on purchase cost (d) for used buy back product which negative impact on retailer's total profit. So, for obtaining more profit, retailer should be reduced value of d.

6. Selling price discount (p_0) facility on used buyback product is more effective to gain the retailer's total profit. This finding implies that retailer's gives to more price discount on used buyback product during resell to customers, increases total profit with increases selling price and ordering quantity.

7. Optimal selling price increase when system parameters a, α, d, h, h_u and p_0 increases but if parameters $b, C, \epsilon, \beta, A, \tau$ increase then selling price decrease. Admittedly, p is highly positive sensitive to a, α, d, p_0 and strongly negative sensitive to b, C, ϵ.

8. When the value of the parameters a, α, p_0 increase, the optimal total profit will increase, However, for increasing in parameter $b, \beta, C, A, d, \epsilon, h, \tau, \theta$ then total profit will decrease.

9. It is noted that replenishment cycle time T is positively related to system parameters b, A, τ and negatively related to a, α, β. However, not much effect in cycle time for change in holding cost parameters and remaining others.

10. The higher rate of deterioration of new product (θ) which affects gradually decreases the retailer's total profit. Retailer's total profit may gradually increases due to increase the rate of deterioration (γ) of used buyback product. This finding implies that the higher selling price and higher rate of deterioration of new product may negative effects on total profit but lower selling price, higher ordering quantity and higher rate of deterioration of used buyback product may positive impacts on retailer's total profit.

7. CONCLUSION

This study is an attempt to formulate inventory system in order to maximize the retailer's total profit who sells the new product as well as collets used products from the customers and resell them (a product like plastic bags, water bottles, mobile phones, marker pens, battery, etc). It is assumed that items are deteriorating and its demand rate (of new product as well as used products) is price sensitive. We presented mathematical formulation of the scenario and the optimal selling price, replenishment time, ordering quantity of new product, and optimal quantity of used product are determined using classical optimization. The numerical example has been solved to validate the proposed model. The sensitivity analysis of various key parameters on the optimal solutions is carried out for authentication of optimal strategies. The possible extension of this model is to be considering rework of used buyback product and again sell it, stock dependent demand, advertisement dependent demand, trade credit policy, shortages etc., case may consider.

REFERENCES

Bakker, M., Riezebos, J., & Teunter, R. H. (2012). Review of inventory systems with deterioration since 2001. *European Journal of Operational Research*, *221*(2), 275-284.

Chen, C. K., Weng, T. C., & Lo, C. C. (2016). Optimal replenishment quantity for new products and return rate of used products for a retailer. *Applied Mathematical Modelling*, *40*(23-24), 9754-9766.

Covert, R. P., & Philip, G. C. (1973). An eoq model for items with weibull distribution deterioration. *AIIE transactions*, *5*(4), 323–326.

Cárdenas-Barrón, L. E., & Sana, S. S. (2015). Multi-item eoq inventory model in a two-layer supply chain while demand varies with promotional effort. *Applied Mathematical Modelling*, *39*(21), 6725-6737.

Daniel, S. E., Pappis, C. P., & Voutsinas, T. G. (2003). Applying life cycle inventory to reverse supply chains: a case study of lead recovery from batteries. *Resources, Conservation and Recycling*, *37*(4), 251–281.

Dey, B. K., Sarkar, B., Sarkar, M., & Pareek, S. (2019). An integrated inventory model involving discrete setup cost reduction, variable safety factor, selling price dependent demand, and investment. *RAIRO-Operations Research*, *53*(1), 39-57.

Ghare, P. M. (1963). A model for an exponentially decaying inventory. *Journal of Industrial Engineering*, *14*, 238-243.

Giri, B. C., & Masanta, M. (2020). Developing a closed-loop supply chain model with price and quality dependent demand and learning in production in a stochastic environment. *International Journal of Systems Science: Operations & Logistics*, *7*(2), 147-163.

González-Torre, P. L., & Adenso-Díaz, B. (2006). Reverse logistics practices in the glass sector in spain and belgium. *International Business Review*, *15*(5), 527-546.

Govindan, K., Soleimani, H., & Kannan, D. (2015). Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. *European Journal of Operational Research*, *240*(3), 603-626.

Goyal, S. K., & Giri, B. C. (2001). Recent trends in modeling of deteriorating inventory. *European Journal of operational research*, *134*(1), 1-16.

Jaggi, C. K., Tiwari, S., & Goel, S. K. (2017). Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities. *Annals of Operations Research*, *248*(1-2), 253-280.

Janssen, L., Claus, T., & Sauer, J. (2016). Literature review of deteriorating inventory models by key topics from 2012 to 2015. *International Journal of Production Economics*, *182*, 86-112.

Kannan, G., Sasikumar, P., & Devika, K. (2010). A genetic algorithm approach for solving a closed loop supply chain model: A case of battery recycling. *Applied Mathematical Modelling*, *34*(3), 655–670.

Koh, S. G., Hwang, H., Sohn, K. I., & Ko, C. S. (2002). An optimal ordering and recovery policy for reusable items. *Computers & Industrial Engineering*, *43*(1-2), 59-73.

Mabini, M. C., Pintelon, L. M., & Gelders, L. F. (1992). Eoq type formulations for controlling repairable inventories. *International Journal of Production Economics*, *28*(1), 21-33.

Mondal, B., Bhunia, A. K., & Maiti, M. (2003). An inventory system of ameliorating items for price dependent demand rate. *Computers & Industrial Engineering*, *45*(3), 443-456.

Mukhopadhyay, S., Mukherjee, R. N., & Chaudhuri, K. S. (2004). Joint pricing and ordering policy for a deteriorating inventory. *Computers & Industrial Engineering*, *47*(4), 339-349.

Nagurney, A., & Toyasaki, F. (2005). Reverse supply chain management and electronic waste recycling: a multitiered network equilibrium framework for e-cycling. *Transportation Research Part E: Logistics and Transportation Review*, *41*(1), 1-28.

Pati, R. K., Vrat, P., & Kumar, P. (2008). A goal programming model for paper recycling system. *Omega*, *36*(3), 405-417.

Philip, G. C. (1974). A generalized eoq model for items with weibull distribution deterioration. *AIIE Transactions*, *6*(2), 159-162.

Richter, K., & Dobos, I. (2004). Production-inventory control in an eoq-type reverses logistics system. *Supply chain management and reverse logistics*, 139-160.

Schrady, D. A. (1967). A deterministic inventory model for reparable items. *Naval Research Logistics Quarterly*, *14*(3), 391-398.

Shah, N., & Vaghela, C. (2017). Retailer's replenishment and pricing decisions for non-instantaneous deterioration and price-dependent demand. *International Journal of Industrial Engineering & Production Research*, *28*(2), 101-111.

Shah, N. H., Chaudhari, U., & Cárdenas-Barrón, L. E. (2020). Integrating credit and replenishment policies for deteriorating items under quadratic demand in a three echelon supply chain. *International Journal of Systems Science: Operations & Logistics*, *7*(1), 34-45.

Shah, N. H., & Shukla, K. T. (2009). Deteriorating inventory model for waiting time partial backlogging. *Applied Mathematical Sciences*, *3*(9), 421-428.

Shah, N. H., & Vaghela, C. R. (2018). Retailer's optimal pricing and replenishment policy for new product and optimal take-back quantity of used product. *Yugoslav Journal of Operations Research*, *28*(3), 345-353.

Shaikh, A. A., & Cárdenas-Barrón, L. E. (2020). An eoq inventory model for non-instantaneous deteriorating products with advertisement and price sensitive demand under order quantity dependent trade credit. *Investigación Operacional*, *41*(2), 168-187.

Shukla, K. T., & Suthar, M. S. (2016). A framework for an inventory model for deteriorating items with expiration under trapezoidal-type demand and partial backlogging. *Logforum*, *12*(2), 269-281.

Sundararajan, R., Palanivel, M., & Uthayakumar, R. (2019). n eoq model of non-instantaneous deteriorating items with price, time-dependent demand and backlogging. *Journal of Control and Decision*, 1-30.

Suthar, M. S., & Shukla, K. T. (2019). Pricing and ordering policy for deteriorating inventory system with price sensitive ramp type demand, pre and post deterioration discounts. *Investigación Operacional*, *40*(2), 267-274.

Tadikamalla, P. R. (1978). An eoq inventory model for items with gamma distributed deterioration. *AIIE transactions*, *10*(1), 100-103.

Whitin, T. M. (1955). Inventory control and price theory. *Management science*, *2*(1), 61-68.

APPENDIX: A

To show second order derivative of $\pi(p,T)$ with respect to T is less than zero.

$$\begin{aligned}\frac{\partial^2\pi(p,T)}{\partial T^2} = & -\frac{2Cap^{-b}}{T}\psi_1 - \frac{hap^{-b}e^{-T\epsilon}}{T(\epsilon-\theta)\theta}\psi_2 - \frac{hap^{-b}e^{-T\epsilon}}{T(\epsilon-\theta)\theta\epsilon}\psi_3 - \left(\frac{2A}{T^3}-\eta_1\right) \\ & -\frac{h_u}{T\gamma^2}\psi_4 - \left(\frac{h_u}{2T\gamma}+\frac{h_u}{T^3+\gamma^3}\right)\psi_5 - \frac{h_u}{2T\gamma^3}\psi_6 + \left\{\left(-\frac{h_u}{T^2\gamma^2}\psi_7 - \frac{h_u}{T^2\gamma^3}\psi_8\right)+\eta_2\right\} \quad \text{(A.1)} \\ & -\frac{ap^{-b+1}}{T}\psi_9 - \frac{C(1-d)}{(T-\tau)}\psi_{10}.\end{aligned}$$

Since Eq. (A.1) satisfied following conditions,
$a > 0, 0 < b < 1, 0 < \epsilon < 1, 0 < \beta < 1, p \geq 0, T > \tau, p > C$ and $\epsilon \neq \theta, 0 < \gamma < 1.$

And

$$\begin{aligned}
\psi_1 &= \frac{1}{T^2(\epsilon-\theta)} - e^{T(\epsilon-\theta)}\left(\frac{(\epsilon-\theta)}{2}+\frac{1}{T}+\frac{1}{T^2(\epsilon-\theta)}\right) > 0, \\
\psi_2 &= \left(e^{T\epsilon}\theta - e^{T\theta}\epsilon + \epsilon - \theta\right)\left(\epsilon + \frac{2}{T} + \frac{1}{T^2\epsilon}\right) > 0, \\
\psi_3 &= \theta\epsilon^2 e^{T\epsilon} - \theta^2\epsilon e^{T\theta} > 0, \\
\psi_4 &= \left[\begin{array}{l} -\gamma^3\alpha\beta(T^2-\tau^2) + 2\alpha\beta\gamma^2(T-\tau) + 2\gamma^3 pp_0(T-\tau) \\ -2\gamma^3(p-\alpha)(T-\tau) + 2\alpha\beta\gamma(e^{\gamma(T-\tau)}-1) \end{array}\right] > 0, \\
\psi_5 &= \left[\begin{array}{l} \gamma^2\alpha\beta(T^2-\tau^2) - 2\gamma^2 pp_0(T-\tau) + 2\gamma^2(p-\alpha)(T-\tau) \\ +2\alpha\beta\gamma(\tau - Te^{\theta(T-\tau)}) + 2(e^{\gamma(T-\tau)}-1)(\gamma pp_0 + \alpha\beta + \alpha\gamma - p\gamma) \end{array}\right] > 0, \\
\psi_6 &= \left[\begin{array}{l} \gamma^4\alpha\beta(T^2-\tau^2) - 2\gamma^4 pp_0(T-\tau) - 2\gamma^3\alpha\beta(2T-\tau) \\ +2\gamma^4(p-\alpha)(T-\tau) + 2\gamma^3(\alpha-p) + 2\gamma^3 pp_0 \end{array}\right] > 0, \\
\psi_7 &= \left[\begin{array}{l} -\gamma^2\alpha\beta(T^2-\tau^2) + 2\gamma^2 pp_0(T-\tau) + 2\alpha\beta\gamma(Te^{\gamma(T-\tau)}-\tau) \\ +2\gamma^2(\alpha-p)(T-\tau) + 2(1-e^{\gamma(T-\tau)})(\gamma pp_0 + \alpha\beta + \alpha\gamma - p\gamma) \end{array}\right] < 0, \\
\psi_8 &= \left[\begin{array}{l} \gamma^3\alpha\beta(T^2-\tau^2) - 2\gamma^3 pp_0(T-\tau) - 2\alpha\beta\gamma^2(T-\tau) \\ +2\gamma^3(p-\alpha)(T-\tau) + 2\alpha\beta\gamma(e^{\gamma(T-\tau)}-1) \end{array}\right] < 0, \\
\psi_9 &= \left[\left(\epsilon + \frac{2}{T} + \frac{2}{T^2\epsilon}\right)e^{-T\epsilon} - \frac{2}{T^2\epsilon} + \alpha\beta p(1-p_0)\right] > 0, \\
\psi_{10} &= \left[\begin{array}{l} \frac{2}{(T-\tau)^2}\left(\left(\frac{-\alpha\beta\tau}{\gamma}+\frac{\alpha\beta}{\gamma^2}+\frac{pp_0}{\gamma}+\frac{\alpha}{\gamma}-\frac{p}{\gamma}\right)+e^{\gamma(T-\tau)}\left(\frac{\alpha\beta T}{\gamma}-\frac{\alpha\beta}{\gamma^2}-\frac{pp_0}{\gamma}-\frac{\alpha}{\gamma}+\frac{p}{\gamma}\right)\right) \\ +e^{\theta(T-\tau)}\gamma^2\left(\frac{2\alpha\beta}{\gamma^2}+\frac{\alpha\beta T}{\gamma}-\frac{\alpha\beta}{\gamma^2}-\frac{pp_0}{\gamma}-\frac{\alpha}{\gamma}+\frac{p}{\gamma}\right) \\ -\frac{2}{(T-\tau)}\left(\left(\frac{\alpha\beta}{\gamma}\right)e^{\gamma(T-\tau)}+e^{\gamma(T-\tau)}\gamma\left(\frac{\alpha\beta T}{\gamma}-\frac{\alpha\beta}{\gamma^2}-\frac{pp_0}{\gamma}-\frac{\alpha}{\gamma}+\frac{p}{\gamma}\right)\right) \end{array}\right] > 0, \\
\eta_1 &= \frac{2hap^{-b}e^{-T\epsilon}}{T(\epsilon-\theta)\theta}\left((e^{T\epsilon}\epsilon\theta - e^{T\theta}\theta\epsilon)\left(1+\frac{1}{T\epsilon}\right)\right) \text{ and } \frac{2A}{T^3} > \eta_1, \\
\eta_2 &= \frac{2p(1-p_0)}{T}\left[\begin{array}{l} \frac{\left(\frac{-\alpha\beta}{2}(T^2-\tau^2)+\alpha(T-\tau)-p(1-p_0)(T-\tau)\right)}{T^2} \\ -\frac{(-\alpha\beta T+\alpha-p(1-p_0))}{T} \end{array}\right] < 0 \text{ and} \\
&\left(-\frac{h_u}{T^2\lambda^2}\psi_7 - \frac{h_u}{T^2\lambda^3}\psi_8\right) + \eta_2 < 0,
\end{aligned}$$

It is shown that,

$$\begin{aligned}\frac{\partial^2\pi(p,T)}{\partial T^2} = & -\frac{2Cap^{-b}}{T}\psi_1 - \frac{hap^{-b}e^{-T\epsilon}}{T(\epsilon-\theta)\theta}\psi_2 - \frac{hap^{-b}e^{-T\epsilon}}{T(\epsilon-\theta)\theta\epsilon}\psi_3 - \left(\frac{2A}{T^3}-\eta_1\right) \\ & -\frac{h_u}{T\gamma^2}\psi_4 - \left(\frac{h_u}{2T\gamma}+\frac{h_u}{T^3+\gamma^3}\right)\psi_5 - \frac{h_u}{2T\gamma^3}\psi_6 + \left\{\left(-\frac{h_u}{T^2\gamma^2}\psi_7 - \frac{h_u}{T^2\gamma^3}\psi_8\right)+\eta_2\right\} \quad \text{(A.1)} \\ & -\frac{ap^{-b+1}}{T}\psi_9 - \frac{C(1-d)}{(T-\tau)}\psi_{10} < 0\end{aligned}$$

Hence, the Eq. (18) has a unique solution and this satisfies the sufficient condition for the maximum. Competed the proof.