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Study of Reliability Measures of A Two Unit System with Inspection and On-Line/Off-Line Repairs Using the Regenerative Point Technique

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Abstract: In this paper, we analysed a system model which consists of two identical unit with the inspection of each unit. Each unit has three modes – normal (N), partial failure (P), and total failure (F). A unit can't enter into F-mode directly without passing through the P-mode. A single repair facility is always available with the system that inspects the partially failed unit first and decides whether to repair it on-line or off-line. A unit can't enter into F-mode during its on-line repair. The failure and inspection rates are taken as constants while the repair time distribution as general. Various important measures of system effectiveness are obtained by using the regenerative point technique.
Keyword — Reliability, Availability, M.T.S.F., Busy period, and Profit analysis.

1. INTRODUCTION

Two unit standby redundant systems have widely been studied in the literature of reliability due to their prevalence in modern business and industrial systems. Several researchers have analyzed the two unit cold standby systems models working in the field of reliability theory. They have considered the concept like random inspection of the units, random shocks, allowed downtime, imperfect and slow switching devices, a random check of standby units, etc. Arora (2005) discussed a two-unit standby redundant system with constant repair time and a single repairman. Yusuf (2016) studied a parallel system with a supporting device and two types of preventive maintenance with different operative conditions of the system. Some authors including R. Gupta and Varshney (2006) analyzed two unit cold standby system models assuming the three modes of each unit – Normal, Partial failure, and Total failure. They considered a single repair facility that repairs a unit failed partially or totally under the following assumptions that the repair of a partially failed unit may be performed during its operation and the partially failed unit operates to run the system while the other (standby) unit is available in its normal mode.

In the real-world, the situations arise in many cases when the repair of a partially failed unit is not possible during its operation. Keeping this fact into consideration we in the present paper considered a two identical unit cold standby system with on-line and off-line repairs of a partially failed unit i.e. a partially failed unit first goes for inspection to detect the fault before its repair. If a minute fault is detected after inspection, then the partially failed unit goes for on-line repair otherwise for off-line repair. The model of the system is developed by assuming a system consists of two identical units. Initially, one unit of the system is operative and the other is kept as a cold standby. Each unit of the system has three modes – normal (N), partial failure (P), and total failure (F). If a unit fails partially, it is first inspected for on-line/off-line repair. The inspection is performed during its operation. In inspection, if a minute fault is detected after inspection, the failed unit goes for on-line repair otherwise it goes for off-line repair. The partially failed unit during the inspection/on-line repair may fail totally. A unit can't enter into F-mode without passing through P-mode. A single repair facility is always available with the system for inspection and to repair (on-line, off-line, and total failed unit) on a first come first serve basis. The switching device, used to detect the failed unit and to switch the standby unit into operation, is perfect and instantaneous. After each type of repair, the repaired unit works as good as new. The failure and repair times of the units are assumed to be independent and uncorrelated random variables. The failure time distributions of both the units and time to inspection are taken as exponential with different parameters while all the repair time distributions are general. The system failure occurs when it breaks down in any way. The behavior of mean time to system failures (MTSF) and profit analysis has been observed graphically for a particular case.

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2. NOTATIONS

The following notations have been used:

α_1/α_2	: Constant failure rate of a unit from N to P-mode/ P to F-mode.
θ	: Constant rate of inspection of a unit.
p/q	: Probability that the partially failed unit goes for on-line/off-line repair.
$G_1(\cdot), G_2(\cdot)$	: c.d.f. of time for on-line/off-line repair of a partially failed unit.
$H(\cdot)$	: c.d.f. of time to repair of a total failed unit.
q_{ij}	: p.d.f. of transition time from state S_i to S_j.
p_{ij}	: Steady-state transition probability from state S_i to S_j , such that, $p_{ij} = \int q_{ij}(u)du.$
$q_{ij}^{(k)}$	: p.d.f. of transition time from state S_i to S_j via state S_k.
$p_{ij}^{(k)}$	: Steady-state transition probability from state S_i to S_j via state S_k.
$q_{ij}^{(k,l)}$	: p.d.f. of transition time from state S_i to S_j via state S_k and S_l.
$p_{ij}^{(k,l)}$	: Steady-state transition probability from state S_i to S_j via state S_k and S_l.
$Z_i(t)$	: Probability that the system sojourns in the state S_i up to time t.
Ψ_i	: Mean sojourn time in the state S_i.
$*$	: Symbol for Laplace transform of a function i.e. $f^*(s) = \int_0^\infty e^{-st} f(t)dt.$
©	: Symbol for ordinary convolution. Let $f(t)$ and $g(t)$ be two functions of non–negative variable 'T' then the convolution (or ordinary convolution) of the functions $f(t)$ and $g(t)$ is given by $A(t)©B(t) = \int_0^t A(u)B(t-u)du.$

3. SYMBOLS FOR THE STATES OF THE SYSTEM

We define the following symbols for the states of the system:

N_O/N_S	: Unit is in N-mode and operative/standby.
P_{OI}	: Unit is in P-mode, operative and under inspection.
P_{1r}/P_{2r}	: Unit is in P-mode and under online/offline repair.
P_{wI}	: Unit is in P-mode and waiting for inspection.
F_r/F_{wr}	: Unit is in F-mode and under repair/waiting for repair.

Using the above symbols, the possible states of the system are defined as under
Up states: In these states, system is in working condition;

$S_0 \equiv (N_O, N_S)$; $S_1 \equiv (P_{OI}, N_S)$; $S_2 \equiv (P_{1r}, N_S)$;
$S_3 \equiv (P_{2r}, N_O)$; $S_4 \equiv (F_r, N_O)$; $S_5 \equiv (P_{2r}, P_{wI})$;
$S_6 \equiv (F_r, P_{wI})$.

Failed States: In the following states, system is not in working condition;

$$S_7 \equiv (P_{2r}, F_{wr}); \qquad S_8 \equiv (F_r, F_{wr}).$$

The transition diagram of the system model, along with transition rates, is shown in Figure – 1. From the figure, we observe that the epochs of entrance from S_3 to S_5, S_4 to S_6, S_5 to S_7 and S_6 to S_8 are non-regenerative as the future probabilistic behaviour of the system at these epochs depends upon the previous states. The all other entrance epochs are regenerative.

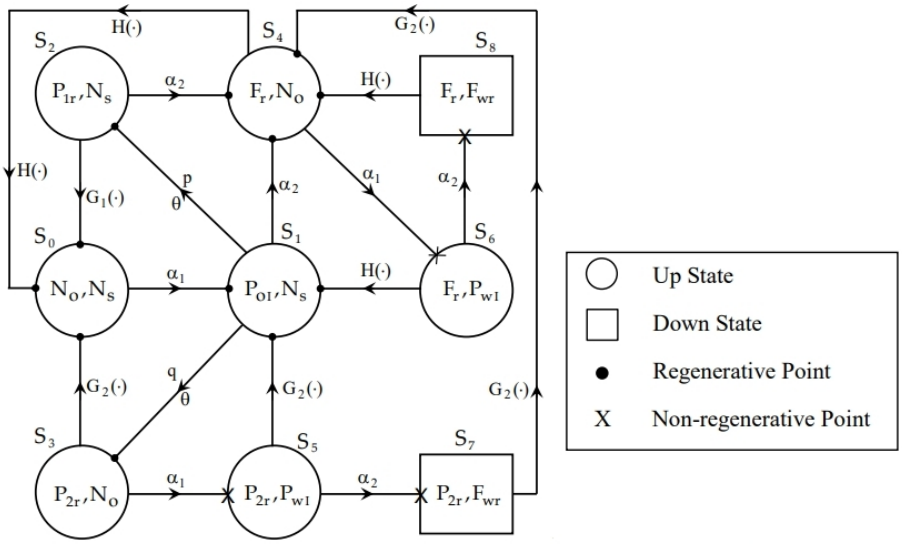

Figure 1: State Transition Diagram

4. RELIABILITY MEASURES

4.1 Transition Probabilities and Mean Sojourn Times

The steady state transition probabilities are given by,

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t)\ , \qquad p_{ij}^{(k)} = \lim_{t\to\infty} Q_{ij}^{(k)}(t) \text{ and } \qquad p_{ij}^{(k,l)} = \lim_{t\to\infty} Q_{ij}^{(k,l)}(t).$$

$$p_{01}(t) = \int_0^\infty q_{01}(t)dt = \int_0^\infty \alpha_1 e^{-\alpha_1 t}dt = 1 - e^{-\alpha_1 t}.$$

Similarly,

$$p_{12} = p\theta/(\alpha_2+\theta); \qquad p_{13} = q\theta/(\alpha_2+\theta); \qquad p_{14} = \alpha_2/(\alpha_2+\theta);$$

$$p_{20} = G_1(\alpha_2); \qquad p_{24} = 1 - G_1(\alpha_2); \qquad p_{30} = G_2(\alpha_1);$$

$$p_{31}^{(5)} = \frac{\alpha_1}{\alpha_1-\alpha_2}\left[G_2(\alpha_2) - G_2(\alpha_1)\right]; \qquad p_{34}^{(5,7)} = 1 - \frac{\alpha_1}{\alpha_1-\alpha_2}G_2(\alpha_2) + \frac{\alpha_2}{\alpha_1-\alpha_2}G_2(\alpha_1);$$

$$p_{40} = H(\alpha_1); \qquad p_{41}^{(6)} = \frac{\alpha_1}{\alpha_1-\alpha_2}\left[H(\alpha_2) - H(\alpha_1)\right];$$

$$p_{44}^{(6,8)} = 1 - \frac{\alpha_1}{\alpha_1-\alpha_2}H(\alpha_2) + \frac{\alpha_2}{\alpha_1-\alpha_2}H(\alpha_1).$$

Thus, we observe the following relations,

$$p_{01} = 1; \qquad p_{12}+p_{13}+p_{14} = 1; \qquad p_{20}+p_{24} = 1;$$

$$p_{30} + p_{31}^{(5)} + p_{34}^{(5,7)} = 1; \qquad p_{40} + p_{41}^{(6)} + p_{44}^{(6,8)} = 1.$$

Mean sojourn time, Ψ_i, is defined as the expected time for which the system stays in state S_i before transiting to any other state. If T_i is the sojourn time in state S_i, then mean sojourn time in state S_i is $\Psi_i = \int_0^\infty P(T_i > t)dt$.

The mean sojourn time for state S_0, denoted by Ψ_0, is obtained as:

$$\Psi = \int_0^\infty P(T_i > t)dt = \int_0^\infty e^{-\alpha_1 t}dt = \frac{1}{\alpha_1}.$$

Similarly, the mean sojourn time for various states are as follows:

$$\Psi_0 = \frac{1}{\alpha_1}; \qquad \Psi_1 = \frac{1}{(\alpha_2+\theta)}; \qquad \Psi_2 = \int e^{-\alpha_2 t}\bar{G}_1(t)dt;$$
$$\Psi_3 = \int e^{-\alpha_1 t}\bar{G}_2(t)dt; \qquad \Psi_4 = \int e^{-\alpha_1 t}\bar{H}(t)dt; \qquad \Psi_5 = \int e^{-\alpha_2 t}\bar{G}_2(t)dt;$$
$$\Psi_6 = \int e^{-\alpha_2 t}\bar{H}(t)dt; \qquad \Psi_7 = \int \bar{G}_2(t)dt; \qquad \Psi_8 = \int \bar{H}(t)dt.$$

4.2 Reliability and MTSF

To determine reliability, $R_i(t)$, of the system, the reliability of the system when the system starts initially from state $S_i \in E$, we assume the failed states S_7 and S_8 of the system as absorbing. By simple probabilistic arguments, we see that $R_0(t)$ is the sum of the following contingencies –

(i) The system remains up in state S_0 without making any transition to any other state up to time t. The probability of this contingency is $e^{-\alpha_1 t} = Z_0(t)$, say.

(ii) The system first enters to the state S_1 from S_0 during $(u, u+du)$, $u \leq t$ and then starting from S_1, it remains up continuously during the remaining time $(t–u)$, the probability of this contingency is

$$\int_0^t q_{01}(u)duR_1(t-u) = q_{01}(t)©R_1(t).$$

Thus, we have,

$$R_0(t) = Z_0(t) + q_{01}(t)©R_1(t).$$

Similarly,

$$R_1(t) = Z_1(t) + q_{12}(t)©R_2(t) + q_{13}(t)©R_3(t) + q_{14}(t)©R_4(t)$$
$$R_2(t) = Z_2(t) + q_{20}(t)©R_0(t) + q_{24}(t)©R_4(t)$$
$$R_3(t) = Z_3(t) + q_{35}(t)©Z_5(t) + q_{30}(t)©R_0(t) + q_{31}^{(5)}(t)©R_1(t)$$
$$R_4(t) = Z_4(t) + q_{46}(t)©Z_6(t) + q_{40}(t)©R_0(t) + q_{41}^{(6)}(t)©R_1(t)$$

where

$$Z_0(t) = e^{-\alpha_1 t}; \qquad Z_1(t) = e^{-(\alpha_2+\theta)t}; \qquad Z_2 = e^{-\alpha_2 t}\bar{G}_1(t);$$
$$Z_3 = e^{-\alpha_1 t}\bar{G}_2(t); \quad Z_4 = e^{-\alpha_1 t}\bar{H}(t); \quad Z_5 = e^{-\alpha_2 t}\bar{G}_2(t) \;\; and \;\; Z_6 = e^{-\alpha_2 t}\bar{H}(t).$$

Taking Laplace Transform of the relations and solving for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

where,

$$N_1(s) = \left(1 - q_{12}^* q_{24}^* q_{41}^{(6)*} - q_{13}^* q_{31}^{(5)*} - q_{14}^* q_{41}^{(6)*}\right) Z_0^* + q_{01}^* Z_1^* + \; q_{01}^* q_{12}^* Z_2^*$$
$$+ q_{01}^* q_{13}^* \left(Z_3^* + q_{35}^* Z_5^*\right) + q_{01}^* \left(q_{12}^* q_{24}^* + q_{14}^*\right)\left(Z_4^* + q_{46}^* Z_6^*\right)$$

and

$$D_1(s) = 1 - q_{01}^* \left[q_{12}^* q_{20}^* + q_{13}^* q_{30}^* + \left(q_{12}^* q_{24}^* + q_{14}^*\right) q_{40}^*\right] - q_{12}^* q_{24}^* q_{41}^{(6)*} - \; q_{13}^* q_{31}^{(5)*} - q_{14}^* q_{41}^{(6)*}$$

Taking the inverse Laplace transform of $R_0(s)$, to get the reliability of the system when it initially starts from state S_0. Now mean time to system failure (MTSF) is given by,

$$E(T_0) = \lim_{s\to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)} = \frac{N_1}{D_1}$$

where,

$$N_1 = \left(1 - p_{12}p_{24}p_{41}^{(6)} - p_{13}p_{31}^{(5)} - p_{14}^{*}p_{41}^{(6)*}\right)\Psi_0 + \Psi_1 + p_{12}\Psi_2 \\ + p_{13}\left(\Psi_3 + p_{35}\Psi_5\right) + \left(p_{12}p_{24} + p_{14}^{*}\right)\left(\Psi_4 + p_{46}\Psi_6\right)$$

and

$$D_1 = 1 - p_{12}p_{20} - p_{13}p_{30} - \left(p_{12}p_{24} + p_{14}\right)p_{40} - p_{12}p_{24}p_{41}^{(6)} - p_{13}p_{31}^{(5)} - q_{14}^{*}q_{41}^{(6)*}$$

4.3 System Availability Analysis

4.3.1 When a unit in N-mode and operative

Let us define $A_i^N(t)$ as the probability that the system is up due to a unit in Normal (N) at epoch t, when it initially starts from state $S_i \in E$. Using the definition of $A_i(t)$ and probabilistic concepts, $A_i^N(t)$ is the sum of following contingencies:

(iii) The system remains up in state S_0 without making any transition to any other state up to time t. The probability of this event is $e^{-\alpha_1 t} = Z_0(t)$, say.

(iv) The system transit to the state S_1 from S_0 during $(u, u+du)$, $u \le t$ and then remains up in state S_1 for the remaining time (t–u), the probability of this contingency is

$$\int_0^t q_{01}(u)du A_1(t-u) = q_{01}(t) © A_1(t).$$

Therefore,

$$A_0^N(t) = Z_0(t) + q_{01}(t) © A_1^N(t).$$

Similarly,

$$A_1^N(t) = q_{12}(t) © A_2^N(t) + q_{13}(t) © A_3^N(t) + q_{14}(t) © A_4^N(t)$$
$$A_2^N(t) = q_{20}(t) © A_0^N(t) + q_{24}(t) © A_4^N(t)$$
$$A_3^N(t) = Z_3(t) + q_{30}(t) © A_0^N(t) + q_{31}^{(5)}(t) © A_1^N(t) + q_{34}^{(5,7)}(t) © A_4^N(t)$$
$$A_4^N(t) = Z_4(t) + q_{40}(t) © A_0^N(t) + q_{41}^{(6)}(t) © A_1^N(t) + q_{44}^{(6,8)}(t) © A_4^N(t)$$

Using the technique of Laplace transform, we can obtain the values of $A_0^N(t)$ in terms of their Laplace transforms i.e. $A_0^{N*}(s)$.
In the long run, the availability of the system due to a unit in N-mode is given by,

$$A_0^N = \lim_{t\to\infty} A_0^N(t) = \lim_{s\to 0} sA_0^{N*}(s) = \frac{N_2}{D_2}.$$

Where, in terms of

$$n_1 = \int te^{-\alpha_1 t}dG_2(t) + \frac{\alpha_1}{\alpha_1-\alpha_2}\left[\int t\left(e^{-\alpha_2 t} - e^{-\alpha_1 t}\right)dG_2(t)\right] + \frac{\alpha_1\alpha_2}{\alpha_1-\alpha_2}\int t\left[\frac{1-e^{-\alpha_2 t}}{\alpha_2} - \frac{1-e^{-\alpha_1 t}}{\alpha_1}\right]dG_2(t)$$
$$n_2 = \int te^{-\alpha_1 t}dH(t) + \frac{\alpha_1}{\alpha_1-\alpha_2}\left[\int t\left(e^{-\alpha_2 t} - e^{-\alpha_1 t}\right)dH(t)\right] + \frac{\alpha_1\alpha_2}{\alpha_1-\alpha_2}\int t\left[\frac{1-e^{-\alpha_2 t}}{\alpha_2} - \frac{1-e^{-\alpha_1 t}}{\alpha_1}\right]dH(t)$$
$$N_2 = \left[1 - p_{44}^{(6,8)} - p_{12}p_{24}p_{41}^{(6)} - p_{13}\left\{p_{31}^{(5)}\left(1-p_{44}^{(6,8)}\right) + p_{34}^{(5,7)}p_{41}^{(6)}\right\} - p_{14}p_{41}^{(6)}\right]\Psi_0 \\ + p_{13}\left(1-p_{44}^{(6,8)}\right)\Psi_3 + \left(p_{12}p_{24} + p_{13}p_{34}^{(5,7)} + p_{14}\right)\Psi_4$$

and

$$D_2 = \left[\left(p_{12}p_{20} + p_{13}p_{30}\right)\left(1-p_{44}^{(6,8)}\right) + p_{40}\left(p_{12}p_{24} - p_{13}p_{34}^{(5,7)} + p_{14}\right)\right]\Psi_0 \\ + \left(1-p_{44}^{(6,8)}\right)\left(\Psi_1 + p_{12}\Psi_2 + p_{13}n_1\right) + \left(p_{12}p_{24} + p_{13}p_{34}^{(5,7)} + p_{14}\right)n_2$$

4.3.2 When a unit is in P-mode

Let us define $A_i^P(t)$ as the probability that the system is up due to a unit in preventive maintenance (P) mode at epoch t, when it initially starts from state $S_i \in E$. Using the definition of $A_i^P(t)$ and probabilistic concepts, the recurrence relations, as obtained in previous case, for $A_i^P(t)$ are given as follows:

$$\begin{aligned}
A_0^P(t) &= q_{01}(t)©A_1^P(t)\\
A_1^P(t) &= Z_1(t) + q_{12}(t)©A_2^P(t) + q_{13}(t)©A_3^P(t) + q_{14}(t)©A_4^P(t)\\
A_2^P(t) &= Z_2(t) + q_{20}(t)©A_0^P(t) + q_{24}(t)©A_4^P(t)\\
A_3^P(t) &= q_{30}(t)©A_0^P(t) + q_{31}^{(5)}(t)©A_1^P(t) + q_{34}^{(5,7)}(t)©A_4^P(t)\\
A_4^P(t) &= q_{40}(t)©A_0^P(t) + q_{41}^{(6)}(t)©A_1^P(t) + q_{44}^{(6,8)}(t)©A_4^P(t)
\end{aligned}$$

Using the technique of Laplace transform, we can obtain the values of $A_0^P(t)$ in terms of their Laplace transforms i.e. $A_0^{P*}(s)$.
In the long run, the availability of the system due to a unit in P-mode is given by

$$A_0^P = \frac{N_3}{D_2}.$$

Where, $N_3 = (\Psi_1 + p_{12}\Psi_2)\left(1 - p_{44}^{(6,8)}\right)$ and D_2 as defined above.

4.4 Busy Period of Repair Facility

4.4.1 When repair facility is busy in the inspection of a unit

Let $B_i^i(t)$ be the probability that the repair facility is busy in the inspection of a partially failed unit at time t, when the system initially starts from the state $S_i \in E$. Using elementary probabilistic arguments, we have the following recursive relations for $B_i^i(t)$:

$$\begin{aligned}
B_0^i(t) &= q_{01}(t)©B_1^i(t)\\
B_1^i(t) &= Z_1(t) + q_{12}(t)©B_2^i(t) + q_{13}(t)©B_3^i(t) + q_{14}(t)©B_4^i(t)\\
B_2^i(t) &= q_{20}(t)©B_0^i(t) + q_{24}(t)©B_4^i(t)\\
B_3^i(t) &= q_{3,0}(t)©B_0^i(t) + q_{31}^{(5)}(t)©B_1^i(t) + q_{34}^{(5,7)}(t)©B_4^i(t)\\
B_4^i(t) &= q_{40}(t)©B_0^i(t) + q_{41}^{(6)}(t)©B_1^i(t) + q_{44}^{(6,8)}(t)©B_4^i(t)
\end{aligned}$$

Taking Laplace Transform (L.T.) of above relations and solving them for $B_0^{i*}(s)$, we get

$$B_0^{i*}(s) = \frac{N_4(s)}{D_2(s)}.$$

Where, $N_4(s) = q_{01}^*\left(1 - q_{44}^{(6,8)}\right)Z_1^*$ and $D_2(s)$ is the same as in case of availability analysis.

4.4.2 When repair facility is busy in the repair of a totally failed unit

Let $B_i^F(t)$ be the probability that the repair facility is busy in the repair of a totally failed unit at time t, when the system initially starts from the state S_i^E. Using elementary probabilistic arguments, we have

$$\begin{aligned}
B_0^F(t) =& q_{01}(t)©B_1^F(t)\\
B_1^F(t) =& q_{12}(t)©B_2^F(t) + q_{13}(t)©B_3^F(t) + q_{14}(t)©B_4^F(t)\\
B_2^F(t) =& q_{20}(t)©B_0^F(t) + q_{24}(t)©B_4^F(t)\\
B_3^F(t) =& q_{30}(t)©B_0^F(t) + q_{31}^{(5)}(t)©B_1^F(t) + q_{34}^{(5,7)}(t)©B_4^F(t)\\
B_4^F(t) =& Z_4(t) + q_{46}(t)©Z_6(t) + q_{46}(t)©q_{68}(t)©Z_8(t) + q_{40}(t)©B_0^F(t)\\
&+ q_{41}^{(6)}(t)©B_1^F(t) + q_{44}^{(6,8)}(t)©B_4^F(t)
\end{aligned}$$

Taking L.T. of above relations and solving them for $B_0^{F*}(s)$, we get

$$B_0^{F*}(s) = \frac{N_5(s)}{D_2(s)}.$$

Where, $N_5(s) = q_{01}^*\left(q_{12}^*q_{24}^* + q_{13}^*q_{34}^{(5,7)*} + q_{14}^*\right)(Z_4^* + q_{46}^*Z_6^* + q_{46}^*q_{68}^*Z_8^*)$ and $D_2(s)$ is the same as in case of availability analysis.

4.4.3 When repair facility is busy in the on-line repair of a partially failed unit

Let $B_i^1(t)$ be the probability that the repair facility is busy in the on-line repair of a partially failed unit at time t, when the system initially starts from the state $S_i \in E$. By using elementary probabilistic arguments, we have

$$\begin{aligned}
B_0^1(t) &= q_{01}(t)©B_1^1(t)\\
B_1^1(t) &= q_{12}(t)©B_2^1(t) + q_{13}(t)©B_3^1(t) + q_{14}(t)©B_4^1(t)\\
B_2^1(t) &= Z_2(t) + q_{20}(t)©B_0^1(t) + q_{24}(t)©B_4^1(t)\\
B_3^1(t) &= q_{30}(t)©B_0^1(t) + q_{31}^{(5)}(t)©B_1^1(t) + q_{34}^{(5,7)}(t)©B_4^1(t)\\
B_4^1(t) &= q_{40}(t)©B_0^1(t) + q_{41}^{(6)}(t)©B_1^1(t) + q_{44}^{(6,8)}(t)©B_4^1(t)
\end{aligned}$$

Taking L.T. of above relations and solving them for $B_0^{1*}(s)$, we get

$$B_0^{1*}(s) = \frac{N_6(s)}{D_2(s)}.$$

Where, $N_6(s) = q_{01}^*q_{12}^*\left(1 - q_{44}^{(6,8)}\right)Z_2^*$ and $D_2(s)$ is the same as in case of availability analysis.

4.4.4 When repair facility is busy in the off-line repair of a partially failed unit

Let $B_i^2(t)$ be the probability that the repair facility is busy in the off-line repair of a partially failed unit at time t, when the system initially starts from the state $S_i \in E$. By using elementary probabilistic arguments, we have

$$\begin{aligned}
B_0^2(t) =& q_{01}(t)©B_1^2(t)\\
B_1^2(t) =& q_{12}(t)©B_2^2(t) + q_{13}(t)©B_3^2(t) + q_{14}(t)©B_4^2(t)\\
B_2^2(t) =& q_{20}(t)©B_0^2(t) + q_{24}(t)©B_4^2(t)\\
B_3^2(t) =& Z_3(t) + q_{35}(t)©Z_5(t) + q_{35}(t)q_{57}(t)©Z_7(t) + q_{30}(t)©B_0^2(t)\\
&+ q_{31}^{(5)}(t)©B_1^2(t) + q_{34}^{(5,7)}(t)©B_4^2(t)\\
B_4^2(t) =& q_{40}(t)©B_0^2(t) + q_{41}^{(6)}(t)©B_1^2(t) + q_{44}^{(6,8)}(t)©B_4^2(t)
\end{aligned}$$

Taking L.T. of above relations and solving them for $B_0^{2*}(s)$, we get

$$B_0^{2*}(s) = \frac{N_7(s)}{D_2(s)}.$$

Where, $N_7(s) = q_{01}^*q_{13}^*\left(1 - q_{44}^{(6,8)}\right)(Z_3^* + q_{35}^*Z_5^* + q_{35}^*q_{57}^*Z_7^*)$ and $D_2(s)$ is the same as in case of availability analysis.

Now to obtain the steady-state probabilities that the repair facility will be busy in the inspection of a unit, repairing of a totally failed unit, on-line and off-line repair of a unit respectively, we use the results:

$$Z_i^*(0) = \Psi_i \text{ and } q_{ij}(0) = p_{ij}.$$

We observe that

$$B_0^i = \frac{N_4}{D_2}, \qquad B_0^F = \frac{N_5}{D_2}, \qquad B_0^1 = \frac{N_6}{D_2} \text{ and } B_0^2 = \frac{N_7}{D_2}.$$

Where,

$$\begin{aligned}
N_4 &= \left(1 - p_{44}^{(6,8)}\right)\Psi_1\\
N_5 &= \left(p_{12}p_{24} + p_{13}p_{34}^{(5,7)} + p_{14}\right)(\Psi_4 + p_{46}\Psi_6 + p_{46}p_{68}\Psi_8)\\
N_6 &= p_{12}\left(1 - p_{44}^{(6,8)}\right)\Psi_2\\
N_7 &= p_{13}\left(1 - p_{44}^{(6,8)}\right)(\Psi_3 + p_{35}\Psi_5 + p_{35}p_{57}\Psi_7)
\end{aligned}$$

and D_2 is the same as defined in case of availability analysis.

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5. PROFIT ANALYSIS

The net expected profit earned by the system during $(0, t)$ is given by,

$$P(t) = K_0\mu_{up}^N(t) + K_1\mu_{up}^P(t) - K_2\mu_b^i(t) - K_3\mu_b^F(t) - K_4\mu_b^1(t) - K_5\mu_b^2(t).$$

The expected profit per unit time in steady-state is

$$P = K_0A_0^N + K_1A_0^P - K_2B_0^i - K_3B_0^F - K_4B_0^1 - K_5B_0^2.$$

Where, K_0 and K_1 are the revenues made by the system per unit time due to N and P–mode of a unit. K_2, K_3, K_4 and K_5 are the amounts paid per unit of time for inspection, repair a unit in F–mode, on-line repair, and off-line repair respectively.

6. GRAPHICAL PRESENTATION OF RELIABILITY MEASURES

To study the behavior of the system the value of reliability and profit function is obtained by assuming all the repair time distributions as negative exponential i.e.

$$G_1(t) = 1 - e^{-\lambda_1 t}; \qquad G_2(t) = 1 - e^{-\lambda_2 t}; \qquad H(t) = 1 - e^{-\beta t}.$$

We have the following changes in steady-state transition probabilities and mean sojourn times-

$$p_{20} = \frac{\lambda_1}{\lambda_1 + \alpha_2}; \quad p_{24} = \frac{\alpha_2}{\lambda_1 + \alpha_2}; \quad p_{30} = \frac{\lambda_2}{\lambda_2 + \alpha_2}; \quad p_{3,1}^{(5)} = \frac{\alpha_1\lambda_2}{(\lambda_2 + \alpha_1)(\lambda_2 + \alpha_2)};$$

$$p_{3,4}^{(5,7)} = \frac{\alpha_1\alpha_2}{(\lambda_2 + \alpha_1)(\lambda_2 + \alpha_2)}; \quad p_{40} = \frac{\beta}{\alpha_1 + \beta}; \quad p_{4,1}^{(6)} = \frac{\alpha_1\beta}{(\alpha_1 + \beta)(\alpha_2 + \beta)};$$

$$p_{4,4}^{(6,8)} = \frac{\alpha_1\alpha_2}{(\alpha_1 + \beta)(\alpha_2 + \beta)}; \quad \Psi_2 = \frac{1}{(\alpha_2 + \lambda_1)}; \quad \Psi_3 = \frac{1}{(\alpha_1 + \lambda_2)};$$

$$\Psi_4 = \frac{1}{(\alpha_1 + \beta)}; \quad \Psi_5 = \frac{1}{(\alpha_2 + \lambda_2)}; \quad \Psi_6 = \frac{1}{(\alpha_2 + \beta)};$$

$$\Psi_7 = \frac{1}{\lambda_2}; \quad \Psi_8 = \frac{1}{\beta}$$

$$n_1 = \frac{\lambda_2}{(\lambda_2 + \alpha_1)^2}\left[1 + \frac{\alpha_1(\alpha_1 + \alpha_2)}{(\lambda_2 + \alpha_2)^2} + \frac{\alpha_2}{\alpha_1 - \alpha_2}\right] - \frac{\alpha_1\lambda_2}{(\alpha_1 - \alpha_2)(\lambda_2 + \alpha_2)^2} + \frac{1}{\lambda_2}$$

$$n_2 = \frac{\beta}{(\alpha_1 + \beta)^2}\left[1 + \frac{\alpha_1(\alpha_1 + \alpha_2)}{(\alpha_2 + \beta)^2} + \frac{\alpha_2}{\alpha_1 - \alpha_2}\right] - \frac{\alpha_1\beta}{(\alpha_1 - \alpha_2)(\alpha_2 + \beta)^2} + \frac{1}{\beta}$$

7. CONCLUSION

For a more concrete study of system behaviour of the model, we plot the curves for MTSF and profit function in Figure–2 and Figure–3 w.r.t. α_1 for different values of λ_2 while the other parameters are kept fixed as: $\alpha_2 = 0.009$, $\lambda_1 = 0.02$, $\theta = 0.09$, $\beta = 0.06$, $p = 0.5$, $q = 0.5$, $K_0 = 5000$, $K_1 = 3000$, $K_2 = 500$, $K_3 = 1000$, $K_4 = 800$, $K_5 = 500$.

α_1	$\lambda_2 = 0.01$	$\lambda_2 = 0.02$	$\lambda_2 = 0.03$
0.01	684.03	1183.61	1733.92
0.02	456.93	729.96	1024.10
0.03	380.46	578.02	787.09
0.04	341.83	501.69	668.38
0.05	318.43	455.69	597.03
0.06	302.70	424.89	549.40
0.07	291.37	402.81	515.33
0.08	282.81	386.20	489.74
0.09	276.11	373.24	469.82
0.1	270.72	362.84	453.87

Table 1: Variation in the values of the MTSF of the system

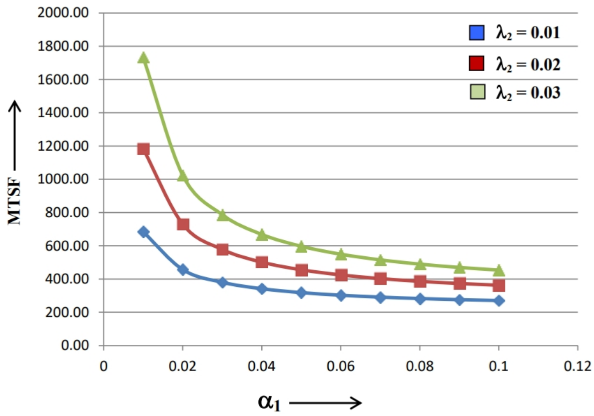

Figure 2

The graph in Figure 2 and data in Table 1 represents the behavior of MTSF with respect to α_1 for three different values of $\lambda_2 (= 0.01, 0.02,$ and $0.03)$. It is clear from the graph that the MTSF decreases α_1 increases from 0.01 to 0.10. We also observe that an increment in the repair rate λ_2 (off-line repair rate of a partially failed unit) corresponds to the increment in MTSF also. Further, MTSF decreases significantly in the beginning, and after that, it decreases approximately in a constant manner.

α_1	$\lambda_2 = 0.01$	$\lambda_2 = 0.02$	$\lambda_2 = 0.03$
0.01	4234.04	4476.98	4754.33
0.02	3780.94	4084.25	4443.76
0.03	3441.94	3710.78	4147.77
0.04	3184.19	3419.91	3875.17
0.05	2978.46	3193.91	3636.46
0.06	2791.24	3015.85	3430.25
0.07	2638.03	2873.01	3252.23
0.08	2532.94	2756.41	3097.95
0.09	2448.43	2659.68	2963.46
0.1	2388.54	2578.29	2845.47

Table 2: Variation in the values of the profit function of the system

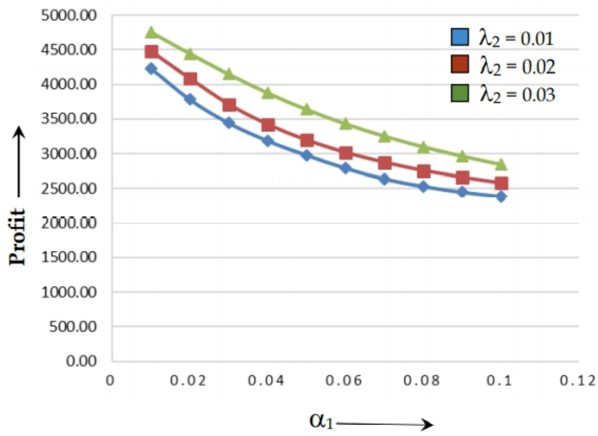

Figure 3

Figure–3 represents the behavior of profit function with respect to α_1 for same three different values of λ_2. Profit function also predicts the same trends as that for MTSF.

CONFLICT OF INTEREST

The work is original and has not been submitted anywhere for publication.

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