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# Reliability Analysis of Multi-Workstation Computer Network Configured as Series-Parallel System via Gumbel - Hougaard Family Copula

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**Abstract:** The purpose of this paper is to investigate the dependability of a computer system composed of four workstations, three hubs and two routers. Subsystem 1 consists of workstations $A_1$, $A_2$, $A_3$, and $A_4$ that are parallel and linked to Subsystem 2 (hubs $B_1$, $B_2$ and $B_3$), while Subsystem 3 router consists of $C_1$ and $C_2$ that are also parallel to each other. Each subsystem is assigned a human operator ($H_1$, $H_2$, and $H_3$). To assess the system's reliability, a system of first order partial differential equations is derived from the system's transition diagram and solved using the supplementary variables technique and Laplace transforms. It is assumed that workstation, hub, and router failure times follow an exponential distribution, whereas repair times follow a general distribution and the Gumbel-Hougaard family copula distribution. Reliability measures of testing system effectiveness, such as reliability, availability, MTTF, and cost function, are developed and investigated. Tables and graphs show some of the most important findings.
**Keyword —** Reliability, network, series-parallel, computer, workstation, Gumbel-Hougaard family copula, hub and routers.

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## 1. INTRODUCTION

There are two types of failures in a computer network: hardware and software. A number of strategies for improving computer system efficiency have been proposed by researchers, designers, and engineers. Redundancy is a strategy used to improve reliability, availability, and mean failure times, which leads to improved system health, product quality, increased output, and revenue mobilization. Using parallel units, standby units, and fault tolerance units improves system reliability. One of the most important aspects of developing stochastic models for computer networks was the technique of unit-wise redundancy. In computer networks, the unit-wise redundancy technique in cold standby mode has also been used. Each computer system has programs that run on many different computers that are linked via a network, which has become very complicated and difficult to rely on. The ability of a system to perform its intended function under specified conditions for a specified period of time is defined as reliability. Many researchers have proposed various types of studies / mathematical models in order to improve the reliability of computer systems and have declared better performance in their operations. Wu (2014), for example, talked about modeling distributed file systems for practical performance. Malik (2013) investigated various computer system models with cold standby redundancy units and various repair policies. However, it has been demonstrated that component-wise redundancy outperforms unit-wise redundancy in terms of reliability. Ashish Kumar and Malik (2012) and Malik and Munday (2014) discussed computer system modeling, with preventive maintenance taking precedence over software replacement and hardware repair taking precedence over replacement. Kumar, Saini, and Malik (2015) investigated the performance of a computer system with hardware fault detection. Monika Gahlot, Singh, Ayagi, and Goel (2018) used a copula linguistic approach to compare the performances of (k-out-of-n:G/F) types of repairable systems with different types of failures and two types of repair and concluded that copula repair policy outperforms general repair policy. Garg (2015) addressed the prediction of uncertain behavior in critical engineering systems operating in a hazy environment.

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Garg (2016a) discussed a method for analyzing the reliability of industrial systems using fuzzy Kolmogorov differential equations. Garg (2016b) used credibility theory and various types of intuitionistic fuzzy numbers to analyze the reliability of a series-parallel system. Niwas and Garg (2014) investigated the dependability and profitability of an industrial system based on a cost-free warranty policy. Yusuf, Sani, and Yusuf (2019) investigated the profitability of a series-parallel system. Yusuf (2016) presents a parallel system reliability model with two types of preventive maintenance. Isa, U. A Ali, Yusuf, and Bashir Yusuf (2020) Cost–benefit analysis of three different series–parallel dynamo configurations was performed. Lado, Singh, Ismail, and Yusuf (2018) used Copula Linguistics to assess the performance of a repairable system in series configuration under various types of failure and repair policies. Abdul Kareem L. and Singh (2019) discuss the cost assessment of a complex repairable system with two subsystems in series using the Gumbel Hougaard family copula. Monika Gahlot, Singh, Ayagi, and Ibrahim Abdullahi (2019) used the Copula Approach to perform a stochastic analysis of a two-unit complex repairable system with a switch and a human failure.

Workstations, hubs and routers were observed in this computer network research as three subsystems connected in a series-parallel arrangement. The workstation was considered subsystem 1, the hub was considered subsystem 2, and the router was considered subsystem 3. In the past, researchers have presented excellent work on the analysis of the reliability of complex repairable systems and have declared improved performance of the repairable system by their operations. There is still a need for further research into the new types of models in order to provide a justified and satisfactory assessment. As a result, this paper examined a series-parallel Multi-Workstation Computer Network Configuration. Consisting of three subsystems (one, two, and three) and three human operators. The system's performance is investigated using an additional variable technique and Laplace transformations. For various failure and repair values, various reliability measures such as availability, reliability, mean time to system failure (MTTF), MTTF sensitivity, and cost analysis have been computed. The present work was done on a CBT center (computer base test) that comprises of computer network containing workstation, hub and router to see the effectiveness of the network in the center. Abdul Kareem L. and Singh (2019) analyzed the cost assessment of complex repairable system consisting two subsystems in series configuration using gumbel hougaard family copula in their work one human operator was considered for the two subsystems which in our on case each of the three subsystem has its own human operator attached to it.

From the previous research of computer network, little or no attention is paid on the reliability analysis of multi-workstation computer network, in this research work reliability analysis of multi-workstation computer network configured as series-parallel system, is studied.

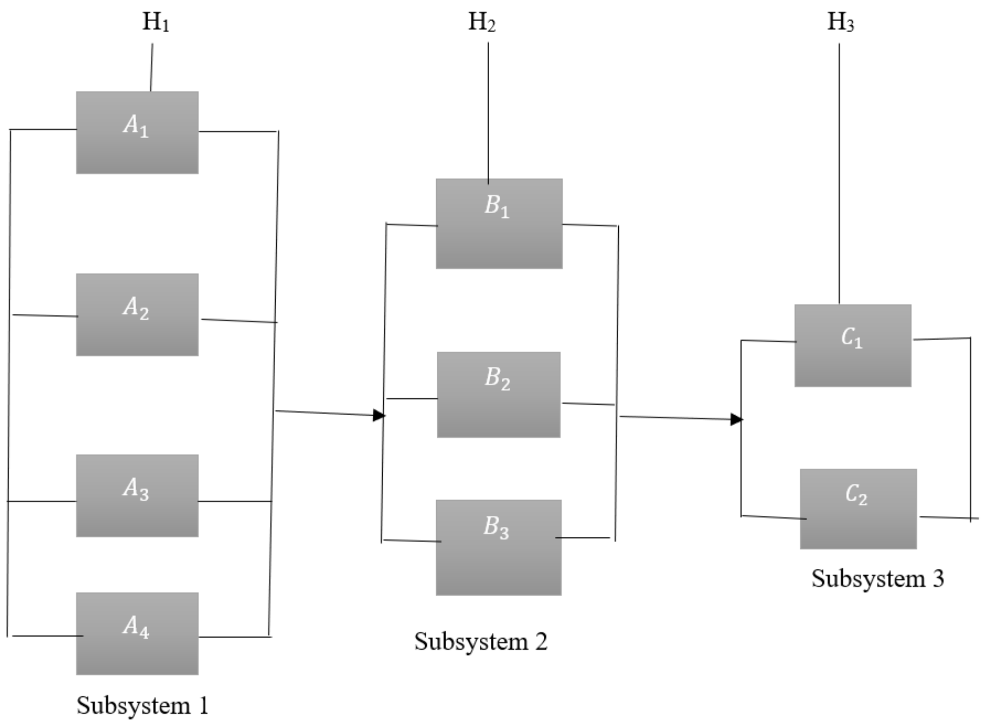
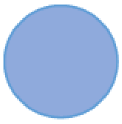
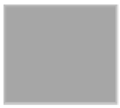



Figure 1: The relationship between lead time and crashing cost

## 2. NOTATIONS, ASSUMPTIONS, AND DESCRIPTION OF THE SYSTEM

### 2.1 Notations

| | |
|---|---|
|  | Indicate full working state |
|  | Indicate partial working state |
|  | Indicate failure state |
| $t$ : | Time variable. |
| $s$ : | Laplace transform variable for all expressions. |
| $\beta_1$ : | Failure rate of workstation (Subsystem 1). |
| $\beta_2$ : | Failure rate of hub (Subsystem 2). |
| $\beta_3$ : | Failure rate of router (Subsystem 3). |
| $\beta_{H_1}$ : | Failure rate as results of human error 1. |
| $\beta_{H_2}$ : | Failure rate as results of human error 2. |
| $\beta_{H_3}$ : | Failure rate as results of human error 3. |
| $\Phi(x)$ : | Repair rate of the unit of Subsystem 1. |
| $\Phi(y)$ : | Repair rate of the unit of Subsystem 2. |
| $\Phi(z)$ : | Repair rate of the unit of Subsystem 3. |
| $\mu_0(x)/\mu_0(y)/\mu_0(z)$ : | Repair rates for complete failed states. |
| $p_i(t)$ : | The probability that the system is in $S_i$ state at instants for $i = 0$ to 12. |
| $\bar{P}(s)$ : | Laplace transformation of state transition probability $p(t)$. |
| $P_i(x,t)$ : | The probability that a system is in state $S_i$ for $i = 1......12,$ the system under repair and elapse repair time is $(x,t)$ with repair variable $x$ and time variable $t$. |
| $P_i(y,t)$ : | The probability that a system is in state $S_i$ for $i = 1......12,$ the system under repair and elapse repair time is $(y,t)$ with repair variable $x$ and time variable $t$. |
| $P_i(z,t)$ : | The probability that a system is in state $S_i$ for $i = 1......12,$ the system under repair and elapse repair time is $(z,t)$ with repair variable $z$ and time variable $t$. |
| $P_{H_1}(x,t)$ : | Probability that the system is in state $S_i$ for $i = 12$ state, the system is running under repair and elapse repair time is $(x,t)$ with repair variable $y$ and time variable $t$. |
| $E_p(t)$ : | Expected profit during the time interval $[0,t)$. |
| $K_1, K_2$ : | Revenue and service cost per unit time, respectively. |
| $\mu_0(x)$ : | The expression of joint probability (failed state $S_i$ to good state $S_0$) according to Gumbel-Hougaard family copula definition $\mu_0(x) = c_\theta(u_1, u_2(x)) = \exp\left(x^\theta + \{\log\phi(x)^\theta\}^{\frac{1}{\theta}}\right), 1 \le \theta \le \infty.$ Where $\mu_1 = \phi(x),$ and $2 = e^x$. |

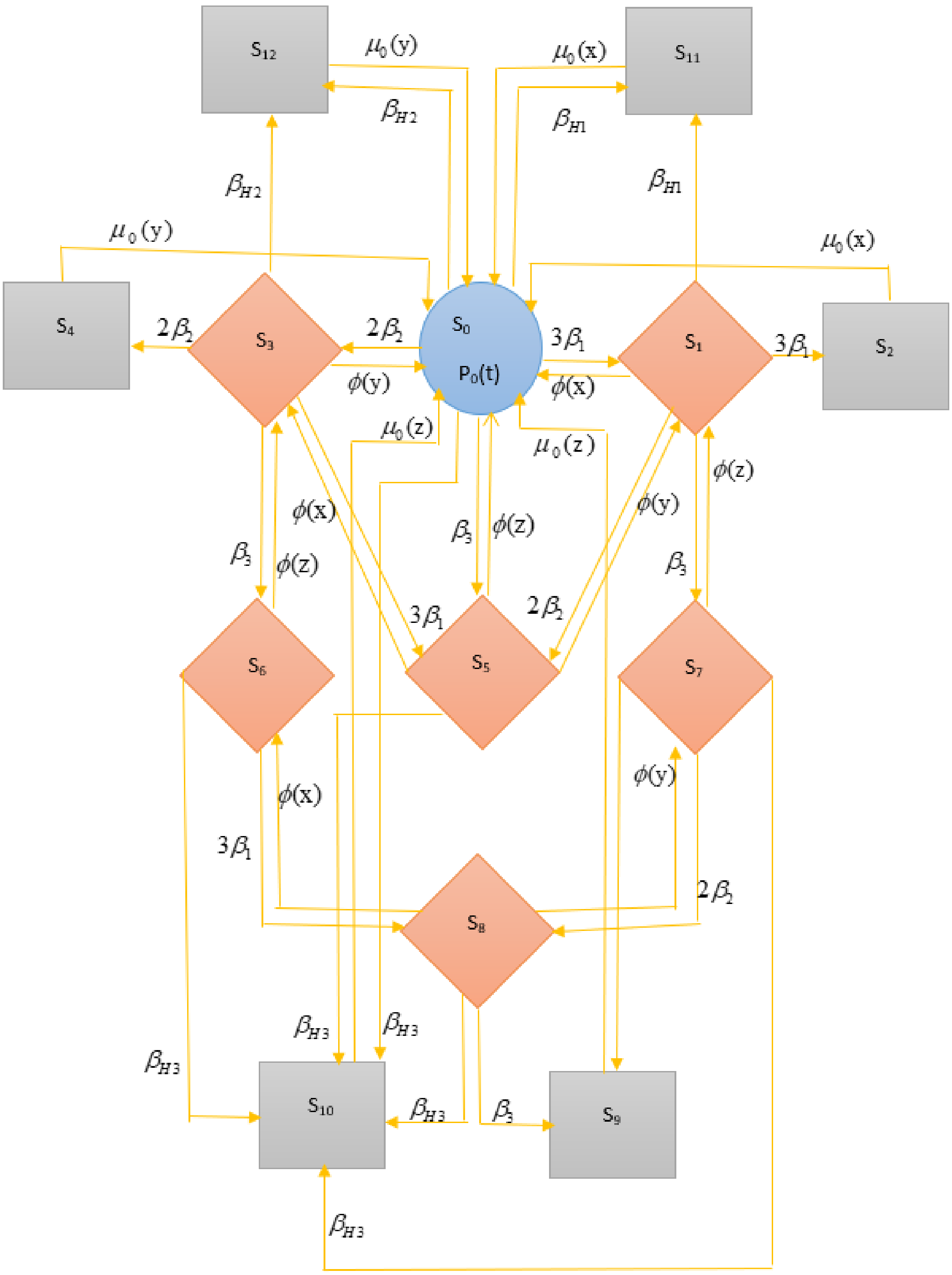



Figure 2: State transition diagram

### 2.2 Assumptions

1. Computer systems have redundant standby units.
2. Repair is immediate.
3. Switching from standby to operation is perfect.
4. Each failure is repairable.
5. Workstation are identical to each other.
6. Hub and routers are identical to each other.
7. Each computer system failed independent of the other.
8. Computer system works simultaneously and independently.
9. Both subsystems are initially in good working order.
10. For operational mode, three units of subsystem A and two out of three units of subsystem B are required.
11. If two subsystem A units or two out of three subsystem B units fail, the system will be rendered inoperable.
12. All failure rates are assumed to be constant and to follow an exponential distribution.
13. It is assumed that a repaired system works as well as a new system and that no damage occurs during the repair process.
14. Gumbel-Hougaard family copula distribution is used to repair the entire failed system.
15. The failed unit is ready to perform the task as soon as it is repaired.

### 2.3 Description of the system:

- √ Subsystem 1: Workstation $A_1$, Workstation $A_2$, Workstation $A_3$ and Workstation $A_4$ connected in series with hub.
- √ Subsystem 2: Hub $B_1$, Hub $B_2$ and Hub $B_3$ connected to router.
- √ Subsystem 3: Consist of Router $C_1$, Router $C_2$ and Router $C_3$.
- √ Each of the Subsystem is attached to its Human Operator.

This study focuses solely on computer systems configured in a series-parallel configuration with three subsystems, 1, 2, and 3. Subsystem 1 has three active parallel units, whereas Subsystem 2 has two out of three active parallel units. Initially, the system is in perfect working order, with all subsystems functioning properly. When a unit from subsystem 1, 2, or 3 fails, the system enters minor partial failure and remains operational while the failed unit is immediately sent for repair. System failure occurs when two units of subsystem 1 fail, or when two units of subsystem 2 and two units of subsystem 3 fail, or when human failure occurs, which is likely in all states. Minor / complete failed states are repaired using general distribution, whereas a complete failed state is deployed using a Gumbel-Hougaard family copula distribution. The different system states are described in Table 1. The system described above is a classic example of a workstation-router computer network with a geographically separated hub, typically in a cloud computing environment that provides similar services to workstations on different continents.

Table 1: States of the system

| State | Subsystem 1 | | | | Subsystem 2 | | | Subsystem 3 | | Systems' Status |
|---|---|---|---|---|---|---|---|---|---|---|
| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $B_1$ | $B_2$ | $B_3$ | $C_1$ | $C_2$ | |
| $S_0$ | Good | Good | Good | standby | Good | Good | standby | Good | standby | Operational |
| $S_1$ | Failed | Good | Good | Good | Good | Good | standby | Good | standby | Operational |
| $S_2$ | Failed | Failed | Idle | Idle | Idle | Idle | standby | Idle | standby | Down |
| $S_3$ | Good | Good | Good | standby | Failed | Good | Good | Good | standby | Operational |
| $S_4$ | Idle | Idle | Idle | Idle | Failed | Failed | Idle | Idle | standby | Down |
| $S_5$ | Failed | Good | Good | Good | Failed | Good | Good | Good | standby | Operational |
| $S_6$ | Good | Good | Good | standby | Good | Good | standby | Failed | Good | Operational |
| $S_7$ | Failed | Good | Good | Good | Good | Good | standby | Failed | Good | Operational |
| $S_8$ | Failed | Good | Good | Good | Failed | Good | Good | Failed | Good | Operational |
| $S_9$ | Idle | Idle | Idle | standby | Idle | Idle | standby | Failed | Failed | Down |
| $S_{10}$ | System failure due to Human operator | | | | | | | | | |
| $S_{11}$ | System failure due to Human operator | | | | | | | | | |
| $S_{12}$ | System failure due to Human operator | | | | | | | | | |

## 3. FORMULATION OF MATHEMATICAL MODEL

### 3.1 Formulation of Mathematical Model

For the probability of considerations, the following steps of differential difference equation are derived.

$$\left(\frac{\delta}{\delta t}+3\beta_1+2\beta_2+\beta_3+\beta_{H_1}+\beta_{H_2}+\beta_{H_3}\right)P_0(t)=\int_0^\infty\phi(x)p_1(x,t)dx+\int_0^\infty\phi(y)p_3(y,t)dy+$$
$$\int_0^\infty\phi(z)p_5(z,t)dz+\int_0^\infty\mu_0(x)p_{H_1}(x,t)dx+$$
$$\int_0^\infty\mu_0(y)p_{H_2}(y,t)dy+\int_0^\infty\mu_0(z)p_{H_3}(z,t)dz+\int_0^\infty\mu_0(x)p_2(x,t)dx+$$
$$\int_0^\infty\mu_0(y)p_4(y,t)dy+\int_0^\infty\mu_0(z)p_9(z,t)dz \tag{1}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+3\beta_1+2\beta_2+\beta_3+\beta_{H_1}+\phi(x)\right)P_1(x,t)=0 \tag{2}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+\mu_0(x)\right)P_2(x,t)=0 \tag{3}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y}+3\beta_1+2\beta_2+\beta_3+\beta_{H_2}+\phi(y)\right)P_3(y,t)=0 \tag{4}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y}+\mu_0(y)\right)P_4(y,t)=0 \tag{5}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta z}+\beta_{H_3}+\phi(z)+\phi(y)\right)P_5(z,t)=0 \tag{6}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta z}+3\beta_1+\beta_{H_3}+\phi(z)\right)P_6(z,t)=0 \tag{7}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta z}+2\beta_2+\beta_{H_3}+\phi(z)\right)P_7(z,t)=0 \tag{8}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta z}+\beta_3+\beta_{H_3}+\phi(x)+\phi(y)\right)P_8(z,t)=0 \tag{9}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta z}+\mu_0(z)\right)P_9(z,t)=0 \tag{10}$$

$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta z}+\mu_0(z)\right)P_{H_1}(z,t)=0 \tag{11}$$
$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta x}+\mu_0(x)\right)P_{H_2}(x,t)=0 \tag{12}$$
$$\left(\frac{\delta}{\delta t}+\frac{\delta}{\delta y}+\mu_0(y)\right)P_{H_3}(y,t)=0 \tag{13}$$

*Boundary Conditions*

$$P_1(0,t)=3\beta_1P_0(t) \tag{14}$$
$$P_2(0,t)=9\beta_1^2P_0(t) \tag{15}$$
$$P_3(0,t)=2\beta_2P_0(t) \tag{16}$$
$$P_4(0,t)=4\beta_2^2P_0(t) \tag{17}$$
$$P_5(0,t)=(\beta_3+12\beta_1\beta_2)P_0(t) \tag{18}$$
$$P_6(0,t)=2\beta_2\beta_3P_0(t) \tag{19}$$
$$P_7(0,t)=3\beta_1\beta_3P_0(t) \tag{20}$$
$$P_8(0,t)=6\beta_1\beta_2\beta_3P_0(t) \tag{21}$$
$$P_9(0,t)=6\beta_1\beta_2\beta_3^2P_0(t) \tag{22}$$
$$P_{H_1}(0,t)=\left(2\beta_2\beta_3\beta_{H_3}+6\beta_1\beta_2\beta_3\beta_{H_3}+\beta_{H_3}+\beta_3\beta_{H_3}+12\beta_1\beta_2\beta_{H_3}+3\beta_1\beta_3\beta_{H_3}\right)P_0(t) \tag{23}$$
$$P_{H_2}(0,t)=\left(3\beta_1\beta_{H_1}+\beta_{H_1}\right)P_0(t) \tag{24}$$
$$P_{H_3}(0,t)=\left(2\beta_2+\beta_{H_2}\right)P_0(t) \tag{25}$$

### 3.2 Solution of the Model

By taking the Laplace transformation of equations (1) to (25) we obtain the following results

$$\left(s+3\beta_1+2\beta_2+\beta_3+\beta_{H_1}+\beta_{H_2}+\beta_{H_3}\right)\bar{P}_0(s)=1+\int_0^\infty\phi(x)p_1(x,s)dx+\int_0^\infty\phi(y)p_3(y,s)dy+$$
$$\int_0^\infty\phi(z)p_5(z,s)dz+$$
$$\int_0^\infty\mu_0(x)p_{H_1}(x,s)dx+\int_0^\infty\mu_0(y)p_{H_2}(y,s)dy+\int_0^\infty\mu_0(z)p_{H_3}(z,s)dz+$$
$$\int_0^\infty\mu_0(x)p_2(x,s)dx+\int_0^\infty\mu_0(y)p_4(y,s)dy+\int_0^\infty\mu_0(z)p_9(z,s)dz \tag{26}$$
$$\left(s+\frac{\delta}{\delta x}+3\beta_1+2\beta_2+\beta_3+\beta_{H_1}+\phi(x)\right)\bar{P}_1(x,s)=0 \tag{27}$$
$$\left(s+\frac{\delta}{\delta x}+\mu_0(x)\right)\bar{P}_2(x,s)=0 \tag{28}$$
$$\left(s+\frac{\delta}{\delta y}+3\beta_1+2\beta_2+\beta_3+\beta_{H_2}+\phi(y)\right)\bar{P}_3(y,s)=0 \tag{29}$$
$$\left(s+\frac{\delta}{\delta y}+\mu_0(y)\right)\bar{P}_4(y,s)=0 \tag{30}$$
$$\left(s+\frac{\delta}{\delta z}+\beta_{H_3}+\phi(x)+\phi(y)+\phi(z)\right)\bar{P}_5(z,s)=0 \tag{31}$$
$$\left(s+\frac{\delta}{\delta z}+3\beta_1+\beta_{H_3}+\phi(z)\right)\bar{P}_6(z,s)=0 \tag{32}$$
$$\left(s+\frac{\delta}{\delta z}+2\beta_2+\beta_{H_3}+\phi(z)\right)\bar{P}_7(y,s)=0 \tag{33}$$
$$\left(s+\frac{\delta}{\delta z}+\beta_3+\beta_{H_3}+\phi(x)+\phi(y)\right)\bar{P}_8(z,s)=0 \tag{34}$$
$$\left(s+\frac{\delta}{\delta z}+\mu_0(z)\right)\bar{P}_9(z,s)=0 \tag{35}$$

$$\left(s+\frac{\delta}{\delta z}+\mu_0(z)\right)\bar{P}_{10}(z,s)=0 \tag{36}$$
$$\left(s+\frac{\delta}{\delta x}+\mu_0(x)\right)\bar{P}_{11}(x,s)=0 \tag{37}$$
$$\left(s+\frac{\delta}{\delta y}+\mu_0(y)\right)\bar{P}_{12}(y,s)=0 \tag{38}$$

$$\bar{P}_1(0,s)=3\beta_1\bar{P}_0(s) \tag{39}$$
$$\bar{P}_2(0,s)=9\beta_1^2\bar{P}_0(s) \tag{40}$$
$$\bar{P}_3(0,s)=2\beta_2\bar{P}_0(s) \tag{41}$$
$$\bar{P}_4(0,s)=4\beta_2^2\bar{P}_0(s) \tag{42}$$
$$\bar{P}_5(0,s)=(\beta_3+12\beta_1\beta_2)\bar{P}_0(s) \tag{43}$$
$$\bar{P}_6(0,s)=2\beta_2\beta_3\bar{P}_0(s) \tag{44}$$
$$\bar{P}_7(0,s)=3\beta_1\beta_3\bar{P}_0(s) \tag{45}$$
$$\bar{P}_8(0,s)=6\beta_1\beta_2\beta_3\bar{P}_0(s) \tag{46}$$
$$\bar{P}_9(0,s)=6\beta_1\beta_2\beta_3^2\bar{P}_0(s) \tag{47}$$
$$\bar{P}_{H_1}(0,s)=\left(2\beta_2\beta_3\beta_{H_3}+6\beta_1\beta_2\beta_3\beta_{H_3}+\beta_{H_3}+\beta_3\beta_{H_3}+12\beta_1\beta_2\beta_{H_3}+3\beta_1\beta_3\beta_{H_3}\right)\bar{P}_0(s) \tag{48}$$
$$\bar{P}_{H_2}(0,s)=\left(3\beta_1\beta_{H_3}+\beta_{H_3}\right)\bar{P}_0(s) \tag{49}$$
$$\bar{P}_{H_3}(0,s)=\left(2\beta_2+\beta_{H_2}\right)\bar{P}_0(s) \tag{50}$$

$$\bar{P}_0(s)=\frac{1}{D(s)}$$

$$\bar{P}_1(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_Q\left(s+3\beta_1+2\beta_2+\beta_3+\beta_{H_1}\right)}{s+3\beta_1+2\beta_2+\beta_3+\beta_{H_1}}\right\}3\beta_1 \tag{51}$$
$$\bar{P}_2(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\}9\beta_1^2 \tag{52}$$
$$\bar{P}_3(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_Q\left(s+3\beta_1+2\beta_2+\beta_3+\beta_{H_2}\right)}{s+3\beta_1+2\beta_2+\beta_3+\beta_{H_2}}\right\}2\beta_2 \tag{53}$$
$$\bar{P}_4(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\}4\beta_2^2 \tag{54}$$
$$\bar{P}_5(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_Q\left(s+\beta_{H_3}\right)}{s+\beta_{H_3}}\right\}(\beta_3+12\beta_1\beta_2) \tag{55}$$
$$\bar{P}_6(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\}(2\beta_2\beta_3) \tag{56}$$
$$\bar{P}_7(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_Q\left(s+2\beta_2+\beta_{H_3}\right)}{s+2\beta_2+\beta_{H_3}}\right\}(3\beta_1\beta_3) \tag{57}$$
$$\bar{P}_8(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_Q\left(s+\beta_3+\beta_{H_3}\right)}{s+\beta_3+\beta_{H_3}}\right\}(6\beta_1\beta_2\beta_3) \tag{58}$$
$$\bar{P}_9(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\}\left(6\beta_1\beta_2\beta_3^2\right) \tag{59}$$
$$\bar{P}_{H_1}(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\}(2\beta_2\beta_3\beta_{H_3}+6\beta_1\beta_2\beta_3\beta_{H_3}+\beta_{H_3}+\beta_3\beta_{H_3}+12\beta_1\beta_2\beta_{H_3}+ 3\beta_1\beta_3\beta_{H_3}) \tag{60}$$
$$\bar{P}_{H_2}(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\}(3\beta_1\beta_{H_1}+\beta_{H_1}) \tag{61}$$
$$\bar{P}_{H_3}(s)=\frac{1}{D(s)}\left\{\frac{1-\bar{S}_{\mu_0}(s)}{s}\right\}(2\beta_2+\beta_{H_2}) \tag{62}$$

where:

$$D(s) = s + 3\beta_1 + 2\beta_2 + \beta_3 + \beta_{H_1} + \beta_{H_2} + \beta_{H_3} - \left( \begin{array}{l} 3\beta_1 \left\{ \bar{S}_Q\left(s + 3\beta_1 + 2\beta_2 + \beta_3 + \beta_{H_1}\right) \right\} + 2\beta_2 \left\{ \bar{S}_Q\left(s + 3\beta_1 + 2\beta_2 + \beta_3 + \beta_{H_2}\right) \right\} \\ +2\beta_2 \left\{ \bar{S}_Q\left(s + 3\beta_1 + 2\beta_2 + \beta_3 + \beta_{H_2}\right) \right\} + (\beta_3 + 12\beta_1\beta_2) \left\{ \bar{S}_Q\left(s + \beta_{H_3}\right) \right\} \\ + [2\beta_2\beta_3\beta_{H_3} + 6\beta_1\beta_2\beta_3\beta_{H_3} + \beta_{H_3} + \beta_3\beta_{H_3} + 12\beta_1\beta_2\beta_{H_3} + 3\beta_1\beta_3\beta_{H_3}] \left\{ \bar{S}_{\mu_0}(s) \right\} \\ (3\beta_1\beta_{H_1} + \beta_{H_1}) \left\{ \bar{S}_{\mu_0}(s) \right\} + (2\beta_2 + \beta_{H_2}) \left\{ \bar{S}_{\mu_0}(s) \right\} + 9\beta_1^2 \left\{ \bar{S}_{\mu_0}(s) \right\} + 4\beta_2^2 \left\{ \bar{S}_{\mu_0}(s) \right\} \\ 6\beta_1\beta_2\beta_3^2 \left\{ \bar{S}_{\mu_0}(s) \right\} \end{array} \right) \quad (63)$$

$$\bar{P}_{UP}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_3(s) + \bar{P}_5(s) + \bar{P}_6(s) + \bar{P}_7(s) + \bar{P}_8(s) \quad (64)$$

## 4. ANALYTICAL ANALYSIS OF THE MODEL FOR PARTICULAR CASES

### 4.1 Availability Analysis

If the repair follows Gumbel-Hougaard family copula distribution, and by setting $\bar{S}_{\mu_0}(s) = \bar{S}_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{\frac{1}{\theta}}}(S) = \frac{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{\frac{1}{\theta}}}{s + \exp[x^\theta + \{\log\phi(x)\}^\theta]^{\frac{1}{\theta}}}$, $\bar{S}\beta_i(s) = \frac{\beta_i}{s+\beta_i}$, $i = 1, 2$ and $\bar{S}_{\phi s}(s) = \frac{\phi s}{S + \phi s}$ and taking the values of different parameter as: $\beta_1 = 0.001$, $\beta_2 = 0.002$, $\beta_3 = 0.003$ and $\beta_{H_1} = 0.004$, $\beta_{H_2} = 0.005$, $\beta_{H_3} = 0.006$, $\theta(y) = 1$ and $\mu = 1$ and $\phi(x) = 1$ in equation (62), then taking the inverse Laplace transform, one can obtain the expression for availability as:

$$\begin{aligned} P_{up}(t) &= -0.00001571463750e^{-(1.008000000t)} + 0.006979904697e^{-(2.737437444t)} \\ &- 0.0002588679930e^{-(1.022846536t)} - 7.71765584410^{-7}e^{-(1.014405834t)} \\ &- 0.00001244234496e^{-(1.007505634t)} + 0.9932967803e^{0.003895449043t} \\ &+ 0.00001111192777e^{-(1.007000000t)} \end{aligned} \quad (65)$$

For different values of time $t = 0$, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 units of time, one may obtain different values of $P_{up}(s)$ using equation (62) as shown in table 2 and Figure 3.

Table 2: Variation of availability with respect of time

| Time(t) | Availability |
|---|---|
| 0 | 1.00000 |
| 0.01 | 0.99985 |
| 0.02 | 0.99971 |
| 0.03 | 0.99957 |
| 0.04 | 0.99944 |
| 0.05 | 0.99931 |
| 0.06 | 0.99919 |
| 0.07 | 0.99907 |
| 0.08 | 0.99895 |
| 0.09 | 0.99884 |

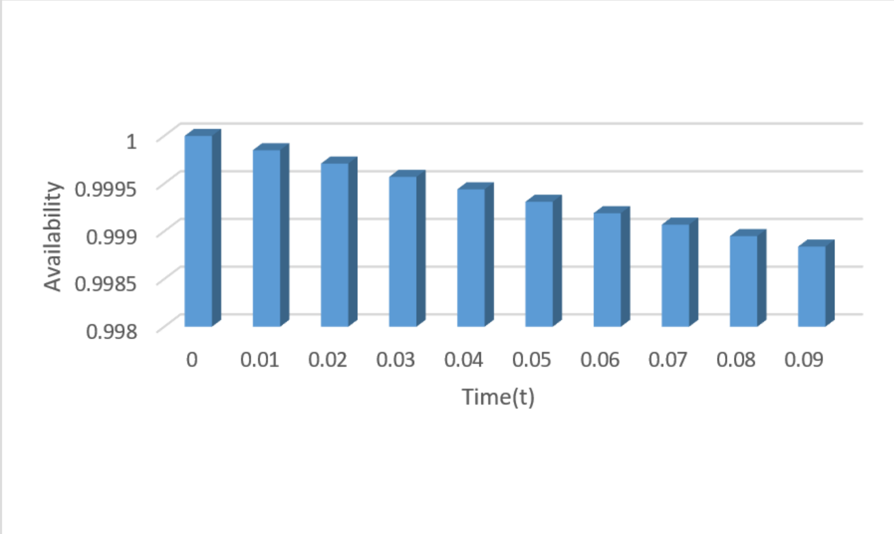



Figure 3: Availability against time $t$

### 4.2 Reliability analysis

Setting all the repairs rates $\phi(x)$, $\phi(y)$, $\mu_0(x)$ and $\mu_0(y)$ in equation (62) to zero with the same values of failure rates as: $\beta_1 = 0.001$, $\beta_2 = 0.002$, $\beta_3 = 0.003$, $\beta_{H_1} = 0.004$, $\beta_{H_2} = 0.005$, and $\beta_{H_3} = 0.006$, then taking inverse Laplace transform, one may get the expression for reliability for system as:

$$R(t) = 0.0005294117647e^{-(0.008000000000t)} + 0.2727272727e^{-(0.01400000000t)} + 0.0006686666667e^{-(0.007000000000t)} + 0.4000000000e^{-(0.01500000000t)} + 0.1669167541e^{-(0.02500000000t)} + 0.1591578947e^{(-(0.006000000000t)} \quad (66)$$

For various values of time $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ units of time, one may obtain different values of $R(t)$ with the help of equation (62) as shown in table 3 and graphical representation in figure 4.

Table 3: Reliability variation for copula repair

| Time(t) | Reliability |
|---|---|
| 0 | 1.00000 |
| 1 | 0.98517 |
| 2 | 0.97059 |
| 3 | 0.95625 |
| 4 | 0.94215 |
| 5 | 0.92829 |
| 6 | 0.91466 |
| 7 | 0.90126 |
| 8 | 0.88808 |
| 9 | 0.87512 |

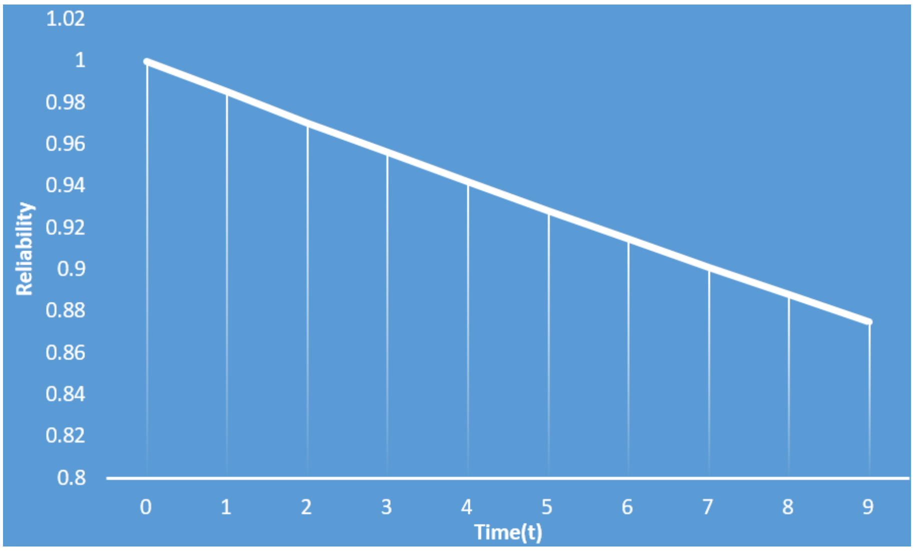



Figure 4: Reliability against time $t$

### 4.3 Mean Time to Failure (MTTF)

Setting all the repairs to zero in equation (62) and as $s \to 0$, we obtain MTTF expression as:

$$MTTF = \lim_{x \to 0} \frac{1}{3\beta_1 + 2\beta_2 + \beta_3 + \beta_{H_1} + \beta_{H_2} + \beta_{H_3}} \left( \frac{1 + \frac{3\beta_1}{3\beta_1+2\beta_2+\beta_3+\beta_{H_1}} + \frac{3\beta_2}{3\beta_1+2\beta_2+\beta_3+\beta_{H_2}} + \frac{(\beta_3+12\beta_1\beta_2)}{\beta_{H_2}}}{\frac{2\beta_2\beta_3}{3\beta_1+\beta_{H_3}} + \frac{6\beta_1\beta_2\beta_3}{\beta_3+\beta_{H_3}}} \right) \tag{67}$$

Setting $\beta_1 = 0.001$, $\beta_2 = 0.002$, $\beta_3 = 0.003$, $\beta_{H_1} = 0.004$, $\beta_{H_2} = 0.005$ and, $\beta_{H_3} = 0.006$ and varying $\beta_1, \beta_2, \beta_3, \beta_{H_1}, \beta_{H_2} and, \beta_{H_3}$ one by one respectively as $0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09$ in the above equation, one may obtain the variation of MTTF with respect to failure rates as shown in table 4 and figure 5.

Table 4: Reliability variation for copula repair

| Failure<br>$\beta_1/\beta_2/$<br>$\beta_3/\beta_{H_1}/$<br>$\beta_{H_2}/\beta_{H_3}$ | MTTF$\beta_1$<br>$\beta_2 = 0.002$,<br>$\beta_3 = 0.003$,<br>$\beta_{H_1} = 0.004$,<br>$\beta_{H_2} = 0.005$,<br>$\beta_{H_3} = 0.006$ | MTTF$\beta_2$<br>$\beta_1 = 0.001$,<br>$\beta_3 = 0.003$,<br>$\beta_{H_1} = 0.004$,<br>$\beta_{H_2} = 0.005$,<br>$\beta_{H_3} = 0.006$ | MTTF$\beta_3$<br>$\beta_1 = 0.001$,<br>$\beta_2 = 0.002$,<br>$\beta_{H_1} = 0.004$,<br>$\beta_{H_2} = 0.005$,<br>$\beta_{H_3} = 0.006$ | MTTF$\beta_{H_1}$<br>$\beta_1 = 0.001$,<br>$\beta_2 = 0.002$,<br>$\beta_3 = 0.003$,<br>$\beta_{H_2} = 0.005$,<br>$\beta_{H_3} = 0.006$ | MTTF$\beta_{H_2}$<br>$\beta_1 = 0.001$,<br>$\beta_2 = 0.002$,<br>$\beta_3 = 0.003$,<br>$\beta_{H_1} = 0.004$,<br>$\beta_{H_3} = 0.006$ | MTTF$\beta_{H_3}$<br>$\beta_1 = 0.001$,<br>$\beta_2 = 0.002$,<br>$\beta_3 = 0.003$,<br>$\beta_{H_1} = 0.004$,<br>$\beta_{H_2} = 0.005$ |
|---|---|---|---|---|---|---|
| 0.01 | 8.60658 | 8.89540 | 9.88372 | 10.98378 | 11.23277 | 23.66482 |
| 0.02 | 8.21386 | 8.60658 | 8.24838 | 9.55490 | 10.65786 | 20.95430 |
| 0.03 | 7.80824 | 8.29807 | 8.30658 | 9.54909 | 10.66714 | 18.26715 |
| 0.04 | 7.42503 | 7.99149 | 7.95483 | 9.96962 | 10.79607 | 14.60658 |
| 0.05 | 7.07455 | 7.69693 | 7.29117 | 8.92048 | 10.60658 | 11.97646 |
| 0.06 | 6.75795 | 7.41873 | 7.11456 | 8.60658 | 9.71955 | 10.38171 |
| 0.07 | 6.47308 | 7.15836 | 7.00463 | 8.53403 | 9.41161 | 9.82874 |
| 0.08 | 6.21675 | 6.91578 | 6.52136 | 8.51065 | 9.40141 | 9.92643 |
| 0.09 | 5.98564 | 6.69026 | 6.50501 | 7.04655 | 8.05054 | 8.88836 |

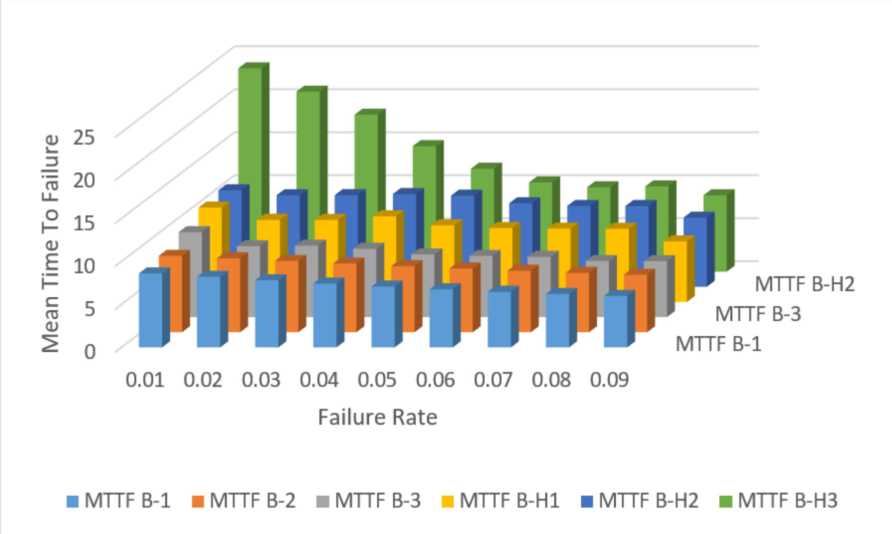



Figure 5: Mean time to failure against failure rate

### 4.4 Cost Benefit Analysis

Let the service facility be always available, then the expected profit during the interval $[0, t)$.

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$$

Where $K_1$ and $K_2$ are the revenue generated and service cost per unit time in the interval $[0, t)$. For the same set of parameters as in (62), one can obtain (67).

$$\begin{aligned} E_p(t) = K_1[&0.00001558991815e^{-(1.008000000t)} - 0.002549795142e^{-(2.737437444t)} \\ &+ 0.0002530858578e^{-(1.022846536t)} + 7.60805546010^{-7}e^{-(1.014405834t)} \\ &+ 0.00001234965298e^{-(1.007505634t)} + 254.9890319e^{0.0038954490 43t} \\ &- 0.00001103468498e^{-(1.007000000t)}] - K_2.t \end{aligned} \quad (68)$$

Setting $K_1 = 1$ and $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2,$ and $0.1$ respectively and varying $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ units of time, one may obtain the results for expected profit as shown in table 5 and figure 6.

Table 5: Cost computation for different value of time

| Time($t$) | $E_p(t)$ $K_2 = 0.1$ | $E_p(t)$ $K_2 = 0.2$ | $E_p(t)$ $K_2 = 0.3$ | $E_p(t)$ $K_2 = 0.4$ | $E_p(t)$ $K_2 = 0.5$ | $E_p(t)$ $K_2 = 0.6$ |
|---|---|---|---|---|---|---|
| 0 | -0.00005 | -0.00005 | -0.00005 | -0.00005 | -0.00005 | -0.00005 |
| 1 | 0.89739 | 0.79739 | 0.69739 | 0.59739 | 0.49739 | 0.39739 |
| 2 | 1.79660 | 1.59660 | 1.39660 | 1.19660 | 0.99660 | 0.79660 |
| 3 | 2.69961 | 2.39961 | 2.09961 | 1.79961 | 1.49961 | 1.19961 |
| 4 | 3.60653 | 3.20653 | 2.80653 | 2.40653 | 2.00653 | 1.60653 |
| 5 | 4.51739 | 4.01739 | 3.51739 | 3.01739 | 2.51739 | 2.01739 |
| 6 | 5.43220 | 4.83220 | 4.23220 | 3.63220 | 3.03220 | 2.4322 |
| 7 | 6.35097 | 5.65097 | 4.95097 | 4.25097 | 3.55097 | 2.85097 |
| 8 | 7.27372 | 6.47372 | 5.67372 | 4.87372 | 4.07372 | 3.27372 |
| 9 | 8.20045 | 7.30045 | 6.40045 | 5.50045 | 4.60045 | 3.70045 |

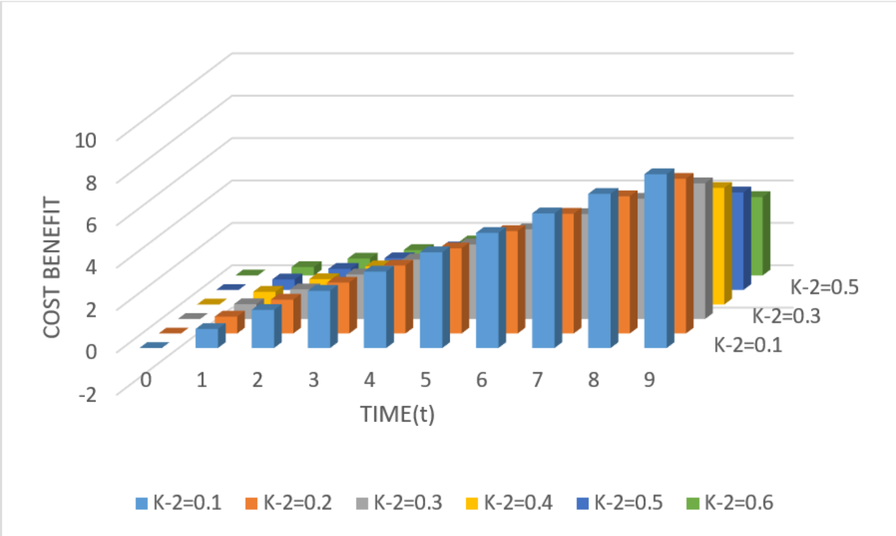



Figure 6: Cost Benefit against time $t$

## 5. CONCLUSIONS THROUGH RESULT DISCUSSION

Table 2 and Figure 3 show a simple description of how the performance of the repairable series system changes with respect to time t as failure rates are varied. When failure rates are at lower values $\beta_1 = 0.001$, $\beta_2 = 0.002$, $\beta_3 = 0.003$, $\beta_{H_1} = 0.004$, $\beta_{H_2} = 0.005$ and $\beta_{H_3} = 0.006$, system availability decreases gradually with the passage of time and eventually becomes steady to the value zero after a long interval of time. As a result, the future behavior of the repairable device can be accurately predicted at any point for any given set of parametric values. This is evident from the platform's graphical design. It has been observed that when the repair is performed, the system performance is far superior to when the repair is not performed. Tables 2 and 3 show that the corresponding availability values are greater than the corresponding reliability values. To improve system performance, this simulation suggests that regular repairs be performed.

Furthermore, Table 4 and figure 5 yields the MTTF of the system with respect to variation in $\beta_1, \beta_2, \beta_3, \beta_{H_1}, \beta_{H_2}$ and $\beta_{H_3}$ respectively when other parameters are kept constant. The variation in MTTF corresponding to $\beta_1$ and $\beta_2$ are almost close but the variation in MTTF corresponding to $\beta_3, \beta_{H_1}, \beta_{H_2}$ and $\beta_{H_3}$ are higher than $\beta_1$ and $\beta_2$.

Table 4 and Figure 5 show the trends of the cost function against time t when the revenue cost per unit time $K_1$ is fixed at 1, and the service cost $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2, 0.1$. This table and figure show that the cost rises over time as the service cost $K_2$ falls. The computed cost in the table shows that $K_2 = 0.1$ is the maximum and $K_2 = 0.6$ is the minimum. Finally, it has been discovered that as service costs decrease, the cost increases with time variation. In general, the cost function is higher for low service cost ($K_2 = 0.1$) than for high service cost ($K_2 = 0.6$). This study will serve as a guide for engineers, computer system designers, reliability engineers, maintenance managers etc as they design more critical systems to improve efficiency and lower operational costs. As a result, existing work should include repairs and replacement for partial and total failure under the free renewal warranty. More research could be done to address the existing gap, computer networking using switch, hubs, routers, data base server should be solved using genetic algorithm, particle swamp optimization and so on.

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