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Comparative study of Economic Order Quantity (EOQ) model for time – sensitive holding cost with constant and exponential time - dependent Demand with and without deterioration

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Abstract: In the study of EOQ models the consideration of constant demand over infinite planning horizon is valid in the maturity stage of the commodity life cycle and for a limited period. In other stages of an item life cycle, demand for a commodity may rise of fall according to the situation. Several research papers were published with stable and fluctuating demand. In this study three models are discussed viz. (i) invariable demand (ii) exponential time – sensitive demand and (iii) exponential time – dependent under deterioration. Holding cost is considered linearly time sensitive. Mathematical models are discussed for these three models for finding optimal solution. Bases on the optimal solution numerical examples and sensitivity analysis are discussed. The comparison of these models is also conversed. Mathematica software 7.0 software is used to find the numerical outcomes.
Keyword — Comparative; time – dependent; deterioration; demand; time sensitive holding cost

1. INTRODUCTION

One of the most important jobs for supervising of inventories that every administrator must do competently and effectively in any association. At present, all associations are concerned in a worldwide aggressive competitive market and then these associations are taking seriously the activities connected to handle their inventories. In this study we consider three cases, viz. (i) Constant demand (ii) Stock dependent demand and stock dependent demand with deterioration. A number of research papers were published with constant demand rate. Tripathi and Uniyal (2020) developed a deterministic inventory model for deteriorating item with constant demand rate over a finite planning horizon. Ghare and Scharder (1963) established a model for decaying inventory system. Aggarwal (1978) presented and order level inventory model with constant rate of deterioration. Hou and Lin (2009) designed a cash flow oriented EOQ model with deteriorating item under trade credit. Chen and Teng (2014) presented the retailer's optimal replenishment cycle time for deteriorating item under trade credit financing. Goyal (1985) developed the economic order quantity model under permissible delay in payment. Chen, Barron, and Teng (2014) established and inventory model to obtain the optimal solution to the inventory problem with conditionally trade credits. Hariga, Gumus, and Goyal (2013) presented a cost efficient heuristic to solve the problem. Zhang, C., and H. (2014) presented the buyer's inventory strategy under advance payment, containing all payment in advance and fractional delayed payment. Wee, Huang, Wang, and Chen (2014) established an EPQ model with two back ordering costs and partial backlogging. Several relation articles in this direction is presented by Aggrawal and Jaggi (1995), Jamal, Sarkar, and Wang (1997), Chung and Huang (2009), Teng (2002) etc.

In real life, demand rate of any product is not always constant; it is (i) price dependent(ii) time dependent (iii) price sensitive etc. In most cases, demands of items are uncertain. Brill and Chauch (1995) presented a model that incorporates variations in the demand rate at random time points into the inventory planning decision. Halkin (2017) studied a case study of an EOQ model with variable parameters. Ouyang and Chang (2001) presented a stochastic inventory model with a fuzzy back order rate. Tripathi, Pareek, and Kaur (2017) designed an EOQ model with exponential demand under variable deterioration. Tripathi and Mishra (2016) developed a model with linear time linked demand with variable holding cost. Sarkar (2012) established an EOQ model for finite replenishment rate where demand and deterioration both are time- sensitive. Hossen, Hakin, Ahmed, and Uddin (2016) proposed a fuzzy EOQ model for deteriorating items with price and time linked demand. Goh (1994) presented the unbroken, infinite horizon, single

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EOQ scheme and inventory sensitive demand rate. Tripathi, Singh, and Rao (2019) established an EOQ model by means of quantity reduction, pricing and partial back ordering in which the item in stock dependent over time. Tripathi (2018) considered an EPQ model in which demand rate is stock-dependent under inflation. Sarkar and Sarkar (2013) developed an EOQ model allowing for stock - linked demand and time changeable backordering rate and deterioration rate. Several related articles with variable demand are Sarkar, Saren, and Wee (2013), Mandal and Phaujdere (1989), C. T. Yang (2014), Chang, Teng, and Goyal (2010), Soni and Shah (2008), Hwang and Hahn (2000), H. L. Yang, Teng, and Chern (2010), Goyal and Chang (2009), Change and Dye (1999), Tripathi, Singh, and Aneja (2018) .

In real life holding cost is in fluctuating state.The availability of literature containing variable holding costs are limited. The present study fill the gap of variable demand. Alfares (2014) considered a production – inventory system with stock – dependent demand and variable holding cost. Alfares (2012) established an EPQ production inventory model. Several related articles in this area are Alfares and Ghaithan (2018), Muhlemann and Valtis – Spanopoulos (1980), Pervin, Roy, and Weber (2018), Alfares (2007) and others.

The remainder of the study is framed as follows: In section 2, notation and assumption are specified. The mathematical formulation, optimal solution with sensitivity analysis is discussed in section 3. Based on the optimal solution some constructive results are obtained. The managerial insights is offered in section 5 followed by conclusion and future research direction.

2. ASSUMPTION AND NOTATIONS

The following assumption being made throughout the manuscript:

- The demand rate is constant for model I , exponential time sensitive for models II and III respectively
- Constant deterioration in considered for model III
- Shortages are not permitted
- The holding cost/ time is time dependent
- The models are assumed for single item only

In addition, the following notations are used in the whole study:

$q(t)$	: inventory level any instant 't'
Q	: order quantity
$h(t) = h + \gamma.t$	: holding cost per unit time
C_0	: ordering cost per order
T	: cycle time
θ	: deterioration rate
OC and HC	: ordering and holding cost respectively
TC	: total cost per cycle time
T^*	: optimal T
Q^*, OC^*, HC^* and TC^*	: optimal $Q, OC, HC,$ and TC respectively

3. MATHEMATICAL FORMULATION

In this study, three models are considered. In the first model demand rate is considered stock dependent. In the second model deterioration and stock-sensitive demand is assumed. In the third model deterioration and exponential demand is considered.

3.1 Model I: Constant demand

In this model, it is assumed that demand rate for the item is stock-dependent. The inventory of commodities, decrease due to purchases and stock-linked demands $[0, T]$. Therefore, the differential equation of the state is:

$$\frac{dq(t)}{dt} = -\alpha \tag{1}$$

with the boundary conditions, $q(0) = Q$ and $q(T) = 0$ (2)

The solution (1) is:

$$q(t) = \alpha(T - t) \quad \text{(using (2))} \tag{3}$$

and

$$Q = q(0) = \alpha T \tag{4}$$

Total cost contains OC and HC:

$$\text{(i) } OC = \frac{C_0}{T} \tag{5}$$

$$\text{(ii)} HC = \frac{1}{T}\int_0^T (h + \gamma t)q(t)dt = \frac{\alpha T}{6}(3h + \gamma T) \tag{6}$$

$$TC = OC + HC = \frac{C_0}{T} + \frac{\alpha T}{6}(3h + \gamma T) \tag{7}$$

Optimality Condition
The twice differentiation of (7) w.r.t. 'T' are:

$$\frac{d(TC)}{dT} = 6C_0 - 3\alpha hT^2 - 2\alpha\gamma T^3 \text{ and } \frac{d^2(TC)}{dT^2} = \frac{2C_0}{T^3} + \frac{\alpha\gamma}{3} > 0 \text{ (i.e. } TC^* \text{ is minimum)}$$

The condition of minimization is also shown by the following graph:

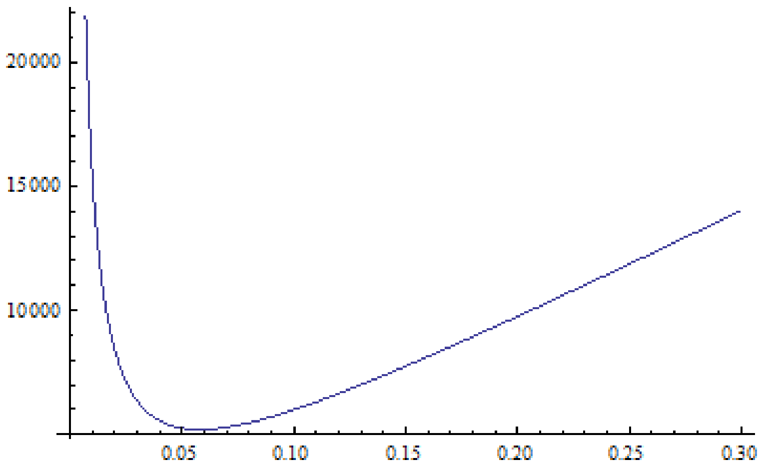

Figure 1: Graph between T (x- axis , 0.0 – 0.30) and TC (y- axis)

$$T^* \text{ is obtained by solving } \frac{d(TC)}{dT} = 0 \Rightarrow \alpha T^2(2\gamma T + 3h) - 6C_0 = 0. \tag{8}$$

Example 1 : Let us consider the parameters, $\alpha = 4500, C_0 = 150, h = 20, \gamma = 0.5$ in appropriate units. Substituting these values in (8). This gives $T^* = 0.0577073$ yrs, corresponding $Q^* = 259.68$, $OC^* = \$2599.32$, $HC^* = \$2598.08$ and $TC^* = \$5197.4$.

Sensitivity Analysis
It is reasonable to study the sensitivity study with respect to constraints over a known optimum solution. It is imperative to get the belongings on dissimilar scheme parameters, such as holding cost, ordering cost, etc. In the following Table 1, keeping all parameters same, as discussed in the example.1, changing one parameter at a time.

Table 1: The effect of parameter on T^*, Q^*, OC^*, HC^* and TC^*

Parameters	Optimal value				
	T^*	Q^*	OC^*	HC^*	TC^*

C_0	130	0.0537243	241.76	2419.76	2418.68	4838.44
	140	0.0557514	250.88	2511.15	2509.98	5021.13
	160	0.0595989	268.20	2684.61	2683.29	5367.90
	170	0.0614322	276.44	2767.28	2765.86	5533.14
	180	0.0632123	284.46	2847.55	2846.05	5693.60
h	22	0.0550253	247.61	2726.02	2724.89	5450.91
	24	0.0526854	237.08	2847.09	2846.05	5693.14
	26	0.0506205	227.79	2963.23	2962.26	5925.49
	28	0.0487808	219.51	3074.98	3074.08	6149.06
	30	0.0471281	212.08	3182.81	3181.98	6364.79
α	4600	0.0570769	262.55	2628.03	2626.79	5254.82
	4700	0.0564667	265.39	2656.43	2655.19	5311.62
	4800	0.0558757	268.20	2684.53	2683.28	5367.81
	4900	0.0553029	270.98	2712.34	2711.09	5423.43
	5000	0.0547473	273.74	2739.86	2738.62	5478.48
γ	0.55	0.0577045	229.67	2599.45	2598.08	5197.53
	0.60	0.0577017	259.66	2599.58	2598.07	5197.65
	0.65	0.0576990	259.65	2599.69	2598.08	5197.78
	0.70	0.0576962	259.63	2599.82	2598.51	5198.33
	0.75	0.0576934	259.62	2599.95	2598.08	5198.03

3.2 Model II: Exponential time- associated Demand

In this model stock-dependent demand and deterioration both are measured. The greater part of items in the universe deteriorates over time. Daily utilizable product like, bread, milk, green vegetable etc. deteriorate over time. The differential equation of state is:

$$\frac{dq(t)}{dt} = -\alpha e^{\beta t},\ 0 < t < T \tag{9}$$

with the boundary conditions, $q(0) = Q$ and $q(T) = 0$ (10)

The solution of (9) is:

$$q(t) = \frac{\alpha}{\beta}(e^{\beta T} - e^{\beta t}) \quad \text{(using } q(T) = 0\text{)} \tag{11}$$

$$Q = q(0) = \frac{\alpha}{\beta}(e^{\beta T} - 1) \approx \alpha T\left(1 + \frac{\beta T}{2} + \frac{\beta^2 T^2}{6}\right),\ \text{(Approx.)} \tag{12}$$

Total cost contains OC and HC:

$$HC = \frac{1}{T}\int_0^T (h + \gamma t)q(t)dt = \frac{\alpha T}{2}\left\{h(1 + \beta T) + \frac{\beta\gamma T^2}{2}\right\}. \tag{13}$$

Therefore,

$$TC = \frac{C_0}{T} + \frac{\alpha T}{2}\left\{h(1 + \beta T) + \frac{\beta\gamma T^2}{2}\right\}. \tag{14}$$

Optimality Condition
Differentiating (14) w.r.t. 'T', twice

$$\frac{d(TC)}{dT} = \frac{C_0}{T^2} + \frac{\alpha}{2}\left\{h(1 + 2\beta T) + \frac{3\beta\gamma T^2}{2}\right\} \tag{15}$$

and

$$\frac{d^2(TC)}{dT^2} = \frac{2C_0}{T^3} + \frac{\alpha}{2}\{2h\beta + 3\beta\gamma T\} > 0.\ \text{(i.e. } TC^* \text{ is minimum)} \tag{16}$$

It can also be shown by the following graph:

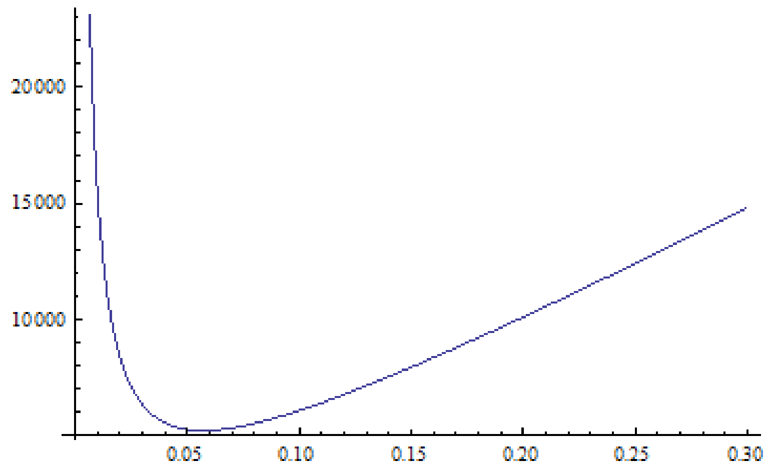

Figure 2: Graph between T (x- axis , 0.0 – 0.30) and TC (y- axis)

T^* is obtained on solving

$$\frac{d(TC)}{dT} = 0 \Rightarrow -\frac{C_0}{T^2} + \frac{\alpha}{4}\{2h(1 + 2\beta T) + 3\beta\gamma T^2\} = 0$$

or

$$\alpha T^2\{2h(1 + 2\beta T) + 3\beta\gamma T^2\} - 4C_0 = 0. \quad (17)$$

Example 2 : Let us consider the parameters, $\alpha = 4500$, $C_0 = 150$, $h = 20$, $\beta = 0.2$ in proper units. Putting these values in (17). This gives $T^* = 0.0570862$ yrs, corresponding $Q^* = 258.36$, $OC^* = \$2627.61$, $HC^* = \$2598.22$ and $TC^* = \$5225.83$.

Sensitivity Analysis

It is reasonable to study the sensitivity with respect to constraints over a known optimum solution. It is imperative to get the belongings on dissimilar scheme parameters, such as holding cost, ordering cost, etc. In the following Table 2, keeping all parameters same, discussed in example 2, varying one parameter at a time.

Table 2: The effect of parameter on T^*, Q^*, OC^*, HC^* and TC^*

Parameters		Optimal value				
		T^*	Q^*	OC^*	HC^*	TC^*
C_0	130	0.0531851	240.610	2444.29	2418.81	4863.10
	140	0.0551713	249.646	2537.55	2510.12	5047.67
	160	0.0589371	266.786	2714.76	2683.45	5398.21
	170	0.0607296	274.950	2799.29	2766.05	5565.34
	180	0.0624690	282.874	2881.43	2846.25	5727.68
h	22	0.0544577	246.399	2754.43	2725.04	5479.47
	24	0.0521628	235.961	2875.61	2846.20	5721.81
	26	0.0501364	226.749	2991.84	2962.40	5954.24
	28	0.0483298	218.539	3103.68	3074.22	6177.90
	30	0.0467059	211.161	3211.59	3182.10	6393.69
α	4600	0.0564692	261.231	2656.32	2626.94	5283.26
	4700	0.0558717	264.070	2684.72	2655.34	5340.06
	4800	0.0552930	266.879	2712.82	2683.44	5396.26
	4900	0.0547319	269.660	2740.63	2711.24	5451.87
	5000	0.0541876	272.410	2768.16	2738.76	5506.92
β	0.22	0.0570233	258.221	2630.50	2598.27	5228.77
	0.24	0.0569608	258.084	2633.39	2598.30	5231.69
	0.26	0.0568985	257.956	2636.27	2598.34	5234.61
	0.28	0.0568366	257.664	2639.14	2598.38	5237.52
	0.30	0.0567751	257.676	2642.00	2598.43	5240.43

γ	0.55	0.0570862	258.360	2627.61	2598.23	5225.84
	0.60	0.0570861	258.360	2627.61	2598.23	5225.84
	0.65	0.0570860	258.359	2627.61	2598.23	5225.84
	0.70	0.0570860	258.359	2627.61	2598.23	5225.84
	0.75	0.0570859	258.358	2627.62	2598.23	5225.85

3.3 Model III: Exponential Demand under deterioration

In most of EOQ model demand is considered invariable. While in real situation demand is always in dynamic state. The model is developed for deteriorating inventory in which demand is an exponential function of time. The differential equation of state is:

$$\frac{dq(t)}{dt} + \theta q(t) = -\alpha e^{\beta t},\ 0 < t < T \tag{18}$$

with the boundary conditions, $q(0) = Q$ and $q(T) = 0$ (19)

The solution Eq.(18) is:

$$q(t) = \frac{\alpha}{\beta+\theta}\left\{e^{(\beta+\theta)T}e^{-\theta t} - e^{\beta t}\right\} \quad \text{(using } q(T)=0\text{)} \tag{20}$$

$$Q = q(0) = \frac{\alpha}{\beta+\theta}\left\{e^{(\beta+\theta)T} - 1\right\} = \alpha T\left\{1 + \frac{(\theta+\beta)}{2}\right\} \text{ (approx.)} \tag{21}$$

Total cost contains ordering cost and holding cost:
Holding Cost

$$= \frac{1}{T}\int_0^T (h+\gamma t)q(t)dt = \frac{\alpha}{(\beta+\theta)T}\left[(\beta+\theta)h\left\{\frac{1}{\beta\theta} + \frac{T}{\theta} + \frac{(\beta+\theta)T^2}{2\theta}\right\} - \frac{(h+\gamma T)(\beta+\theta)}{\beta\theta}\right.$$
$$\left.\left(1+\beta T+\frac{\beta^2T^2}{2}\right) + \gamma T\left\{\left(1+\beta T+\frac{\beta^2T^2}{2}\right)\left(\frac{1}{\theta}+\frac{T}{2}+\frac{T^2\theta}{6}\right) + \left(\frac{1}{\beta}+\frac{T}{2}+\frac{\beta T^2}{6}\right)\right\}\right] \tag{22}$$

Therefore

$$TC = \frac{C_0}{T} + \frac{\alpha}{(\beta+\theta)T}\left[(\beta+\theta)h\left\{\frac{1}{\beta\theta} + \frac{T}{\theta} + \frac{(\beta+\theta)T^2}{2\theta}\right\} - \frac{(h+\gamma T)(\beta+\theta)}{\beta\theta}\right.$$
$$\left.\left(1+\beta T+\frac{\beta^2T^2}{2}\right) + \gamma T\left\{\left(1+\beta T+\frac{\beta^2T^2}{2}\right)\left(\frac{1}{\theta}+\frac{T}{2}+\frac{T^2\theta}{6}\right) + \left(\frac{1}{\beta}+\frac{T}{2}+\frac{\beta T^2}{6}\right)\right\}\right]. \tag{23}$$

Optimality Condition
The differential of (23), w.r.t. 'T' , twice are:

$$\frac{d(TC)}{dT} = -\frac{C_0}{T^2} + \alpha\left[h\left\{\frac{(\beta+\theta)}{2\theta} - \frac{1}{\beta\theta T^2}\right\} - \frac{1}{\beta\theta}\left\{(h+\gamma T)\left(\frac{\beta^2}{2} - \frac{1}{T^2}\right) + \right.\right.$$
$$\left.\gamma\left(\frac{1}{T}+\beta+\frac{\beta^2T}{2}\right)\right\} + \frac{\gamma}{(\beta+\theta)}\left\{\frac{(3+2T\theta)}{6}\left(1+\beta T+\frac{\beta^2T^2}{2}\right) + \right.$$
$$\left.\left.\beta(1+\beta T)\left(\frac{1}{\theta}+\frac{T}{2}+\frac{T^2\theta}{6}\right) + \frac{1}{6}(3+2\beta T)\right\}\right] \tag{24}$$

and

$$\frac{d^2(TC)}{dT^2} = \frac{2C_0}{T^3} + \alpha\left[\frac{2h}{\beta\theta T^3} - \frac{1}{\beta\theta}\left\{\frac{2}{T^3}(h+\gamma T) + 2\gamma\left(\frac{\beta^2}{2} - \frac{1}{T^2}\right)\right\} + \right.$$
$$\left.\frac{\gamma}{(\beta+\theta)}\left\{\frac{1}{3}\beta(1+\beta T)(3+2T\theta) + \frac{\theta}{3}\left(1+\beta T+\frac{\beta^2T^2}{2}\right) + \beta^2\left(\frac{1}{\theta}+\frac{T}{2}+\frac{T^2\theta}{6}\right) + \frac{\beta}{3}\right\}\right] > 0. \tag{25}$$

(i.e. TC^* is minimum)
It can also be shown graphically as follows:

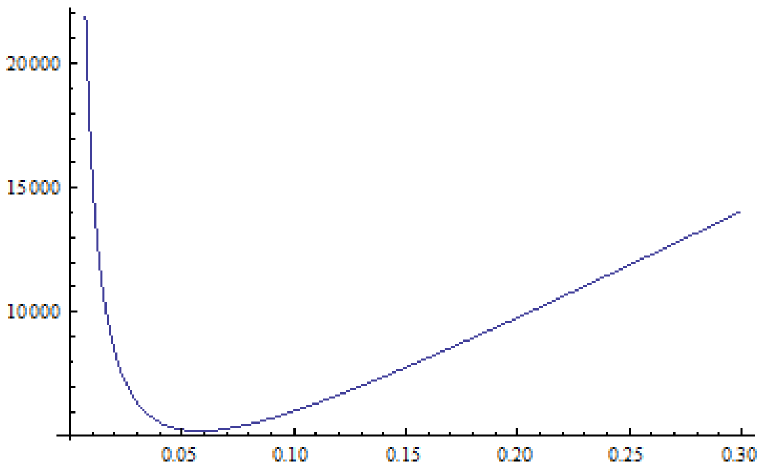

Figure 3: Graph between T (x- axis , 0.0 – 0.30) and TC (y- axis)

Optimal cycle time T is calculated on putting $\dfrac{d(TC)}{dT} = 0$

$$\Rightarrow -\frac{C_0}{T^2} + \alpha\left[h\left\{\frac{(\beta+\theta)}{2\theta} - \frac{1}{\beta\theta T^2}\right\} - \frac{1}{\beta\theta}\left\{(h+\gamma T)\left(\frac{\beta^2}{2} - \frac{1}{T^2}\right) + \gamma\left(\frac{1}{T} + \beta + \frac{\beta^2 T}{2}\right)\right\} + \frac{\gamma}{(\beta+\theta)}\left\{\frac{(3+2T\theta)}{6}\left(1+\beta T+\frac{\beta^2T^2}{2}\right) + \beta(1+\beta T)\left(\frac{1}{\theta}+\frac{T}{2}+\frac{T^2\theta}{6}\right) + \frac{1}{6}(3+2\beta T)\right\}\right] = 0. \tag{26}$$

Example 3 : Let us consider the parameters, $\alpha = 4500$, $C_0 = 150$, $h = 20$, $\beta = 0.2$, $\theta = 0.05$, $\gamma = 0.5$ in appropriate units. Substituting these values in (26). This gives $T^* = 0.0577066$ years, corresponding $Q^* = 260.055$, $OC^* = \$2599.36$, $HC^* = \$2598.06$ and $TC^* = \$5197.42$.

Sensitivity Analysis

It is reasonable to study the sensitivity with respect to constraints over a known optimum solution. It is imperative to get the belongings on dissimilar scheme parameters, such as holding cost, ordering cost, etc. In the following Table 3, keeping all parameters same, discussed in example 3, varying one parameter at a time.

Table 3: The effect of parameter on T^*, Q^*, OC^*, HC^* and TC^*

Parameters		Optimal value				
		T^*	Q^*	OC^*	HC^*	TC^*
C_0	130	0.0537238	242.082	2419.78	2418.67	4838.45
	140	0.0557508	251.299	2511.17	2509.97	5021.14
	160	0.0595981	268.591	2684.65	2683.27	5367.92
	170	0.0614314	276.866	2767.31	2765.86	5533.17
	180	0.0632114	284.901	2847.59	2846.04	5693.63
h	22	0.0550247	247.952	2726.05	2724.87	5450.92
	24	0.0526849	237.395	2847.12	2846.04	5693.16
	26	0.0506202	228.079	2963.24	2962.26	5925.50
	28	0.0487806	219.781	3074.99	3074.09	6149.08
	30	0.0471279	212.326	3182.83	3181.98	6364.81
α	4600	0.0570762	262.926	2628.07	2626.77	5254.84
	4700	0.0564661	265.766	2656.44	2655.20	5311.64
	4800	0.0558751	268.575	2684.56	2683.27	5367.83
	4900	0.0553023	271.356	2712.36	2711.09	5423.45
	5000	0.0547967	274.359	2737.39	2741.11	5478.50
β	0.3	0.0577063	260.053	2599.37	2598.06	5197.43
	0.4	0.0577059	260.052	2599.39	2598.05	5197.44
	0.5	0.0577055	260.050	2599.41	2598.04	5197.45

	0.6	0.0577052	260.048	2599.42	2598.05	5197.47
	0.7	0.0577048	260.047	2599.44	2598.04	5197.48
γ	0.55	0.0577038	260.042	2599.48	2598.07	5197.55
	0.60	0.0577009	260.029	2599.61	2598.07	5197.68
	0.65	0.0576981	260.016	2599.74	2598.06	5197.80
	0.70	0.0576953	260.003	2599.87	2598.06	5197.93
	0.75	0.0576924	259.991	2560.00	2598.06	5198.06
θ	0.02	0.0577066	259.830	2599.36	2598.06	5197.42
	0.07	0.0577066	260.205	2599.36	2598.06	5197.42
	0.12	0.0577067	260.581	2599.35	2598.07	5197.42
	0.17	0.0577067	260.958	2599.35	2598.06	5197.42
	0.22	0.0577067	261.336	2599.35	2598.06	5197.42

The following inferences can be made from table 1- 3:

- On increasing C_0 ; T^*, OC^*, HC^* and TC^* are increasing
- On increasing h; T^* and Q^* are decreasing , while OC^*, HC^* and TC^* are increasing
- On increasing α, T^* is decreasing while Q^*, OC^*, HC^* and TC^* are rising
- An increase of θ, β and γ; insignificant change in the optimal values

4. COMPARISON OF OPTIMAL CYCLE TIME, LOT – SIZE, ORDERING COST, HOLDING COST AND TOTAL COSTS

In the present competitive erne the comparison of inventories is a natural phenomenon. The comparison of all the three models (Model I, II and III) is given in the following Table.

Table 4: A comparative study Model I ,II and III

	T^*	Q^*	OC^*	HC^*	TC^*
Model I	0.0577073	259.680	2599.32	2598.08	5197.40
Model II	0.0570862	258.360	2627.61	2598.22	5225.83
Model III	0.0577066	260.055	2599.36	2598.06	5197.42

From the Table given above it could be deciphered that T^*, Q^* and TC^* obtained from model II is superior to model I and III. It is also seen that optimal setup cost and holding cost fluctuates. It means that optimal costs like: HC^* and TC^* of model II is better to compare the HC^* and TC^* of model I and III.

5. RESULT BASED ON OPTIMAL SOLUTION

ResultI : T^* having only one positive root.
Proof : From Eqs.(8) ,(17) and (26), we get

$$2\alpha\gamma T^3 + 3\alpha h T^2 - 6C_0 = 0, \tag{27}$$

$$3\alpha\beta\gamma T^4 + 4\alpha\beta h T^3 + 2\alpha h T^2 - 4C_0 = 0, \tag{28}$$

and

$$\alpha\left\{4\beta^2\theta\gamma T^5 + 3\beta\gamma(3\beta + 2\theta)T^4 + 4\gamma(\beta+\theta)T^3 + 6h(\beta+\theta)T^2\right\} - 12(\beta+\theta)C_0 = 0. \tag{29}$$

Since only one sign change in (27) ,(28) and (29) ,by Descartes' rule, there exist only positive root.
ResultII : T^* is increasing function of C_0.
Proof : On differentiating (27), (28) and (29) with respect to C_0, we get

$$\frac{dT^*}{dC_0} = \frac{3}{\alpha T(\gamma T + 3h)} > 0, \tag{30}$$

$$\frac{dT^*}{dC_0} = \frac{1}{\alpha T\left\{h(1+3\beta T) + 3\beta\gamma T^2\right\}} > 0, \tag{31}$$

and

$$\frac{dT^*}{dC_0} = \frac{\theta + \beta}{\alpha T\left\{5\gamma\theta\beta^2 T^3 + 3\beta\gamma(3\beta + 2\theta)T^2 + 3\gamma(\theta + \beta)T + 3h(\theta + \beta)\right\}} > 0. \tag{32}$$

Since $\dfrac{dT^*}{dC_0} > 0.$

Therefore, T^* is increasing function of C_0.
ResultIII : The optimal T is a decreasing function of h.
Proof : The differential of (27) ,(28) and (29) with respect to 'h', gives

$$\frac{dT^*}{dh} = -\frac{3T}{2(h + \gamma T)} < 0, \tag{33}$$

$$\frac{dT^*}{dh} = -\frac{T(1 + 2\beta T)}{2\left\{h(1 + 3\beta T) + 3\beta\gamma T^2\right\}} < 0, \tag{34}$$

and

$$\frac{dT^*}{dh} = -\frac{3(\theta + \beta)}{\left\{5\gamma\theta\beta^2 T^3 + 3\beta\gamma(3\beta + 2\theta)T^2 + 3\gamma(\theta + \beta)T + 3h(\theta + \beta)\right\}} < 0. \tag{35}$$

From (33) - (35), it is seen that

$$\frac{dT^*}{dh} < 0$$

Therefore, T^* is decreasing function of h.

6. CONCLUSION AND FUTURE RESEARCH

In our proposed model some realistic features are included. The effect of deterioration in one of the inventory model is examined. We have developed these inventory control models for three different situations i.e. the demand rate for model (i) I is constant (ii) II is exponential time – dependent and (iii) III exponential time dependent under deterioration. Mathematics formulations are derived for finding optimal solutions. The projected model illustrated through three numerical example and sensitivity analyses is executed. Based on optimal solution some constructive results are also obtained. Comparison of all these models is discussed. Our studies show that the model III gives the minimum total cost.

A number of expected extensions of the proposed models that can be presented as like: (i) variable decay and Weibull deterioration (ii) to assume a quadratic time – associative carrying cost (iii) to comprise fall in the purchasing cost/ unit and (iv) to study the case of inflation and shipment charges. One can also be extend the model into more practical concern, such as allowable shortages or finite replenishment rate.

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