International Journal of
Operations Research

International Journal of Operations Research Vol. 19, No. 2, 27-40 (2022)

Reliability and Performance Analysis of Two Unit Active Parallel System Attended by Two Repairable Machines

Muhammad Auwal Lawan[1], U. A. Ali[2], Abdulkareem Lado Ismal[3], Muhammad Salihu Isa[4], Anas Sani Maihulla[5], Ibrahim Yusuf[6*]

[1]Department of Mathematics, Kano University of Science and Technology, Wudil, Kano, Nigeria
[2]Department of Mathematics, Federal University, Dutse, Nigeria
[3]Department of Mathematics, Kano State College of Education and Preliminary Studies, Kano, Nigeria
[4]Department of Mathematics, Yusuf Maitama Sule University, Kano
[5]Department of Mathematics, Sokoto State University, Sokoto
[6*]Department of Mathematical Sciences, Bayero University, Kano, Nigeria

Received October 2021; Revised January 2022; Accepted June 2022

Abstract: Human error or error is one of the factors leading to the breakdown of production equipment. Human error caused by human operators has led to disruptions in the production process, output, product quality, production loss, lower revenue generation and high maintenance costs. To avoid the risk of system downtime, the present paper introduce the use of repair machines as a medium of repair there is need to replace human work in terms of system maintenance particularly partial failure. The present paper focus on the performance evaluation of an active dissimilar parallel system attended by two repair machines set aside to handle unit failure. At failure of a unit, the repair machines will engage in the repair of the failed unit. The system of linear differential difference equation is analyzed to obtained expressions of reliability measures of determining the strength of system such as availability, mean time to failure (MTTF) and profit function. Numerical examples such as surface plots and sensitivity analysis are presented to illustrate the obtained results and to analyze the effect of various system parameters. On the basis of numerical experiments, it is observed that the system's best availability, MTTF and profit results may be achieved on a regular basis when repair machines are deployed.
Keyword — Performance, parallel system, service station, availability, mean time to failure

---

1. INTRODUCTION

In the analysis of system effectiveness, the system performance evaluation is of paramount importance. The strength of the system can be determined by computing the corresponding measures such as reliability, availability, mean time to failure, profit and benefit-cost analysis. Where adequate maintenance facilities such as regular inspection, online and offline preventive maintenance to the system, introducing fault tolerant components in the system, invoking perfect repair at system or unit failure, replacement of worn-out part, are put in place, the system performance will undoubtedly attain its peak. This will pave way to high system reliability.

To achieve high system reliability and availability, the system must be maintained at the highest order. To achieve this end, numerous researchers have designed different types of mathematical models to study and compare their reliability, availability and mean time to failure.

Researchers such as; Aggarwal AK, Kumar, and Singh (2017) used RAMD analysis to model the efficiency of serial processes in a sugar refinery system. Yusuf, Lado, Singh, Ali, and Sufi (2020) published a paper on performance analysis of a multi computer system with three subsystems in series and a Copula repair policy. Gahlot, Singh, Ayagi, and Goel (2018) used Copula linguistics to evaluate the efficiency of repairable systems in series configurations under various forms of failure and repair policies. Singh, Poonia, and Abdullahi (2020) investigated the performance analysis of a complex repairable device with two subsystems in series and a defective imperfect switch. Chopra and Ram (2017) have provided stochastic analysis of two non-identical unit parallel system incorporating waiting time. Sanusi, Yusuf, and Mamuda (2020) evaluated the efficiency of an industrial that was designed as a series-parallel system. Ram and Kumar (2015) have presented performability analysis of a system under 1-out-of-2: G scheme with perfect reworking. Abubakar

*Corresponding author's e-mail: iyusuf.mth@buk.edu.ng

http://doi.org/10.6886/IJOR.202206_19(2).0001

and Singh (2019) have assessed the Performance of an industrial system using copula linguistic approach. Gulati, Singh, and Rawal (2018) used Copula to measure the Performance of two unit's redundant system under various failure and repair policies. Singh and Singh (2015) have analyzed the performance of three-unit redundant system with switch and human failure. Kumar, Barak, and Devi (2016) studied the performance analysis of a redundant system with Weibull failure and repair laws. Lado, Singh, Kabiru, and Yusuf (2018) reported on the performance and cost of a repairable complex system with two subsystems linked in sequence. Kakkar, Chitkara, and Bhatti (2016) investigated reliability analysis of two dissimilar parallel unit repairable system with failure during preventive maintenance. Saini and Kumar (2019) used RAMD analysis to investigate the performance of evaporation system in sugar industry. Pundir, Patawa, and Gupta (2018) proposed a stochastic forecast for two non-identical unit parallel systems with a repair priority. Gulati, Singh, Rawal, and Goel (2016) used copula linguistic approach to analyzed the performance of complex system in series connection under different failure and repair policies. Malik and Tewari (2018) published a paper on the output modeling and management of priorities decisions for a coal-fired thermal power plant's water flow system. The Performance measurement and management for maintenance: a literature review has been presented by Parida, Kumar, Galar, and Stenström (2015). Kumar, Garg, and Tiwari (2014) provided a performance modeling and availability simulation of a brewery malt mill system. Singh, Gulati, Rawal, and Goel (2016) have used Copula to analyzed the Performance of a complex system connected in series under various forms of failure and repair discipline. Chauhan and Malik (2017) used Weibull failure laws to evaluate the reliability and MTSF of a parallel system. Shim, Kim, and Lee (2017) addressed the availability of a redundant system with two parallel active components under Markovian assumptions. Kakkar, Chitkara, and Bhatti (2015) analyzed the reliability of two unit's parallel repairable industrial system. Shim et al. (2017) has studied availability of a redundant system with two parallel active components. Temraz (2019) presented a study on availability and reliability of a parallel system under imperfect repair and replacement: analysis and cost optimization.

Researchers above have previously presented works that have been praised for their contributions to the analysis of repairable complex systems by calculating the performance of a complex system under various failure and repair discipline. Researchers above have presented their research work on some reliability measures of system strength and effectiveness and proclaimed a better performance of the system. Little is known on reliability and performance evaluation of systems attended by repair machines in which the partial failure is rectified by repair machines. Still reliability analysis of system attended by two repair machines is required.

As a result, a structure for evaluating the performance of a two-unit active parallel system with repairable service stations has been presented in this paper. It is also necessary to have enough information on failure and repair so as to determine system availability, reliability, and calculate exact performance rates. The rest of this paper is organized as follows: The notions used in the study are given in section 2. Section 3 captures the description of the system. The model formulation and solution are presented in section 4. Section 5 gave the results analysis of the study and the paper is concluded in section 6.

2. NOTATIONS

Table 1: Notation used

Notation	Meaning
S_i	State of the system $i = 0, 1, 2, \ldots, 7$.
$P(t)$	Probability row vector
$P_i(t)$	Probability that the system is in state i at time $t \geq 0$
$A_v(\infty)$	Steady-state availability
$P_f(\infty)$	Profit function
$\alpha_0/\alpha_1/\alpha_2$	Repair rate of unit A/unit B/repair machine
$\beta_0/\beta_1/\beta_2$	Failure rate of unit A/unit B/repair machine
B_{T1}/B_{T3}	Busy period probability of repairman due to partial failure of units' subsystem A/subsystem B
B_{T2}/B_{T4}	Busy period probability of repairman due to complete failure of unit A/unit B
C_0	Revenue generated
C_{T1}/C_{T3}	Cost due repair of partial failure of unit A/unit B
C_{T2}/C_{T4}	Cost due repair of complete failure of unit A/unit B

3. SYSTEM DESCRIPTION

The system in this study is a parallel system consisting of two dissimilar units A and B in active parallel. Two repairable machines are set aside to evoke repair at the failure of unit A or B or both.

It is assumed that units and repair machines fail independent of the other. Primary units (A and B) fail with exponential failure time distribution with parameter β_0 and β_1 with exponential repair time with parameters α_0 and α_1 respectively. System failure occur only whenever unit A and B have failed. On the other hand, repair machines are systems themselves that are liable to failure. Each of the repair machine fail with exponential failure time distribution and parameter β_2, and exponential repair time with parameter α_2.

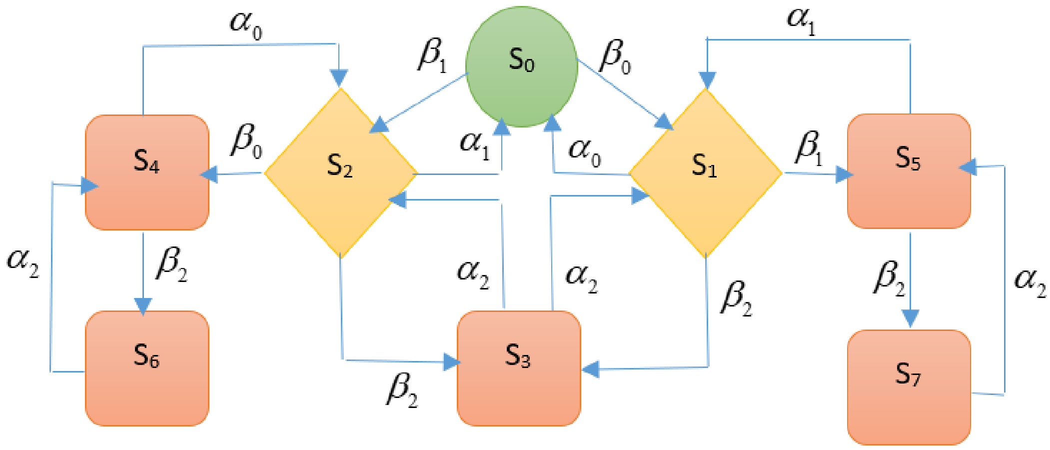

Figure 1: Transition diagram of the model

Table 2: State description of the system

State	Description
S_0	Initial state, unit A and B are working, service stations R_{m1} and R_{m2} are idle. The system is operational.
S_1	Unit A has failed and is attended by R_{m1} for repair, unit B is working, R_{m2} is idle. The system is operational.
S_2	Unit B has failed and is attended by R_{m2}, unit A is working, R_{m1} is idle. The system is operational.
S_3	Unit A and B have failed and are attended by both R_{m1} and R_{m2} for repairs. The system is down.
S_4	Previously unit B has failed and is attended by R_{m2}, suddenly unit A has failed is attended by R_{m1} for repair. The system is down.
S_5	Previously unit A has failed and is attended by R_{m1}, suddenly unit B has failed is attended by R_{m2} for repair. The system is down.
S_6	R_{m1} has failed on the process of repairing unit A, unit B previously failed and is attended by R_{m2} for repair. The system is down.
S_7	R_{m2} has failed on the process of repairing unit B, unit A previously failed and is attended by R_{m1} for repair. The system is down.

4. FORMULATION OF RELIABILITY MODELS

To estimate the performance of the system, as well as the profit function, due to partial and complete failure. The probability of the system being within s_i at $t > 0$ is define as $p_i(t), i = 0, 1, 2, 3, \ldots, 7$. Define $p(t) = [p_1(t), p_2(t), \ldots, p(t)]$ at time t to be the row vector of these probabilities. In this analysis, the initial condition is:

$$p_i(0) = \begin{cases} 1 & , i = 0 \\ 0 & , i = 1, 2, 3, \ldots, 7 \end{cases} \tag{1}$$

The difference-differential equations resulting from Figure 1 are given by:

$$\left.\begin{aligned}
\frac{d}{dt}p_0(t) &= -(\beta_0+\beta_1)p_0(t)+\alpha_0 p_0(t)+\alpha_1 p_2(t)\\
\frac{d}{dt}p_1(t) &= -(\alpha_0+\beta_1+\beta_2)p_1(t)+\beta_0 p_0(t)+\alpha_2 p_3(t)+\alpha_1 p_5(t)\\
\frac{d}{dt}p_2(t) &= -(\alpha_1+\beta_0+\beta_2)p_2(t)+\beta_1 p_0(t)+\alpha_2 p_3(t)+\alpha_0 p_5(t)\\
\frac{d}{dt}p_3(t) &= -2\alpha_2 p_3(t)+\beta_2 p_1(t)+\beta_2 p_2(t)\\
\frac{d}{dt}p_4(t) &= -(\alpha_0+\beta_2)p_4(t)+\beta_0 p_2(t)+\alpha_2 p_6(t)\\
\frac{d}{dt}p_5(t) &= -(\alpha_1+\beta_2)p_5(t)+\beta_1 p_1(t)+\alpha_2 p_7(t)\\
\frac{d}{dt}p_6(t) &= -\alpha_2 p_6(t)+\beta_2 p_4(t)\\
\frac{d}{dt}p_7(t) &= -\alpha_2 p_7(t)+\beta_2 p_5(t)
\end{aligned}\right\} \tag{2}$$

which are written in the following format

$$\begin{pmatrix} p_0'(t)\\ p_1'(t)\\ p_2'(t)\\ p_3'(t)\\ p_4'(t)\\ p_5'(t)\\ p_6'(t)\\ p_7'(t)\end{pmatrix} = \begin{pmatrix}
-(\beta_0+\beta_1) & \alpha_0 & \alpha_1 & 0 & 0 & 0 & 0 & 0\\
\beta_0 & -(\alpha_0+\beta_1+\beta_2) & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0\\
\beta_1 & 0 & -(\alpha_1+\beta_0+\beta_2) & \alpha_2 & \alpha_0 & 0 & 0 & 0\\
0 & \beta_2 & \beta_2 & -2\alpha_2 & 0 & 0 & 0 & 0\\
0 & 0 & \beta_0 & 0 & -(\alpha_0+\beta_2) & 0 & \alpha_2 & 0\\
0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2\\
0 & 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 & 0\\
0 & 0 & 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2
\end{pmatrix}\begin{pmatrix} p_0(t)\\ p_1(t)\\ p_2(t)\\ p_3(t)\\ p_4(t)\\ p_5(t)\\ p_6(t)\\ p_7(t)\end{pmatrix}$$

Expressions for steady state availability, busy period of repairman due to partial failure and complete failure of unit A and B and service stations are

$$A_T(\infty) = p_0(\infty)+p_1(\infty)+p_2(\infty) \tag{3}$$
$$B_{P1}(\infty) = p_1(\infty)+p_4(\infty) \tag{4}$$
$$B_{P2}(\infty) = p_2(\infty)+p_5(\infty) \tag{5}$$
$$B_{P3}(\infty) = p_3(\infty)+p_6(\infty) \tag{6}$$
$$B_{P4}(\infty) = p_3(\infty)+p_7(\infty) \tag{7}$$

In the steady state, the derivatives of states probabilities become zero and therefore (2) becomes

$$\begin{pmatrix}
-(\beta_0+\beta_1) & \alpha_0 & \alpha_1 & 0 & 0 & 0 & 0 & 0\\
\beta_0 & -(\alpha_0+\beta_1+\beta_2) & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0\\
\beta_1 & 0 & -(\alpha_1+\beta_0+\beta_2) & \alpha_2 & \alpha_0 & 0 & 0 & 0\\
0 & \beta_2 & \beta_2 & -2\alpha_2 & 0 & 0 & 0 & 0\\
0 & 0 & \beta_0 & 0 & -(\alpha_0+\beta_2) & 0 & \alpha_2 & 0\\
0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2\\
0 & 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 & 0\\
0 & 0 & 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2
\end{pmatrix}\begin{pmatrix} p_0(\infty)\\ p_1(\infty)\\ p_2(\infty)\\ p_3(\infty)\\ p_4(\infty)\\ p_5(\infty)\\ p_6(\infty)\\ p_7(\infty)\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\0\end{pmatrix}$$

Using the following normalizing condition

$$\sum_{j=0}^{7} p_j(\infty) = 1 \tag{8}$$

To compute the state probabilities $p_i(t)\ i = 0, 1, 2, \ldots, 7$, (7) is substituted in the last of (6) to give

$$\begin{pmatrix} -(\beta_0+\beta_1) & \alpha_0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -(\alpha_0+\beta_1+\beta_2) & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0 \\ \beta_1 & 0 & -(\alpha_1+\beta_0+\beta_2) & \alpha_2 & \alpha_0 & 0 & 0 & 0 \\ 0 & \beta_2 & \beta_2 & -2\alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & 0 & -(\alpha_0+\beta_2) & 0 & \alpha_2 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2 \\ 0 & 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{9}$$

The terms for the availability of the steady-state, busy period due to partial and complete failure of both units and their service stations in (3) to (7) are presented in equation (9) after solving (8) with the MATLAB software tool to obtain $p_i(t)$.

$$p_0(\infty) = \frac{\alpha_0\alpha_1\alpha_2(\alpha_1\beta_2 + 2\alpha_1\alpha_0 + \alpha_0\beta_2)}{\Delta_0 + \Delta_1 + \Delta_2}$$

$$p_1(\infty) = \frac{\alpha_0\alpha_1\alpha_2(2\alpha_1\beta_0 + \beta_1\beta_2 + \beta_0\beta_2)}{\Delta_0 + \Delta_1 + \Delta_2}$$

$$p_2(\infty) = \frac{\alpha_0\alpha_1\alpha_2(2\alpha_0\beta_1 + \beta_1\beta_2 + \beta_0\beta_2)}{\Delta_0 + \Delta_1 + \Delta_2}$$

$$p_3(\infty) = \frac{\alpha_0\alpha_1\beta_2(\alpha_1\beta_0 + 2\alpha_0\beta_1 + \beta_1\beta_2 + \beta_0\beta_2)}{\Delta_0 + \Delta_1 + \Delta_2}$$

$$p_4(\infty) = \frac{\alpha_1\alpha_2\beta_0(2\alpha_0\beta_1 + \beta_1\beta_2 + \beta_0\beta_2)}{\Delta_0 + \Delta_1 + \Delta_2}$$

$$p_5(\infty) = \frac{\alpha_0\alpha_2\beta_1(2\alpha_1\beta_0 + \beta_1\beta_2 + \beta_0\beta_2)}{\Delta_0 + \Delta_1 + \Delta_2}$$

$$p_6(\infty) = \frac{\alpha_1\beta_0\beta_2(2\alpha_0\beta_1 + \beta_1\beta_2 + \beta_0\beta_2)}{\Delta_0 + \Delta_1 + \Delta_2}$$

$$p_7(\infty) = \frac{\alpha_1\beta_0\beta_2(2\alpha_0\beta_1 + \beta_1\beta_2 + \beta_0\beta_2)}{\Delta_0 + \Delta_1 + \Delta_2}$$

$$A_T(\infty) = \frac{\alpha_0\alpha_1\alpha_2(\alpha_1\beta_2 + \alpha_0\beta_2 + 2\alpha_0\alpha_1 + 2\alpha_1\beta_0 + 2\beta_1\beta_2 + 2\beta_0\beta_2 + 2\alpha_0\beta_1)}{\Delta_0 + \Delta_1 + \Delta_2}$$

Because the equipment's are prone to partial and complete failure, the repairman is busy performing preventive maintenance on the broken items. Defines C_0, C_1, C_2, C_3, and C_4 as the money generated when the system is in good operating order and no income when it is in bad functioning order, as well as the cost of each repair due to partial or complete failure. The predicted total system profit per unit time is given by the predicted overall system benefit per unit time expended in steady condition:

$$P_T(\infty) = C_0 * A_V(\infty) - C_1 * B_{P1}(\infty) - C_2 * B_{P2}(\infty) - C_3 * B_{P3}(\infty) - C_4 * B_{P4}(\infty) \tag{10}$$

The explicit expression for the MTTF is computed using:

$$\text{MTTF} = P(0)(-Q^{-1})[1, 1, 1]^T \tag{11}$$

Thus, the MTTF expression for system is:

$$\text{MTTF} = \frac{(\alpha_0 + \beta_1 + \beta_2)(\alpha_1 + \beta_0 + \beta_2) + \beta_0(\alpha_1 + \beta_0 + \beta_2) + \beta_1(\alpha_1 + \beta_0 + \beta_2)}{\beta_0^2(\beta_1 + \beta_2) + \beta_1(\alpha_0 + \beta_1)(\beta_0 + \beta_2) + \beta_2^2(\beta_0 + \beta_1) + \beta_0\beta_1(\alpha_1 + \beta_2) + \beta_0\beta_2(\alpha_1 + \beta_1)} \tag{12}$$

Where,

$$T = \begin{pmatrix} -(\beta_0+\beta_1) & \alpha_0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -(\alpha_0+\beta_1+\beta_2) & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0 \\ \beta_1 & 0 & -(\alpha_1+\beta_0+\beta_2) & \alpha_2 & \alpha_0 & 0 & 0 & 0 \\ 0 & \beta_2 & \beta_2 & -2\alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & 0 & -(\alpha_0+\beta_2) & 0 & \alpha_2 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & -(\alpha_1+\beta_2) & 0 & \alpha_2 \\ 0 & 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_2 & 0 & -\alpha_2 \end{pmatrix}$$

$$M = \begin{pmatrix} -(\beta_0+\beta_1) & \beta_0 & \beta_1 \\ \alpha_0 & -(\alpha_0+\beta_1+\beta_2) & 0 \\ \alpha_1 & 0 & -(\alpha_1+\beta_0+\beta_2) \end{pmatrix}$$

$$\Delta_0 = 2\alpha_0\alpha_1^2\alpha_2\beta_0 + \alpha_0\alpha_1^2\alpha_2\beta_2 + 2\alpha_1^2\alpha_2^2\alpha_2 + \alpha_0\alpha_1^2\beta_0\beta_2 + \alpha_1\alpha_2\beta_0\beta_2^2 + \alpha_0\alpha_1\alpha_2\beta_1\beta_2 + 4\alpha_0\alpha_1\alpha_2\beta_0\beta_1 +$$
$$\Delta_1 = 2\alpha_0\alpha_1\alpha_2\beta_0\beta_2 + 2\alpha_0\alpha_1\alpha_2\beta_1\beta_2 + 2\alpha_0^2\alpha_1\alpha_2\beta_1 + \alpha_0^2\alpha_1\alpha_2\beta_2 + \alpha_1\beta_0^2\beta_2^2 + \alpha_1\beta_0\beta_1\beta_2^2 + 4\alpha_0\alpha_1\beta_0\beta_1\beta_2 +$$
$$\Delta_2 = \alpha_0\alpha_1\beta_0\beta_2^2 + \alpha_0\alpha_1\beta_1\beta_2^2 + \alpha_0^2\alpha_1\beta_1\beta_2 + \alpha_0\alpha_2\beta_0\beta_1\beta_2 + \alpha_2\alpha_0\beta_1^2\beta_2 + \alpha_0\beta_0\beta_1\beta_2^2 + \alpha_0\beta_1^2\beta_2^2$$

5. NUMERICAL ANALYSIS

This section presents numerical analysis for the established models in terms of availability, mean time to failure (MTTF), and benefit function. The following parameters are fixed in the simulation for consistency in the model analysis:

$$\alpha_0 = 0.5, \alpha_1 = 0.5, \alpha_2 = 0.4, \beta_0 = 0.2, \beta_1 = 0.1, \beta_2 = 0.3, C_0 = 2500000, C_1 = 250, C_2 = 250, C_3 = 350,$$
$$\text{and } C_4 = 350$$

Figures 2 and 5 show the effect of unit A's failure rate(β_0) and repair rate(α_0) on system availability and profit. The availability and profit decrease with respect to failure rate(β_0) and increase with respect to repair rate(α_0), as shown in these graphs. The value of system availability and profit in terms of α_0 is higher than the value of system availability and profit in terms of β_0. This sensitivity analysis illustrates what it takes to keep the system running.

Figures 3 and 6 depict the system's availability and profit trends in relation to unit B's failure(β_1) and repair(α_1) rates, respectively. When the rate of repair α_1 rises, so does availability and profit, whereas as the rate of failure β_1 rises, so does the availability and profit. As a result, preventive and substantial maintenance is critical for maximizing the system availability and profit.

For different values of service station, figures 4 and 7 present the impact of failure rate(β_2) and repair rate (α_2) on system's availability and benefit function. These graphs show a growing pattern of availability and benefit for repair and a declining pattern of availability and profit for failure. According to this report, normal system repair can result in higher system availability and revenue.

Figure 11 shows the mean time to failure of the system versus the failure(β_0) and repair(α_0) rates of unit A. It has been discovered that MTTF rises with the rate of repair and falls with the rate of failure. Figure 12 presents the system mean time to failure in comparison to unit B's failure(β_1) and repair(α_1) rates. MTTF rises with the rate of repair and falls with the rate of failure, according to the data. Figures 11 and 12 indicate that as the failure rate of unit A increases, the distance in the figure, i.e. figure 11 narrows, indicating that failure of unit A will impact the entire system.

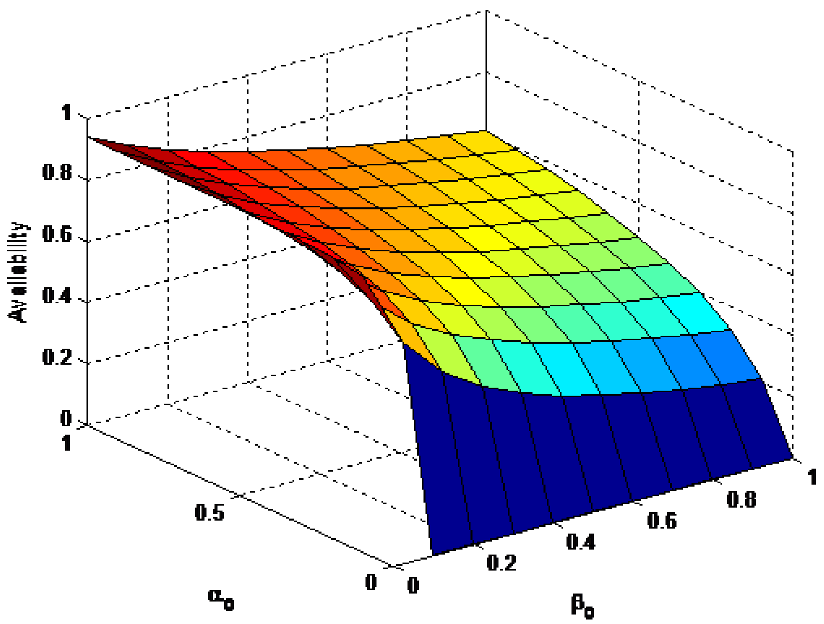

Figure 2: Surface plot of availability against α_0 and β_0

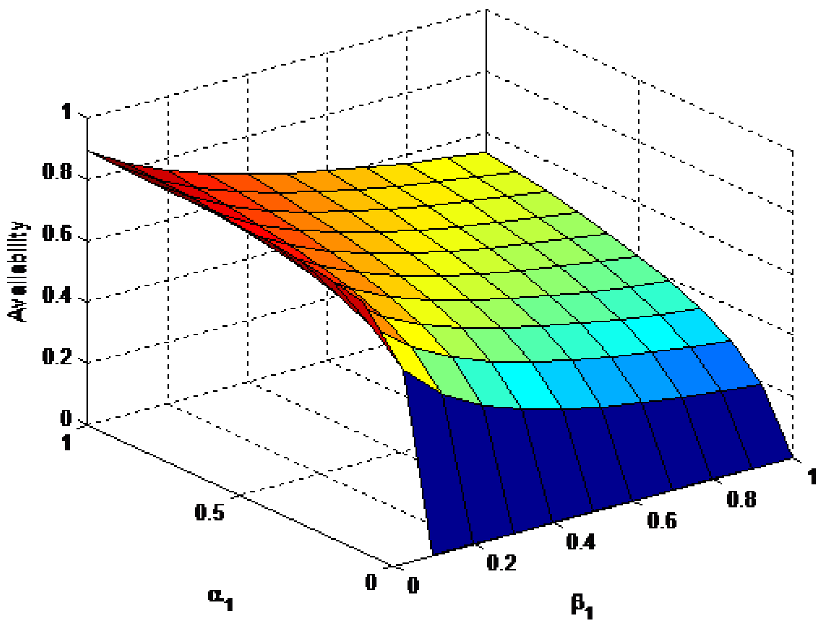

Figure 3: Surface plot of availability against α_1 and β_1

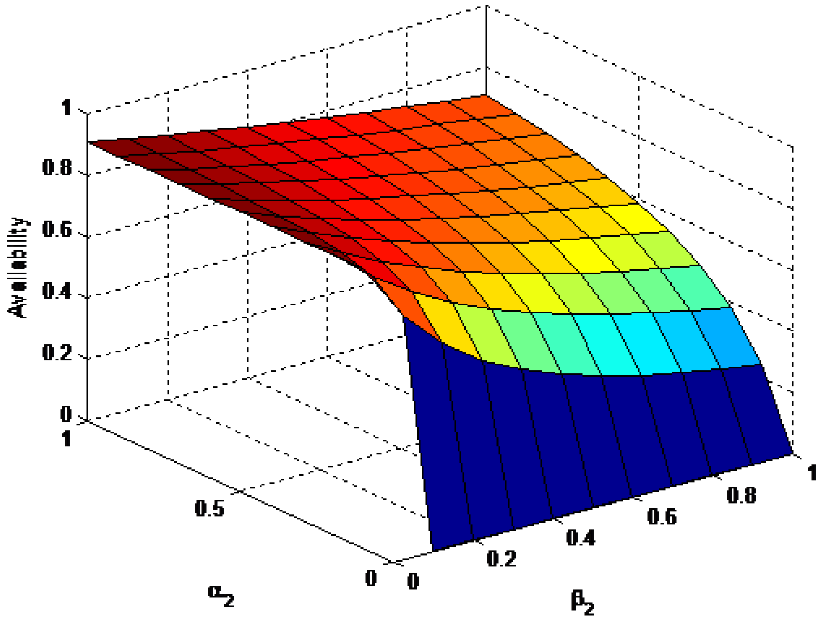

Figure 4: Surface plot of availability against α_2 and β_2

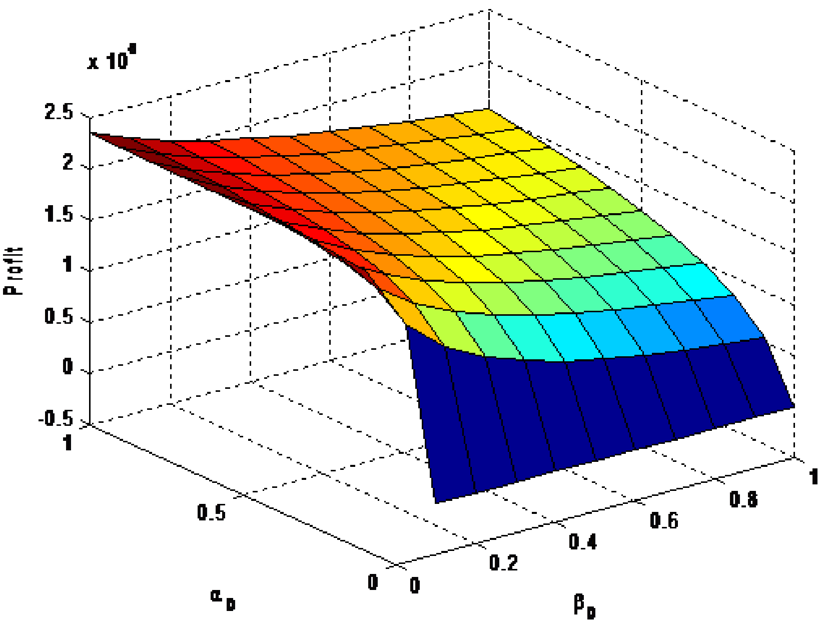

Figure 5: Surface plot of profit against α_0 and β_0

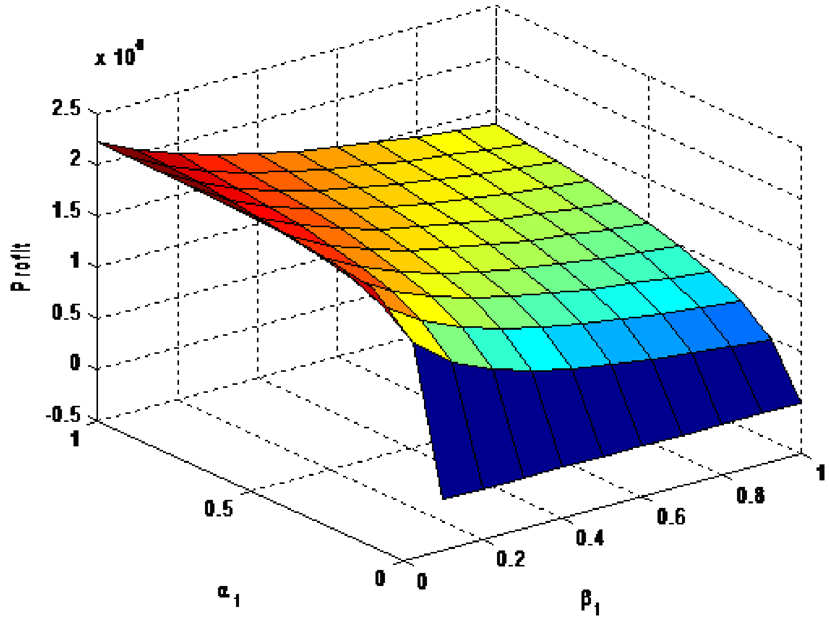

Figure 6: Surface plot of profit against α_1 and β_1

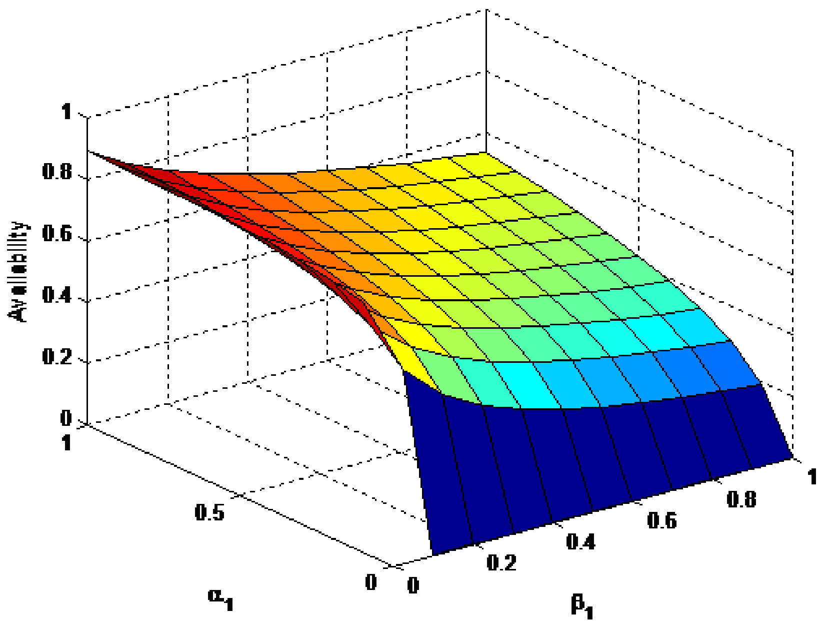

Figure 7: Surface plot of profit against α_2 and β_2

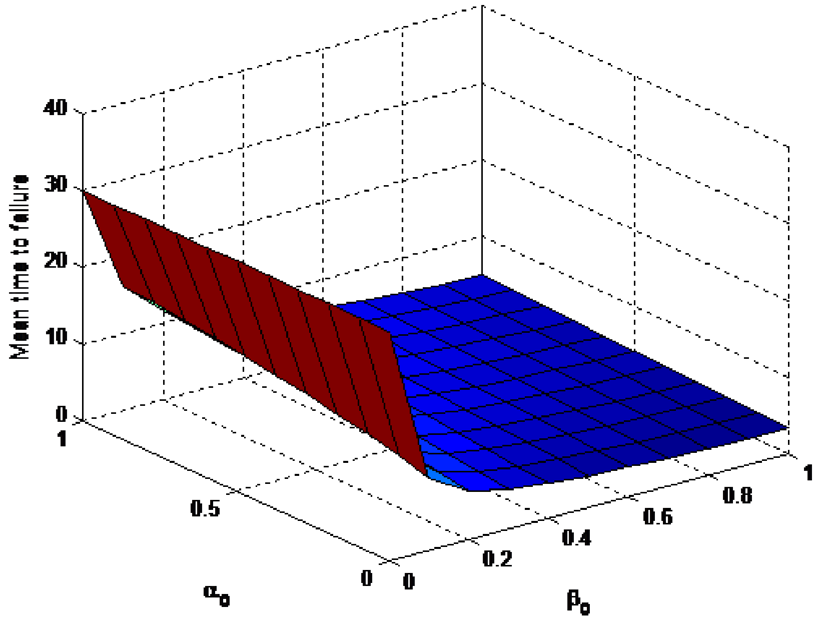

Figure 8: Surface plot of MTTF against α_0 and β_0

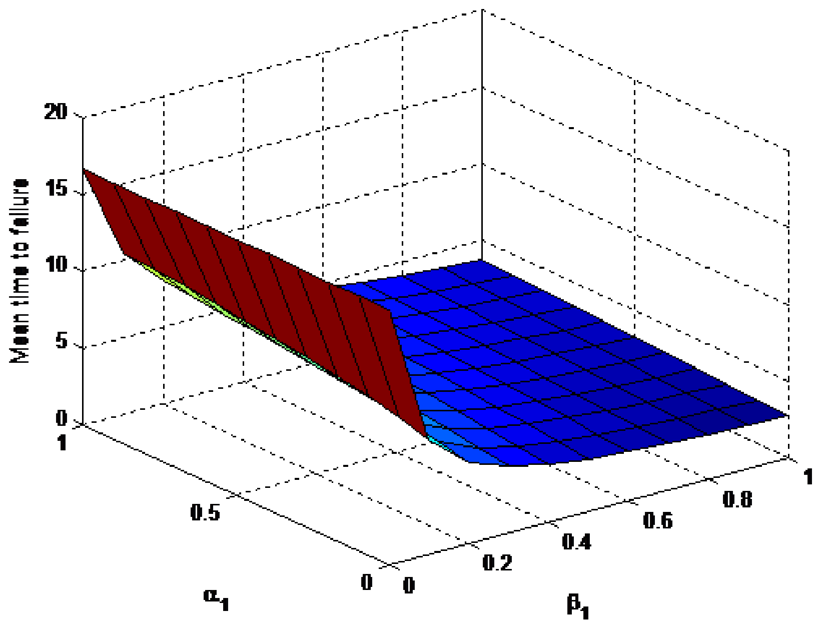

Figure 9: Surface plot of MTTF against α_1 and β_1

Table 3: Variation of Availability, Profit and MTTF with respect to repair rate of unit A

α_0	$A_V(\infty)$			$P_F(\infty)*10^6$			MTTF		
	$\beta_0=0.02$	$\beta_0=0.05$	$\beta_0=0.08$	$\beta_0=0.02$	$\beta_0=0.05$	$\beta_0=0.08$	$\beta_0=0.02$	$\beta_0=0.05$	$\beta_0=0.08$
0.0	0.0000	0.0000	0.0000	-0.0012	-0.0012	-0.0012	19.8554	13.6290	10.6642
0.1	0.8284	0.7456	0.6811	2.0700	1.8629	1.7015	21.1170	15.0000	11.8830
0.2	0.8680	0.8105	0.7630	2.1692	2.0253	1.9063	22.0652	16.1184	12.9199
0.3	0.8874	0.8420	0.8032	2.2177	2.1040	2.0069	22.8093	17.0482	13.8130
0.4	0.8993	0.8615	0.8284	2.2476	2.1529	2.0700	23.3955	17.8333	14.5902
0.5	0.9075	0.8750	0.8460	2.2680	2.1866	2.1141	23.8801	18.5052	15.2726
0.6	0.9135	0.8849	0.8591	2.2829	2.2114	2.1469	24.2842	19.0865	15.8767
0.7	0.9180	0.8925	0.8693	2.2943	2.2305	2.1724	24.6264	19.5946	16.4151
0.8	0.9216	0.8986	0.8774	2.3033	2.2457	2.1928	24.9199	20.0424	16.8980
0.9	0.9245	0.9035	0.8841	2.3106	2.2581	2.2094	25.1744	20.4400	17.3337

Table 4: Variation of Availability, Profit and MTTF with respect to failure rate of unit A

β_0	$A_V(\infty)$			$P_F(\infty)*10^6$			MTTF		
	$\alpha_0=0.5$	$\alpha_0=0.7$	$\alpha_0=0.9$	$\alpha_0=0.5$	$\alpha_0=0.7$	$\alpha_0=0.9$	$\alpha_0=0.5$	$\alpha_0=0.7$	$\alpha_0=0.9$
0.0	0.9316	0.9365	0.9395	2.3283	2.3405	2.3481	30.0000	30.0000	30.0000
0.1	0.8284	0.8549	0.8719	2.0700	2.1364	2.1790	13.7500	14.8750	15.7955
0.2	0.7554	0.7934	0.8186	1.8873	1.9824	2.0456	9.5200	10.4444	11.2414
0.3	0.6997	0.7445	0.7751	1.7480	1.8602	1.9366	7.5806	8.3333	9.0000
0.4	0.6551	0.7044	0.7385	1.6363	1.7596	1.8450	6.4706	7.1004	7.6678
0.5	0.6180	0.6704	0.7070	1.5435	1.6745	1.7662	5.7530	6.2931	6.7857
0.6	0.5863	0.6410	0.6795	1.4644	1.6010	1.6975	5.2518	5.7241	6.1589
0.7	0.5588	0.6151	0.6551	1.3956	1.5364	1.6365	4.8824	5.3019	5.6909
0.8	0.5345	0.5921	0.6333	1.3348	1.4789	1.5818	4.5990	4.9763	5.3282
0.9	0.5128	0.5714	0.6135	1.2805	1.4271	1.5323	4.3750	4.7177	5.0391

Table 5: Variation of Availability, Profit and MTTF with respect to repair rate of unit B

α_1	$A_V(\infty)$			$P_F(\infty)*10^6$			MTTF		
	$\beta_1=0.03$	$\beta_1=0.06$	$\beta_1=0.09$	$\beta_1=0.03$	$\beta_1=0.06$	$\beta_1=0.09$	$\beta_1=0.03$	$\beta_1=0.06$	$\beta_1=0.09$
0.0	0.0000	0.0000	0.0000	-0.0027	-0.0027	-0.0027	14.6142	10.9160	8.8727
0.1	0.7876	0.7165	0.6616	1.9678	1.7898	1.6524	15.4160	11.7481	9.6338
0.2	0.8358	0.7844	0.7421	2.0887	1.9597	1.8540	16.0541	12.4444	10.2894
0.3	0.8596	0.8180	0.7827	2.1483	2.0441	1.9556	16.5740	13.0356	10.8601
0.4	0.8745	0.8393	0.8086	2.1854	2.0972	2.0205	17.0058	13.5439	11.3614
0.5	0.8847	0.8541	0.8270	2.2111	2.1345	2.0666	17.3700	13.9854	11.8052
0.6	0.8923	0.8652	0.8408	2.2300	2.1622	2.1012	17.6815	14.3726	12.2009
0.7	0.8981	0.8737	0.8516	2.2446	2.1836	2.1283	17.9508	14.7150	12.5559
0.8	0.9027	0.8806	0.8604	2.2562	2.2008	2.1501	18.1861	15.0198	12.8761
0.9	0.9065	0.8862	0.8675	2.2656	2.2149	2.1681	18.3933	15.2929	13.1665

Table 6: Variation of Availability, Profit and MTTF with respect to failure rate of unit B

β_1	$A_V(\infty)$			$P_F(\infty) * 10^6$			MTTF		
	$\alpha_1 = 0.6$	$\alpha_1 = 0.8$	$\alpha_1 = 1.0$	$\alpha_1 = 0.6$	$\alpha_1 = 0.8$	$\alpha_1 = 1.0$	$\alpha_1 = 0.6$	$\alpha_1 = 0.8$	$\alpha_1 = 1.0$
0.0	0.9227	0.9270	0.9298	2.3063	2.3169	2.3239	23.3333	23.3333	23.3333
0.1	0.8333	0.8540	0.8681	2.0822	2.1341	2.1694	11.6340	12.3077	12.8649
0.2	0.7691	0.7985	0.8192	1.9215	1.9952	2.0471	8.1600	8.7407	9.2414
0.3	0.7201	0.7545	0.7793	1.7988	1.8850	1.9472	6.4986	6.9816	7.4074
0.4	0.6810	0.7185	0.7460	1.7009	1.7948	1.8636	5.5274	5.9363	6.3019
0.5	0.6488	0.6882	0.7175	1.6202	1.7190	1.7923	4.8918	5.2449	5.5639
0.6	0.6215	0.6622	0.6928	1.5518	1.6540	1.7304	4.4444	4.7545	5.0370
0.7	0.5979	0.6396	0.6710	1.4928	1.5972	1.6759	4.1130	4.3892	4.6425
0.8	0.5771	0.6195	0.6516	1.4409	1.5470	1.6273	3.8580	4.1068	4.3363
0.9	0.5587	0.6016	0.6342	1.3946	1.5021	1.5837	3.6559	3.8823	4.0920

Table 7: Variation of Availability, Profit and MTTF with respect to repair rate of the service station

α_2	$A_V(\infty)$			$P_F(\infty) * 10^6$			MTTF		
	$\beta_2 = 0.1$	$\beta_2 = 0.3$	$\beta_2 = 0.5$	$\beta_2 = 0.1$	$\beta_2 = 0.3$	$\beta_2 = 0.5$	$\beta_2 = 0.1$	$\beta_2 = 0.3$	$\beta_2 = 0.5$
0.0	0.0000	0.0000	0.0000	-0.0049	-0.0049	-0.0049	14.9057	9.5200	7.5566
0.1	0.7186	0.5047	0.3883	1.7951	1.2594	0.9679	14.9057	9.5200	7.5566
0.2	0.8020	0.6481	0.5433	2.0041	1.6186	1.3560	14.9057	9.5200	7.5566
0.3	0.8343	0.7159	0.6266	2.0850	1.7884	1.5648	14.9057	9.5200	7.5566
0.4	0.8514	0.7554	0.6787	2.1279	1.8873	1.6952	14.9057	9.5200	7.5566
0.5	0.8621	0.7813	0.7143	2.1546	1.9521	1.7844	14.9057	9.5200	7.5566
0.6	0.8693	0.7995	0.7402	2.1727	1.9978	1.8492	14.9057	9.5200	7.5566
0.7	0.8745	0.8131	0.7598	2.1858	2.0318	1.8985	14.9057	9.5200	7.5566
0.8	0.8785	0.8235	0.7753	2.1958	2.0581	1.9372	14.9057	9.5200	7.5566
0.9	0.8816	0.8319	0.7877	2.2036	2.0789	1.9684	14.9057	9.5200	7.5566

Table 8: Variation of Availability, Profit and MTTF with respect to failure rate of the service station

β_2	$A_V(\infty)$			$P_F(\infty) * 10^6$			MTTF		
	$\alpha_2 = 0.3$	$\alpha_2 = 0.6$	$\alpha_2 = 0.9$	$\alpha_2 = 0.3$	$\alpha_2 = 0.6$	$\alpha_2 = 0.9$	$\alpha_2 = 0.3$	$\alpha_2 = 0.6$	$\alpha_2 = 0.9$
0.0	0.9091	0.9091	0.9091	2.2723	2.2723	2.2723	23.8462	23.8462	23.8462
0.1	0.8343	0.8693	0.8816	2.0850	2.1727	2.2036	14.9057	14.9057	14.9057
0.2	0.7706	0.8329	0.8560	1.9255	2.0815	2.1393	11.3953	11.3953	11.3953
0.3	0.7159	0.7995	0.8319	1.7884	1.9978	2.0789	9.5200	9.5200	9.5200
0.4	0.6683	0.7687	0.8092	1.6692	1.9206	2.0221	8.3529	8.3529	8.3529
0.5	0.6266	0.7402	0.7877	1.5648	1.8492	1.9684	7.5566	7.5566	7.5566
0.6	0.5898	0.7137	0.7674	1.4726	1.7829	1.9175	6.9864	6.9784	6.9784
0.7	0.5571	0.6891	0.7482	1.3905	1.7212	1.8693	6.5396	6.5396	6.5396
0.8	0.5277	0.6661	0.7299	1.3171	1.6636	1.8234	6.1951	6.1951	6.1951
0.9	0.5013	0.6446	0.7125	1.2510	1.6098	1.7798	5.9175	5.9175	5.9175

The surface plots of availability, profit and MTTF with respect to α_0 and β_0 are depicted in Figures 2, 5 and 8. It is clear from these figures that availability, profit and MTTF decreases as β_0 increase and increases with increase in α_0.

It is evident from these plots that availability, profit and MTTF can be enhance through preventive maintenance that would prevent the occurrence of unit A failure.

Figures 3, 4 and 9 displayed the impact of α_1 and β_1 on availability, profit and MTTF. From the figure it is observed that both availability, profit and MTTF decreases with increase in β_1 and increases with increase in α_1. To improve the system availability, profit and MTTF, some maintenance actions such as online and offline preventive maintenance, regular inspection, used of fault tolerance unit, introduction of redundant units can be employed.

Figures 4 and 7 reflects the effect of α_2 and β_2 on availability and profit. The figures have shown that availability and profit increases with increase in α_2 and decreases with increase in β_2. This implies that absent or failure of repair machine retarded the system performance and reliability of system. This suggested that redundant and fault tolerance repair machine be introduce to help in reducing and maximizing the reliability of the system.

Tables 3 and 4 show how availability, benefit, and MTTF change as a function of unit A's repair and failure rates respectively. The availability, benefit, and MTTF show an increasing trend with respect to repair and a decreasing pattern with respect to failure, as shown in Table 3 and 4, respectively.

Tables 5 and 6 respectively show the relationship between availability, profit, and MTTF and unit B's repair and failure rates. Tables 5 and 6 show that when it comes to repair, availability, profit, and MTTF all show an increasing trend, whereas when it comes to loss, they show a decreasing pattern.

The effects of service station repair and failure on system availability, profit, and MTTF are shown in Tables 7 and 8 respectively. The system availability, profit, and MTTF increase with respect to repair and decrease with respect to failure, as can be seen in these tables. For all units, it is also clear that availability, profit, and MTTF increase with higher repair values and decrease with higher failure values. This indicates that minimizing the occurrence of failure at early stage or engaging in perfect repair will yield higher availability, profit and the system's expected life time.

6. CONCLUSION

In this paper, the reliability characteristics for different failure values and repair rates are thoroughly studied so as to evaluate the performance of the system under consideration. Numerical experiments were used to obtain and validate the fundamental formulations for system characteristics such as system availability, busy repairman time owing to partial and complete failure, and profit function. Analysis of the effect of various system parameters on mean time to failure, profit function and availability was performed. System/unit/repair machine failure is viewed in different scenario. Failure of the system/repair machine is viewed as partial or complete These are the main contributions of this study. Based on the findings of the numerical tests, it is clear that the system's optimum availability, MTTF and profit can be attained when the entire system is fixed on a regular basis and redundant and fault tolerance repair machines are used. System failure will lower manufacturing performance and this may have negative effect on any industry. Introduction of a repair machine into maintenance activities has become necessary due to the challenges faced by repairers in certain manufacturing and industrial applications that are hazardous to humans, such as nuclear power, highways, power line, aerospace, diagnosis of ailment such as covid-19, ebola disease, etc. In some developed countries, robots have been introduced to manage maintenance activities due to ground or hazardous tasks that are harmful to human health. These repair machines are system also which is likely to fail. In order to improve the efficiency of the system, repair machines are put in place to carry out any kind of maintenance activity that would be hazardous to repairers. Future studies would, however, necessitate further investigation. Understanding availability measurements can help engineers and designers to develop more vital systems in the service of humankind.

REFERENCES

Abubakar, M. I., & Singh, V. V. (2019). Performance assessment of an industrial system (African Textile Manufactures Ltd.) through copula linguistic approach. *Operation Research and Decisions*, *29*(4), 1-18.

Aggarwal AK, Kumar, S., & Singh, V. (2017). Performance modeling of the serial processes in refining system of a sugar plant using RAMD analysis. *International Journal of Systems Assurance Engineering and Management*, *8*(2), 1910-1922.

Chauhan, S. K., & Malik, S. C. (2017). Evaluation of reliability and MTSF of a parallel system with Weibull failure laws. *Journal of Reliability and Statistics*, *10*(1), 137-146.

Chopra, G., & Ram, M. (2017). Stochastic analysis of two non-identical unit parallel system incorporating waiting time. *International Journal of Quality* & *Reliability Management*, *34*(6), 817-832. Retrieved from `https://doi.org/10.1108/IJQRM-06-2016-0075`

Gahlot, M., Singh, V. V., Ayagi, H. I., & Goel, C. K. (2018). Performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics. *International Journal of Reliability and Safety*, *12*(4), 348-374.

Gulati, J., Singh, V. V., & Rawal, D. K. (2018). Performance assessment of two unit's redundant system under different failure and repair policies using copula. *International Journal of Reliability and application*, *19*(2), 109-124.

Gulati, J., Singh, V. V., Rawal, D. K., & Goel, C. K. (2016). Performance analysis of complex system in series configuration under different failure and repair discipline using copula. *International Journal of Reliability, Quality and Safety Engineering*, *23*(2), 812-832.

Kakkar, M. K., Chitkara, A. K., & Bhatti, J. (2015). Reliability analysis of two-unit parallel repairable industrial system. *Decision Science Letters*, *4*, 525-536.

Kakkar, M. K., Chitkara, A. K., & Bhatti, J. (2016). Reliability analysis of two dissimilar parallel unit repairable system with failure during preventive maintenance. *Management Science Letters*, *6*, 285-298.

Kumar, A., Barak, M. S., & Devi, K. (2016). Performance analysis of a redundant system with Weibull failure and repair laws. *Revista Investigacion Operacional*, *37*(3), 247-257.

Kumar, A., Garg, R. K., & Tiwari, P. C. (2014). Performance modeling and availability simulation of a malt mill system of brewery industry. *International Journal of Current Engineering and Technology*, *4*(3), 1452-1459.

Lado, A. I., Singh, V. V., Kabiru, H. I., & Yusuf, I. (2018). Performance and cost assessment of repairable complex system with two subsystems connected in series configuration. *International Journal Reliability and Applications*, *19*(1), 27-42.

Malik, S., & Tewari, P. C. (2018). Performance modeling and maintenance priorities decision for water flow system of a coal based thermal power plant. *International Journal of Quality & Reliability Management*, *35*(4), 996-1010.

Parida, A., Kumar, U., Galar, D., & Stenström, C. (2015). Performance measurement and management for maintenance: a literature review. *Journal of Quality in Maintenance Engineering*, *21*(1), 2-33.

Pundir, P. S., Patawa, R., & Gupta, P. K. (2018). Stochastic outlook of two non-identical unit parallel system with priority in repair. *Cogent Mathematics* & *Statistics*, *5*, 1467208. Retrieved from `https://doi.org/10.1080/25742558.2018.1467208`

Ram, M., & Kumar, A. (2015). Performability analysis of a system under 1-out-of-2: G scheme with perfect reworking. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, *37*(3), 1029-1038.

Saini, M., & Kumar, A. (2019). Performance analysis of evaporation system in sugar industry using RAMD analysis. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, *41*, 175. Retrieved from `https://doi.org/10.1007/s40430-019-1681-3`

Sanusi, A., Yusuf, I., & Mamuda, B. Y. (2020). Performance evaluation of an industrial configured as series-parallel system. *Journal of Mathematical and Computational Science*, *10*, 692-712.

Shim, J., Kim, C. K., & Lee, Y. (2017). Availability of a redundant system with two parallel active components under Markovian Assumptions. *International Journal of New Technology and Research*, *3*(2), 17-18.

Singh, V. V., Gulati, J., Rawal, D. K., & Goel, C. K. (2016). Performance analysis of complex system in series configuration under different failure and repair discipline using copula. *International Journal of Reliability Quality and Safety Engineering*, *23*(2).

Singh, V. V., Poonia, P. K., & Abdullahi, A. H. (2020). Performance analysis of a complex repairable system with two subsystems in series configuration with an imperfect switch. *Journal of Mathematical and Computational Science*, *10*(2), 359-383. Retrieved from `https://doi.org/10.28919/jmcs/4399`

Singh, V. V., & Singh, N. P. (2015). Performance analysis of three-unit redundant system with switch and human failure using copula distribution. *Mathematics in Engineering, Science* & *Aerospace*, *6*(2), 295-308.

Temraz, N. S. Y. (2019). Availability and reliability of a parallel system under imperfect repair and replacement: analysis and cost optimization. *International Journal of System Assurance Engineering and Management*, *10*, 1002-1009. Retrieved from `https://doi.org/10.1007/s13198-019-00829-2`

Yusuf, I., Lado, A. I., Singh, V. V., Ali, U. A., & Sufi, N. A. (2020). Performance analysis of multi computer system consisting of three subsystems in series configuration using copula repair policy. *SN Computer Science*, *1*, 241. Retrieved from `https://doi.org/10.1007/s42979-020-00258-0`