International Journal of
Operations Research

International Journal of Operations Research Vol. 19, No. 2, 41-50 (2022)

Production-Inventory Model with Non-Linear Price-Stock Dependent Demand Rate and Preservation Technology Investment under Complete Backlog

Priyanka Singh[1], Uttam Kumar Khedlekar[1], R P S Chandel[2]

[1]Department of Mathematics and Statistics Dr. Harisingh Gour Vishwavidyalaya, Sagar M.P. India
(A Central University)
[2]Department of Mathematics Govt Model Science College, Jabalpur M.P. India

Received February 2021; Revised April 2022; Accepted June 2022

Abstract: In this paper, a production inventory model for deteriorating items with price-stock dependent demand rate under complete backlog is developed. The deterioration rate is controlled by investment in preservation technology and optimum preservation cost is obtained. Under these general assumptions, we first proved that the optimal sales price and optimal preservation investment cost not only exists but is unique, for any given number of the production cycle. Next, we have shown that the total profit is a concave function of the selling price and production cycle when the preservation cost is given. Numerical results demonstrated the proposed model and further shown that the effects of different system parameters on the optimal variables and the optimal expected profit.
Keyword — Inventory, Pricing, Deteriorating items, Price-stock dependent demand, Backlogging, Preservation technology

1. INTRODUCTION

Deteriorating inventory had been elaborated in the past decades (Dave and Patel (1981), Kang and Kim (1983), Wee (1997), Lodree Jr. and Uzochukwu (2008), Bhunia, Kundu, Sannigrahi, and Goyal (2009), Mahata (2012), Chang, Teng, and Goyal (2010)), and they commonly centralized on constant or variable deterioration rate and ways to reduce the effect of deterioration. Deteriorating items are items that deteriorate with time, resulting in a decreasing utility, quality, marginal value and quantity from the original ones. Such items include medicines, fruits, vegetables, fashion goods, blood, electronic equipment, etc. Misra (1975) proposed a production lot size model for an inventory system with deteriorating items. He analyzed the model for both the varying and constant rate of deterioration. Wee (1993) developed a single commodity economic production policy for an ongoing deterioration item with partial back-ordering and finite replenishment. Tripathi and Tomar (2018) presented an EOQ model for deteriorating items with quadratic time-sensitive demand and parabolic-time linked holding cost related to salvage value. In this model, the finest cycle time and order quantity are obtained in terms of theoretical expressions. Sekar and Uthayakumar (2018) proposed and analyzed an EPQ model for a deteriorating item with exponentially increasing demand function and demand dependent production rate. Their model mainly consists of three (beginning, developing and maturity) different stages of production and one decline stage in which determined the number of replenishment cycles under a finite time horizon. Tiwari, Khedlekar, and Khedlekar (2021) found that the total profit for deteriorating items is a concave function of the selling price, ordering frequency, preservation technology investment and time cycle.

The management of inventories is one of the most important tasks that every particular manager must do efficiently and effectively in an organization. Nowadays, there is a huge competition between organizations and thus these organizations are taking seriously the management of inventories. In the last 36 years, the models for inventory management policies involving preservation technology investment have received the attention of several researchers (Hsu, Wee, and Teng (2010), Hsieh and Dye (2013)). Dye and Hsieh (2012) formulated an inventory model with a time-varying rate of deterioration and partial backlogging by considering the amount invested in preservation technology and the replenishment schedule as decision variables.

The first works on optimizing inventory decisions were designed under a reductionist perspective, and it can be traced to the original economic production quantity (EPQ) inventory model developed by Taft (1918). Panda, Saha,

*Corresponding author's e-mail: uvkkcm@yahoo.co.in

http://doi.org/10.6886/IJOR.202206_19(2).0002

and Soumen (2012) developed a single-item perishable inventory model to determine optimal pricing and lot-sizing policy for a retailer in stock and price-sensitive demand environment over a finite horizon. Roy, Ghosh, and Chaudhuri (2013) investigated the optimal production time and optimal cycle time in an economic production quantity model for items with time-proportional deterioration.

The problem of pricing and inventory control faced by a reseller who sells a perishable product with partially backlogged demand is studied by Abad (1996). Abad (2003) discussed a pricing and lot-sizing problem for a perishable good under finite production, exponential decay, partial backordering and lost sale. He used a new approach to modelled backlogging phenomenon without using the backorder cost and the lost sale cost. Goyal and Giri (2003) investigated the production-inventory problem in which the demand, production and deterioration rate of a product is assumed to vary with time and backlogged partially over an infinite planning horizon. Chung, Eduardo Cárdenas-Barrón, and Ting (2013) proposed a new economic production quantity inventory model for deteriorating items under two levels of trade credit to reflect the real business situations. Drake, Pentico, and Toews (2011) jointly planned the production of a final product subject to the conditions of an EPQ model and the production or purchasing of its components to the case where the final product is subject to the partial backordering of unfilled demand.

In the proposed inventory control model, we did an implicit assumption that when there is a shortage, complete back-ordering is assumed. In this paper, we studied the number of production cycles, invest in preservation technology and sales price to maximize the total expected profit, and perform a sensitivity analysis to understand how they depend on cost parameters. In particular, we used price-depend production rate, deterioration and complete backlogging to obtain general results on inventory management. In the end, a numerical example is used to illustrate the proposed model, and concluding remarks are provided.

2. MATHEMATICAL MODEL AND ANALYSIS

2.1 Model assumptions and notations

To develop the proposed mathematical model of production inventory, the notations adopted in this paper are represented in Table 2: Assumptions:

(1) The demand rate depends on the stock quantity displayed in the warehouse/showroom as well as the sales price,

$$f(p,t) = \begin{cases} f(p) + \lambda I(t), & I(t) > 0, 0 < \lambda < 1; \\ f(p), & I(t) \leq 0. \end{cases}$$

where $f(p) = \alpha p^{-\Gamma}$, $0 < \Gamma < 1$ is the known parameter and the price sensitivity of demand.

(2) The deterioration rate of an item follows an exponential function of u with parameter β i,e., $\theta(u) = \theta_0 e^{-\beta u}$,

(3) The production rate in each cycle is finite,

(4) Shortages are allowed and the unsatisfied demand is backlogged completely,

(5) There is no repair of deteriorated items,

(6) The production rate R is a function of the sales price and is given by $R = \frac{f(p)}{\mu}$, where $\mu(> 0)$ is a constant.

Table 1: Notations for parameters and variables

Decision variables	
p	unit sales price per item (\$/unit)
n	number of production cycle (an integer)
u	the preservation investment cost (\$/unit/time unit)
Parameters	
t_1	production length per cycle (weeks)
$t_1 + t_2$	time at which inventory level become zero (weeks)
$\theta(u)$	the deterioration rate of finished item under influence of preservation investment cost
θ_0	the natural deterioration rate
α	the coefficient of price in demand rate
β	the coefficient of preservation investment cost to the deterioration rate
q_p	the quantity produced over $[0, T]$ (units)
λ	parameter of stock dependent consumption rate
$f(p,t)$	demand rate function (function of sales price p and instantaneous stock level $I(t)$) (units/unit time)

$f(p)$	the market demand at price p, (units/unit time)
T	the cycle time (time unit)
R	the rate of production (units)
h	unit stock holding cost per item per week ($/unit/time unit)
Q	inventory level at $t = t_1$
$I_1(t)$	the inventory level that changes with time t during production period (units)
$I_2(t)$	the inventory level that changes with time t during non-production period (units)
$I_3(t)$	the inventory level that changes with time t during shortage period (units)
TEP	the total expected profit ($/time unit)
C_d	the deterioration cost per unit item ($/unit)
C_p	the production cost per unit item($/unit/time unit)
C_h	the inventory holding cost per unit item ($/unit/time unit)
C_s	the shortage cost per unit item ($/unit/time unit)
PDC	the total production cost
HDC	the total holding cost
STC	the total shortage cost
DTC	the total deterioration cost
PIC	the total preservation investment cost

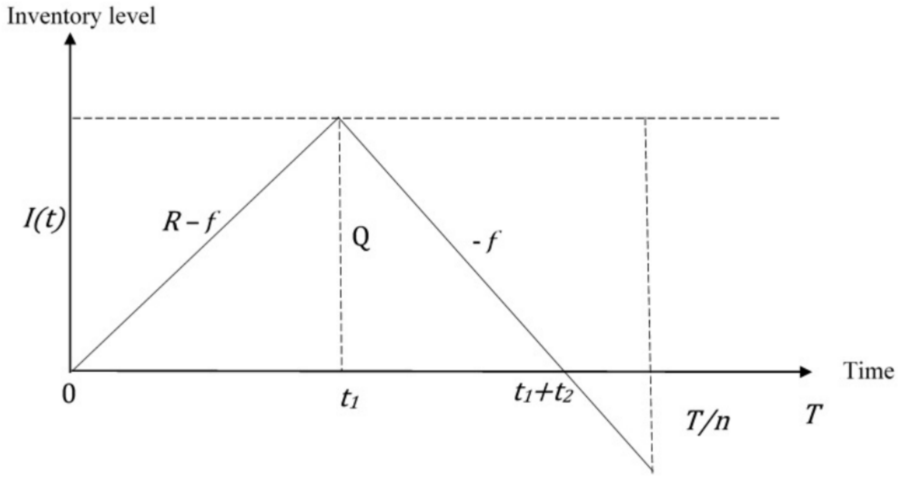

Figure 1: Graphical representation of the production-inventory model

2.2 Model Formulation

The model studies about a production-inventory system for an item. During the time $t = 0$ to $t = t_1$, the production continues with upward direction as depicted in Fig. 1, in presence of demand $f(p, t)$ and deterioration $\theta(u)$. For the time $[t_1, t_1 + t_2]$, there is no production, thus, the holding inventories positions are downstream direction with demand $f(p, t)$ and deterioration $\theta(u)$. The inventory level becomes zero at time $t = t_1 + t_2$. Afterwards, shortages are permitted to happen and whole demand in the period $[t_1 + t_2, T/n]$ is completely backlogged.

The differential equations which describe the inventory level are expressed below:

$$\frac{dI_1(t)}{dt} + (\theta(u) + \lambda)I_1(t) = R(p) - f(p), 0 \le t \le t_1 \tag{2.1}$$

with boundary condition $I_1(0) = 0$ and $I_1(t_1) = Q$.

$$\frac{dI_2(t)}{dt} + (\theta(u) + \lambda)I_2(t) = -f(p), t_1 \le t \le t_1 + t_2 \tag{2.2}$$

with boundary condition $I_2(t_1) = Q$ and $I_2(t_1 + t_2) = 0$.

$$\frac{dI_3(t)}{dt} = -f(p), t_1 + t_2 \le t \le T/n \tag{2.3}$$

with boundary condition $I_3(t_1') = 0$.
The solutions of the above differential Eqs. 2.1, 2.2 and 2.3 using boundary conditions are given by

$$I_i(t) = \begin{cases} \frac{R(p) - f(p)}{\theta(u) + \lambda}\left(1 - e^{-(\theta(u)+\lambda)t}\right), & 0 \le t \le t_1, i = 1; \\ \frac{f(p)}{\theta(u) + \lambda}\left(e^{-(\theta(u)+\lambda)(t_1+t_2-t)} - 1\right), & t_1 \le t \le t_1 + t_2, i = 2; \\ f(p)(t_1 + t_2 - t), & t_1 + t_2 \le t \le T/n, i = 3. \end{cases}$$

The total expected profit is

$$TEP(n, p, u) = Revenue - PDC - HDC - STC - DTC - PIC$$

$$\begin{aligned} TEP(n, p, u) = {} & pf(p)T - nC_pR(p)t_1 - nC_dR(p)t_1 + nC_df(p)(t_1 + t_2) - uT \\ & - nC_sf(p)\left(\frac{(t_1 + t_2)T}{n} - \frac{T^2}{2n^2} - \frac{(t_1 + t_2)^2}{2}\right) \\ & + n(p\lambda - C_h + C_d\lambda)\left[\frac{R(p)}{(\theta(u) + \lambda)^2}\left\{(\theta(u) + \lambda)t_1 - 1 + e^{-(\theta(u)+\lambda)t_1}\right\}\right. \\ & \left. + \frac{f(p)}{(\theta(u) + \lambda)^2}\left\{(e^{(\theta(u)+\lambda)((t_1+t_2)-t_1)} - e^{-(\theta(u)+\lambda)t_1} - (\theta(u) + \lambda)(t_1 + t_2))\right\}\right]. \end{aligned}$$

Taking $e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$. The above equation reduces to

$$\begin{aligned} TEP(n, p, u) = {} & pf(p)T - nC_pR(p)t_1 - nC_dR(p)t_1 + nC_df(p)(t_1 + t_2) - uT \\ & - nC_sf(p)\left(\frac{(t_1 + t_2)T}{n} - \frac{T^2}{2n^2} - \frac{(t_1 + t_2)^2}{2}\right) \\ & + n(p\lambda - C_h + C_d\lambda)\left[\frac{R(p)t_1^2}{6}\{3 - (\theta(u) + \lambda)t_1\} + \frac{f(p)(t_2^2 - t_1^2)}{2}\right. \\ & \left. + \frac{f(p)(\theta(u) + \lambda)(t_2^3 + t_1^3)}{6}\right]. \end{aligned} \tag{2.4}$$

At $t = t_1$, Eqs. 2.1 and 2.2 gives the value of $t_1 + t_2$. Let $t_1 = \frac{\xi T}{n}$, $R(p) = \frac{f(p)}{\mu}$, $f(p) = \alpha p^{-\Gamma}$.

$$\begin{aligned} TEP(n, p, u) = {} & \frac{T}{6}\left[6\alpha p^{-\Gamma+1} - 6u + \frac{3C_sT\alpha p^{-\Gamma}(\xi - \mu)^2}{n\mu^2} - \frac{6C_p\alpha p^{-\Gamma}\xi}{\mu}\right. \\ & \left. + \frac{\alpha p^{-\Gamma}T\xi^2(C_h - C_d\lambda - p\lambda)(\mu - 1)\{3n\mu - T\xi(2\mu - 1)(\lambda + \theta(u))\}}{n^2\mu^3}\right]. \end{aligned} \tag{2.5}$$

In this case, the objective is to maximize the total expected profit function to obtain the optimal number of the production cycle, optimal sales price and optimal preservation cost.

Theorem 2.1 *When the number of production cycles (or replenishments)* n *is fixed, the total expected profit* $TEP(n, p, u)$ *is jointly concave in the sales price* p *and the preservation cost* u.

Proof. For fixed n, the necessary conditions for maximization of $TEP(n, p, u)$ are

$$\frac{\partial TEP(n, p, u)}{\partial p} = 0, \tag{2.6}$$

and

$$\frac{\partial TEP(n, p, u)}{\partial u} = 0. \tag{2.7}$$

By using Eqs. (2.6) and (2.7), we obtain value of u in terms of p:

$$\begin{aligned} u = \Big[& T^2(\theta_0+\lambda)(1-\mu)(1-2\mu)\xi^3\{C_h\Gamma - \{(p^\Gamma)^{\frac{1}{\Gamma}}(\Gamma-1)+C_d\Gamma\}\lambda\} \\ & + 6n^2\mu^2\{-(p^\Gamma)^{\frac{1}{\Gamma}}(\Gamma-1)\mu + C_p\Gamma\xi\} - 3nT\mu\left[C_s\Gamma(\mu-\xi)^2\right. \\ & + \left.\{C_h\Gamma - \{(p^\Gamma)^{\frac{1}{\Gamma}}(\Gamma-1)+C_d\Gamma\}\lambda\}(\mu-1)\xi^2\right]\Big] / [\theta_0 T^2\beta\{C_h\Gamma \\ & - \{(p^\Gamma)^{\frac{1}{\Gamma}}(\Gamma-1)+C_d\Gamma\}\lambda\}(1-\mu)(1-2\mu)\xi^3\Big] \end{aligned} \tag{2.8}$$

Consider $\theta(u) = \theta_0 e^{-\beta u} \approx \theta_0(1-\beta u + \frac{\beta^2 u^2}{2})$. For any given feasible n, taking the second partial derivative of Eq. 2.5 with respect to p and u yields

$$\begin{aligned} \frac{\partial^2 TEP(n,p,u)}{\partial p^2} = & \frac{p^{-2-\Gamma}T\alpha\Gamma}{12n^2\mu^3}\left[T^2\{\theta_0(2+\beta u(\beta u-2))+2\lambda\}\{-C_h(1+\Gamma)\right. \\ & + (C_d - p + (C_d+p)\Gamma)\lambda\}(1-\mu)(1-2\mu)\xi^3 \\ & + 12n^2\mu^2\{p(\Gamma-1)\mu - C_p(1+\Gamma)\xi\} \\ & + 6nT\mu\{C_s(1+\Gamma)(\mu-\xi)^2 + [C_h(1+\Gamma) \\ & \left. -(C_d - p + (C_d+p)\Gamma)\lambda](\mu-1)\xi^2\}\right] \end{aligned} \tag{2.9}$$

$$\frac{\partial^2 TEP(n,p,u)}{\partial u^2} = -\frac{\theta_0 p^{-\Gamma}T^3\alpha\beta^2(C_h - C_d\lambda - p\lambda)(1-\mu)(1-2\mu)\xi^3}{6n^2\mu^3} \tag{2.10}$$

and

$$\frac{\partial^2 TEP(n,p,u)}{\partial u\partial p} = \frac{\theta_0 p^{-1-\Gamma}T^3\alpha\beta(\beta u-1)\{C_h\Gamma + (p-(C_d+p)\Gamma)\lambda\}(1-\mu)(1-2\mu)\xi^3}{6n^2\mu^3} \tag{2.11}$$

The sufficient conditions for maxima of total expected profit is that $TEP(n,p,u)$ should be concave function

$$\frac{\partial^2 TEP(n,p,u)}{\partial p^2} < 0, \ \frac{\partial^2 TEP(n,p,u)}{\partial u^2} < 0 \tag{2.12}$$

and

$$\begin{aligned} \Bigg[& \left(\frac{\partial^2 TEP(n,p,u)}{\partial p^2}\bigg|_{(p,u)=(p^*,u^*)}\right)\left(\frac{\partial^2 TEP(n,p,u)}{\partial u^2}\bigg|_{(p,u)=(p^*,u^*)}\right) - \\ & \left(\frac{\partial^2 TEP(n,p,u)}{\partial p\partial u}\bigg|_{(p,u)=(p^*,u^*)}\right)^2\Bigg] > 0, \end{aligned} \tag{2.13}$$

because $0 < \Gamma < 1$ and $0 < \mu < \frac{1}{2}$. Also $(C_d - p + (C_d+p)\Gamma) < 0$.
Solving Eqs. 2.6 and 2.7 the optimal values p^* and u^* of p and u can be found. By putting these optimal values in Eq. 2.5 the optimal value $TEP^*(n,p,u)$, the total expected profit can be obtained.

Theorem 2.2 *For fix* u *and* $0 < \mu < \frac{1}{2}$*, the total expected profit,* $TEP(n,p,u)$*, is a strictly concave function of* (n,p).

Proof.

$$\begin{aligned} \frac{\partial^2 TEP(n,p,u)}{\partial p^2} = & \frac{p^{-2-\Gamma}T\alpha\Gamma}{6n^2\mu^3}\left[T^2(\theta(u)+\lambda)\{-C_h(1+\Gamma) + (C_d - p + (C_d+p)\Gamma)\lambda\}\right. \\ & (1-\mu)(1-2\mu)\xi^3 + 6n^2\mu^2\{p(\Gamma-1)\mu - C_p(1+\Gamma)\xi\} \\ & \left. + 3nT\mu\{C_s(1+\Gamma)(\mu-\xi)^2 + [C_h(1+\Gamma) - (C_d - p + (C_d+p)\Gamma)\lambda](\mu-1)\xi^2\}\right] < 0, \end{aligned} \tag{2.14}$$

because $0 < \Gamma < 1$ and $0 < \mu < \frac{1}{2}$. Also $(C_d - p + (C_d+p)\Gamma) < 0$.

$$\begin{aligned} \frac{\partial^2 TEP(n,p,u)}{\partial n^2} = & \frac{p^{-\Gamma}T^2\alpha}{n^4\mu^3}\left[-T(\theta(u)+\lambda)(C_h - C_d\lambda + p\lambda)(1-\mu)(1-2\mu)\xi^3\right. \\ & \left. + n\mu\{C_s(\mu-\xi)^2 - (C_h - C_d\lambda + p\lambda)(1-\mu)\xi^2\}\right] < 0, \end{aligned} \tag{2.15}$$

provided $(C_h - C_d\lambda + p\lambda) > 0$.

$$\frac{\partial^2 TEP(n,p,u)}{\partial p \partial n} = \frac{p^{-1-\Gamma}T^2\alpha}{6n^3\mu^3}\left[2T(\theta(u)+\lambda)\{-C_h\Gamma + (p(\Gamma-1)+C_d\Gamma)\lambda\}\right.$$
$$\left.(1-\mu)(1-2\mu)\xi^3 + 3n\mu\{C_s\Gamma(\mu-\xi)^2 + [C_h\Gamma + \{p-(C_d+p)\Gamma\}\lambda](\mu-1)\xi^2\}\right]. \quad (2.16)$$

Thus, the Hessian matrix is

$$H = \frac{\partial^2 TEP(n,p,u)}{\partial p^2}\frac{\partial^2 TEP(n,p,u)}{\partial n^2} - \left[\frac{\partial^2 TEP(n,p,u)}{\partial p \partial n}\right]^2 > 0. \quad (2.17)$$

From the above analysis, we have obtained that, for any given u, the point (n^*, p^*) maximizing the total expected profit. As a consequence, $TEP(n,p,u)$ is a concave function of n and p, for a given u. (Also, shown in Fig. 3)

Corollary 2.1 *The total expected profit $TEP(n,p,u)$ is decreasing in θ_0, while the selling price p is increasing in α.*
This corollary states that to reach a maximum profit, the producer should reduce the natural deterioration rate and higher sales price when it obtains a higher coefficient of price in demand rate.

3. NUMERICAL EXAMPLE

In this section, a numerical example is provided to validate the proposed model. The input data are given in Table 2. Results for numerical examples are given in Table 3.

Table 2: Input parameters of Example 1.

T (weeks)	α (units)	Γ (units)	C_p (\$/unit)	C_d (\$/unit)
203	140	0.4	5	0.1
C_h (\$/unit/week)	C_s (\$/unit/week)	λ	μ	ξ
0.02	0.1	0.001	0.2	0.6
θ_0	β			
0.08	0.8			

Table 3: Optimum results of Example 1.

n	p	u	Total Demand	Total Expected Profit
7	19.70	8.90	14809.19	90123.4
	(\$/unit)	(\$/unit/week)	(units)	(\$)

4. SENSITIVITY ANALYSIS

This section introduces sensitivity analyses for parameters α, C_p, C_h, β and μ, to show the overall effect of value changes on the selling price, the preservation cost and the total profit. This sensitivity analysis is performed by changing the parameter values by $-10\%, -5\%, +5\%$, and $+10\%$ and keeping other parameters unchanged. Table 4 and 5 presents the results of the sensitivity analysis of Example 1.
(1) *Sensitivity analysis of demand scale α:* As the demand scale α increases under the complete backordering, Table 4 shows that p, u and TEP increase. This implies that when α is relatively high, the producer has to invest more in the preservation technology to fulfill the high demand rate and hence, the producer tends to increase the sales price.
(2) *Sensitivity analysis of production cost C_p:* In the proposed model, as production cost C_p increases, u increases, while TEP decreases; p almost remains unchanged, which implies that the sales cost per unit is very less sensitive to production cost per unit. It reveals that a higher production cost weakens the ability of the producer to coordinate between workers and as the result of this production decreases, hence the demand rate decreases and the total expected profit also decreases. Due to the high production cost of the item, it is necessary to preserve the item for a long time without any deterioration, so high investment is needed in preservation techniques, otherwise, it will lead to a high loss to the producer.
(3) *Sensitivity analysis of holding cost C_h:* As seen from Table 4, when the unit inventory holding cost C_h increases, p and TEP increase, whereas u decreases. If the holding cost C_h is higher, the producer is inclined to avoid too much inventory by reducing the produced quantity, this also reduces preservation technology investment. Accordingly, the retailer (or producer) could gain more profits by setting a relatively higher sales price. Here, p, u and TEP are highly

sensitive to changes in C_h.
(4) *Sensitivity analysis of sensitive parameter of investment to the deterioration rate* β*:* As the investment cost coefficient β increases, p and TEP increase, while u decreases. Here, p is less sensitive to β; u and TEP are highly sensitive to β.
(5) *Sensitivity analysis of* μ*:* When the production rate parameter μ increases, p and EPR decrease, while u increase. The high value of parameter μ leads to a lower production rate, this will reduce the profit value.
(6) *Sensitivity analysis of deterioration cost* C_d*:* As the deterioration cost C_d increases, p and TEP decrease, while u increases. u and p is very less sensitive to change in C_d, whereas TEP is less sensitive to C_d.
(7) *Sensitivity analysis of* ξ*:* As the ξ increases, p and TEP increase, while u decreases.
(8) *Sensitivity analysis of* θ_0*:* As the θ_0 increases, p and TEP increase, while u decreases.
(9) *Sensitivity analysis of shortage cost* C_s*:* From Table 6, when the unit shortage cost C_s increases, TEP increases, while u decreases and p almost remain unchanged. Facing a large shortage cost, the retailer will charge a high selling price and bring down the preservation technology investment cost in order to extract more profits.
(10) *Sensitivity analysis of* λ*:* As the λ increases, p, TEP, and u decrease.

As seen from Fig. 2 and Table 6, as the price-sensitive parameter Γ increases, the curve representing the total profit function $TEP(n,p,u)$ decreases abruptly, which indicates that larger value of parameter Γ decreases the demand rate of the product. Fig. 3 represents that $TEP(n,p,u)$ is jointly concave in n and p, for a feasible u.

Table 4: The sensitivity analysis of Example 1.

Parameter	% changes	p	u	Total Expected Profit
α	−10%	19.68	8.9025	80930.30
	−5%	19.69	8.9031	85526.80
	+5%	19.71	8.9043	94719.90
	+10%	19.72	8.9048	99316.50
C_p	−10%	19.70	8.643	103064.00
	−5%	19.70	8.773	96593.60
	+5%	19.70	9.034	83653.10
	+10%	19.70	9.165	77182.90
C_h	−10%	17.71	9.08	76138.70
	−5%	18.70	8.99	83251.10
	+5%	20.69	8.83	96778.40
	+10%	21.69	8.76	103236.00
β	−10%	19.68	9.89	89922.60
	−5%	19.69	9.37	90028.30
	+5%	19.71	8.48	90209.40
	+10%	19.72	8.10	90287.70
μ	−10%	19.76	6.47	94293.30
	−5%	19.73	7.59	91785.60
	+5%	19.65	10.43	89085.40
	+10%	19.60	12.20	88504.40

Table 5: The sensitivity analysis of Example 1.

Parameter	% changes	p	u	Total Expected Profit
C_d	-10%	19.71	8.902	90191.00
	-5%	19.70	8.903	90157.20
	+5%	19.69	8.904	90089.60
	+10%	19.68	8.905	90055.80
ξ	-10%	19.62	11.53	88648.60
	-5%	19.66	10.09	89131.90
	+5%	19.73	7.92	91634.30
	+10%	−	−	−
θ_0	-10%	19.68	9.75	89950.80
	-5%	19.69	9.31	90041.60
	+5%	19.71	8.54	90197.40

	+10%	19.72	8.21	90264.60
C_s	-10%	19.70	9.00	85119.70
	-5%	19.70	8.95	87621.50
	+5%	19.70	8.85	92625.20
	+10%	19.70	8.80	95127.00
λ	-10%	21.89	9.47	104497.00
	-5%	20.74	9.17	970520.00
	+5%	18.76	8.66	83649.40
	+10%	17.90	8.44	77577.90

Table 6: Sensitivity analysis for Γ

Γ	p	u	Total Profit
0.4	19.70	8.90	90123.40
0.5	19.63	8.44	66453.90
0.6	19.53	7.97	48927.20
0.7	19.41	7.50	35955.30
0.8	19.25	7.04	26360.60
0.9	19.03	6.58	19269.70

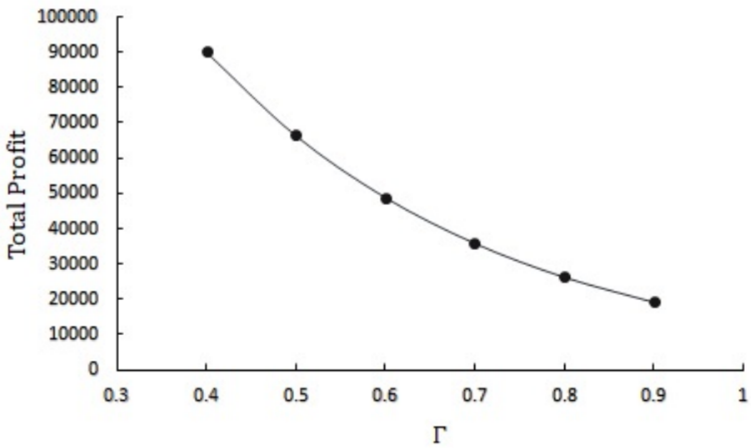

Figure 2: Graphical representation of Total Profit w.r.t Γ

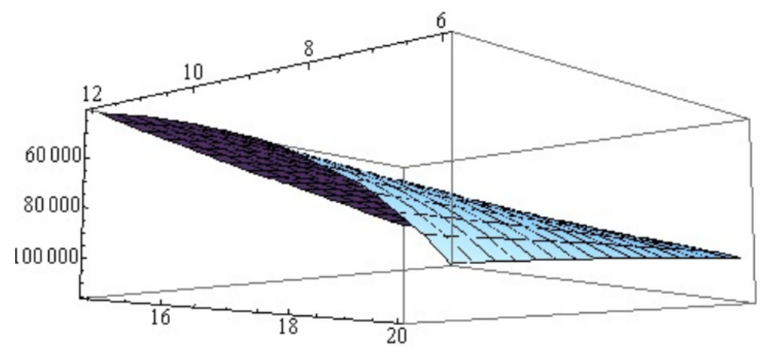

Figure 3: Graphical representation of $TEP(n, p, u)$ for fix u = 8.90

4.1 Managerial Insights

This study brings out recommendations on how the investment of preservation can improve stock within the framework of the production cycle, where generally shortage is allowed. However, the management can decide the optimum selling price, based on the situation of more or fewer products necessity. The producer can increase or decrease the production rate based on market demand. In that case, the chances of the deteriorating item will be reduced.

The selection of a number of replenishment cycles should be taken care of as major tasks are done by management. Thus, the management can get the proper quantity of stock of products with maximum profits.

5. CONCLUDING REMARKS

In this paper, we studied a production inventory system with preservation invested deterioration rate and complete backlogging. The producer invests in the preservation technology to reduce the deteriorating rate. We also found that if the retailer or manufacturer can effectively reduce the deteriorating rate of the item by improving the preservation technology investment then the total profit will be increased. The optimal number of production cycle n^*, the sales price p^* and the preservation cost u^* and the total expected profit $TEP(n, p, u)^*$ have been obtained. Further, sensitivity analysis has also been performed to validate the obtained findings. Then, numerical simulations and sensitivity analysis of the corresponding solutions with respect to demand scale, production cost, holding cost, deterioration cost, preservation cost coefficient, shortage cost are given to verify the effectiveness of the proposed model, and meanwhile, we provided some managerial insights.

The main findings are summarized as follows. First, increased demand scale tends to invest more on preservation cost to fulfil the increased demand rate. Second, when the parameter inversely proportional to the production rate increases, the total profit decreases due to less production and demand rate. Third, the total expected profit decreases with an increase in the price-dependent parameter.

In future research, we encourage extending this inventory model by considering some features such as variable production cost, probabilistic demand rate, multiple products and allowable delay in payment.

ACKNOWLEDGMENT

The authors would like to thank the editor and anonymous reviewers for their constructive comments. This paper was revised in light of their comments.

REFERENCES

Abad, P. L. (1996). Optimal pricing and lot-sizing under conditions of perishability and partial backordering. *Management Science*, *42*(8), 1093-1104.

Abad, P. L. (2003). Optimal pricing and lot-sizing under conditions of perishability, finite production and partial backordering and lost sale. *European Journal of Operational Research*, *144*, 677-685.

Bhunia, A. K., Kundu, S., Sannigrahi, T., & Goyal, S. K. (2009). An application of tournament genetic algorithm in a marketing oriented economic production lot-size model for deteriorating items. *International Journal of Production Economics*, *119*(1), 112-121.

Chang, C. T., Teng, J. T., & Goyal, S. K. (2010). Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. *International Journal of Production Economics*, *123*(1), 62-68.

Chung, K. J., Eduardo Cárdenas-Barrón, L., & Ting, P. S. (2013). An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade credit. *International Journal of Production Economics*, *155*, 310-317.

Dave, U., & Patel, L. K. (1981). (t, s_i) policy inventory model for deteriorating items with time proportional demand. *Journal of the Operational Research Society*, *40*, 137-142.

Drake, M. J., Pentico, D. W., & Toews, C. (2011). Using the EPQ for coordinated planning of a product with partial backordering and its components. *Mathematical and Computer Modelling*, *53*(1-2), 359-375.

Dye, C. Y., & Hsieh, T. P. (2012). An optimal replenishment policy for deteriorating items with effective investment in preservation technology. *European Journal of Operational Research*, *218*(1), 106-112.

Goyal, S. K., & Giri, B. C. (2003). The production-inventory problem of a product with time varying demand, production and deterioration rates. *European Journal of Operational Research*, *147*, 549-557.

Hsieh, T. P., & Dye, C. Y. (2013). A productioninventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. *Journal of Computational and Applied Mathematics*, *239*, 25-36.

Hsu, P., Wee, H., & Teng, H. (2010). Preservation technology investment for deteriorating inventory. *International Journal of Production Economics*, *124*(2), 388-394.

Kang, S., & Kim, I. T. (1983). A study on the price and production level of the deteriorating inventory system. *International Journal of Production Research*, *21*, 899-908.

Lodree Jr., E. J., & Uzochukwu, B. M. (2008). Production planning for a deteriorating item with stochastic demand and consumer choice. *International Journal of Production Economics*, *116*(2), 219-232.

Mahata, G. C. (2012). An epq-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. *Expert systems with Applications*, *39*, 3537-3550.

Misra, R. B. (1975). Optimum production lot size model for a system with deteriorating inventory. *International Journal of Production Research*, *13*, 495–505.

Panda, S., Saha, S., & Soumen, N. (2012). Retailers optimal pricing and economic order quantity in stock and price sensitive demand environment. *International Journal of Operational Research*, *15*(4), 406-423.

Roy, T. S., Ghosh, S. K., & Chaudhuri, K. S. (2013). An economic production quantity model for items with time proportional deterioration under permissible delay in payments. *International Journal of Mathematics in Operational Research*, *5*(3), 301-316.

Sekar, T., & Uthayakumar, R. (2018). A manufacturing inventory model for exponentially increasing demand with preservation technology and shortage. *International Journal of Operations Research*, *15*(2), 61-70.

Singh, P., & Khedlekar, U. K. (2020). Production planning for controllable deteriorating items with price and stock dependent demand rate. *International Journal of Nonlinear Analysis and Applications*, *Accepted*.

Taft, E. W. (1918). The most economical production lot. *Iron Age*, *101*, 1410-1412.

Tiwari, R. K., Khedlekar, N. K., & Khedlekar, U. K. (2021). Optimal pricing policy for stock dependent demand with investment in preservation technology. *International Journal of Nonlinear Analysis and Applications*, *12*(2), 249-264.

Tripathi, R. P., & Tomar, S. S. (2018). Establishment of eoq model with quadratic time-sensitive demand and parabolic-time linked holding cost with salvage value. *International Journal of Operations Research*, *15*(3), 135-144.

Wee, H. M. (1993). Economic production lot size model for deteriorating items with partial back-ordering. *Computers and Industrial Engineering*, *24*, 449-458.

Wee, H. M. (1997). A replenishment policy for items with a price-dependent demand and a varying rate of deterioration. *Production Planning* & *Control*, *8*, 494-499.